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AN ANALYTICAL INVESTIGATION OF THE INDIRECT MEASUREMENT METHOD OF ESTIMATING THE ACOUSTIC IMPEDANCE OF A TIME-VARYING SOURCE

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This paper presents an analytical investigation into the relationship between the characteristics of a linear, time-variant acoustic source, and the effective source impedance and source strength which follow if one then assumes the source to be time invariant. It thus models the indirect measurement method of obtaining the effective source characteristics of an actual time-varying acoustic source such as the intake or exhaust system of an internal combustion engine. It is shown that the effective source impedance and source strength have no physical meaning, bear no resemblance to the real source, and are dependent on the load system used to evaluate them. It is found that the effective source resistance is frequently negative; hence time variance of the source alone is a sufficient reason for the implausible negative resistance values as observed with the indirect measurement method. It is also shown that the meaningless effective source characteristics do, nevertheless, sometimes give a fairly accurate estimate of the noise output when the acoustic source is used with a different load, again as observed by experimentation.

1. INTRODUCTION

There are various techniques available by which to model the acoustic characteristics of the exhaust or intake system of an internal combustion (IC) engine. The most widespread of these is linear frequency-domain modelling [1] which, relative to other techniques, is very quick and enables one to make realistic representations of the complex internals which are typical of commercial mufflers. The main drawback of the technique is the difficulty in accurate characterization of the source, which is both non-linear and time variant. A schematic of the problem for the exhaust side of the engine is shown in Figure 1(a), where everything downstream of the exhaust valves is regarded as an acoustic load, Z_l on the engine. The equivalent electrical network for a single frequency component of the linear acoustic, time-invariant representation of the engine-exhaust model is shown in Figure 1(b).

The requirements by way of source data are different for different noise measures, but always include the source impedance Z_s and sometimes also the source strength P_s for each frequency ω_n. The impedance is complex, consisting of a resistance and a reactance, and

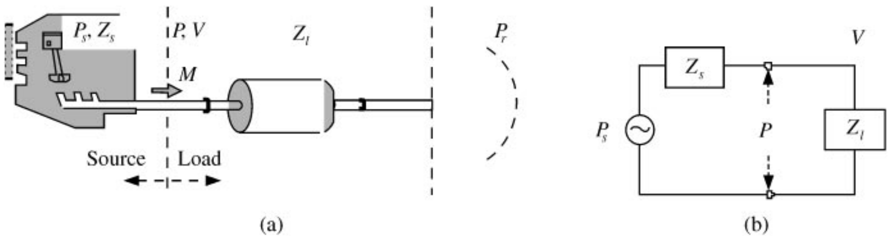

Figure 1. Source-load model. (a) Acoustic source-load system; (b) electro-acoustic circuit.

characterizes not only the form of the discharge of the source into the system, but also the manner in which acoustic waves travelling towards the source are reflected by the source. The source data is generally obtained by experimentation, using either a direct or an indirect method. The direct method requires the use of a second source, placed somewhere in the load section, of much greater magnitude than the primary source such that sound output from the latter is negligible. The impedance of the effectively passive source termination is then measured by conventional techniques [2–7]. The problem for an IC-engine source is to provide a dominant second source and, even if this is possible, the resultant sound level will be so high as to make non-linear effects significant. Thus, indirect methods are generally used for IC-engine sources, and have the additional benefit of yielding the source strength as well as its impedance.

Indirect methods make use of measurements relating to two or more varying loads on the same source. If phase information is retained then at least two loads are required [8], else a minimum of three or four loads [9, 10] are used. In order to reduce the effect of measurement errors a larger number of loads [11–13] are often used which enables some sort of averaging on the over-determined system. However, even these multi-load methods give mostly negative resistance values for IC-engine sources, which is physically implausible. Despite this, if the measured source strength and impedance are used to compute the radiated sound level from a different load, the predictions are often in quite good agreement with experimental results. The possible cause of the negative resistance measurements has been attributed to various factors [14], in particular the fact that an IC-engine source is actually non-linear and time variant.

In this paper, an analytical investigation is made of the discharge from a linear, time-variant source, to determine whether time variance alone can account for the negative resistance values. Specifically, it is not the intention to model accurately an IC-engine source, which is also non-linear. The source is made time variant by letting the discharge occur through a valve whose open area is time dependent, the valve being considered as part of the source. In particular, the valve is assumed to be completely closed over part of the cycle, as is the case for an IC-engine. In the first instance, a simple idealized linear equation of discharge through the valve is assumed. Since the purpose is to determine the relationship between the actual characteristics of a given linear time-variant source and the source characteristics that would be determined if that same source were assumed to be both linear and time invariant, it does not matter that the actual source used is unrealistic.

It is shown that the source strength and impedance which follow from the assumption of time invariance have no physical meaning whatsoever and bear no resemblance to the actual characteristics of the time-varying source. In particular, the source resistance is frequently found to be negative. It is also shown, however, that if these estimated source characteristics are used to determine the noise output from a different load, then in some cases a reasonable estimate of the noise output from the actual source can be obtained. Thus the key features of the indirect experimental evaluation of the characteristics of a time-varying source are replicated. In addition, it is found that the values of the effective source strength and impedance are dependent upon the actual load systems which are used in their evaluation, even though the idealized time-variant source is made independent of load. Previously, such variations have been attributed solely to experimental error, but the current work implies that the multi-load methods such as are used in experimentation are not simply reducing experimental error, but are averaging the effective source characteristics as determined from different load combinations.

Finally, a slightly more realistic and hence more complicated linear equation of discharge through the valve is introduced, which includes inertial effects. It is found that all of the above comments, *vis-à-vis* the relationship between the actual source characteristics and the effective source characteristics assuming time invariance, remain unaltered. This section serves to reinforce the assertion that the relationship is not dependent upon the precise nature of the time-variant source.

2. IDEALIZED LINEAR TIME-VARIANT SOURCE

Consider a source of pressure $P_s(t)$ which discharges into a system through a valve; see Figure 1. Let the pressure on the downstream side of the valve be $P(t)$ and the velocity of flow through the valve be $U(t)$. Both pressures are assumed to be relative to atmospheric pressure. Throughout this paper all the sources are assumed to be constant pressure sources: i.e., $P_s(t)$ remains the same whatever system is attached to the valve outlet.

In this first case of an idealized time-varying linear source, it is assumed that the flow through an open valve is governed by the equation

$$\bar{\rho}\bar{c}\,U(t) = C_D[P_s(t) - P(t)], \tag{1}$$

where C_D is a constant non-dimensional discharge coefficient and $\bar{\rho}$, $\bar{c}$ are the mean density and speed of sound of the gas flow through the valve respectively. Equation (1) is a linearized, pseudo-steady version of the known non-linear equation which governs steady flow through a valve [15].

The time-variant nature of the source is introduced by letting the valve area $\tilde{A}(t)$ vary periodically with time, with period T. In particular, the valve is assumed to be closed over some portion of the cycle, at which time of course the velocity of discharge will be zero. Thus, from equation (1), the equation of discharge throughout a cycle can be written as

$$\bar{\rho}\bar{c}\,V(t) = C_D\tilde{A}(t)[P_s(t) - P(t)], \qquad \tilde{A}(t)\left\{\begin{matrix} \neq 0, & 0 \leqslant t \leqslant \tau \\ = 0, & \tau \leqslant t \leqslant T \end{matrix}\right\}, \tag{2}$$

where $V(t)$ is the volume velocity of the flow through the valve. Let $A(t) = \tilde{A}(t)/A_{max}$ be a non-dimensional valve area, where A_{max} is the maximum open area of the valve at any time, and let $v(t) = (\bar{\rho}\bar{c}/A_p)V(t)$, where A_P is the constant area of the pipe into which the source exhausts. Equation (2) can then be re-written as

$$v(t) = CA(t)[P_s(t) - P(t)], \qquad \left\{\begin{matrix} 0 < A(t) \leqslant 1, \ 0 \leqslant t \leqslant \tau \\ A(t) = 0, \ \tau \leqslant t \leqslant T \end{matrix}\right\}, \tag{3}$$

where $C = C_D A_{max}/A_P$. This idealized source is seen to be a particular simplification of the more general linear, time-variant source as considered by Bodén [16].

The variables can each be expanded as complex Fourier series,

$$v(t)=\sum_{j=-\infty}^{j=+\infty} v_j \mathrm{e}^{\mathrm{i}\omega_j t}, \quad P_s(t)=\sum_{j=-\infty}^{j=+\infty} S_j \mathrm{e}^{\mathrm{i}\omega_j t}, \quad P(t)=\sum_{j=-\infty}^{j=+\infty} P_j \mathrm{e}^{\mathrm{i}\omega_j t}, \quad CA(t)=\sum_{j=-\infty}^{j=+\infty} A_j \mathrm{e}^{\mathrm{i}\omega_j t}, \tag{4a–d}$$

where the frequency component $\omega_j = 2\pi j/T$.

If only the first $N+1$ harmonics are non-negligible, then equation (3) results in the matrix system

$$\begin{bmatrix} v_{-N} \\ \cdots \\ v_0 \\ \cdots \\ v_N \end{bmatrix} = \begin{bmatrix} A_0 & \cdots & A_{-N} & \cdots & A_{-2N} \\ \cdots & A_0 & \cdots & \cdots & \cdots \\ A_N & \cdots & A_0 & \cdots & A_{-N} \\ \cdots & \cdots & \cdots & A_0 & \cdots \\ A_{2N} & \cdots & A_N & \cdots & A_0 \end{bmatrix} \begin{bmatrix} S_{-N}-P_{-N} \\ \cdots \\ S_0 - P_0 \\ \cdots \\ S_N - P_N \end{bmatrix}, \tag{5}$$

or

$$\{\mathbf{v}\} = [\mathbf{A}]\{\mathbf{S}-\mathbf{P}\}, \tag{6}$$

where $[\mathbf{A}]$ is a non-dimensional admittance matrix. Its inverse is the non-dimensional impedance matrix of the source [16].

The conventional time-invariant source model as represented by Figure 1(b) is characterized by the equation

$$S_j(\omega_j) - P_j(\omega_j) = Z_{s_j}(\omega_j)V(\omega_j) = \zeta_{s_j}(\omega_j)v(\omega_j), \tag{7}$$

where ζ_{s_j} is the non-dimensional source impedance for frequency component ω_j. Comparison of equations (5) and (7) indicates that they can be equivalent only if the admittance matrix is diagonal, which from equation (5) implies that the valve area $A(t) = A_0$, a constant. The admittance matrix is thus not only diagonal, but has constant coefficients, as of course will its inverse the impedance matrix

The non-dimensional load impedance ζ_l where

$$\zeta_{l_j}(\omega_j) = P_j(\omega_j)/v_j(\omega_j) \tag{8}$$

can be used to re-write equation (6) as

$$\{[\mathbf{A}][\boldsymbol{\zeta}_l] + [\mathbf{I}]\}\{\mathbf{v}\} = [\mathbf{A}]\{\mathbf{S}\}, \tag{9}$$

where the load impedance matrix $[\boldsymbol{\zeta}_l]$ is a diagonal matrix whose jth term is ζ_{jj}, $-N \leqslant j \leqslant N$. For a given geometry of the exhaust system together with the mean temperature and flow conditions, the components of the load impedance matrix $[\zeta_l]$ follow from conventional transfer matrix theory [1]. Furthermore, for a known time history of the source pressure and valve motion, the components of $[\mathbf{A}]$ and $\{\mathbf{S}\}$ follow. Thus, equation (9) can be solved for the frequency components of $\{\mathbf{v}\}$ and hence those of $\{\mathbf{P}\}$ follow from equation (8). The values for the components of acoustic pressure and velocity can then be calculated at any point within the exhaust system or in free space, following radiation from the tailpipe orifice.

3. EVALUATION OF SOURCE IMPEDANCE

It was shown in section 2 that equation (9) enables one to calculate the acoustic properties anywhere within a source–load system when the characteristics of the load and time-varying source are known. This process can be repeated for different loads and the results can be used to determine the effective impedance of the source, in the same manner as is done when using experimental measurements in the indirect method. For the ith load, with impedance matrix $[\zeta_l^i]$, let $\{\mathbf{P}^i\}$ and $\{\mathbf{v}^i\}$ be the calculated acoustic pressure and volume velocity vectors that follow from equations (8) and (9). Fourier decomposition then gives the relevant frequency components to insert in equation (7), for an assumed linear time-invariant source, namely

$$S_j(\omega_j) - P_j^i(\omega_j) = \zeta_{s_j}(\omega_j) v_j^i(\omega_j). \tag{10}$$

Thus, on the assumption that the source strength remains constant whatever the load, it follows from equation (10) that

$$\zeta_{s_j}(\omega_j) = \frac{P_j^2(\omega_j) - P_j^1(\omega_j)}{v_j^1(\omega_j) - v_j^2(\omega_j)}, \tag{11}$$

where ζ_s is a non-dimensional source impedance.

Only two different loads are required to give the components of the source impedance and source strength. Unlike the experimental case, no advantage is gained by using the four-load method and calculating only the radiated pressure, since the phase relationship to the source is known precisely and the in-duct values are known as accurately as the radiated values. Likewise there is no need to simulate any of the multi-load methods, whose purpose is to reduce the effect of errors in the measurements, since all values are known precisely relative to the assumed model.

4. RESULTS FOR AN IDEALIZED LINEAR TIME-VARIANT SOURCE

By way of example, a source of constant pressure $P_s = 10\,\mathrm{Pa}$ was assumed with a valve whose non-dimensional open area is given by

$$A(t) = \left\{ \begin{array}{ll} \frac{1}{2}\left[1 + \cos\left(\frac{6\pi t}{T} - \pi\right)\right], & 0 \leqslant t \leqslant T/3 \\ 0, & T/3 \leqslant t \leqslant T \end{array} \right\}, \qquad A(t+T) = A(t). \tag{12}$$

These conditions form a simplistic model of the exhaust cycle of a four-stroke engine, since the cylinder pressure is approximately constant over the exhaust stroke and the exhaust valve is generally timed to open before BDC and close after TDC, extending the open period beyond $T/4$. The valve was assumed to have a maximum open area of $100\,\mathrm{mm}^2$ and an exhaust pipe of radius 20 mm was used. The gas temperature of flow through the valve and exhaust was taken as 600 K. The value of the discharge coefficient C_D was evaluated as that which would give the same steady flow velocity from the linear model of equation (1) as is obtained from the actual non-linear model, for the same pressure difference of 10 Pa.

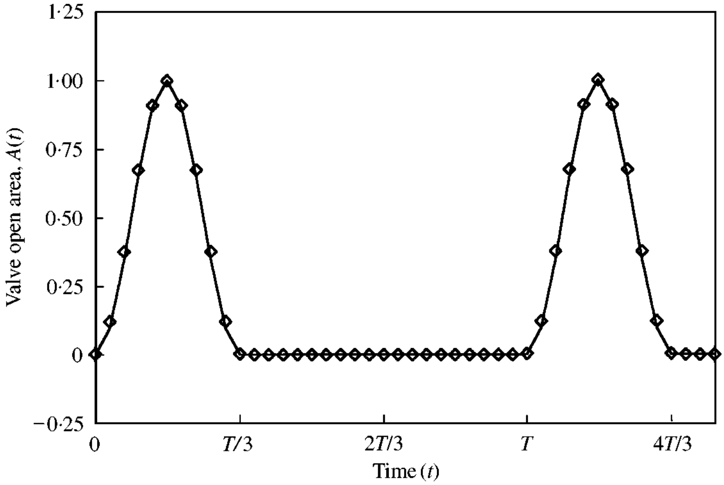

Figure 2. Time history of the non-dimensional valve open area. ——, Exact; ◇, calculated for $N = 10$.

TABLE 1

The A_j coefficients which form the admittance matrix, see equation (5), the diagonal terms $\zeta_{j,j}$ and the elements of the first row $\zeta_{1,j}$ of the impedance matrix

j	A_j	$\zeta_{j,j}$	$\zeta_{1,j}$
0	1·36259	$0·221 \times 10^5$	$0·221 \times 10^5$
1	1·26771	$0·544 \times 10^6$	$-0·107 \times 10^6$
2	1·01417	$0·256 \times 10^7$	$0·211 \times 10^6$
3	0·68129	$0·399 \times 10^7$	$-0·178 \times 10^6$
4	0·36220	$0·397 \times 10^7$	$-0·142 \times 10^5$
5	0·12677	$0·481 \times 10^7$	$0·136 \times 10^6$
6	0·00000	$0·490 \times 10^7$	$-0·566 \times 10^5$
7	− 0·03622	$0·495 \times 10^7$	$-0·380 \times 10^5$
8	− 0·02305	$0·487 \times 10^7$	$0·655 \times 10^4$
9	0·00000	$0·481 \times 10^7$	$0·181 \times 10^5$
10	0·01114	$0·482 \times 10^7$	$0·340 \times 10^5$
11	0·00823	$0·481 \times 10^7$	$-0·322 \times 10^5$
12	0·00000	$0·487 \times 10^7$	$-0·388 \times 10^5$
13	− 0·00488	$0·495 \times 10^7$	$0·418 \times 10^5$
14	− 0·00387	$0·490 \times 10^7$	$0·221 \times 10^5$
15	0·00000	$0·481 \times 10^7$	$-0·339 \times 10^5$
16	0·00257	$0·397 \times 10^7$	$-0·843 \times 10^4$
17	0·00213	$0·399 \times 10^7$	$0·248 \times 10^5$
18	0·00000	$0·256 \times 10^7$	$-0·789 \times 10^4$
19	− 0·00152	$0·544 \times 10^6$	$-0·308 \times 10^4$
20	− 0·00130	$0·221 \times 10^5$	$0·172 \times 10^4$

Results have been obtained by using only the first 10 acoustic harmonics, i.e. $N = 10$, and a fundamental frequency of 25 Hz. Figure 2 shows the precise time history of the valve open area together with the calculated values from the Fourier expansion followed by its inverse, using $N = 10$. Table 1 gives the coefficients for A_j, $0 \leqslant j \leqslant 20$, as required in the admittance matrix of equation (5) for $N = 10$. $A_j = A_j^*$, of course, and the coefficients are all real. It is seen that many of the off-diagonal terms are of similar magnitude to the diagonal terms, and

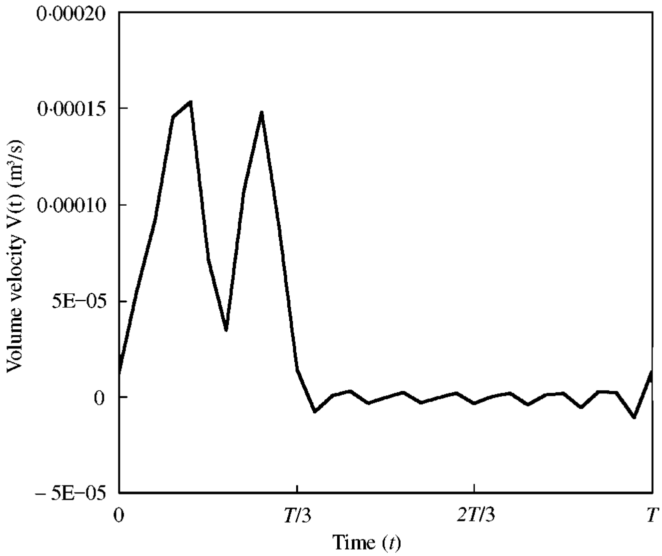

Figure 3. Time history of the outlet volume velocity, calculated for $N = 10$.

are certainly not negligible. Table 1 also lists the diagonal elements and the elements of the first row of the impedance matrix $[A]^{-1}$. Both the diagonal and off-diagonal elements are extremely large and, if sufficient harmonics were used, they would all become infinite, since the valve closed condition corresponds to $Z_s(t) \to \infty$ over part of each cycle. In other words only the admittance matrix can be used since for sufficient harmonics it is singular and its inverse does not exist. Thus, in all cases where the valve is completely closed over part of the cycle, impedance is not a useful concept and one needs to use the admittance instead, at least for a constant pressure source such as is assumed here. To be precise, the time-variant source assumed here is a constant pressure source when the valve is open and a constant (zero) velocity source when the valve is closed. The zero admittance of a closed valve then ensures that the source equation (6) for constant source pressure is valid throughout the cycle. For a source that could be assumed to have constant velocity with load throughout the cycle, one would switch to a parallel rather than series representation of the source, as used by Wang [17, 18], and hence be able to retain the use of impedance. In relating his time-variant model of an intake system to its time-invariant counterpart, Wang [17, 18] assumed that only the diagonal and steady flow elements were of significance and the results here show that to be an invalid assumption.

The time history of the volume velocity for discharge into a pipe of length 1·7 m, calculated from Fourier coefficients as evaluated from equation (9), is shown in Figure 3. There is a small but noticeable discharge over the time when the valve is closed, due to the use of a finite number of harmonics. The corresponding time history of the flow velocity through the valve is shown in Figure 4. The velocity has been set to zero over the time when the valve is closed. The velocity becomes very large when the valve is only fractionally open and is limited in the figure by the size of the time interval for evaluation. In reality, the flow becomes choked [19] at these conditions, but that is a non-linear phenomena which a linear model cannot replicate. Since the time interval of large velocities is very small, and the corresponding volume velocity is also very small, it is hopefully of minor significance. It is seen from Figure 4 that the chosen source pressure was small enough to keep the velocity of

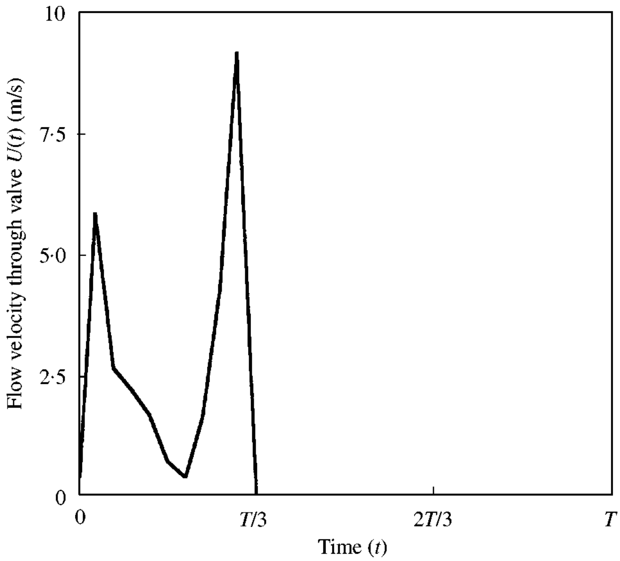

Figure 4. Time history of the velocity of flow through the valve, calculated for $N = 10$.

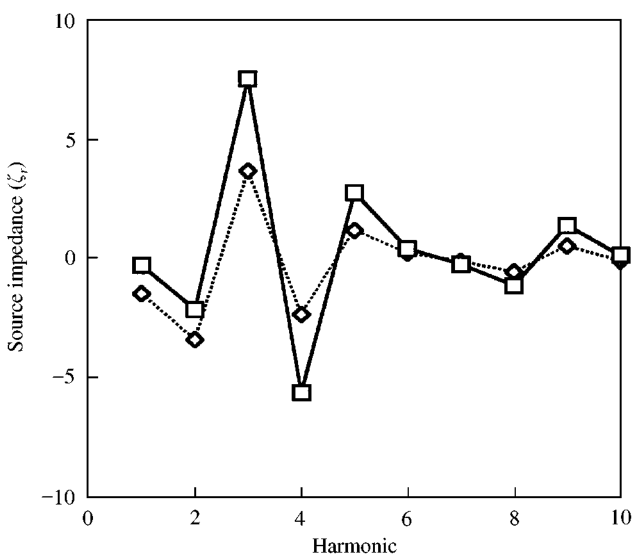

Figure 5. The effective impedance of the source, if it were time-invariant. ——, Resistance; ······, ◇ reactance.

flow well below 10 m/s for the majority of the time. Ingard and Ising [20] found in their experiments that a velocity of 10 m/s marked the transition from linear to non-linear behaviour of a valve.

A second load, a pipe of length 1·1 m, was then used in order to evaluate an effective source impedance from the two-load method, equation (11). The results are shown in Figure 5. The values are complex and their magnitude bears no relation to the terms in the impedance matrix, in Table 1. In particular, note that the real part of many of the model values is negative. Figure 6 shows the corresponding values of admittance, the more rational measure to use, as explained earlier. Once again the two-load method gives

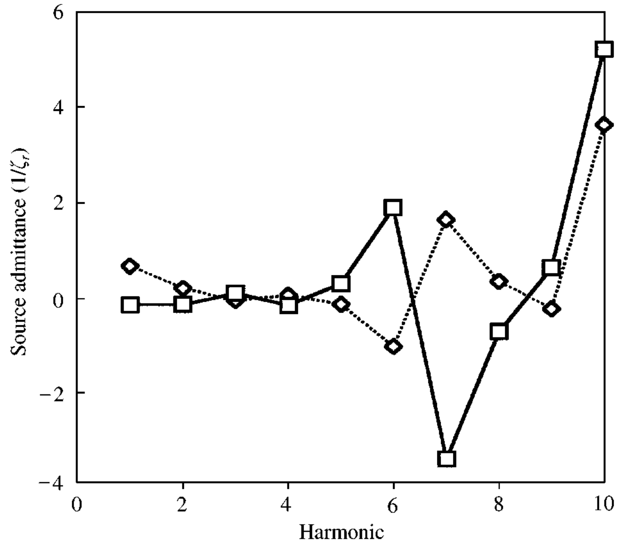

Figure 6. The effective admittance of the source, if it were time invariant. ——, Real part; ······, ◇ imaginary part.

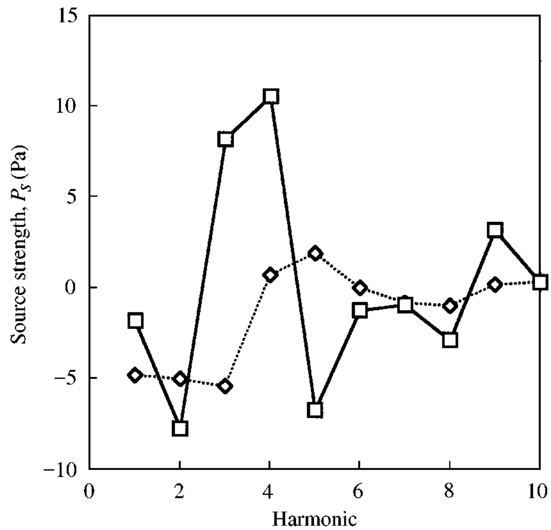

Figure 7. The effective strength of the source, if it were time invariant. ——, Real part; ······, ◇ imaginary part.

complex values instead of the real values in the admittance matrix, and the magnitude of the values bear no relation either; see Table 1. The two-load method yields an effective source strength as well as impedance, and these values are shown in Figure 7. The exact value is 10 Pa, real and constant. In summary, the effective source properties as measured by the two-load method, which would have been precise for a time-invariant source, bear no relationship whatsoever to the actual properties of a linear time-variant source.

Figure 8 shows the sound pressure level in the exhaust duct, just downstream of the valve, as calculated for a third load, a pipe of length 1·4 m. A comparison is shown between the exact values, which follow from equations (8) and (9), and the values obtained when one assumes a linear time-invariant source, equations (8) and (10), with the source properties as

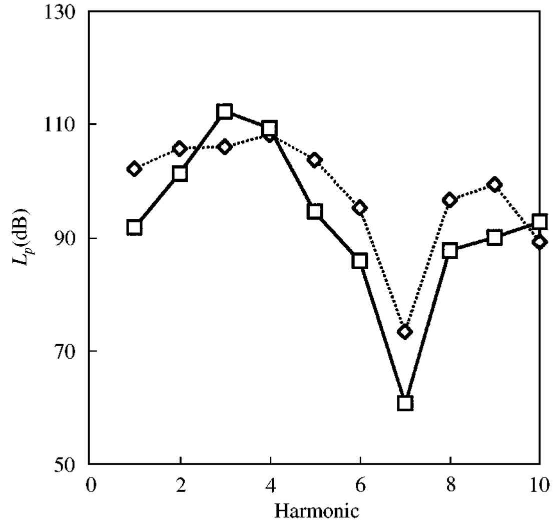

Figure 8. The sound level at the inlet to a pipe of length 1·4 m. ——, Exact; ······, ◇ calculated from effective time-invariant source characteristics.

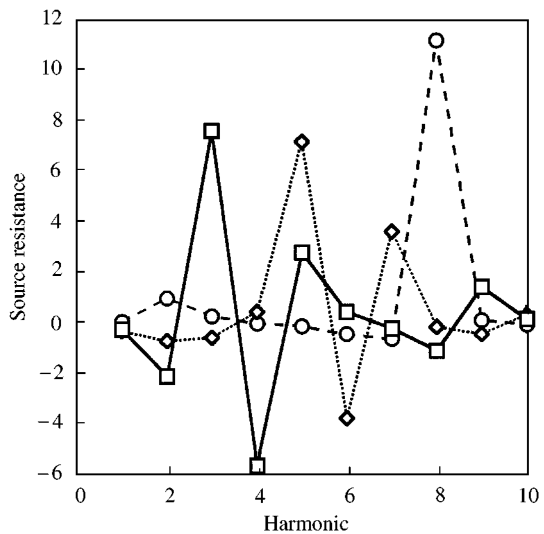

Figure 9. The effective resistance of the source, if it were time invariant, as calculated from different load combinations. ——, 1·1 m, 1·7 m; ······, ◇ 0·9 m, 0·3 m; – – – –, ○ 1·9 m, 2·3 m

determined from the above two-load analysis. The agreement is seen to be quite acceptable, despite the observations above.

The effective source properties as measured by the two-load method will actually be different for different pairs of loads, even when the actual ideal time-variant source is unaltered by the load. Figures 9 and 10 give some indication of the variation in source resistance and reactance, respectively, for different load pairs.

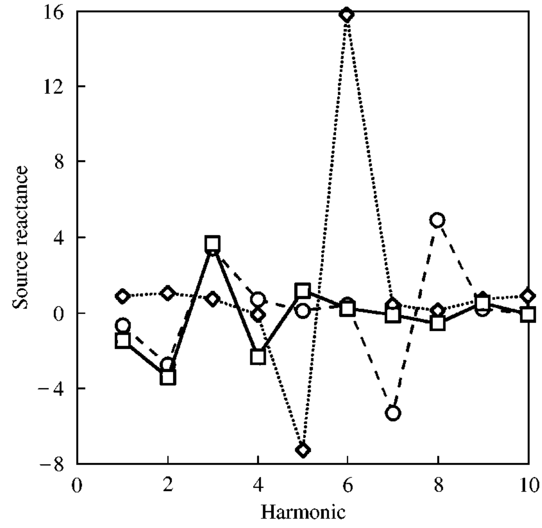

Figure 10. The effective reactance of the source, if it were time invariant, as calculated from different load combinations. ——, 1·1 m, 1·7 m; ······, ◇ 0·9 m, 0·3 m; – – – –, ○ 1·9 m, 2·3 m

5. IDEALIZED, INERTIAL, LINEAR TIME-VARIANT SOURCE

With reference to experimental results [20] when using a harmonic source pressure, Ingard [21] showed that the unsteady flow through an orifice could be approximated by the equation

$$[P_s(t) - P(t)] = \frac{\bar{\rho} U(t)|U(t)|}{2C_D^2} + \bar{\rho} l \frac{\mathrm{d}U(t)}{\mathrm{d}t}, \tag{13}$$

where l is the thickness of the orifice including the mass end corrections. At low velocities, the non-linear term is negligible, while at high velocities the linear term is negligible. The critical velocity for change of regime is about 10 m/s. If one wishes to use a linear expression which is valid only at low velocities but includes the possibility of a steady state flow, then it is necessary to use an equation of the form

$$[P_s(t) - P(t)] = \frac{\bar{\rho}\bar{c} U(t)}{C_D} + \bar{\rho} l \frac{\mathrm{d}U(t)}{\mathrm{d}t}, \tag{14}$$

where the first term on the right-hand side is a linear approximation to the non-linear term in equation (13) which is valid at low flow velocities. Comparison with equation (1) shows that the idealized source neglected the inertial term, due to the pseudo-steady approximation, whereas in reality the inertial term generally dominates the behaviour. Equation (14) still represents a particular simplification of the more general linear, time-variant source as considered by Bodén [16].

A pressure source is assumed to be connected to a system through a valve whose open area is time varying, as in section 2, and the efflux at any time instant is now considered to be given by equation (14), such that

$$\left[1 + \frac{C_D l}{\bar{c}} \frac{\mathrm{d}}{\mathrm{d}t}\right] v(t) = CA(t)[P_s(t) - P(t)], \qquad \left\{\begin{matrix} 0 < A(t) \leqslant 1, \ 0 \leqslant t \leqslant \tau \\ A(t) = 0, \ \tau \leqslant t \leqslant T \end{matrix}\right\}. \tag{15}$$

The effective thickness of the orifice has been assumed to be constant, although more precisely it would be time varying since the end corrections are dependent upon the time-varying open area.

Fourier expansion of all variables, as in equation (4), followed by a restriction to the lowest N harmonics leads to the finite matrix equation

$$\begin{bmatrix} 1-iND & 0 & 0 & 0 & 0 \\ 0 & 1-ijD & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 \\ 0 & 0 & 0 & 1+ijD & 0 \\ 0 & 0 & 0 & 0 & 1+iND \end{bmatrix} \begin{bmatrix} v_{-N} \\ \cdots \\ v_0 \\ \cdots \\ v_N \end{bmatrix}$$

$$= \begin{bmatrix} A_0 & \cdots & A_{-N} & \cdots & A_{-2N} \\ \cdots & A_0 & \cdots & \cdots & \cdots \\ A_N & \cdots & A_0 & \cdots & A_{-N} \\ \cdots & \cdots & \cdots & A_0 & \cdots \\ A_{2N} & \cdots & A_N & \cdots & A_0 \end{bmatrix} \begin{bmatrix} S_{-N}-P_{-N} \\ \cdots \\ S_0-P_0 \\ \cdots \\ S_N-P_N \end{bmatrix}, \qquad (16)$$

where $D = 2\pi l C_D/\bar{c}T$. It follows from equation (8) that

$$\{[\mathbf{A}][\zeta_l] + [\mathbf{D}]\}\{\mathbf{v}\} = [\mathbf{A}]\{\mathbf{S}\}, \qquad (17)$$

where $[\mathbf{D}]$ is the diagonal matrix on the left-hand side of equation (16). The admittance matrix is now $[\mathbf{D}]^{-1}[\mathbf{A}]$ and hence its diagonal terms are no longer constant, but are frequency varying. The impedance matrix is now $[\mathbf{A}]^{-1}[\mathbf{D}]$ and all of its terms will again tend to infinity if sufficient harmonics are taken.

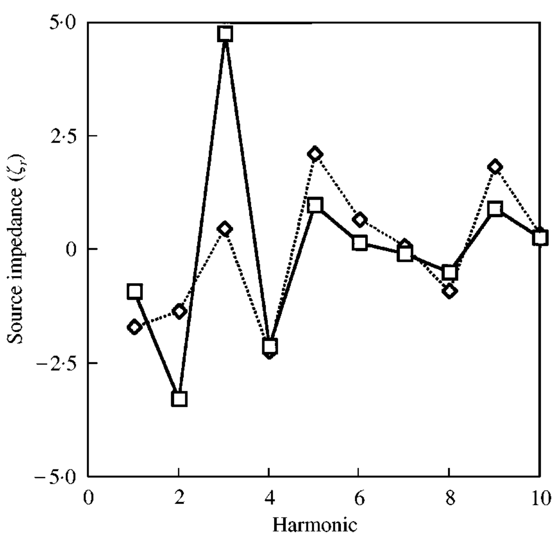

Figure 11. The effective impedance of the revised, inertial source, if it were time invariant. ——, Resistance; ······, ◇ reactance.

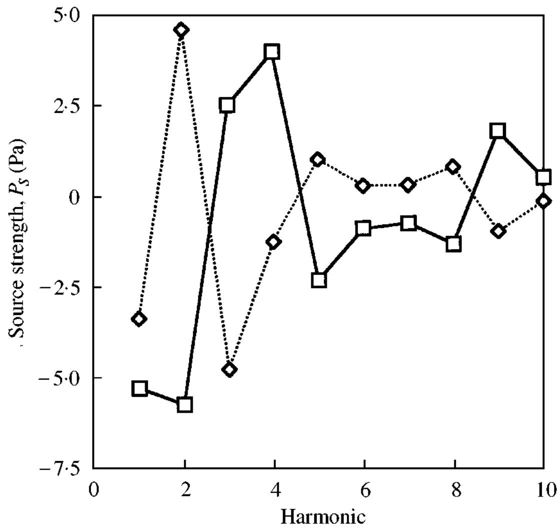

Figure 12. The effective source strength of the revised, inertial source, if it were time invariant. ——, Resistance; ······, ◇ reactance.

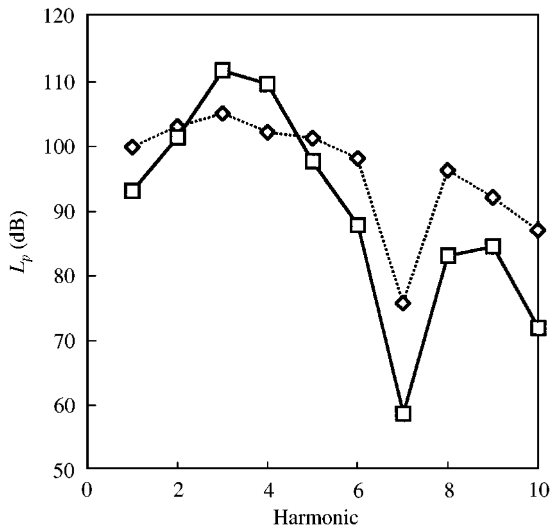

Figure 13. The sound level at the inlet to a pipe of length 1·4 m, for the revised inertial source. ——, Exact; ······, ◇ calculated from effective time-invariant source characteristics.

Results have been obtained for the same source pressure and operating conditions as were used in section 4, for a load pair of 1·1 and 1·7 m pipes. The effective source impedance of an assumed time-invariant source is shown in Figure 11. Once again the source resistance is frequently negative, and the impedance values bear no relation to the elements of the impedance matrix. The same is true for the effective admittance and the terms of the admittance matrix, of course, but these are not shown. Figure 12 shows the effective source pressure and again this bears no relationship whatsoever to the actual source pressure, which has a real, constant value of 10 Pa. Finally, Figure 13 shows a comparison between the exact sound level at the inlet to a pipe of length 1·4 m, and that predicted from a time-invariant source with the effective characteristics as given in Figures 11 and 12. The comparison is not as good as for the idealized source, but may still be regarded as acceptable.

6. CONCLUSIONS

An analytical model has been presented of the experimental multiple-load procedure for determining the characteristics of an acoustic source. The actual acoustic sources considered were linear and time variant. The multiple-load technique evaluates the effective source impedance and strength of an assumed linear, time-invariant acoustic source. It has been shown that these effective source characteristics, as given by the multiple-load method, have no physical meaning, bear no resemblance to the real source, and are dependent upon the acoustic load system used to evaluate them. It was found that the effective source resistance was frequently negative, as has been observed from experimental studies on non-linear, time-variant, IC-engine sources. Thus, it has been shown that time variance of the acoustic source alone is sufficient reason for the observed negative resistance values, although non-linearity is also likely to be a contributing factor.

The fact that the effective source characteristics are dependent upon the load systems used to evaluate them, even though the ideal linear, time-variant acoustic sources were independent of load, throws new light upon the advantages of using an over-determined set of multiple loads in the experimental procedure. It has been thought that the benefit arose solely from reducing experimental error, whereas in reality the method also averages the different effective properties as measured by many different load combinations.

Despite the fact that the effective source strength and impedance have no physical meaning or relevance to the real source, it has been shown that in some cases they can be used to give a fairly accurate estimate of the noise output when the actual source is used with different loads. Again, this is in accordance with experimental observations.

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