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DEVELOPMENT OF A BLOCKED PRESSURE CRITERION FOR APPLICATION OF THE PRINCIPLE OF ACOUSTIC RECIPROCITY

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The principle of acoustic reciprocity can be applied in practice to estimate the sound radiation of a vibrating surface, from an inverse measurement of the Green function. This principle is especially useful in evaluating and ranking the sound radiation from individual vibrating components of a complex mechanical system. Usually, a blocked pressure assumption is needed in order to simplify the formulation of the reciprocity technique. This paper examines the validity of the blocked pressure hypothesis in the context of the reciprocity method, and suggests a "blocked pressure criterion" to rigorously quantify the impact of this hypothesis on the sound radiation prediction. This criterion is tested numerically on a system consisting of two coplanar, simply supported thin baffled plates that are mechanically uncoupled. It is shown that the numerical results support the conclusions obtained from the blocked pressure criterion.

1. INTRODUCTION

The prediction of the acoustic response from a vibrating body is of interest in the reduction of structure-borne noise. In many practical situations, it is desirable to estimate the sound radiated by a specific component of a complex mechanical system, and rank the various individual components in terms of their sound radiation. Such an analysis allows the most noise-contributing sources to be identified, and efficient noise reduction measures to be implemented.

Numerical methods such as the finite element method or the boundary element method are commonly used to predict the acoustic radiation of vibrating structures. However, for complex mechanical systems, these methods are not well adapted to the estimation of the sound radiation from individual components of the system. On the other hand, experimental methods such as the reciprocity method have been recently developed to specifically achieve this goal. For a structure which is part of a complex mechanical system, it is possible to calculate the exact acoustic response using the surface velocity field and the Green function. It has been shown that the principle of acoustic reciprocity can be used to provide an estimate of the Green function. Developed originally by Helmholtz [1] for acoustics, the principle of reciprocity has been extended to elastic solids by Rayleigh and Lyamshev [1, 2]. The required Green function is calculated from inverse measurements, using a controlled sound source insonifying the structure (when the structure is passive and assumed to be in a rigid state) and measuring the resulting sound pressure field on the

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surface of the structure considered. The measured Green function is then used together with the experimentally determined vibration of the structure (when active) to derive the sound pressure radiated at a given location in space. This approach is correctly derived by Holland *et al.* [3]; some general applications are proposed by Mason *et al.* [4, 5]. An extension of the principle of reciprocity to mechano-acoustical systems is proposed by ten Wolde [6].

In practice, a basic assumption when measuring the Green function using an inverse measurement is that the measured surface acoustic pressure is not affected by the dynamic response of the global mechanical system excited by the acoustic point source. Such a surface acoustic pressure is called “blocked pressure”, because the structural motion is considered as being blocked [3]. The validity of this hypothesis should obviously be strongly dependent on the mechanical properties of the structure, such as the mass, stiffness and damping. However, the blocked pressure hypothesis has not been thoroughly examined in the past, in the context of the reciprocity method.

The aim of this paper is to propose a criterion called “blocked pressure criterion” to verify the applicability of the acoustic reciprocity method when the blocked pressure hypothesis is done.

In section 1, the principle of acoustic reciprocity is reviewed and an exact expression of the sound pressure radiated by a vibrating surface is derived from the reciprocity technique. In section 2, the principle of reciprocity is derived for an elastic solid medium from the dynamic equilibrium equations. The theoretical developments involved in these two preliminary sections are used to derive an analytical formulation of an original blocked pressure criterion in section 3. This criterion is tested numerically on a system consisting of two coplanar, simply supported thin baffled plates that are mechanically uncoupled. It is shown that the numerical results support the conclusions obtained from the criterion.

2. THE PRINCIPLE OF ACOUSTIC RECIPROCITY

In this section, the principle of acoustic reciprocity is rigorously derived for a fluid subjected to different boundary conditions and excitation sources. The exact, direct expression of the acoustic field radiated by a vibrating surface with a given surface velocity field is recalled. An alternative expression for the same acoustic field is also proposed, based on the principle of reciprocity. These expressions will be used in section 3 of this paper to support the development of the blocked pressure criterion for the principle of reciprocity.

A fluid domain Ω bounded by a surface S_Ω, as shown in Figure 1, contains a set of three-dimensional bodies described by their closed surfaces S_k. The fluid domain also contains sound sources q_i radiating at the angular frequency ω. A unit normal vector $\mathbf{n}$ pointing into Ω is defined on the bounding surfaces S_Ω and S_k.

In the following, two different independent acoustic fields, labelled (1) and (2), are considered in Ω, depending on the selection of active sound sources q_i and boundary conditions over S_k. The superscripts identify the state (1) or (2) for each variable. The sound pressure is expressed in each case by the Helmholtz equation for a non-dissipative fluid,

$$\text{State (1): } (\nabla^2 + k^2)\hat{p}^{(1)} = -\mathrm{j}\omega\rho_0\hat{q}^{(1)}, \tag{1}$$

$$\text{State (2): } (\nabla^2 + k^2)\hat{p}^{(2)} = -\mathrm{j}\omega\rho_0\hat{q}^{(2)}, \tag{2}$$

where k is the wavenumber, $k = \omega/c$, c is the speed of sound, $\mathrm{j}^2 = -1$, ρ_0 is the fluid density, and $\hat{q}^{(1)}$ and $\hat{q}^{(2)}$ designate acoustic sources in states (1) and (2). The symbol $\hat{}$ denotes complex variables. Equations (1) and (2) are multiplied by $\hat{p}^{(2)}$ and $\hat{p}^{(1)}$, respectively, and

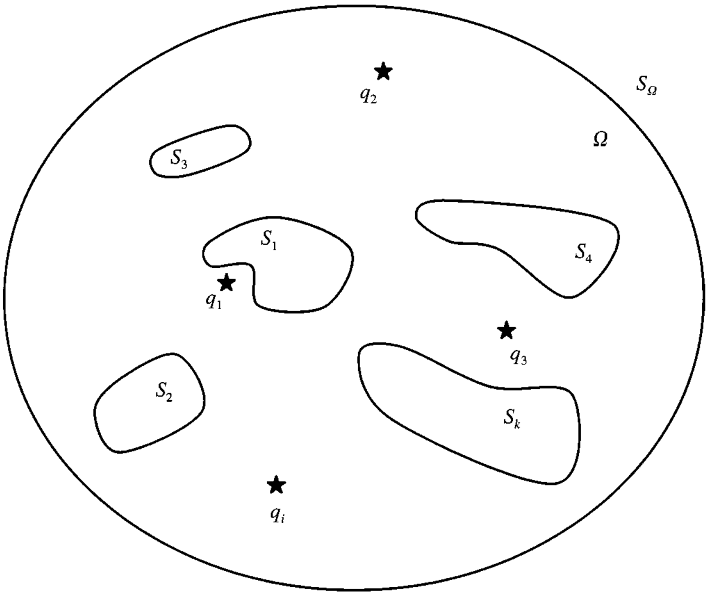

Figure 1. Volume of fluid enclosed in a surface S_Ω, containing structures S_k and sound sources q_i.

they are then integrated on the volume Ω and subtracted to give

$$\iiint_\Omega (\nabla^2 \hat{p}^{(1)}\hat{p}^{(2)} - \nabla^2 \hat{p}^{(2)}\hat{p}^{(1)})\,\mathrm{d}\Omega = -\mathrm{j}\omega\rho_0 \iiint_\Omega (\hat{q}^{(1)}\hat{p}^{(2)} - \hat{q}^{(2)}\hat{p}^{(1)})\,\mathrm{d}\Omega. \tag{3}$$

The Green theorem is applied to the left-hand side of equation (3),

$$-\iint_{\sum S_k + S_\Omega} \left(\frac{\partial}{\partial n}\hat{p}^{(1)}\hat{p}^{(2)} - \frac{\partial}{\partial n}\hat{p}^{(2)}\hat{p}^{(1)}\right)\mathrm{d}\Omega = -\mathrm{j}\omega\rho_0 \iiint_\Omega (\hat{q}^{(1)}\hat{p}^{(2)} - \hat{q}^{(2)}\hat{p}^{(1)})\,\mathrm{d}\Omega, \tag{4}$$

where $\partial/\partial n$ represents the derivative in the direction of the unit normal vector $\mathbf{n}$. In the limiting case where S_Ω is a sphere of infinite radius $|r_{S_\Omega}| \to \infty$, the Sommerfeld radiation condition applies,

$$\lim_{|r_{S_\Omega}| \to \infty} \left\{ \iint_{S_\Omega} \left(\frac{\partial}{\partial n}\hat{p}^{(1)}\hat{p}^{(2)} - \frac{\partial}{\partial n}\hat{p}^{(2)}\hat{p}^{(1)}\right)\mathrm{d}\Omega \right\} = 0. \tag{5}$$

The pressure gradient in relationship (4) is expressed as a function of the surface velocity on S_i,

$$\frac{\partial}{\partial n}\hat{p} = -\mathrm{j}\omega\rho_0\hat{w}, \tag{6}$$

where w is the normal component of the surface velocity. A general formulation of the principle of reciprocity applied to a fluid medium results,

$$\iint_{\sum S_k} (\hat{w}^{(1)}\hat{p}^{(2)} - \hat{w}^{(2)}\hat{p}^{(1)})\,\mathrm{d}S = \iiint_\Omega (-\hat{q}^{(1)}\hat{p}^{(2)} + \hat{q}^{(2)}\hat{p}^{(1)})\,\mathrm{d}\Omega. \tag{7}$$

The volume source configurations $\hat{q}^{(1)}$ and $\hat{q}^{(2)}$ are now specified. It is assumed that there is no sound source in state (1), that is $\hat{q}^{(1)} = 0$, and there is one single-point source at location $\mathbf{r}_0$ in state (2), $\hat{q}^{(2)} = \hat{Q}^{(2)}\delta(\mathbf{r} - \mathbf{r}_0)$. Applying equation (7) results in

$$\iint_{\sum S_k} (\hat{w}^{(1)}\hat{p}^{(2)} - \hat{w}^{(2)}\hat{p}^{(1)})\,\mathrm{d}S = \hat{Q}^{(2)}\hat{p}^{(1)}(\mathbf{r}_0). \tag{8}$$

The sound pressure field radiated by the vibrating surfaces S_k in state (1) can therefore be expressed as

$$\hat{p}^{(1)}(\mathbf{r}_0) = \sum_k \hat{p}_k^{(1)}(\mathbf{r}_0), \tag{9}$$

where $\hat{p}_k^{(1)}$, the contribution of the surface S_k to the total pressure field radiated by all the surfaces is

$$\hat{p}_k^{(1)}(\mathbf{r}_0) = \frac{1}{\hat{Q}^{(2)}} \iint_{S_k} (\hat{w}^{(1)}\hat{p}^{(2)} - \hat{w}^{(2)}\hat{p}^{(1)})\,\mathrm{d}S. \tag{10}$$

The volume flow $\hat{Q}^{(2)}$ injected in state (2) by the point source in the fluid is

$$\hat{Q}^{(2)} = \iiint_{\Omega} \hat{q}^{(2)}\,\mathrm{d}\Omega. \tag{11}$$

If it is now assumed that $\hat{w}^{(2)} = 0$, the contribution is simply [3–5]

$$\hat{p}_k^{(1)}(\mathbf{r}_0) = \iint_{S_k} \frac{\hat{p}^{(2)}}{\hat{Q}^{(2)}}\,\hat{w}^{(1)}\,\mathrm{d}S. \tag{12}$$

Equation (12) suggests a simple experimental procedure to estimate the sound field of each individual vibrating surface S_k at location $\mathbf{r}_0$ using the principle of reciprocity: (1) the transverse velocity $\hat{w}^{(1)}$ is measured over the surface S_k when it is active (corresponding to state (1)); (2) an acoustic point source of strength $\hat{Q}^{(2)}$ is introduced in the fluid domain at point $\mathbf{r}_0$ with the surface S_k inactive (corresponding to state (2)) and the resulting sound pressure $\hat{p}^{(2)}$ is measured over S_k; (3) the sound pressure radiated by S_k at point $\mathbf{r}_0$ in state (1) is estimated using equation (12). It is to be noted that the application of equation (12) requires that S_k is perfectly rigid in state (2) ($\hat{w}^{(2)} = 0$). The validity of this assumption is investigated in detail in the following. In a first instance, the principle of reciprocity is extended to an elastic solid in the next section.

3. THE PRINCIPLE OF RECIPROCITY APPLIED TO AN ELASTIC SOLID

The principle of reciprocity applied to an elastic solid medium is derived in this section from the equations of dynamic equilibrium.

A three-dimensional elastic solid V with a bounding surface S is considered, as shown in Figure 2. It is submitted to various harmonic disturbing forces with angular frequency ω: point forces $\mathbf{f}_P$, line forces $\mathbf{f}_L$, surface forces $\mathbf{f}_S$, volume forces $\mathbf{f}_V$, moments $\mathbf{m}$.

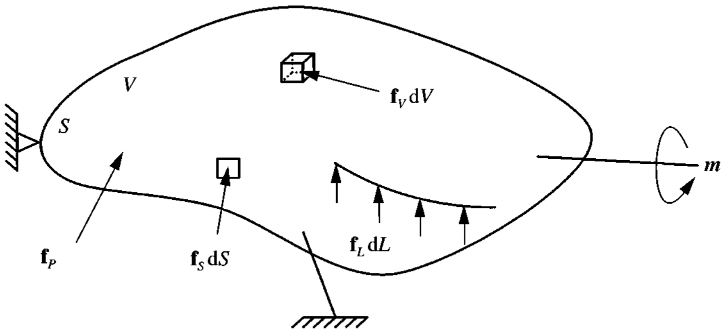

Figure 2. Description of the structure and disturbing forces.

For Hookean solids, the stress tensor $\boldsymbol{\sigma}$ is related to the strain tensor $\boldsymbol{\varepsilon}$ by

$$\hat{\boldsymbol{\sigma}} = \mathbf{C}(1 + \mathrm{j}\eta)\hat{\boldsymbol{\varepsilon}}, \tag{13}$$

where $\mathbf{C}$ and η are the tensor of the elastic constants and the structural loss factor respectively. Assuming that a modal basis is available, the displacement of the structure can be expressed as

$$\mathbf{u} = \sum_{i=1}^{\infty} \mathbf{u}_i = \sum_{i=1}^{\infty} \Phi_i \mathbf{X}_i, \tag{14}$$

where the $\mathbf{X}_i$ are the eigenfunctions and Φ_i are the modal displacements.

Again, two distinct independent states (1) and (2) are considered, corresponding to two distinct excitation configurations. Virtual works of internal and external forces in state (1) along virtual displacement $\delta\mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ corresponding to state (2) are detailed in the following.

The work of the distributed inertia forces in state (1) along virtual displacement $\delta\mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ in state (2) takes the following form (where the orthogonality of the eigenfunctions $\mathbf{X}_i$ is invoked):

$$\delta W_I^{(12)} = \sum_{i=1}^{\infty} \iiint_V -\rho_S \left(\frac{\partial^2}{\partial t^2} \mathbf{u}^{(1)} \right) \cdot \mathbf{u}_i^{(2)}\,\mathrm{d}V = \omega^2 \sum_{i=1}^{\infty} \rho_S \Phi_i^{(1)} \Phi_i^{(2)} \iiint_V \mathbf{X}_i^2\,\mathrm{d}V, \tag{15}$$

where ρ_S is the mass density of the solid. The above result can be expressed as

$$\delta W_I^{(12)} = \omega^2 \sum_i \Phi_i^{(1)} M_i^* \Phi_i^{(2)}, \tag{16}$$

where the modal mass M_i^* is defined by

$$M_i^* = \iiint_V \rho_S \mathbf{X}_i^2\,\mathrm{d}V. \tag{17}$$

The work of the elastic forces in state (1) along the virtual displacement $\delta \mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ in state (2) is

$$\delta W_E^{(12)} = -\,\delta U^{(12)} = -\sum_{i=1}^{\infty} \frac{\partial}{\partial \Phi_i^{(1)}} \left(\iiint\limits_V \frac{1}{2} (\boldsymbol{\varepsilon}^{\mathrm{T}} \mathbf{C} \boldsymbol{\varepsilon})^{(1)} \, \mathrm{d}V \right) \Phi_i^{(2)}, \tag{18}$$

where U is the strain energy in the structure. Introducing the strain–displacement relationship $\boldsymbol{\varepsilon} = \mathbf{L}\mathbf{u}$ and assuming the orthogonality property $\iiint_V (\boldsymbol{\varepsilon}_i^*)^{\mathrm{T}} \mathbf{C} \boldsymbol{\varepsilon}_j^* \, \mathrm{d}V = 0$ for $i \neq j$, where $\boldsymbol{\varepsilon}_i^*$ is the modal strain tensor,

$$\boldsymbol{\varepsilon}_i^* = \mathbf{L}\mathbf{X}_i, \tag{19}$$

equation (18) is expressed as

$$\delta W_E^{(12)} = -\sum_i \Phi_i^{(1)} K_i^* \Phi_i^{(2)}, \tag{20}$$

where K_i^* is the modal stiffness,

$$K_i^* = \iiint\limits_V (\boldsymbol{\varepsilon}_i^*)^{\mathrm{T}} \mathbf{C} \boldsymbol{\varepsilon}_i^* \, \mathrm{d}V. \tag{21}$$

The work of the damping forces in state (1) along the virtual displacement $\delta \mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ in state (2) is

$$\delta W_D^{(12)} = -\sum_i \frac{\partial}{\partial t} \Phi_i^{(1)} C_i^* \Phi_i^{(2)}, \tag{22}$$

where C_i^* is the modal damping [8],

$$C_i^* = \frac{K_i^* \eta}{\omega}. \tag{23}$$

The work done by the external forces in state (1) along virtual displacement $\delta \mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ in state (2) is given by

$$\delta W_F^{(12)} = \sum \mathbf{f}_P^{(1)} \cdot \mathbf{u}^{(2)} + \int\limits_L \mathbf{f}_L^{(1)} \, \mathrm{d}L \cdot \mathbf{u}^{(2)} + \iint\limits_S \mathbf{f}_S^{(1)} \, \mathrm{d}S \cdot \mathbf{u}^{(2)} + \iiint\limits_V \mathbf{f}_V^{(1)} \, \mathrm{d}V \cdot \mathbf{u}^{(2)} + \sum \mathbf{m}^{(1)} \cdot (\nabla \times \mathbf{u})^{(2)}. \tag{24}$$

The virtual work principle states that

$$\delta W_I^{(12)} + \delta W_E^{(12)} + \delta W_D^{(12)} + \delta W_F^{(12)} = 0. \tag{25}$$

In the same manner, if one considers the work done by external and internal forces in state (2) along the virtual displacement $\delta \mathbf{u}^{(2)} = \mathbf{u}^{(1)}$ in state (1), the virtual work principle states that

$$\delta W_I^{(21)} + \delta W_E^{(21)} + \delta W_D^{(21)} + \delta W_F^{(21)} = 0. \tag{26}$$

Equations (16) and (20) show that $\delta W_I^{(12)} = \delta W_I^{(21)}$ and $\delta W_E^{(12)} = \delta W_E^{(21)}$; similarly, $\delta W_D^{(12)} = \delta W_D^{(21)}$. Therefore, the subtraction of equations (25) and (26) results in

$$\delta W_F^{(12)} = \delta W_F^{(21)}, \tag{27}$$

that is, the work exerted by external forces in state (1) along the virtual displacement $\delta \mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ in state (2) is equal to the work exerted by external forces in state (2) along the virtual displacement $\delta \mathbf{u}^{(2)} = \mathbf{u}^{(1)}$ in state (1). The principle of reciprocity for an elastic solid is well known, and states that for a concentrated force, the ratio of the response at an arbitrary location to the applied force is unchanged when the force location and the response location are permuted.

4. DEVELOPMENT OF THE BLOCKED PRESSURE CRITERION

4.1. THEORETICAL ANALYSIS

The pressure radiated by a set of vibrating surfaces was derived in section 1 and is given by the exact expression provided by relationship (10). For practical applications using the principle of acoustic reciprocity [3–6], state (1) assumes that the mechanical system operates in its normal condition, that is, the vibrating surfaces of the system are active and they radiate in a fluid without sound sources. The objective here is to derive the pressure field created by a given individual vibrating surface which belongs to the set of vibrating surfaces. In state (2), the whole mechanical system is passive, the sound field is created by an acoustic point source in the volume and the resulting sound pressure is calculated on the surface of the structure considered. Equation (12) can then be used to estimate the pressure field radiated by the surface considered. This expression is valid if the velocity field of the surface is assumed to be zero in state (2). However, in a real situation, the flexibility of the mechanical system induces a non-zero surface velocity in response to the point source. This is not taken into account by equation (12), but for many applications this approximation would still yield accurate values.

This section uses results presented in the two previous sections to develop a criterion to precisely evaluate the difference between the approximate sound pressure field calculated by equation (12) and the exact sound pressure field calculated by equation (10). The criterion applies here to the case of an individual structure which is mechanically uncoupled from other structures (that is, no power is transmitted between structures through mechanical junctions).

Let us consider an individual structure k, described by its surface S_k. The approximate equation (12) can be derived from the exact equation (10) provided that the following condition is satisfied:

$$\left| \iint_{S_k} \hat{w}^{(2)} \hat{p}^{(1)} \, \mathrm{d}S \right| \ll \left| \iint_{S_k} \hat{w}^{(1)} \hat{p}^{(2)} \, \mathrm{d}S \right|. \tag{28}$$

Dividing both sides by $\mathrm{j}\omega$ yields

$$\left| \iint_{S_k} \hat{u}^{(2)} \hat{p}^{(1)} \, \mathrm{d}S \right| \ll \left| \iint_{S_k} \hat{u}^{(1)} \hat{p}^{(2)} \, \mathrm{d}S \right|, \tag{29}$$

where $\hat{u}$ is the normal displacement to the surface. Expression (29) states that the work of the surface acoustic pressures in state (1) along the displacement in state (2) is negligible compared to the reciprocal effect. Assuming complex notation, this last equation is written as

$$|\delta \widehat{W}_A^{(12)}| \ll |\delta \widehat{W}_A^{(21)}|, \tag{30}$$

where $\delta \widehat{W}_A$ denotes the work of surface acoustic pressures. On the other hand, the principle of reciprocity given by equation (27) and applied to an the individual structure k states that in the present cases,

$$\delta \widehat{W}_M^{(12)} + \delta \widehat{W}_A^{(12)} = \delta \widehat{W}_A^{(21)}, \tag{31}$$

where $\delta \widehat{W}_M$ denotes the work of the mechanical forces applied to the structure. In our case, mechanical forces exist only in state (1), since state (2) corresponds to an acoustic point source excitation. The term $\delta \widehat{W}_A^{(12)}$ in expression (31) represents the work of surface acoustic pressures radiated by all the vibrating surfaces, calculated on S_k. It is also worth remembering that the studied structure S_k is mechanically uncoupled from its surroundings.

Combining relationships (30) and (31) results in

$$|\delta \widehat{W}_A^{(12)}| \ll |\delta \widehat{W}_M^{(12)} + \delta \widehat{W}_A^{(12)}|. \tag{32}$$

Applying equation (25) to introduce the works of the distributed inertia forces, the internal elastic forces and the dissipative forces, expression (32) becomes

$$|\delta \widehat{W}_A^{(12)}| \ll |\delta \widehat{W}_I^{(12)} + \delta \widehat{W}_E^{(12)} + \delta \widehat{W}_D^{(12)}|. \tag{33}$$

The various terms of expression (33) are now developed; the work of the acoustic disturbing pressures in state (1) along the displacement in state (2) is expressed by

$$\delta \widehat{W}_A^{(12)} = -\iint_{S_k} \hat{u}^{(2)} \hat{p}^{(1)} \, \mathrm{d}S_k = -\iint_{S_k} \sum_i (\hat{\Phi}_i^{S_k})^{(2)} (\mathbf{X}_i^{S_k}(\mathbf{r}_k) \cdot \mathbf{n}) \hat{p}^{(1)}(\mathbf{r}_k) \, \mathrm{d}S_k, \tag{34}$$

where $\mathbf{X}_i^{S_k}$ and $\Phi_i^{S_k}$ are the eigenfunctions and modal displacements of the structure S_k respectively. In the absence of any volume source, the surface acoustic pressure $\hat{p}^{(1)}(\mathbf{r}_k)$ is expressed by the Helmholtz–Huygens equation,

$$\hat{p}^{(1)}(\mathbf{r}_k) = -\iint_{S'} G(\mathbf{r}_k, \mathbf{r}') \frac{\partial}{\partial n} \hat{p}^{(1)}(\mathbf{r}') \, \mathrm{d}S' = \mathrm{j}\omega\rho_0 \iint_{S'} G(\mathbf{r}_k, \mathbf{r}') \hat{w}^{(1)}(\mathbf{r}') \, \mathrm{d}S', \tag{35}$$

where G is the Green function, which is the solution of the inhomogeneous Helmholtz equation, and which satisfies the homogeneous Neumann condition on S' [8], that is

$$\frac{\partial}{\partial n} G = 0 \text{ on } S', \tag{36}$$

where $S' = \sum S_k$ is the total surface of the mechanical system, and $\mathbf{r}_k$ and $\mathbf{r}'$ are position vectors, describing S_k and S' respectively. Expanding $\hat{w}^{(1)}(\mathbf{r}')$ in relationship (35) over the eigenfunctions of S' and inserting equation (35) into equation (34) gives

$$\delta \hat{W}_A^{(12)} = \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \rho_0 \omega^2 \sum_j \sum_p \iint_{S_k} \iint_{S_p} (\mathbf{X}_i^{S_k}(\mathbf{r}_k)\cdot\mathbf{n}) G(\mathbf{r}_k, \mathbf{r}_p) (\hat{\Phi}_i^{S_p})^{(1)} (\mathbf{X}_j^{S_p}(\mathbf{r}_p)\cdot\mathbf{n})\, \mathrm{d}S_k\, \mathrm{d}S_p, \qquad (37)$$

that is,

$$\delta \hat{W}_A^{(12)} = \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \left(-\mathrm{j}\omega \sum_j \sum_p Z_{ij}^{S_k S_p} (\hat{\Phi}_j^{S_p})^{(1)} \right), \qquad (38)$$

where $Z_{ij}^{S_k S_p}$ is the intermodal radiation impedance of mode j of S_p on mode i of S_k,

$$Z_{ij}^{S_k S_p} = \mathrm{j}\omega\rho_0 \iint_{S_k} \iint_{S_p} (\mathbf{X}_i^{S_k}(\mathbf{r}_k)\cdot\mathbf{n}) G(\mathbf{r}_k, \mathbf{r}_p) (\mathbf{X}_j^{S_p}(\mathbf{r}_p)\cdot\mathbf{n})\, \mathrm{d}S_k\, \mathrm{d}S_p. \qquad (39)$$

Inserting the expressions of the work of distributed inertia forces, internal elastic forces and damping forces given by expressions (16), (20) and (22), equation (33) finally becomes

$$\left| \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \left(-\mathrm{j}\omega \sum_j \sum_p Z_{ij}^{S_k S_p} (\hat{\Phi}_j^{S_p})^{(1)} \right) \right| \ll \left| \sum_i (\hat{\Phi}_i^{S_k})^{(2)} (-\omega^2 (M_i^{S_k})^* + (K_i^{S_k})^* (1 + \mathrm{j}\eta_{S_k})) (\hat{\Phi}_i^{S_k})^{(1)} \right|, \qquad (40)$$

where $(M_i^{S_k})^*$, $(K_i^{S_k})^*$ and η_{S_k} are the modal mass, the modal stiffness and the loss factor of the structure S_k respectively. Using the expression of the modal mass given by expression (17), the above condition is written as

$$\left| \omega \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \sum_j \sum_p Z_{ij}^{S_k S_p} (\hat{\Phi}_j^{S_p})^{(1)} \right| \ll \left| \rho_{S_k} \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \left(\iiint_V (\mathbf{X}_i^{S_k})^2\, \mathrm{d}V \right) (-\omega^2 + (\omega_i^{S_k})^2 (1 + \mathrm{j}\eta_{S_k})) (\hat{\Phi}_i^{S_k})^{(1)} \right|, \qquad (41)$$

that is,

$$A_{S_k}(\omega) = \frac{|\omega \sum_i (\hat{\Phi}_i^{S_k})^{(2)} \sum_j \sum_p Z_{ij}^{S_k S_p} (\hat{\Phi}_j^{S_p})^{(1)}|}{|\rho_{S_k} \sum_i (\hat{\Phi}_i^{S_k})^{(2)} (\iiint_V (\mathbf{X}_i^{S_k})^2\, \mathrm{d}V)(-\omega^2 + (\omega_i^{S_k})^2 (1 + \mathrm{j}\eta_{S_k}))(\hat{\Phi}_i^{S_k})^{(1)}|} \ll 1, \qquad (42)$$

where $\omega_i^{S_k} = \sqrt{(K_i^{S_k})^*/(M_i^{S_k})^*}$ is the angular natural frequency of mode i for the structure S_k. Equation (42) furnishes a criterion for the blocked pressure approximation in the principle of reciprocity. The quantity A_{S_k} essentially depends on structural parameters and on the radiation impedance coefficients. For complex systems, the modal displacements, modal masses and angular natural frequencies can be calculated from a finite element analysis. The intermodal radiation impedance characterizes the coupling that occurs through the fluid, between each individual structure S_k and all structures (including S_k). If one states that the geometry of S_k remains locally approximately plane, the Green function for a semi-infinite medium [8] may be used. The main advantage of the proposed formulation is then to avoid

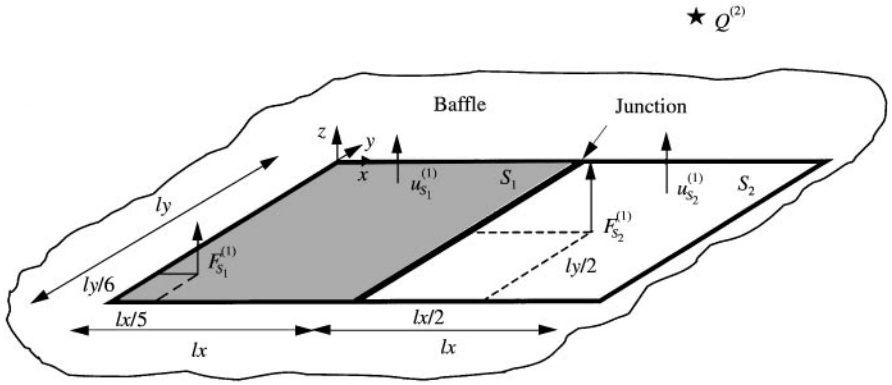

Figure 3. Test case: two coplanar, simply supported, mechanically uncoupled thin baffled plates.

characterizing the Green function from the studied structure to the sound point source $\hat{q}^{(2)}$, which is often difficult to calculate because of the complexity of the vibro-acoustic system.

An approximate blocked pressure criterion can be derived from equation (42) for the structure S_k: at the angular natural frequency $\omega_i^{S_k}$ and considering the corresponding vibrational mode i, one can assume that the contribution of the other modes is negligible; one can also neglect the influence of the surrounding surfaces (light fluid coupling hypothesis). Equation (42) then becomes

$$B_{S_k}(\omega_i^{S_k}) = \frac{|Z_{ii}^{S_k S_k}|}{\omega_i^{S_k} \rho_{S_k} \eta_{S_k} \iiint_V (\mathbf{X}_i^{S_k})^2 \, \mathrm{d}V} \ll 1. \qquad (43)$$

In the following, criterion (42) and its approximate version (43) are tested on two coplanar plates radiating in a semi-infinite fluid.

4.2. APPLICATION TO TWO SIMPLY SUPPORTED THIN BAFFLED PLATES

The blocked pressure criterion formulated in the previous section is numerically tested on the system shown in Figure 3. Two coplanar, adjacent, flexural, simply supported, identical thin baffled plates S_1 and S_2 are assumed, with dimensions $l_x = 0{\cdot}5$ m, $l_y = 0{\cdot}6$ m, thickness $h = 2 \times 10^{-3}$ m, material properties $E = 2 \times 10^{11}$ Pa, $v = 0{\cdot}3$, $\rho_S = 7800\,\mathrm{kg/m^3}$, and loss factor $\eta = 2 \times 10^{-3}$. The two plates are mechanically uncoupled (no power is transmitted through the junction). They are excited in state (1) by concentrated disturbing forces $F_{S_1}^{(1)} = 1$N at ($lx/5$, $ly/6$) on plate S_1 and $F_{S_2}^{(1)} = 2$N at ($lx/2$, $ly/2$) on plate 2. The transverse displacements of the two plates in state (1) are denoted as $u_{S_1}^{(1)}$ and $u_{S_2}^{(1)}$ respectively. In state (2), the mechanical system is passive and is excited by a point source, $Q^{(2)}$ at $\mathbf{r}_0 = (1{\cdot}5$ m, $0{\cdot}9$ m, $1{\cdot}5$ m).

The sound pressure radiated by the plate S_1 at $\mathbf{r}_0$ is calculated using both the exact and the approximate expression of the principle of acoustic reciprocity, and the blocked pressure criterion is tested for this system.

4.2.1. *Plates Radiate in Air*

In a first case, the two plates radiate in air (sound speed $c = 330$ m/s, density $\rho_0 = 1\,\mathrm{kg/m^3}$). The dynamic response of each plate to the point force excitations in state (1) and acoustic excitation in state (2) is detailed in Appendix A. The exact sound pressure

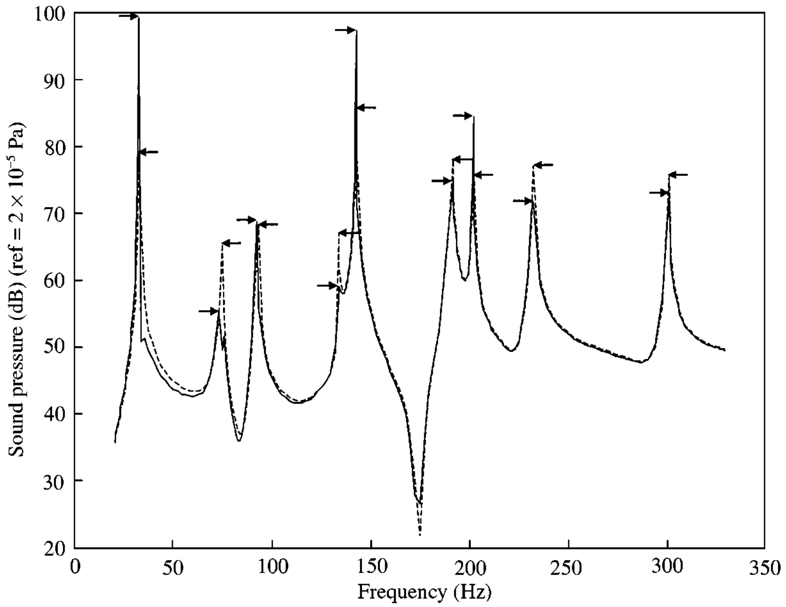

Figure 4. Sound pressure radiated by plate S_1, calculated with the principle of acoustic reciprocity in case of light fluid loading: ——, approximate; - - - - -, exact.

$\{\hat{p}_1^{(1)}\}_{ex}$ radiated by plate S_1 and its approximation $\{\hat{p}_1^{(1)}\}_{ap}$ are derived from the principle of acoustic reciprocity and the blocked pressure assumption in Appendix B. These sound pressures are given by equations (10) and (12) respectively. The results are compared in Figure 4.

Each peak value for both curves is indicated with an arrow. For a specific mode, the peaks may correspond to slightly different frequencies for the two cases due to the fluid loading effect (equations (B5) and (B6) of Appendix B). Due to the light fluid loading, the main differences between exact and approximate sound pressures are restricted to frequencies close to the natural frequencies of plate S_1. The largest difference (20 dB) between the two curves is obtained at the first resonance (33 Hz). The typical discrepancies for peak values are of the order of 5–10 dB. Consequently, the blocked pressure criterion is violated at the natural frequencies, the dynamic response of the mechanical system to the point source excitation is important and neglecting this contribution in equation (12) yields an erroneous estimate of the radiated pressure.

The quantity A_{S_1} establishing the blocked pressure criterion (equation (42)) is plotted in Figure 5.

The term A_{S_1} is much larger than unity at resonance, and usually much smaller than unity off resonance, which is consistent with the observations of Figure 4. The term A_{S_1} is therefore an appropriate indicator of the blocked pressure approximation.

One can also calculate the term B_{S_1} establishing the approximate blocked pressure criterion (equation (43)) at the natural frequencies of plate S_1. The results are listed in Table 1.

As expected, the quantity B_{S_1} succeeds in predicting the accuracy of the blocked pressure assumption. Because the modal coupling is neglected, one remarks that this approximate quantity strongly overestimates the term A_{S_1} from the first to the third natural frequency of the plate S_1. In case of light fluid loading, it appears that the use of the approximate blocked pressure criterion is advantageous.

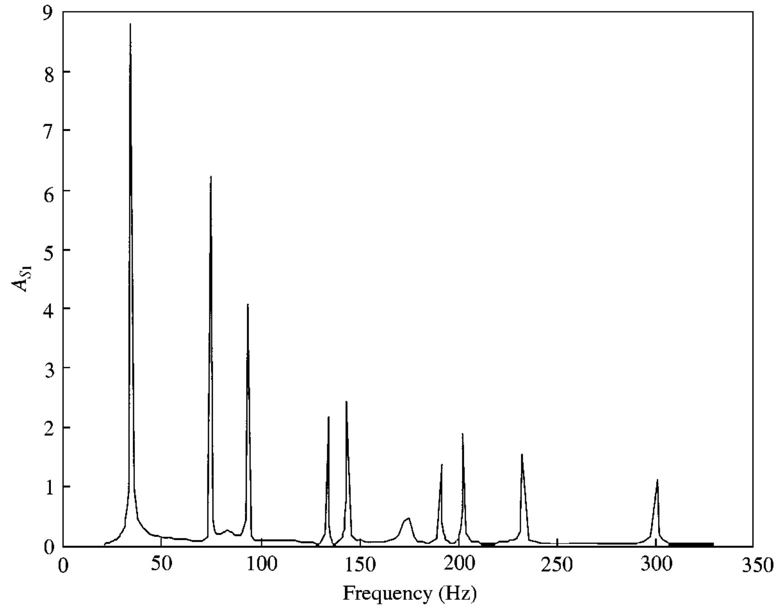

Figure 5. Plot of the term A_{S_1} establishing the blocked pressure criterion in case of light fluid loading.

TABLE 1

Values of the term B_{S_1} establishing the approximate blocked pressure criterion in case of light fluid loading

Modes of plate S_1	1–1	1–2	2–1	2–2	1–3	3–1	2–3	3–2	3–3
Natural frequencies (Hz)	33	75	93	134	143	191	202	232	301
B_{S_1}	50·4	14·5	9·8	4·2	3·2	1·7	1·5	1·2	0·6

4.2.2. *Plates Radiate in Water*

In a second case, the two plates radiate in water (sound speed $c = 1460\,\text{m/s}$, density $\rho_0 = 1000\,\text{kg/m}^3$). As in the previous case, the exact sound pressure $\{\hat{p}_1^{(1)}\}_{ex}$ radiated by plate S_1 and its approximation $\{\hat{p}_1^{(1)}\}_{ap}$ are plotted (Figure 6).

The two curves do not fit over the frequency range (20–200 Hz): the approximate sound pressure strongly overestimates the exact sound pressure, which means that the blocked pressure assumption is violated in this case. Due to heavy fluid loading, the acoustic pressure generated by the vibration of the mechanical system strongly participates in the measured surface acoustic pressure for state (2).

The quantity A_{S_1} establishing the blocked pressure criterion (equation (42)) is plotted in Figure 7.

As expected, the term A_{S_1} is not negligible compared to unity in the frequency range (20–200 Hz). The blocked pressure criterion (equation (42)) is in good agreement with the results of Figure 6. The use of the blocked pressure assumption (equation (12)) is strictly erroneous.

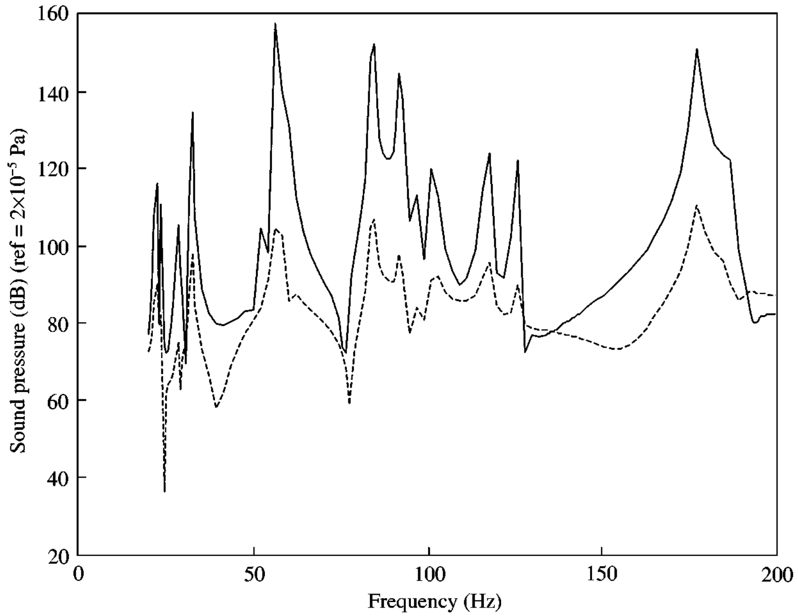

Figure 6. Sound pressure radiated by plate S_1, calculated with the principle of acoustic reciprocity in case of heavy fluid loading: ——, approximate; - - - - -, exact.

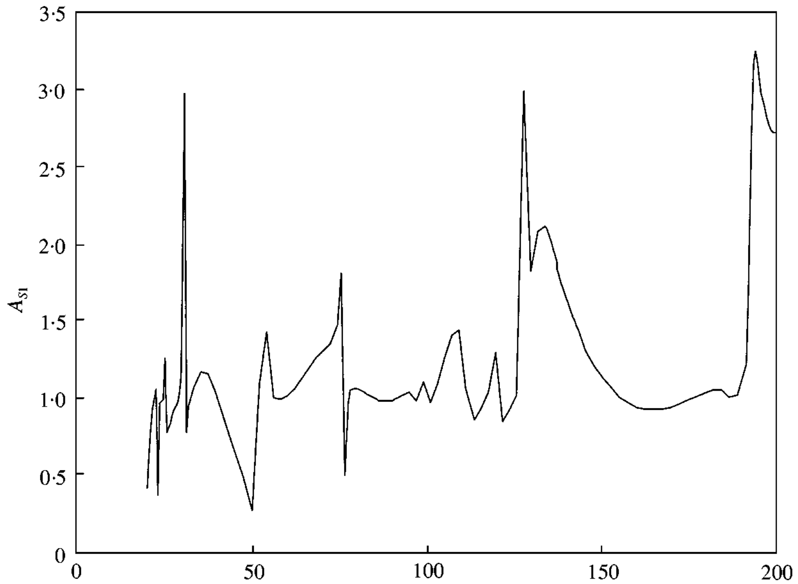

Figure 7. Plot of the term A_{S_1} establishing the blocked pressure criterion in case of heavy fluid loading.

Due to the strong fluid-structure coupling, the resonance of the vibro-acoustic system is not located around natural frequencies of the structure, as was observed previously in the case of light fluid loading: the use of the approximate blocked pressure criterion (equation (43)) is obviously not appropriate in this case.

5. CONCLUSION

The principle of acoustic reciprocity was used to derive an exact expression for the sound pressure radiated by an individual vibrating component located in a complex mechanical system. For a component mechanically uncoupled from the rest of the system, a blocked pressure criterion was developed. An approximate one was also expressed at a given natural frequency of the structure, when the modal coupling is neglected. The criterion indicates in which circumstances the usual reciprocity technique can be used to estimate the radiated sound pressure from the vibrating component. The criterion was numerically tested in the case of two coplanar, simply supported, thin baffled plates; it succeeds in evaluating the accuracy of the usual reciprocity technique in two cases: (1) in case of light fluid loading, the blocked pressure assumption is violated around structural resonance, the use of the approximate criterion seems to be advantageous; (2) in case of heavy fluid loading, the criterion states that the usual reciprocity technique introduces erroneous values in the frequency range studied.

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APPENDIX A: FORCED RESPONSE OF TWO COPLANAR, SIMPLY SUPPORTED THIN BAFFLED PLATES

The transverse displacement fields u_{S_1} and u_{S_2} of the two plates are derived in state (1): the complex displacements of each structure are expressed as modal series:

$$\hat{u}_{S_1}^{(1)} = \sum_i (\hat{\Phi}_i^{S_1})^{(1)} X_i^{S_1}, \quad \hat{u}_{S_2}^{(1)} = \sum_i (\hat{\Phi}_i^{S_2})^{(1)} X_i^{S_2}, \tag{A1, A2}$$

where $X_i^{S_1}$ and $X_i^{S_2}$ designate the plate eigenfunctions; under the assumption of pure bending, the eigenfunctions have a simple analytical expression [7]. The modal displacements $(\hat{\Phi}_i^{S_1})^{(1)}$ and $(\hat{\Phi}_i^{S_2})^{(1)}$ can be found from the following equations of dynamic equilibrium.

For plate S_1:

$$(-\omega^2(\mathbf{M}^{S_1})^* + (\mathbf{K}^{S_1})^*(1 + \mathrm{j}\eta_{S_1}))(\hat{\mathbf{\Phi}}^{S_1})^{(1)} = (\mathbf{F}_{S_1}^*)^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_1S_1}(\hat{\mathbf{\Phi}}^{S_1})^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_1S_2}(\hat{\mathbf{\Phi}}^{S_2})^{(1)}, \tag{A3}$$

where $(\mathbf{F}^*_{S_1})^{(1)}$ is the modal force vector on plate S_1, $\mathbf{Z}^{S_1S_1}$ and $\mathbf{Z}^{S_1S_2}$ are modal radiation impedance matrices from S_1 to S_1 and from S_2 to S_1 respectively (equation (39)). The radiation impedance matrices account for the acoustic loading of S_1 that results from the dynamic responses of both S_1 and S_2. The calculation of $\mathbf{Z}^{S_1S_1}$ and $\mathbf{Z}^{S_1S_2}$ is performed numerically by discretizing the plates into 10×12 elements.

Similarly, for plate S_2:

$$(-\omega^2(\mathbf{M}^{S_2})^* + (\mathbf{K}^{S_2})^*(1+\mathrm{j}\eta_{S_2}))(\hat{\mathbf{\Phi}}^{S_2})^{(1)} = (\mathbf{F}^*_{S_2})^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_2S_2}(\hat{\mathbf{\Phi}}^{S_2})^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_2S_1}(\hat{\mathbf{\Phi}}^{S_1})^{(1)}, \tag{A4}$$

The solution of the system of equations (A3) and (A4) is straightforward:

$$(\hat{\mathbf{\Phi}}^{S_1})^{(1)} = (\mathbf{A}^{S_1} + \omega^2\mathbf{Z}^{S_1S_2}(\mathbf{A}^{S_2})^{-1}\mathbf{Z}^{S_2S_1})^{-1}((\mathbf{F}^*_{S_1})^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_1S_2}(\mathbf{A}^{S_2})^{-1}(\mathbf{F}^*_{S_2})^{(1)}), \tag{A5}$$

$$(\hat{\mathbf{\Phi}}^{S_2})^{(1)} = (\mathbf{A}^{S_2} + \omega^2\mathbf{Z}^{S_2S_1}(\mathbf{A}^{S_1})^{-1}\mathbf{Z}^{S_1S_2})^{-1}((\mathbf{F}^*_{S_2})^{(1)} - \mathrm{j}\omega\mathbf{Z}^{S_2S_1}(\mathbf{A}^{S_1})^{-1}(\mathbf{F}^*_{S_1})^{(1)}), \tag{A6}$$

with

$$\mathbf{A}^{S_1} = -\omega^2(\mathbf{M}^{S_1})^* + (\mathbf{K}^{S_1})^*(1+\mathrm{j}\eta_{S_1}) + \mathrm{j}\omega\mathbf{Z}^{S_1S_1}, \tag{A7}$$

$$\mathbf{A}^{S_2} = -\omega^2(\mathbf{M}^{S_2})^* + (\mathbf{K}^{S_2})^*(1+\mathrm{j}\eta_{S_2}) + \mathrm{j}\omega\mathbf{Z}^{S_2S_2}. \tag{A8}$$

The response of the two plates in state (2) can be obtained from these expressions with the following changes and substitutions:

$$(F^*_{S_1})^{(2)} = -\mathrm{j}\omega\rho_0 Q^{(2)} \iint_{S_1} X_i^{S_1} G \,\mathrm{d}S_1, \quad (F^*_{S_2})^{(2)} = -\mathrm{j}\omega\rho_0 Q^{(2)} \iint_{S_2} X_i^{S_2} G \,\mathrm{d}S_2$$

where G is the Green function for a semi-infinite medium [8], which satisfies the homogeneous Neumann condition on the plane $z = 0$,

$$G(\mathbf{r}_1, \mathbf{r}_0) = \frac{\mathrm{e}^{-\mathrm{j}k|\mathbf{r}_1 - \mathbf{r}_0|}}{2\pi|\mathbf{r}_1 - \mathbf{r}_0|}. \tag{A9}$$

APPENDIX B: SOUND PRESSURE RADIATED BY PLATE S_1

In this section, the exact sound pressure $\{\hat{p}_1^{(1)}\}_{ex}$ radiated by plate S_1, and its approximation $\{\hat{p}_1^{(1)}\}_{ap}$ derived from the principle of acoustic reciprocity are analytically derived. These sound pressures are given by equations (10) and (12) respectively.

The location of the point source defined in state (2) is represented by $\mathbf{r}_0$, and corresponds to the location at which the radiated sound pressure is calculated in state (1). The calculation of $\{\hat{p}_1^{(1)}\}_{ex}(\mathbf{r}_0)$, equation (10), requires that $\hat{p}_1$ and $\hat{p}_2$ be determined over S_1. The sound pressure $\hat{p}^{(1)}(\mathbf{r}_1)$ is given by equation (35):

$$\hat{p}^{(1)}(\mathbf{r}_1) = \mathrm{j}\omega\rho_0 \sum_{p=1,2} \iint_{S_p} G(\mathbf{r}_1, \mathbf{r}_p)\,\hat{w}^{(1)}(\mathbf{r}_p)\,\mathrm{d}S_p, \tag{B1}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ describe surfaces S_1 and S_2 respectively. The sound pressure $\hat{p}^{(2)}(\mathbf{r}_1)$ takes into account the contribution of the point source and the radiation of the entire mechanical system,

$$\hat{p}^{(2)}(\mathbf{r}_1) = \mathrm{j}\omega G(\mathbf{r}_1, \mathbf{r}_0)\hat{Q}^{(2)} + \mathrm{j}\omega\rho_0 \sum_{p=1,2} \iint_{S_p} G(\mathbf{r}_1, \mathbf{r}_p)\, \hat{w}^{(2)}(\mathbf{r}_p)\, \mathrm{d}S_p, \tag{B2}$$

where G is given by equation (A9).
Hence

$$\frac{1}{\hat{Q}^{(2)}} \iint_{S_1} \hat{w}^{(2)}\hat{p}^{(1)}\, \mathrm{d}S_1 = -\frac{\omega^2}{\hat{Q}^{(2)}} \sum_i (\hat{\Phi}_i^{S_1})^{(2)} \sum_j \sum_{p=1,2} Z_{ij}^{S_1 S_p} (\hat{\Phi}_j^{S_p})^{(1)}, \tag{B3}$$

where $Z_{ij}^{S_1 S_p}$ is given by equation (39). Similarly:

$$\frac{1}{\hat{Q}^{(2)}} \iint_{S_1} \hat{w}^{(1)}\hat{p}^{(2)}\, \mathrm{d}S_1 = -\omega^2\rho_0 \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \iint_{S_1} (\mathbf{X}_i^{S_1}(\mathbf{r}_1)\cdot\mathbf{n})\, G(\mathbf{r}_1, \mathbf{r}_0)\, \mathrm{d}S_1$$

$$-\frac{\omega^2}{\hat{Q}^{(2)}} \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \sum_j \sum_{p=1,2} Z_{ij}^{S_1 S_p} (\hat{\Phi}_j^{S_p})^{(2)}. \tag{B4}$$

Finally, the exact sound pressure $\{\hat{p}_1^{(1)}\}_{ex}$ $(\mathbf{r}_0)$ can be expressed as follows:

$$\{\hat{p}_1^{(1)}\}_{ex}(\mathbf{r}_0) = -\omega^2\rho_0 \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \iint_{S_1} (\mathbf{X}_i^{S_1}(\mathbf{r}_1)\cdot\mathbf{n})\, G(\mathbf{r}_1, \mathbf{r}_0)\, \mathrm{d}S_1$$

$$-\frac{\omega^2}{\hat{Q}^{(2)}} \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \sum_j \sum_{p=1,2} Z_{ij}^{S_1 S_p} (\hat{\Phi}_j^{S_p})^{(2)} + \frac{\omega^2}{\hat{Q}^{(2)}} \sum_i (\hat{\Phi}_i^{S_1})^{(2)} \sum_j \sum_{p=1,2} Z_{ij}^{S_1 S_p} (\hat{\Phi}_j^{S_p})^{(1)}. \tag{B5}$$

The first term on the right-hand side of equation (B5) represents the pressure radiated by S_1, assuming that the other surfaces are not radiating. The sum of the two last terms is not necessarily zero. This would hold if S_2 would not exist. These two last terms account for the effect of S_2 on the sound radiation characteristics of S_1.

The sound pressure $\{\hat{p}_1^{(1)}\}_{ap}$ $(\mathbf{r}_0)$ given by the principle of acoustic reciprocity and the blocked pressure assumption is derived from equation (12):

$$\{\hat{p}_1^{(1)}\}_{ap}(\mathbf{r}_0) = -\omega^2\rho_0 \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \iint_{S_1} (\mathbf{X}_i^{S_1}(\mathbf{r}_1)\cdot\mathbf{n})\, G(\mathbf{r}_1, \mathbf{r}_Q)\, \mathrm{d}S_1$$

$$-\frac{\omega^2}{\hat{Q}^{(2)}} \sum_i (\hat{\Phi}_i^{S_1})^{(1)} \sum_j \sum_{p=1,2} Z_{ij}^{S_1 S_p} (\hat{\Phi}_j^{S_p})^{(2)}. \tag{B6}$$

The surface integrals in equations (B5) and (B6) are done numerically by discretizing S_1 into 10×12 elements.