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THE LIFT GENERATED BY VIBRATIONAL MOTIONS

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By applying the reacted pressure on the "deformed" surface, we have calculated theoretically the time-averaged lift forces exerted on a vibrating body in the fluid medium with the non-viscosity and small disturbance assumptions. The lift turns out to be proportional to the product of the amplitudes of translating and deforming vibrations. We could thus deduce the scaling law applicable for the forces generated by the vibrational motions in the fluid medium.

1. INTRODUCTION

In fluid mechanics, the lift force exerted upon an obstacle surrounded by the incoming fluid flow is always linearly related to how much circulation is generated by this inserted obstacle. This relation is known as Kutta–Joukowski law [1], which states that the lift force equals the product of fluid mass density, incoming flow velocity, and the circulation generated. One of the basic assumptions of the Kutta–Joukowski law is that the velocity field must be stationary, i.e., the fluid flow is steady. Unfortunately, there exists no such simple relation or theory for the unsteady flows. In general, unsteady fluid dynamic problems are highly non-linear, and computational fluid dynamics (CFD) is required for practical applications [2]. In spite of theoretical difficulties, however, fluid dynamics theorists have tried to apply the concepts of steady flows to explain the unsteady flow phenomena [3, 4]. Here, we try to derive some theoretical results rather than the numerical computations for simple examples of unsteady flows induced by some vibrating bodies, which will turn out to have useful physical meanings about the lift forces generated by the motions of deformable bodies in the fluid.

2. THE VIBRATIONAL SPHERE

As a sample problem, we consider a sphere of radius a immersed in the exterior infinite fluid medium, as shown in Figure 1. The undisturbed mass density of the fluid is denoted by ρ. The center of the sphere is assumed to oscillate vertically (in Z direction) about the origin with a circular frequency ω and an amplitude $d(t)$. Additional to this vertical oscillation, the radius of this sphere is also varied harmonically at the same frequency and expressed as $a(t) = a_0 + \delta \cos(\omega t + \beta)$, where a_0 is the averaged or undeformed radius, β is the phase angle between the vertical and dilatational oscillations, δ is the amplitude of radius or dilatational oscillation. The question is: what are the pressure distribution and net force exerted on the sphere at various frequencies and phase angles, when the time-averaged net force exerted on the sphere is especially considered.

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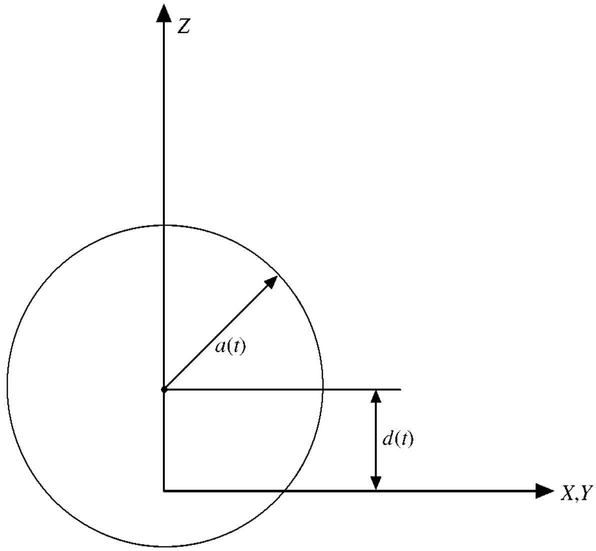

Figure 1. Geometrical description for the sphere problem.

The question stated above is a typical problem of classical fluid mechanics. The most general formalism of fluid dynamics will lead to the Navier–Stokes equations, which is too difficult to be solved exactly even for simple problems. Thus, some assumptions and simplifications are necessary. As in most situations in aerodynamics and hydrodynamics, we will first neglect the viscous effect. This means that we will consider the high Reynolds number flows. Secondly, we will neglect the non-linear convective terms in the Navier–Stokes equations, this can be done by assuming that there is no incoming flow in the far field and there exist the following small disturbance conditions:

$$|d/a_0| \ll 1, \quad |\delta/a_0| \ll 1.$$

It turns out that, under the above assumptions, the Navier–Stokes equations can be reduced to ordinary linear acoustic equations [5]. That is, the problem under consideration will be solved if we can determine the velocity potential φ everywhere in the fluid, which must satisfy the wave equation $c^2 \nabla\varphi = \partial^2\varphi/\partial t^2$ if we denote c as sound speed in the fluid. The velocity field $\mathbf{v}$ equals $-\nabla\varphi$ by definition, and φ relates to the pressure disturbance as $p = \rho\, \partial\varphi/\partial t$. Furthermore, we will specify the boundary conditions compatible with the vibrational motions of the sphere, and the radiation conditions at infinity.

All the considerations mentioned above lead to the standard boundary value problem (BVP) formulation for the acoustic field, and the solution procedures are nothing more than the method of separation of variables for the wave equation. The solution, or the pressure distribution, is just the linear combination of two parts: the sound field generated by the vertically translating motion, plus that generated by the pulsating motion of the sphere [5, 6]. We could calculate the instantaneous physical quantities, such as pressure or velocity fields, everywhere in space and time. But, if we want to calculate the time-averaged force exerted on the deformable sphere, we obtain only zero force, which might violate our physical intuitions.

This zero force contradiction arises from the fact that in an ordinary boundary value formulation of the acoustic or vibrational problems, we usually put the boundary conditions on the "undeformed" boundary. This simplification, of course, is suitable for our small disturbance assumptions, but it can merely lead to compatible results as if the bodies are nearly undeformed. On the other hand, if we want to trace the fluid particle exactly as the fluid deforms, we will get the Lagrangian descriptions [7] for the fluid motion, in which the fluid dynamic equations are much more inconvenient to be solved.

3. LIFT FORCE CALCULATIONS

Our method to overcome the zero force contradiction is quite straightforward: we just apply the pressure calculated by the linear theory to the corresponding instantaneous deformed boundary. For simple body shapes [8], the results are quite simple if we neglect higher order terms, and may be generally expressed as

$$L = \tfrac{1}{2}\rho v_t v_d A C(\xi, \beta), \tag{1}$$

where $v_t = \omega d$ is the velocity amplitude of the translating motion, $v_d = \omega\delta$ is the velocity amplitude of the deforming motion, A is the averaged apparent area viewed from the direction of translational motion, C is the dimensionless lift coefficient which depends on the reduced frequency $\xi = \omega a_0/c$ and the phase angle β defined above. For the present problem, the lift coefficient can be expressed explicitly as

$$C(\xi, \beta) = \frac{8}{3}\,\frac{(\xi^2 + 2)\cos\beta - \xi^3 \sin\beta}{\xi^4 + 4}. \tag{2}$$

For every fixed frequency, we can choose the value of β such that the lift coefficient reaches its maximum. This value of β will be $\beta = \beta_m \equiv -\tan^{-1}(\xi^3/(\xi^2 + 2))$, and the lift coefficient with $\beta = \beta_m$ will be

$$C(\xi, \beta_m) = C_m(\xi) \equiv \frac{8}{3}\,\frac{[(\xi^2 + 2)^2 + \xi^6]^{1/2}}{\xi^4 + 4}. \tag{3}$$

The lift coefficient $C_m(\xi)$ can further be maximized to its maximum value (about 1·696) if $\xi = (\sqrt{5} - 1)^{1/2} \doteqdot 1{\cdot}112$. The meaning of $C_m(\xi)$ is the maximum lift coefficient as the phase angle β is optimally chosen at a given frequency, the corresponding lift is then maximized as

$$L_m = \tfrac{1}{2}\rho v_t v_d A C_m(\xi). \tag{4}$$

4. PHYSICAL INTERPRETATIONS

The phase angle β_m and lift coefficient C_m as defined above are the functions of the reduced frequency ξ, and are plotted in Figure 2. In the low frequency range $\xi \ll 1$, the lift at the given frequency reaches its maximum when the phase angle approximately equals to zero. That is, in order to get the maximum lift force, the sphere should dilate to its maximum size when its center reaches the highest vertical position, and it should contract to its minimum size when its center reaches the lowest vertical position. This is due to the fact that, in the low frequency range the effect of compressibility of the fluid is less important, the reaction of the fluid results from the induced inertia of added mass when the sphere moves in the fluid [7]. At zero frequency, the lift coefficient C_m equals to 4/3. By the present unsteady formulation, it can be shown that, as the frequency approaches zero, the added

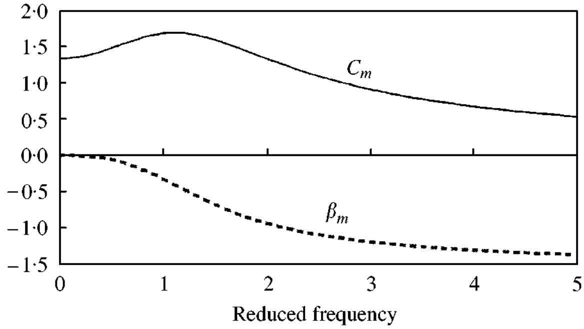

Figure 2. Phase angle β_m and lift coefficient C_m for the sphere problem.

mass of the sphere will be equal to the product of the half of its volume and the density of the fluid, which is compatible with the result of elementary fluid mechanics [1].

On the other hand, for higher frequency such that $\xi \gg 1$, the lift at a given frequency reaches its maximum when the phase angle equals to $-\pi/2$. That is, in order to acquire the maximum lift, the sphere should dilate to its maximum size when its center passes the origin in the downward direction, and it should contract to its minimum size when the center passes the origin in the upward direction. This is due to the fact that, in the high frequency range the effect of compressibility of the fluid is of importance, and the reacted pressure on the surface of the fluid results from the so-called piston effect or reaction localization [6], which states that, in the high frequency limit, the surface pressure depends only on the instantaneous velocity normal to the boundary surface relative to the fluid. The asymptotic behavior of the lift coefficient in a high frequency range is $C_m(\xi) \doteqdot 8/3\xi$.

The physical meaning of equations (1) and (4) are quite clear. To obtain the lift force from vibrational motions, we need not only the simple beating motion, but more importantly the deforming motion. If the body only translates harmonically but does not deform, then the total force exerted on the body will also vary harmonically in time, which will result in no time-averaged lift forces. On the other hand, if the body only deforms harmonically but its center does not translate, then the pressure exerted on the different parts of the body will cancel each other, and we still get no resultant forces. These conclusions will also be true even if we consider the most general cases in which the flow separation, non-linearity and viscous effects are included.

Moreover, we should apply the pressure on the deformed boundary in the theoretical calculations, otherwise we will improperly have the results of no time-averaged lift forces. As shown in the present and following cases, the lift coefficient in the low and medium frequency ranges, turns out to be in the order of unity. The lift coefficient, of course, depends on the geometric configurations (i.e., the shapes of the body, modes of the vibration, etc.) of the problem.

5. VIBRATIONAL CYLINDERS

In order to get a better idea about the characteristics of the lift generated by the vibrational motions, we study the corresponding cylindrical problem [8], i.e., we consider a cylinder of infinite length, which translates laterally in the vertical direction with its radius varied harmonically. Following the same procedures as when we treated the spherical problem, we obtain the phase angle β_m and lift coefficient C_m (which are defined the same way as the spherical problem) as

$$\beta_m = -\tan^{-1}\left(\frac{\operatorname{Im} Q(\xi)}{\operatorname{Re} Q(\xi)}\right), \quad C_m = \frac{\pi}{2}\frac{1}{\xi}\,|Q(\xi)|, \tag{5}$$

where the function $Q(\xi)$ is defined as $Q(\xi) = \mathrm{H}_1(\xi)/\mathrm{H}_1'(\xi)$ and $\mathrm{H}_1(\xi)$ is the first kind Hankel function of order one, $\xi = \omega a_0/c$ and now a_0 is the radius of the cylinder. The phase angle β_m and lift coefficient C_m for this cylinder problem are plotted in Figure 3. At zero frequency, the lift coefficient C_m equals to $\pi/2$. In the high frequency range, the asymptotic behavior of the lift coefficient is $C_m(\xi) \doteqdot \pi/2\xi$. Except for some details, the general trends are similar to the sphere problem, thus we have just the same physical interpretations for the cylinder problem as those given for the sphere problem.

It may be possible to give here more examples or carry out more details and/or modifications, but the general trends are clear: in the low frequency range, the inertial effect of the fluid is the dominant factor; while in the high frequency range, the local compressible effect of the fluid is dominant. Furthermore, for regular bodies with moderate deformations, the lift coefficient may be in the order of unity in the low and medium frequency ranges.

6. LIFT FORCE IN DIMENSIONLESS FORM

As we are considering the time-averaged lift forces for the constant amplitude of vibrations, we could rewrite equation (4) here as

$$L_m = \frac{1}{2}\rho c^2 A\left(\frac{d\delta}{a_0^2}\right)\xi^2 C_m(\xi). \tag{6}$$

We could define the dimensionless lift as $\bar{L}_m = \xi^2 C_m(\xi)$, which is independent of the amplitudes of vibrations. The dimensionless lifts for both the above-mentioned sphere and cylinder problems are plotted in Figure 4. In both cases, there are no lifts at zero frequency, and the lifts increase monotonically as the frequency is increased. The high frequency asymptotic expressions for dimensionless lifts for both the sphere and cylinder cases are $8\xi/3$ and $\pi\xi/2$ respectively. That is, as we beat (with deformation) more quickly, the lifts will be increased without limit. It is obvious that the power consumption increases in any case if we beat more quickly, this will finally limit the beating frequency with limited available energies.

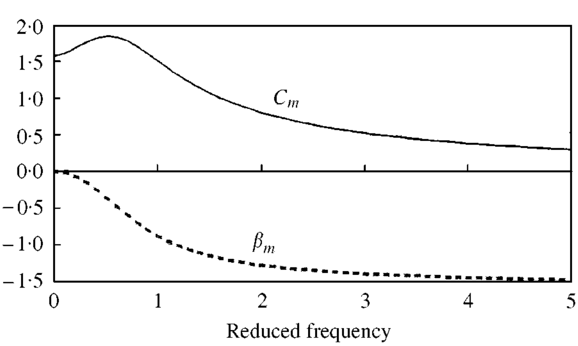

Figure 3. Phase angle β_m and lift coefficient C_m for the cylinder problem.

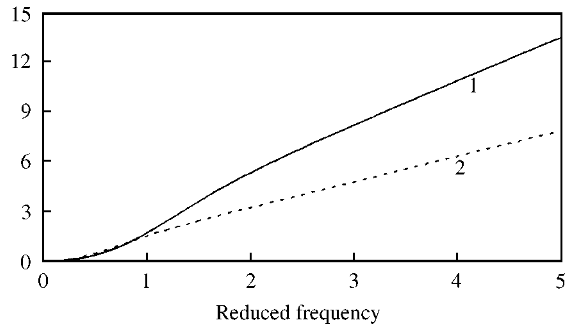

Figure 4. The dimensionless lift for both the sphere and cylinder problems (1 for sphere; 2 for cylinder).

7. DISCUSSION AND CONCLUSIONS

Although we do not consider the effects of flow separation, non-linearity and viscosity, the lift coefficients for simple geometries can be calculated theoretically as shown above, while the results obtained are reasonable even under the present simplifications. In practical instances [2–4], where the lifts generated by vibrational motions are to be evaluated, the situations of the fluid flow may be more complex. In the author's opinion, we can apply equation (1) for cases when the effects of flow separation, non-linearity, viscosity and others should be taken into account, and we need only modify the lift coefficient defined by equation (1) for various situations, in which the coefficient $C(\xi, \beta)$ may be determined numerically or experimentally. The key point is, we can regard equation (1) as the scaling law for various vibrational-lift-generating processes, and we will include all the factors not considered here by numerical or experimental methods.

In conclusion, in order to understand the characteristics of the lift forces generated by the vibrational motions, we have, by applying the reacted pressure on the "deformed" surface, deduced some results about the vibration-generated lift forces in the fluid medium under the non-viscosity and small disturbance assumptions. The present calculations show that the lift is proportional to the product of the amplitudes of translating and deforming vibrations. We could thus define the lift coefficients for the cases in which the deforming motions of the boundary surface are involved. The physical interpretations for these results are reasonable and compatible with the concepts in the elementary fluid dynamics and acoustics. Further applications or extensions of the present theory may be possible in the near future.

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