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DYNAMIC ANALYSIS OF THE VEHICLE–SUBGRADE MODEL OF A VERTICAL COUPLED SYSTEM

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As it is known, track transportation can be divided into track system above and track system below. While the train is moving, the parts above and below are subject to interaction and influence. The shortcoming of dynamic analysis in the past cannot usually reflect how the substructure of track including subgrade affects the running train. The analysis mentioned above indicates that the track damage involves not only the failure in strength of components but also the overdeformation or intolerable settlement of the system below track. So, in connection with the train running, taking track–subgrade as a part of vibration structure of the vehicle model, a vehicle–subgrade model of vertical coupled system has been presented. In this paper, the interactions between the vehicle running quality and the subgrade design parameters have been investigated in systematic concept and from the viewpoint of systematic matching.

1. INTRODUCTION

In recent years, high speed and heavy load technology have been the main trend in the world railway. Some problems about the dynamic interaction and coupled vibration of vehicle–track structure have also constituted a quite active research area. The researches of dynamic interaction relationship between vehicle and bridge system [1–5] and the vehicle–track dynamic model of coupled system [6–8] have all evolved. Some key problems that obstruct the development of high-speed railway, for example, impact coefficient and orientation stability of bridge, rail wear, corrugation, joint impact and irregularity of track, have partly been solved. The model of elastic beam under moving loads [9, 10] studied the interactions between the velocity or the frequency of moving load and the stiffness or damping characteristic of the beam or foundation.

But subgrade dynamic characteristics are not considered as one of the important components of the system, as mentioned above. It is known that track transportation can be divided into track system above and track system below. The track system above is the moving part, namely, locomotive and vehicle. The track system below mainly includes rail, sleeper, ballast and subgrade. They endure repeated load of the train. The track will be damaged or deformed because of factors, such as rail, joint, ballast stiffness, fatigue deformation characteristics of foundation and contact effect between sleeper and ballast, and so on. All these factors would influence the normal use of rail and make the rail surface irregular and affect the normal moving function for the wheel on rail. The analysis mentioned above indicates that the track damage involves not only the failure in strength of components but also the overdeformation or intolerable settlement of the system below track. So while analyzing dynamic interaction between vehicle and track, we cannot ignore the important effect of the participating vibration of the subgrade on the total system. Therefore, in this paper the dynamic interaction of the vehicle–track system will be extended to the vehicle–track–subgrade system.

While the train is moving, the parts above and below the track are subject to interaction and influence. The shortcoming of dynamic analysis in the past cannot usually reflect how the substructure of track including subgrade affects the running train. According to the actual condition of running train, a vehicle–subgrade model of vertical coupled system will be set up in this paper.

2. THE VEHICLE VIBRATION MODEL AND GOVERNING EQUATIONS

According to the different vehicle structures (mainly because of suspension way), the unified vertical coupled model of vehicle–track may be of three different styles. One is the secondary suspension vehicle–track model that mainly reflects the interaction between passenger car and track. A modern freight car, like the British LTF, is also regarded as this model. Another is the conventional primary center suspension freight–track model which is suitable for China's main freight, such as C_{62A}. The third is the primary axis box suspension new freight–track model [6]. The vehicle that has spring suspension installation is a vibrating system with complex degree of freedom. In this paper we mainly study the vehicle–track–subgrade system's vibration caused by vehicle running at high speed. Especially, we conduct research about the influence of the change of the part below track on the vehicle running quality, including degree of comfort, safety and irregularity. In order to simplify the analysis process, practical vehicle analysis is simplified by the primary spring suspension as shown in Figure 1. The main calculating assumptions are given as follows [1–3].

1. The car-body and the wheel of the vehicle are both considered as rigid bodies.
2. The train, consisting of many same or different locomotives or vehicles (they will hereinafter be called vehicles), passes line at even speed.
3. The joint devices linking vehicles do not affect the vehicle's vertical vibration.
4. The wheels of the vehicle always remain in close contact with the track surface.

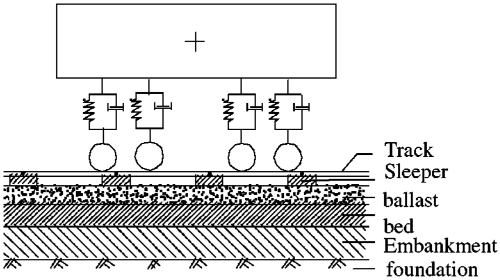

Figure 1. Vertical vehicle–subgrade vibration model.

5. Two degrees of freedom of sink and raise, nod will be considered for each car-body and one degree of freedom of the wheel will be considered.

6. The vehicle that consists of the secondary suspension or the primary axis box suspension is simplified by the primary spring suspension.

Based on the above assumptions, the vehicle calculating model, such as fourth-axis vehicle, is shown in Figure 2. The dynamic balance equations of the i car-body are

$$M_i\ddot{Z}_i + C_{zi}\dot{Z}_i - C_i\sum_{j=1}^{4}\dot{Z}_{wij} + K_{zi}Z_i - k_i\sum_{j=1}^{4}Z_{wij} = 0 \tag{1a}$$

$$J_i\ddot{\theta}_i + C_{\theta i}\dot{\theta}_i - C_i\sum_{j=1}^{4}\eta_j l_{ij}\dot{Z}_{wij} + k_{\theta i}\theta_i - k_i\sum_{j=1}^{4}\eta_j l_{ij}Z_{wij} = 0 \tag{1b}$$

Formula (1a) represents the sink and emerge dynamic balance equation of car-body. Its matrix form is

$$\begin{bmatrix} M_i & 0 \\ 0 & J_i \end{bmatrix}\begin{Bmatrix} \ddot{z}_i \\ \ddot{\theta}_i \end{Bmatrix} + \begin{bmatrix} C_{zi} & 0 \\ 0 & C_{\theta i} \end{bmatrix}\begin{Bmatrix} \dot{z}_i \\ \dot{\theta}_i \end{Bmatrix} + \begin{bmatrix} K_{zi} & 0 \\ 0 & K_{\theta i} \end{bmatrix}\begin{Bmatrix} z_i \\ \theta_i \end{Bmatrix} = \sum_{j=1}^{4}\begin{Bmatrix} k_i z_{wij} + c_i\dot{z}_{wij} \\ \eta_j l_{ij}(k_i z_{wij} + c_i\dot{z}_{wij}) \end{Bmatrix}, \tag{2}$$

where M_i and J_i represent, respectively, the mass and the inertial moment of the i car-body; Z_i and θ_i represent, respectively, the vertical displacement and the angle of the i car-body center of mass; K_{zi} and C_{zi} represent, respectively, the general stiffness and general damping of the sink and rise of the car-body. The springs below are in parallel connection, then $K_{zi} = 4k_i$, $C_{zi} = 4c_i$. $K_{\theta i}$ and $C_{\theta i}$ represent, respectively, the general stiffness and general damping of the nod. According to the primary suspension mode, $K_{\theta i} = 2k_i l_{ij}^2 + 2k_i(l_{ij} + 1/2l_w)^2$, $C_{\theta i} = 2c_i l_{ij}^2 + 2c_i(l_{ij} + 1/2l_w)^2$. k_i and c_i represent, respectively, the primary spring stiffness and the damping of the wheel, respectively. l_{ij} is the distance between the wheel and the center of the car-body. η_{ij} is the symbol function of the wheel. When wheel j is in the front bogie $\eta_j = 1$, and in the back bogie $\eta_j = -1$. Other parameter values and meanings are given in Figure 2 and Table 2.

3. TRACK AND SUBGRADE MODE

In the previous studies of track and subgrade models proposed by Wanming Zhai [6, 7], Chenghui Li [8] and LingQing Li [11], the models are all considered to be the supporting

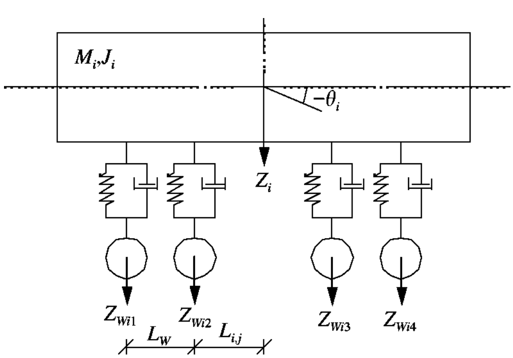

Figure 2. Vehicle calculate model.

model of multi-layer bases track below. Track is considered to be Euler's spring sustaining beam. The mass of sleeper concentrates on the node of the track element. There is a dispersed sustaining system below each sleeper. The ballast will be considered to be loose medium without considering the effect of the vertical vibration among them. So the model of lumped parameters shown in Figure 3 is adopted. The biggest advantage of this model is it that simplifies the computational work of the numerical analysis.

Germany's route model [12] is also the longitudinal dispersed model that is mono-layer spring bearing model of sleeper–track, but the longitudinal plane of the track in dispersed dots is simulated by the method of the imitate approaching fit.

The models mentioned above reflect the physical character and mechanical state of track bases in some aspects, especially that under high-frequent vibration, the link restricting vertical vibration among ballast will be further weakened. Although ballast, embankment and ground constitute a kind of loose, non-continuous medium, the simple longitudinal dispersed model without considering the longitudinal connective effect among them does not conform to the character of their vibration. Besides, failure to consider the subgrade and ground vibration is also one of the main shortcomings of the models mentioned above.

The model of track and subgrade analyzed in this paper is still divided in finite element model by the conventional method shown in Figure 4. The track is divided into beam elements or quadrangle elements. The sleeper, subgrade and foundation are separated in quadrangle elements. The effect of working property, the interaction and the coordination and the parameter on the track and subgrade model in vibration condition will be fully considered. The vibration equation is

$$[m]\{\ddot{\delta}\} + [c]\{\dot{\delta}\} + [k]\{\delta\} = \{f\}. \tag{3}$$

As the finite element model, if the vibration equations of the system of vehicle and subgrade are directly calculated, the calculation work will be very large. So the track and

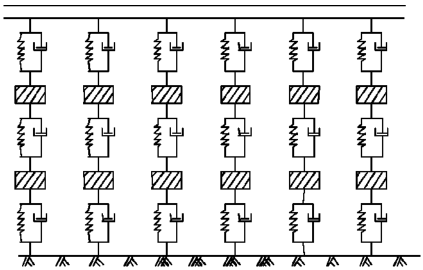

Figure 3. Three layers of bases supporting model below track.

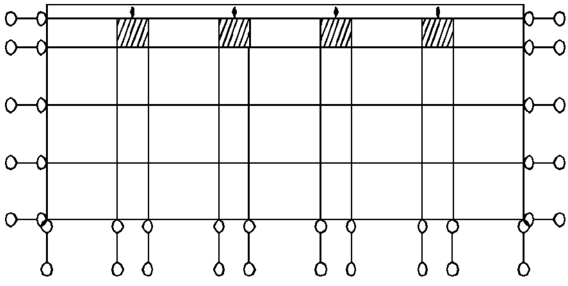

Figure 4. Finite element model.

subgrade model is built by sub-structure principle. Firstly, the every step free frequency and vibration mode is solved. Secondly, by using the normalization of vibration, thousands of coupled equations will be changed into special mode equations. Since the vibration reaction of structure is mainly controlled by the first several low stages of vibration mode, the calculation work will be greatly reduced by using the first several stages of vibration mode.

In practical calculation, the vehicle load acts on subgrade by track and all the structural vibrations affect the vehicle by track too. Every stage of vibration mode can be taken from the joint of the track surface or the position where the wheel is on the track.

The vertical displacement $Z_r(x)$ of any cross-plane of track–subgrade can be summed up by several vibration mode functions. If we calculate by N stages of vibration mode, we have

$$Z_r(x) = \sum_{n=1}^{N} A_n \Phi_n(x), \tag{4}$$

where $\Phi_n(x)$ is the nth stage of vibration mode in some position: A_n the corresponding generalized co-ordinate (i.e., the generalized co-ordinate in some time steps); and x the horizontal position of some cross-plane.

The corresponding nth stage modal equation can be gained by mode analysis as

$$M_n \ddot{A}_n + C_n \dot{A}_n + k_n A_n = F_n \tag{5}$$

or can be written into

$$\ddot{A}_n + 2\xi_n \omega_n \dot{A}_n + \omega_n^2 A_n = F_n, \tag{6}$$

where F_n is the generalized force acting on the nth stage of vibration mode; M_n, k_n and C_n are, respectively, generalized mass, stiffness and damping; ω_n is the circular frequency of the nth stage; and ξ_n the normalization damping coefficient.

The method of the generalized force F_n is defined in the following.

When the wheel j of the car-body i is running through the length of the analyzed track, the force from the wheel acting on the track includes the inertia force of the wheel and the vertical force from the spring and damper. Then the vertical force is

$$P_{j\omega} = P_{is} + k_i z_i + c_i \dot{z}_i + \eta_j l_{ij}(k_i \theta_i + c_i \dot{\theta}_i) - m_{ij} \ddot{z}_{\omega ij}, \tag{7}$$

where P_{is} is the static weight of the wheel, $P_{is} = (1/4M_i + m_{ij})g$; m_{ij} the mass of the wheel j of the car-body i; and $z_{\omega ij}$ the vertical displacement of the wheel j of the car-body i. The other symbols are the same as given above.

If there are N_v car-bodies including N_ω wheels running on the length of the analyzed track, the generalized force F_n corresponding to the nth vibration mode is

$$F_n = \sum_{i=1}^{N_v} \sum_{j=1}^{N_\omega} P_{j\omega} \Phi_n x_{ij}, \tag{8}$$

where x_{ij} is the position of the track surface of the wheel j of the car-body i.

4. THE SOLUTION OF THE COUPLED VIBRATION EQUATION

As mentioned above, though the model of track and subgrade can be transformed into a special mode equation by sub-structure principle and the vibrating analyses of the vehicle–subgrade model can be calculated by several low stages of vibration mode. It will be an important problem as to how to connect the model of track and subgrade and the vehicle–subgrade model, and how to solve them.

In the past, the coupled dynamics of vehicle–track was solved by Hertz non-linear elastic contact theory [6–8] according to wheel/rail contact force. However, in this paper the problem is solved by the displacement compatibility condition of the vehicle–subgrade vibration model.

4.1. THE DISPLACEMENT EQUATION OF THE WHEEL (CONNECTION EQUATION)

According to the assumption mentioned above, while the train is running, the harmonious displacement equation of the wheel's vertical displacement and the dynamic displacement of the track is

$$Z_{\omega ij} = Z_r(x_{ij}) + Z_s(x_{ij}), \tag{9}$$

where $Z_s(x_{ij})$ is the track vertical profile irregularity in x_i position of track surface. It is a simulated value.

Considering (4) and (9), we have

$$Z_{\omega ij} = \sum_{n=1}^{N} A_n \Phi_n(x_{ij}) + Z_s(x_{ij}). \tag{10}$$

4.2. THE DYNAMIC EQUILIBRIUM EQUATION OF VEHICLE–TRACK SYSTEM

The dynamic equation of the vehicle and the mode equation of track–subgrade have been mentioned above respectively. As the dynamic property of the vehicle–track system, they must be combined. If we substitute into the displacement connection equation, the following vertical dynamic equations will be obtained:

$$\begin{bmatrix} M_i & 0 \\ 0 & J_i \end{bmatrix}\begin{Bmatrix} \ddot{z}_i \\ \ddot{\theta}_i \end{Bmatrix} + \begin{bmatrix} C_{zi} & 0 \\ 0 & C_{\theta i} \end{bmatrix}\begin{Bmatrix} \dot{z}_i \\ \dot{\theta}_i \end{Bmatrix} + \begin{bmatrix} K_{zi} & 0 \\ 0 & K_{\theta i} \end{bmatrix}\begin{Bmatrix} z_i \\ \theta_i \end{Bmatrix}$$

$$= \sum_{j=1}^{4} \begin{Bmatrix} \displaystyle\sum_{n=1}^{N} \Phi_n(x_{ij})(k_j A_n + c_j \dot{A}_n) + k_j z_s(x_{ij}) + c_j \dot{z}_s(x_{ij}) \\ \displaystyle\sum_{n=1}^{N} \eta_j l_{ij} \Phi_n(x_{ij})(k_j A_n + c_j \dot{A}_N) + \eta_j l_{ij}(k_j z_s(x_{ij}) + c_j \dot{z}_s(x_{ij})) \end{Bmatrix},$$

$$M_n \ddot{A}_n + c_n \dot{A}_n + K_n A_n = \sum_{i=1}^{N_1} \sum_{j=1}^{N_w} \Phi_n(x_{ij}) \left\{ \left(\frac{1}{4} M_i + m_{ij} \right) g + k_j z_i \right.$$

$$\left. + c_j \dot{z}_i + \eta_j l_{ij}(k_j \theta_i + c_j \dot{\theta}_i) - m_{ij} \left[\sum_{n=1}^{N} \ddot{A}_n \Phi_n(x_{ij}) + \ddot{z}_s(x_{ij}) \right] \right\} \tag{11}$$

The total number of equations is $(2N_V + N)$. N_V is the number of vehicles. Since each car-body has two degrees of freedom of sink, emerge and nod, the number of equations are $2N_V$; N is the number of track–subgrade mode equations. Several representative low-vibration modes will be considered, for example, N is 10. The vibrating equations will be solved by the Newmark-β method.

4.3. SIMULATION OF VERTICAL IRREGULARITY OF THE TRACK SURFACE

While the vehicles are running along the track, the vibration of vehicle and track will be induced because of track irregularity. Different kinds of track irregularity are produced because of such factors as rail structure, joint, weld and subgrade base deformation [13]. They all could be described by random irregularity.

The usual method to measure the vertical irregularity of track is chord measurement [13]. They are described by SIN or COS function; they also accord with the FRA standard. Corresponding to different frequencies, the irregularity value is

$$Z_t(x) = Z_0 \sin(2\pi f_s x), \tag{12}$$

where $Z_t(x)$ is the irregularity value at certain positions; Z_0 the amplitude of vibration; and f_s the corresponding frequency.

From this concept and by the maintenance and management goal of irregularity, the track surface irregularity can be stimulated as

$$Z_x(x) = A_s \sin\left(\frac{2\pi V}{L_s} t + \eta\right), \tag{13}$$

where V is the speed of train; L_s the wavelength corresponding to the management aim; A_s the management aim of irregularity; and η the random number corresponding to different positions (0–1·0).

5. CALCULATING PARAMETERS

The calculating parameters of track–subgrade mainly relate to mechanical properties of material, including elastic modulus, the Poisson ratio, angle of internal friction, adhesion and so on. The calculating parameters of vehicle mainly relate to the vehicle styles. The calculating parameters in different documents are not all the same. The calculating parameters in this paper are chosen according to the on-the-spot survey data and the range of theory calculating. The main calculating parameters of vehicle and subgrade are given in Tables 1 and 2, respectively.

6. THEORETICAL ANALYSIS OF VEHICLE–SUBGRADE

According to the dynamic interaction model of vehicle–subgrade and the dynamic balance equations mentioned above, the following theoretical analysis can be obtained through programming and using the Newmark-β method [17].

(1) The acceleration of the ith car-body is expressed as

$$a_i = \left(\ddot{z}_i + \frac{l_p}{l_u}\ddot{\theta}_i\right), \tag{14}$$

TABLE 1

Basic calculating parameters of vehicle part (power-driven of New trunk in Japan) [14–16]

Parameters	Number
The mass of car-body (t)	36·74
The mass of each wheel (t)	0·8
First suspension spring stiffness (kN/m)	904
First suspension spring damp (kN s/m)	44·2
Fixed spread of axles (m)	2·5
The length of car-body (m)	25·0
Fixed spread of car-body (m)	17·0

TABLE 2

Main calculating parameters of subgrade

Parameters' style		Number					
		1	2	3	4	5	6
Subgrade base stiffness	Surbase	180	180	150	120	100	—
E(MPa), code in Figure is *je*	subbase	150	120	120	80	50	—
Embankment stiffness E(MPa), code is *le*		45	65	—	—	—	—
Foundation E(MPa), code is *de*		40	30	20	10	5	—
Speed of train v(km/h), code is v		180	216	252	288	324	360
Net distance between sleepers b(m), code b		0·1	0·3	0·5	—	—	—

where l_p is the length apart from the center of car-body; and l_u the distance between the center of wheel and the center of car-body.

(2) The jth wheel's reduced load of the ith car-body is written as

$$D_j = \frac{p_{jw} - p_{js}}{p_{js}} = \frac{p_{jw}}{p_{js}} - 1, \tag{15}$$

where p_{jw} is the wheel's vibrating load while the train is running; and p_{js} the average wheel's static load.

(3) The dynamic displacement or dynamic deflection of track surface in the position $x = x_p$ is expressed as

$$z_r(x) = \sum_n^N A_n \phi_n(x)|_{x=x_p}, \tag{16}$$

where A_n is the generalized co-ordinate corresponding to the nth vibration mode; and $\phi_n(x)$ the nth vibration mode function.

7. COMPUTATIONAL RESULTS

The aim of establishment of vehicle–subgrade mode is to solve the interactions between the vehicle's running characters and the subgrade design parameters. As mentioned above, the influences of subgrade design parameters including subgrade stiffness, vehicle's speed, etc., on the vehicle's running quality, including degree of comfort (car-body's vertical acceleration), safety (rate of reducing load of wheel) and the dynamic displacement of track, can be determined. Considering how to control subgrade deformation is the main aspect in design of high-speed railway; in this paper, the relation between the subgrade design parameters and the dynamic deformation of track surface or base surface [17] will be mainly investigated.

7.1. THE RELATION BETWEEN THE ELASTIC DEFORMATION OF BASE SURFACE AND SUBGRADE DESIGN PARAMETERS

7.1.1. *Concerning the problem about elastic deformation of subgrade base*

When acted upon by train loading, the subgrade deformation includes the plastic deformation and the elastic deformation at the same time. The elastic deformation results from the dynamic load of vehicle while the vehicle is running at a high speed. It mainly takes place in the position of subgrade base, especially on base surface. The elastic deformation of subgrade is reflected by the elastic deformation of track surface. The larger the elastic deformation, the lower is the vehicle's speed. When the upper part structure and the technological condition of ballast are defined, the main factor affecting the elastic deformation of track surface is subgrade. The lower the subgrade stiffness, the greater the elastic deformation, and the worse the stability of ballast and upper part structure of track. Then, to what degree should the subgrade stiffness and the corresponding elastic deformation be controlled?

Japan used asphalt concrete to strengthen the subgrade base structure and to control crack in asphalt concrete pavement of highway according to the deformation angle θ standard. If θ is applied to the base surface of railway, the corresponding deflection is just 2·5 mm, which is the controlling elastic deformation of asphalt concrete-strengthened subgrade base.

The base structure form, which is suggested by "high speed parameter research" [18] in China, is the graded gravel–sand cushion. Since it belongs to flexible structure of loose materials, the permitting elastic deformation can certainly be relaxed. But to assure ballast stability and to avoid frequent management, the suggestion value of controlling elastic deformation is 4 mm [19].

7.1.2. *The relation between the elastic deformation of base surface and subgrade design parameters*

Since it is assumed that the wheel of the vehicle remains in close contact with the track surface while the vehicle is running in the vehicle–subgrade coupled dynamic model mentioned above, either the vertical displacement in any position of track surface or the vertical displacement of wheel can be obtained by formula (4). However, the deformation of the track surface reflects the whole process when the vehicles are running on the calculating track while the displacement of any wheel only reflects time step process. Since the deformation produced by track and ballast is very small, the main deformation shown here will be the elastic deformation of base surface.

Figures 5 and 6 show the relation between subgrade stiffness and elastic deformation. The condition of the subgrade and foundation stiffness are as follows: let $le = 1$, $de = 3$, where *iw* and *ip* are the joint codes of the wheel and the track surface respectively. $v = 252$ shows that the vehicle's speed is 252 km/h. The meaning of these symbols is the same below.

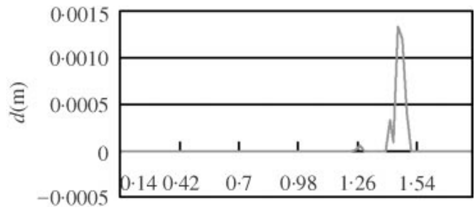

Figure 5. ($v = 252$, $iw = 14$, $je = 1$).

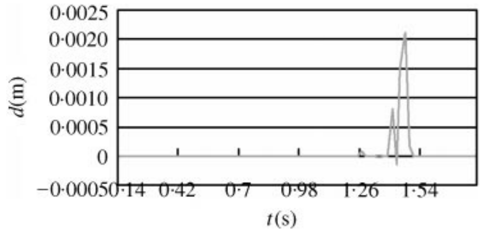

Figure 6. ($v = 252$, $iw = 14$, $je = 5$).

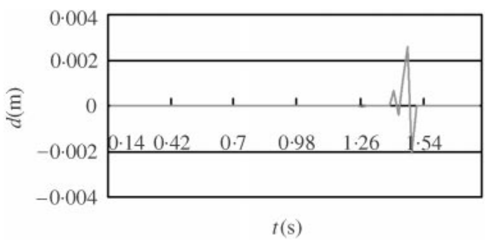

Figure 7. ($v = 252$, $iw = 14$, $de = 1$).

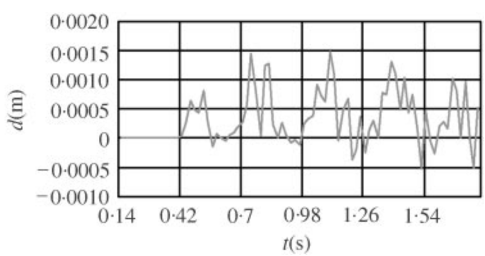

Figure 8. ($v = 252$, $ip = 66$, $de = 2$).

From Figures 5 and 6, it can be seen that the elastic deformation of base surface is in direct proportion to the subgrade base's stiffness. Since the subgrade base stiffness is relatively greater, the influence is not remarkable.

Figures 7–10 show that the foundation stiffness has a remarkable effect on the deformation of base surface. It is not good for controlling the deformation of base surface if the foundation stiffness is too hard or too soft. Of course a soft foundation is more disadvantageous. When the foundation stiffness $de \leqslant 10$ MPa, the foundation should be strengthened.

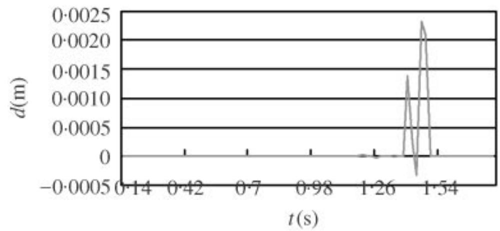

Figure 9. ($v = 252$, $iw = 14$, $de = 4$).

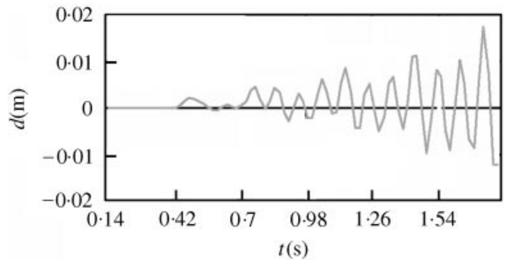

Figure 10. ($v = 252$, $ip = 66$, $de = 5$).

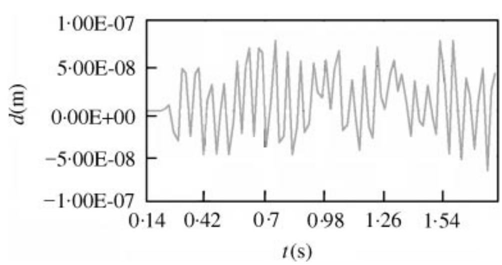

Figure 11. ($v = 252$, $ip = 66$, $b = 0{\cdot}35$, $je = 2$).

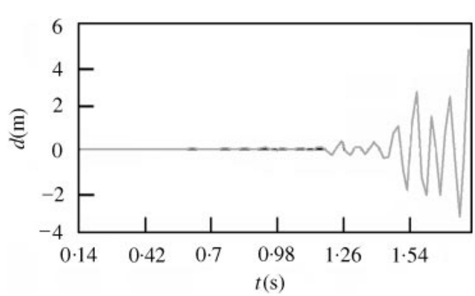

Figure 12. ($v = 252$, $ip = 66$, $b = 0{\cdot}75$, $je = 2$).

From Figures 11 and 12, it can be found that the net distance between sleepers greatly affects the deformation of base surface (when normal distance $b = 0{\cdot}55$ m, the maximum deformation is 2 mm). If the net distance between sleepers is properly reduced, the deformation of base surface will be greatly decreased.

From Figures 13 and 14, it can be found that the effect of vehicle speed on the deformation of base surface is very small.

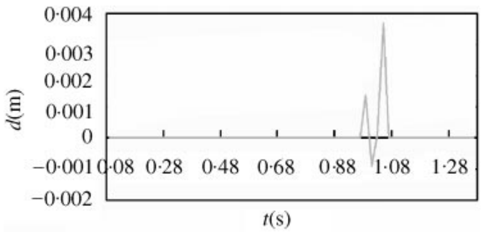

Figure 13. (v = 360, iw = 14, je = 2).

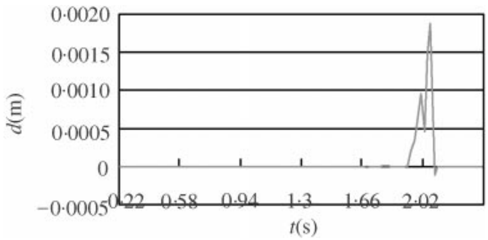

Figure 14. (v = 180, iw = 14, je = 2).

By way of checking the formulation and the computer program, comparisons of the results of this paper with calculated results of other models have been made.

The finite element program of three-dimensional subgrade has been written in the research report [19] (Science and technology brainstorm subject of the National ninth 5-year plan) in order to control the subgrade deformation of high-speed railway. According to different subgrade's cross-plane in different countries, the elastic deformation of base surface can be calculated by the program. Calculated results indicate that the elastic deformations of base surface are in the range of 1·32–2·25 mm under different subgrade heights (5–10 m). According to the lumped parameters model of vehicle-track [12], the maximum elastic displacement such that the value is 2·1 mm, may be deduced by displacement transfer functions (0·02–0·35 mm/mm) on the irregularity of 6 mm/10 m. If the foundation stiffness is given in the range ($De \geqslant 10$ MPa), the results of vehicle–subgrade mentioned above indicate that the elastic deformations of base surface are in the range of 1·2–3·7 mm under the different conditions including different subgrade base stiffnesses, different embankment stiffnesses and different vehicle speeds. Since the model in this paper can calculate the influence of the vehicle's speed, the calculated results of the model in this paper fit the results of other models and model tests well.

7.2. THE RELATION BETWEEN THE SAFETY, DEGREE OF COMFORT OF VEHICLE AND SUBGRADE DESIGN PARAMETERS

According to the suggestion of the eighth five-year plan, the safety of vehicle, the rate of the reduced load of wheel, is defined as $\Delta p/p \leqslant 0{\cdot}8$; the influence of subgrade base, embankment and foundation stiffness is not remarkable. But the influence of net distance between sleepers and the vehicle speed is relatively remarkable. Under good working conditions, $v_{cr} = 288$ km/h is a critical speed. According to the standard in this paper, if the degree of comfort (i.e., car-body's vertical acceleration) is defined as $a_v = 2{\cdot}0\ \mathrm{m/s^2}$, the influence of foundation stiffness is not remarkable. The relation between base stiffness and of degree comfort (at the same time, the foundation and embankment stiffness are moderate values) indicates that there will be a critical value of base stiffness, when the sur-base stiffness is $E = 120$ MPa and sub-base stiffness is $E = 80$ MPa. The value accords with the Germany minimum standard of subgrade base. In addition, the degree of comfort will be affected if the vehicle speed $v \geqslant 288$ km/h.

8. THE CONCLUSIONS

The meanings about establishment of vehicle–subgrade model of the whole vertical coupled dynamic system includes the following: (1) If track–subgrade is regarded as a part of vibration structure of vehicle–subgrade in dynamic analysis, it will perfectly reflect the dynamic property of vehicle–subgrade. (2) For the low-speed railway, the problems of vehicle–subgrade system can solved by dispersing them, respectively, into static, semi-static and dynamic problems of each subsystem. Of course, the dynamic model of vehicle–subgrade system in this paper is a model suitable for high-speed railway. (3) If track–subgrade is regarded as a part of vibration structure of the vehicle model, it can fully reflect the interactions between the vehicle and the sub-structure, and provide a basis of theoretical analysis for the determination of the part design parameters (especially for subgrade design parameter).

In addition, from the results mentioned above, it is easy to make the following conclusions:

(1) The elastic deformation of base surface is direct ratio to the subgrade base stiffness. Since the subgrade base stiffness is relatively greater, the influence is not remarkable.

(2) The foundation stiffness has a remarkable effect on the deformation of base surface. It is not good for controlling the deformation of base surface if the foundation stiffness is too hard or too soft. When the foundation stiffness $de \leqslant 10$ MPa, the foundation should be strengthened.

(3) On properly reducing the net distance between sleepers, the deformation of base surface will be greatly decreased.

(4) The vehicle speed slightly affects the deformation of base surface.

(5) Calculated results indicate that the elastic deformations of base surface are in the range of 1·32–2·25 mm under different subgrade heights (5–10 m).

(6) If the safety of vehicle is defined as $\Delta p/p \leqslant 0{\cdot}8$, the influence of subgrade base, embankment and foundation stiffness is not remarkable. Under good working conditions, $v_{cr} = 288$ km/h is a critical speed.

(7) If the degree of comfort is defined as $a_v = 2{\cdot}0\ \mathrm{m/s^2}$, the influence of foundation stiffness is not remarkable. In addition, the degree of comfort will be affected if the vehicle speed $v \geqslant 288$ km/h.

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