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A VARIANT DESIGN OF THE DYNAMIC VIBRATION ABSORBER

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1. INTRODUCTION

The dynamic vibration absorber (DVA) is a very useful passive device that is used to suppress narrowband vibration. It essentially consists of a mass, a spring, and a damper, which are attached to a primary system subjected to vibration disturbance. In 1928, Ormondroyd and Den Hartog [1] proposed the optimization principle of the damped DVA in terms of minimizing the maximum amplitude response of the primary system. Following this principle, Hahnkamm deduced the relationship for the optimum tuning of DVA [2] and Brock developed the optimum damping [3]. This optimum design method of the dynamic vibration absorber is called the fixed-points theory, which was well documented in the textbook by Den Hartog [4].

In this paper, another form of dynamic vibration absorber is investigated. It is basically an ordinary dynamic absorber. The only difference is that the damping element is not connected to the structure to be controlled but to the earth (a base structure). This is perhaps not a common usage for the dynamic vibration absorber, but it does bring convenience in practice in some cases. A more interesting fact is that, as will be described in the following, given the same mass ratio, this design of dynamic vibration absorber could offer a more effective control result over an ordinary design.

2. ORDINARY DYNAMIC VIBRATION ABSORBER

Figure 1 shows an ordinary dynamic vibration absorber attached to a single-degree-of-freedom (s.d.o.f.) primary system. The primary system is undamped and a viscous damper is assumed for the absorber. The response of the primary system can be described in terms of the ratio of its vibration amplitude to the static deformation of the system under sinusoidal excitation ($f = Fe^{j\omega t}$). With $X_{st} = F/K$, the amplitude response was given as [4]

$$\left|\frac{X_1}{X_{st}}\right| = \sqrt{\frac{p}{q}}, \tag{1}$$

where

$$p = (\gamma^2 - \lambda^2)^2 + (2\zeta\gamma\lambda)^2, \tag{2}$$

$$q = [(1-\lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2 + (2\zeta\gamma\lambda)^2(1 - \mu\lambda^2 - \lambda^2)^2, \tag{3}$$

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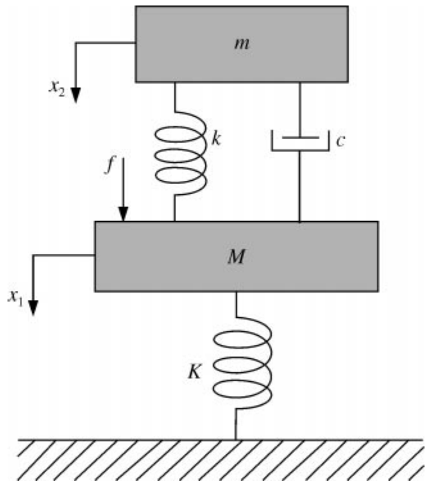

Figure 1. The ordinary dynamic vibration absorber attached to a primary system.

in which the parameters are defined as

$$\lambda = \omega/\sqrt{K/M}, \qquad \gamma = \sqrt{k/m}/\sqrt{K/M},$$

$$\mu = m/M, \qquad \zeta = c/2\sqrt{mk}.$$

The optimum design of this absorber reads as follows [4]:

$$\gamma_{opt} = \frac{k/m}{K/M} = \frac{1}{1+\mu}, \tag{4}$$

$$\zeta_{opt} = \frac{c}{2\sqrt{mk}} = \sqrt{\frac{3\mu}{8(1+\mu)}}, \tag{5}$$

where $\mu = m/M$ is called the mass ratio. Equation (4) determines the optimum tuning frequency of the absorber, and equation (5) defines the optimum damping. Under this condition, the maximum ratio of the vibration amplitude to the static deformation of the primary system reaches minimum, as follows:

$$\left|\frac{X_1}{X_{st}}\right|_{max} = \sqrt{\frac{2+\mu}{\mu}}. \tag{6}$$

It should be pointed out that the above design parameters derived from the fixed-points theory represent an approximate solution. Asami and Nishihara [5] have recently developed a closed-form exact solution to this DVA optimization problem. But in this paper, the fixed-points theory is used. In practice, many problems can be represented by an equivalent s.d.o.f. system. Therefore, this design principle has found a wide variety of applications in the engineering world.

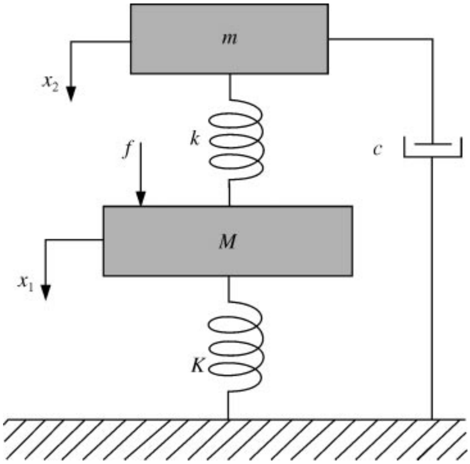

Figure 2. The variant dynamic vibration absorber attached to a primary system.

3. VARIANT FORM OF DYNAMIC VIBRATION ABSORBER

Figure 2 shows the variant form of the dynamic vibration absorber discussed in this paper. The difference compared to the ordinary one is that the damping element is not attached to the primary structure but to the earth (a base structure). This is perhaps not a common layout for applying the dynamic vibration absorber, but it does bring convenience in some cases in practice. Actually, the author had applied this form of DVA in a practical consulting project.

Although the layout changes, it is found that the fixed-points theory still holds in this case. The amplitude response of the primary system in this case reads

$$\left|\frac{X_1}{X_{st}}\right| = \sqrt{\frac{p}{q}}, \tag{7}$$

where

$$p = (\gamma^2 - \lambda^2)^2 + (2\zeta\gamma\lambda)^2, \tag{8}$$

$$q = [(1-\lambda^2)(\gamma^2-\lambda^2) - \mu\gamma^2\lambda^2]^2 + (2\zeta\gamma\lambda)^2(1+\mu\gamma^2-\lambda^2)^2. \tag{9}$$

Following the similar approach of reference [4], the optimum tuning condition and the optimum damping can be deduced as

$$\gamma_{opt} = \sqrt{\frac{1}{1-\mu}}, \tag{10}$$

$$\zeta_{opt} = \sqrt{\frac{3\mu}{8(1-0{\cdot}5\mu)}}. \tag{11}$$

The maximum response amplitude of the primary system reaches minimum under this condition as

$$\left|\frac{X_1}{X_{st}}\right|_{max} = (1-\mu)\sqrt{\frac{2}{\mu}}. \tag{12}$$

For an easy reference, Appendix A gives the details in deducing these relations.

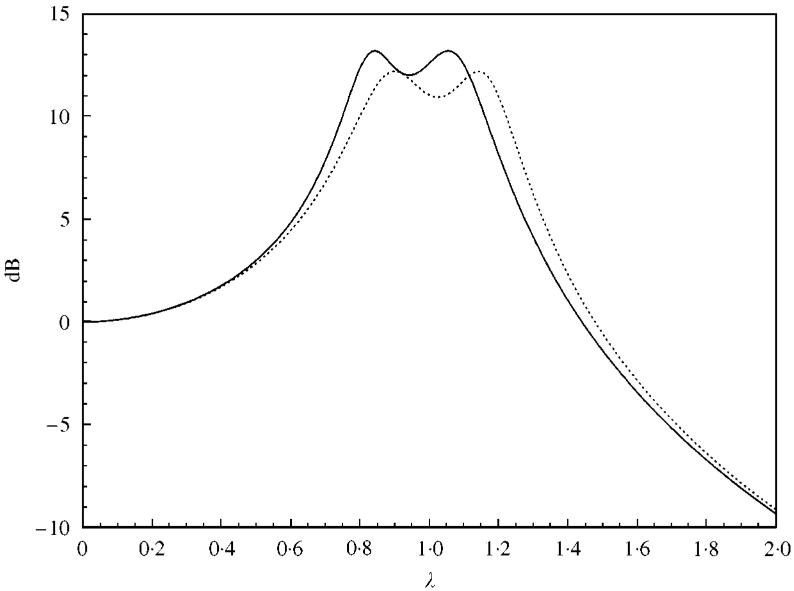

Figure 3. The amplitude response of the primary system with an optimally tuned ordinary absorber (——) and with the new absorber (·····) under the same mass ratio of 0·1.

4. DISCUSSION

Equation (10) suggests that the absorber is tuned to a certain percentage above the operating frequency while an ordinary design (equation (4)) suggests that the absorber is tuned to a certain percentage below the operating frequency. As to the optimum damping, the new type of absorber suggests a damping coefficient that is a little bit higher than the ordinary one (equation (11) versus equation (5)). The most interesting comparison comes between equations (12) and (6). For the same mass ratio, it can be seen that equation (12) will give a lower vibration level than equation (6). For example, for $\mu = 0{\cdot}1$, the difference will be about 1·1 dB; for $\mu = 0{\cdot}2$, the difference will be 2·4 dB. That is to say, without increasing the additional mass, the vibration level can be suppressed more effectively by employing the new design of the dynamic vibration absorber. In Figure 3 is shown the difference of the amplitude response of the primary system with an optimally tuned normal absorber and the new type absorber under the same mass ratio of 0·1.

5. CONCLUDING REMARKS

In this article, a new layout of a dynamic vibration absorber was proposed and its optimum design principles were developed. It is shown that by connecting the damping element of the absorber to the earth (a base structure), the vibration of the primary structure can be a little more effectively suppressed than by using an ordinary DVA of the same mass ratio. Although this layout is not a common usage for the DVA, it is believed that it may bring convenience in certain conditions.

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APPENDIX A

In Figure 2, the motions of the primary system and the dynamic vibration absorber are governed by the following equations:

$$M\ddot{x}_1 + Kx_1 + k(x_1 - x_2) = f, \tag{A1}$$

$$m\ddot{x}_2 + c\dot{x}_2 + k(x_2 - x_1) = 0. \tag{A2}$$

Assuming a harmonic disturbing force $f = F\mathrm{e}^{\mathrm{j}\omega t}$, the responses may be written as

$$x_1 = X_1\mathrm{e}^{\mathrm{j}\omega t}, \qquad x_2 = X_2\mathrm{e}^{\mathrm{j}\omega t}.$$

Therefore, equations (A1) and (A2) become

$$-M\omega^2 X_1 + KX_1 + k(X_1 - X_2) = F, \tag{A3}$$

$$-m\omega^2 X_2 + \mathrm{j}\omega c X_2 + k(X_2 - X_1) = 0. \tag{A4}$$

Solving these equations yields

$$X_1 = \frac{k - m\omega^2 + \mathrm{j}\omega c}{(K - M\omega^2)(k - m\omega^2) - mk\omega^2 + \mathrm{j}(K + k - M\omega^2)\omega c}F. \tag{A5}$$

The magnitude is

$$\begin{aligned} |X_1| &= \left(\frac{[(k - m\omega^2)^2 + (\omega c)^2]\ (K/M)^2}{\{[(K - M\omega^2)(k - m\omega^2) - mk\omega^2]^2 + [(K + k - M\omega^2)\omega c]^2\}\ (K/M)^2}\right)^{1/2} F \\ &= \left(\frac{[(k/m - \omega^2)^2 + (\omega c/m)^2]\ (K/M)^2}{[(K/M - \omega^2)(k/m - \omega^2) - (k/m)(m/M)\omega^2]^2 + [(K/M + k/M - \omega^2)(\omega c/m)]^2}\right)^{1/2} F. \end{aligned} \tag{A6}$$

Multiplying the numerator and denominator inside the root square of equation (A6) with $(K/M)^{-4}$, and introducing the following parameters:

$$X_{st} = F/K, \qquad \lambda = \omega/\sqrt{K/M},$$

$$\gamma = \sqrt{k/m}/\sqrt{K/M}, \qquad \mu = m/M, \qquad \zeta = c/2\sqrt{mk},$$

one obtains

$$\left|\frac{X_1}{X_{st}}\right| = \sqrt{\frac{p}{q}}, \tag{A7}$$

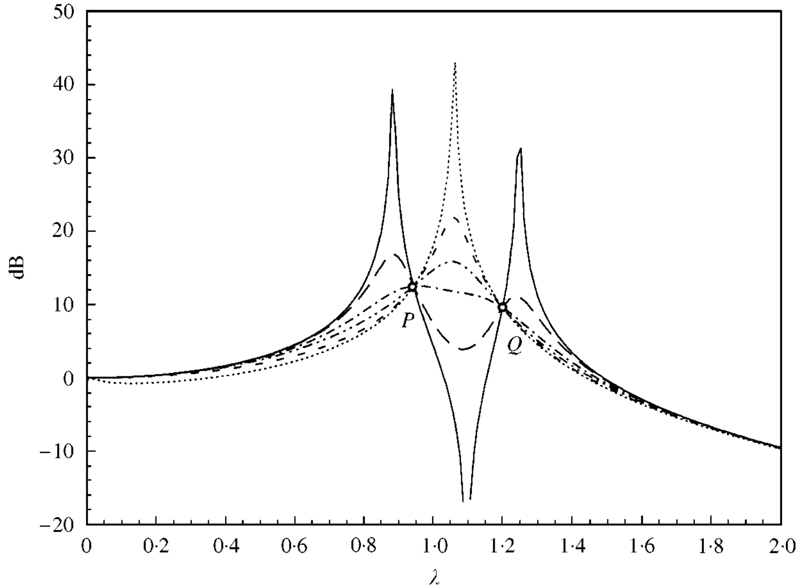

Figure A1. The responses of the primary system under $\mu = 0{\cdot}1$ and $\gamma = 1{\cdot}1$. ——, $\zeta = 0$; — —, $\zeta = 0{\cdot}1$; – · – · – · –, $\zeta = 0{\cdot}25$; – ·· – ·· –, $\zeta = 0{\cdot}4$; – – –, $\zeta = 0{\cdot}8$; ·····, $\zeta = 10$.

where

$$p = (\gamma^2 - \lambda^2)^2 + (2\zeta\gamma\lambda)^2, \tag{A8}$$

$$q = [(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2 + (2\zeta\gamma\lambda)^2(1 + \mu\gamma^2 - \lambda^2)^2. \tag{A9}$$

Equation (A7) defines the response magnitude of the primary system with the absorber. For given μ and γ, the response can be calculated. In Figure A1, the results under $\mu = 0{\cdot}1$ and $\gamma = 1{\cdot}1$ are shown. To demonstrate the characteristics, the results in several cases of damping coefficient (ζ) are given in this figure. It is clearly seen that there exist two common points (P and Q) on all the curves, where the responses are not influenced by the damping. These points are referred to as the fixed points. The optimum condition of the dynamic vibration absorber can be achieved by adjusting the responses at P and Q to the same level (optimum tuning), and meanwhile making P and Q the maximum points on the response curves (optimum damping). This design principle is the well-known fixed-points theory [4]. In the following, this theory will be utilized to develop the optimum design laws of the absorber under study.

Let

$$A = (\gamma^2 - \lambda^2)^2, \qquad B = (2\gamma\lambda)^2,$$

$$C = [(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2, \qquad D = (2\gamma\lambda)^2(1 + \mu\gamma^2 - \lambda^2)^2, \tag{A10}$$

then

$$\frac{p}{q} = \frac{A + B\zeta^2}{C + D\zeta^2} = \frac{B}{D}\frac{A/B + \zeta^2}{C/D + \zeta^2}. \tag{A11}$$

Under the condition $A/C = B/D$,

$$\left|\frac{X_1}{X_{st}}\right| = \sqrt{\frac{B}{D}} = \left|\frac{1}{1 + \mu\gamma^2 - \lambda^2}\right| \tag{A12}$$

will be independent of the damping coefficient ζ. Considering the fact that

$$\left|\frac{X_1}{X_{st}}\right|_{\zeta=0} = \left|\frac{\gamma^2 - \lambda^2}{(1-\lambda^2)(\gamma^2-\lambda^2) - \mu\gamma^2\lambda^2}\right| = \left(\frac{A}{C}\right)^{1/2}, \tag{A13}$$

$$\left|\frac{X_1}{X_{st}}\right|_{\zeta=\infty} = \left|\frac{1}{1+\mu\gamma^2-\lambda^2}\right| = \left(\frac{B}{D}\right)^{1/2}, \tag{A14}$$

the condition $A/C = B/D$ implies the crossing points of curves $|X_1/X_{st}|_{\zeta=0}$ and $|X_1/X_{st}|_{\zeta=\infty}$. By examining equation (A5), it is not difficult to find that the responses at $\zeta = 0$ and ∞ are in opposite phase, therefore, the fixed points can be solved from the following equation:

$$\frac{\gamma^2-\lambda^2}{(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\gamma^2\lambda^2} = \frac{-1}{1+\mu\gamma^2-\lambda^2}. \tag{A15}$$

Manipulating equation (A15) yields

$$\lambda^4 - (1+\gamma^2+\mu\gamma^2)\lambda^2 + (2+\mu\gamma^2)\gamma^2/2 = 0. \tag{A16}$$

Suppose that the roots of equation (A16) are λ_P and λ_Q, then the following equation stands:

$$(\lambda^2-\lambda_P^2)(\lambda^2-\lambda_Q^2) = \lambda^4 - (\lambda_P^2+\lambda_Q^2)\lambda^2 + \lambda_P^2\lambda_Q^2 = 0. \tag{A17}$$

Comparing equations (A16) and (A17), one obtains

$$\lambda_P^2 + \lambda_Q^2 = 1 + \gamma^2 + \mu\gamma^2. \tag{A18}$$

In order to reach the optimum tuning, the responses at P and Q should be the same, therefore

$$\frac{1}{1+\mu\gamma^2-\lambda_P^2} = \frac{-1}{1+\mu\gamma^2-\lambda_Q^2}, \tag{A19}$$

that is

$$\lambda_P^2 + \lambda_Q^2 = 2(1+\mu\gamma^2). \tag{A20}$$

From equations (A18) and (A20) the optimum tuning condition is obtained as

$$\gamma_{opt}^2 = \frac{1}{1-\mu}, \tag{A21}$$

namely,

$$\gamma_{opt} = \sqrt{\frac{1}{1-\mu}}. \tag{A22}$$

The points P and Q under this condition are determined by

$$\lambda_P^2 = \frac{1}{1-\mu}\left(1-\sqrt{\frac{\mu}{2}}\right), \tag{A23}$$

$$\lambda_Q^2 = \frac{1}{1-\mu}\left(1+\sqrt{\frac{\mu}{2}}\right). \tag{A24}$$

The responses at P and Q are

$$\left|\frac{X_1}{X_{st}}\right|_P = \left|\frac{X_1}{X_{st}}\right|_Q = (1-\mu)\sqrt{\frac{2}{\mu}}. \tag{A25}$$

In the above, the optimum tuning condition was deduced. The next step will be to determine the optimum damping in order to make points P and Q the maximum points on the response curve. The condition of points P and Q being the maximum means that the response curve should pass through the two fixed points with a horizontal tangent, that is

$$\frac{\partial}{\partial \lambda^2}\left[\frac{X_1}{X_{st}}\right]^2 = 0. \tag{A26}$$

In another form

$$p'q - pq' = 0, \tag{A27}$$

where, $p' = \partial p/\partial\lambda^2$ and $q' = \partial q/\partial\lambda^2$.

Under the optimum tuning condition

$$\frac{p}{q} = \frac{1}{(1 + \mu\gamma^2 - \lambda^2)^2}, \tag{A28}$$

therefore,

$$(1 + \mu\gamma^2 - \lambda^2)^2\, p' - q' = 0. \tag{A29}$$

From equation (A8) one obtains

$$p' = 4(\zeta\gamma)^2 - 2(\gamma^2 - \lambda^2), \tag{A30}$$

and from equation (A9) and making use of equation (A15) one obtains

$$q' = 2(1 + \mu\gamma^2 - \lambda^2)\,\{(\gamma^2 - \lambda^2)\,[1 - 2\lambda^2 + \gamma^2(1 + \mu)] + 2(\zeta\lambda)^2(1 + \mu\gamma^2 - 3\lambda^2)\}. \tag{A31}$$

Substituting equations (A30) and (A31) into equation (A29),

$$\begin{gathered}(1 + \mu\gamma^2 - \lambda^2)^2\,[4(\zeta\lambda)^2 - 2(\gamma^2 - \lambda^2)] \\ - 2(1 + \mu\gamma^2 - \lambda^2)\,\{(\gamma^2 - \lambda^2)\,[1 - 2\lambda^2 + \gamma^2(1 + \mu)] + 2(\zeta\gamma)^2(1 + \mu\gamma^2 - 3\lambda^2)\} = 0.\end{gathered} \tag{A32}$$

Solving this equation for ζ^2 one obtains

$$\zeta^2 = \frac{\gamma^2 - \lambda^2}{4(\gamma\lambda)^2}\,[2 - 3\lambda^2 + \gamma^2(1 + 2\mu)]. \tag{A33}$$

Substituting equations (A21, A23, A24) into the above equation results in the optimum damping as

$$\zeta^2_{P,Q} = \frac{3}{8}\frac{\mu}{1 \mp \sqrt{\mu/2}}. \tag{A34}$$

Taking an average of ζ_P^2 and ζ_Q^2 produces

$$\zeta_{opt} = \sqrt{\frac{3\mu}{8(1 - 0{\cdot}5\mu)}}. \tag{A35}$$

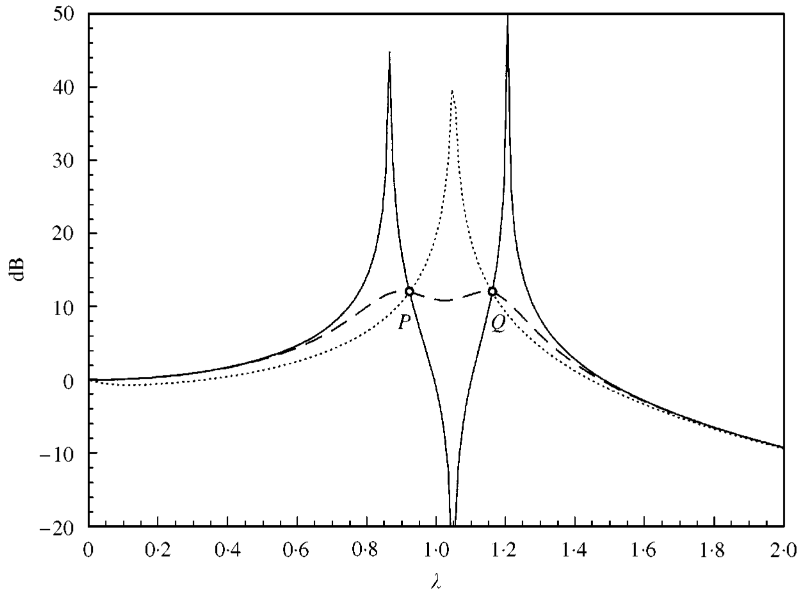

Figure A2. The responses of the primary system under optimum tuning, $\mu = 0{\cdot}1$, $\gamma_{opt} = 1{\cdot}0541$. ——, $\zeta = 0$; — —, $\zeta_{opt} = 0{\cdot}1987$; ········, $\zeta = 10$.

Obviously, with the approximation involved in equation (A35), one cannot exactly make points P and Q as the maximum points of the response curve. Nevertheless, this approximation does not influence the usefulness of the result as a simple design law. Figure A2 shows the response curves under the optimum tuning condition derived herein.