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AN ADDED MASS THEORY FOR THE BASE PLATE IN A PARTIALLY FILLED RECTANGULAR TANK FOR USE WITH FEA

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Undertaking a dynamic finite element analysis on a tank containing a fluid either requires both the fluid and tank to be modelled or the mass of the vessel's walls and base to be adjusted to account for the presence of the fluid. The former generally requires specialized solid elements to model the fluid, which are not available in all finite element software packages. This paper details a set of properties for structural solid elements that allow these elements to accurately emulate water within a vessel. Two fully welded rectangular steel tanks, constructed from plate of different thicknesses, were partially filled with water and dynamically excited. Excellent agreement was found between the dynamic measurements taken from the base of the tanks and the predictions from a finite element model with the fluid modelled using structural solid elements and the derived property set. These experiments show that the fundamental mode of dynamic behaviour of the base of these tanks is primarily dependent on both the depth of fluid in the tank and the thickness of its base. The added mass principle of Greenspon (*Journal of Acoustical Society of America* **33**, 1485–1497 [1]), derived mainly for plates exposed to essentially an infinite body of fluid, could not be used to accurately calculate the adjustment in mass of the base plate to account for the presence of a finite volume of fluid. A modification to the added mass principle of Greenspon has been proposed that fully accounts for this depth and base thickness dependency.

1. INTRODUCTION

It is well known that the dynamics of a container or vessel is affected by the type and amount of liquid surrounding and/or within it. In many instances this change in dynamic behaviour is of little importance, however, there are an increasing number of cases where it is of great importance. A major reason for this is an increase in the number of structures that have been optimized to minimize weight. For example, the work described in this paper started as an investigation into the design of a fuel tank for a locomotive. Here the tank is an integral part of the locomotive's platform and will be vulnerable to excitation from the hard-mounted diesel engine. The tank's dynamic response needs to be known for all fluid levels. It is therefore essential that we not only understand how a fluid affects the dynamics of a container but we can also accurately and readily predict it.

The natural frequencies of vibration of a fluid-filled vessel are lower than its empty counterpart due to coupling between the fluid and the vibrating vessel walls and base [2]. This coupling effectively increases the mass of the submerged plates with its stiffness remaining unchanged. The apparent increase in the mass of these plates can be determined using techniques like the added mass principle and the Rayleigh–Ritz method. Greenspon [1] determined the added mass for a rectangular plate exposed to an infinite body of fluid using a form of the Babayer–Fedenko formula. His work was specific to ocean-going vessels like ships and submarines where one side of the vessel wall was exposed to effectively an infinite volume of fluid. Mazuch *et al.* [3] calculated the added mass by solving the generalized eigenvalue problem with its mass term consisting of both the mass of the plate and an added mass term from the fluid–structure coupling. The added mass term was derived from the pressure field determined from Laplace equation in an incompressible inviscid fluid with boundary conditions consisting of the vibrating plate and a free surface. The Rayleigh–Ritz method has been successfully used by Vaillette [4] to predict mode shapes and associated frequencies of submerged plates. Of the two methods the added mass principle appears to be most robust and stable over a wider range of geometry.

The finite element method has been used successfully by a number of authors, for example, references [3] and [5], to predict the natural frequencies of various shaped vessels containing fluid. Mazuch *et al.* [3] used specially derived low order three-dimensional fluid elements to model the fluid within a cylindrical vessel. Their dynamic predictions were generally in excellent agreement with measurements. Fluid elements are somewhat specialized and are generally not available within most (affordable) commercially available finite element software packages. One of the aims of the work described in this paper was to determine the appropriate properties for structural three-dimensional elements to accurately emulate fluid in a dynamic analysis.

Experimental measurements and finite element modelling undertaken by the authors show the frequency of vibration of the base of a rectangular vessel was dependent on both the depth of fluid within the vessel and the thickness of base. Furthermore, we found that the formulation from reference [1] was inappropriate in predicting the frequencies of plates exposed to a finite volume of fluid. This paper suggests a modification to the added mass formulation of reference [1] to predict the fundamental frequency of vibration of the base of rectangular tanks when partially filled with fluid.

2. BACKGROUND THEORY

As a plate vibrates in a fluid, the fluid immediately surrounding the plate is set into motion. This fluid has coupled its motion with that of the plate during vibration. As a consequence, the fluid will contribute its own stiffness and mass to that of the plate. Given the stiffness of a fluid is negligible, then the coupled fluid essentially only adds mass to the plate and in so doing reduces its frequency of vibration. The effect of this added mass on the natural frequency of the plate has been expressed by [2]:

$$\frac{f_{fluid}}{f_{air}} = \frac{1}{(1 + A_P/M_P)^{1/2}}, \tag{1}$$

where A_P is the added mass of the plate, M_P is the mass of the plate, f_{fluid} and f_{air} are the natural frequencies of the plate exposed to the fluid and in air respectively. Equation (1) will only hold true if the fluid does not modify the plate's vibration mode shape. The magnitude of A_P is a function of several variables including the density of fluid, the geometry of the plate, the plate's boundary conditions and its frequency mode number. Further, equation (1) assumes that only one side of the plate is exposed to fluid; if both sides are exposed to a fluid, then A_P in equation (1) is replaced by $2A_P$ [2].

Greenspon [1] proposed that the added mass for rectangular plates with one side exposed to an infinite body of fluid is given by

$$A_P = \alpha_{ij}\beta\rho a b^2, \tag{2}$$

where a is the width of the plate, b is the length of the plate (such that $b \geqslant a$), ρ is the density of the fluid, β is an aspect ratio dependent factor [2], and α_{ij} is a function of the modal indices and the boundary conditions on the plate. The term α_{ij} is determined by

$$\alpha_{ij} = \frac{\left(\int_A z_{ij}\,\mathrm{d}A\right)^2}{2ab\int_A z_{ij}^2\,\mathrm{d}A} \tag{3}$$

$$\text{with} \quad z_{ij}(x, y) = X_i(x)\,Y_j(y). \tag{4}$$

$X_i(x)$ and $Y_j(y)$ are one-dimensional beam modes that satisfy the plate boundary conditions in the x and y directions; the equations for $X_i(x)$ and $Y_j(y)$ can be found in reference [2].

3. EXPERIMENTAL METHODS

Modal analysis experiments were conducted on two test tanks manufactured from 250 MPa yield strength steel plate. The design of these tanks was based on two chambers of a locomotive fuel tank; Figure 1 shows a typical cross-section through a locomotive fuel tank manufactured by A. Goninan and Company Limited. The test tanks had a 1000 mm square base and were 750 mm deep with a baffle plate welded across the tank bisecting the base—see Figure 2 for a pictorial view of the tank. The thickness of the tank's panels was selected so that the fundamental frequency of vibration was between 0 and 200 Hz. When fabricating the tanks, the out-of-plane distortion of all panels was kept within 1 mm so as not to change the panel's stiffness.

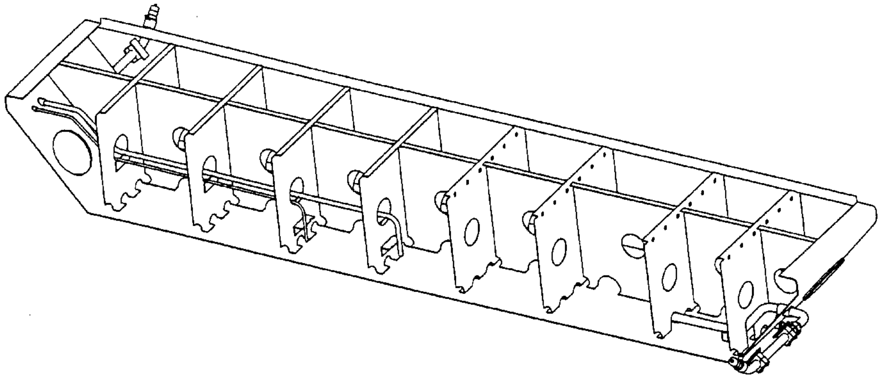

Figure 1. Section through a Goninan locomotive fuel tank.

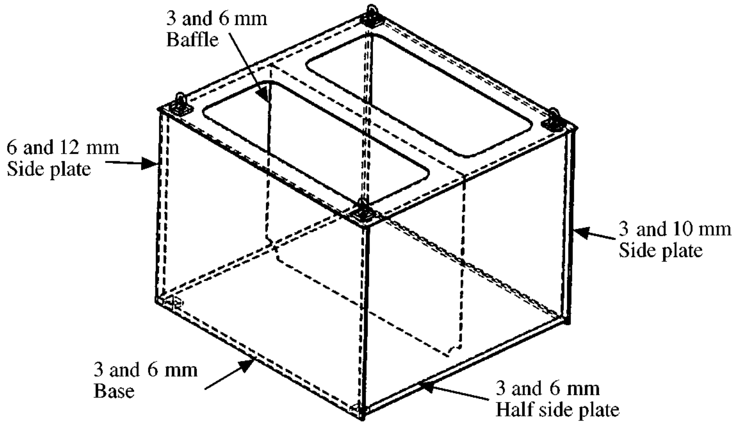

Figure 2. Solid model of the test tank; plate thickness shown for the first and second tank respectively.

The tank was supported by an overhead crane by 10 mm diameter slings attached to the four eye-bolts welded at each corner of the tank. This method of holding the tank was suggested by Bruel and Kjaer [5] to minimize external influences on the natural frequencies. A Bruel and Kjaer 4370 accelerometer was attached to the base plate and the accelerometer signal was analyzed using a Larson Davis_2900 dual channel FFT analyzer. A steel impact hammer similar to a Davidson GK291C hammer was used to excite the tank. Tests were carried out using both water and diesel in the tank.

The frequency of vibration of the 3 mm and the 6 mm base plates from the two tanks were measured at water level of 0, 3·125, 6·25, 12·5, 18·75, 25, 37·5, 50, 75 and 100% of tank capacity. A smaller subset of measurements was taken for the tank with diesel primarily due to safety issues. For all experiments the fluid had coupled over the whole plate and so the mode shape was invariant with fluid depth. The frequencies were determined by analyzing the bode magnitude and phase plots on the dual channel FFT analyzer. As expected the output signal from the accelerometer was slightly polluted with the response of adjacent plates but was sufficiently discernible to obtain accurate readings. Figure 3 shows a typical bode magnitude plot of the frequency response. As can be seen there are several modes evident in the response. The fundamental frequency is the first sharp peek in the bode magnitude plot that corresponds with a simultaneous step change in the phase angle. Other modes in the response correspond with higher order modes.

Figure 4 shows the results of water and diesel tests for both the 3 and 6 mm thick base plate. The frequencies reported here were estimated to have a $\pm 4\%$ error bar. As can be clearly seen in Figure 4, the fundamental frequency of vibration of the base initially decreases rapidly with increasing fluid depth, plateaus followed by a slight rise when the tank is full. For both base plates, this final increase is only 2·5% higher than the lowest value and as such is well within the uncertainty envelope of the measurements. Furthermore, these results are qualitatively similar to the fundamental frequency predictions reported in Chiba [6] for the base of a cylindrical tank with similar fluid/base plate density ratio and base thickness ratio. The minimum frequency of vibration occurred at about 25% and 40% for the 3 and 6 mm base respectively. The change in the frequency of vibration of the base was also a function of the thickness of the base plate and to a much lesser extent, the density of the fluid in the tank. The 3 mm base changed from 62 Hz for an empty tank to 30 Hz with the tank full of water and the 6 mm base changed from 129 to 48 Hz. This represents a 52 and 63% reduction from the fundamental frequency, respectively, for the 3 and 6 mm base.

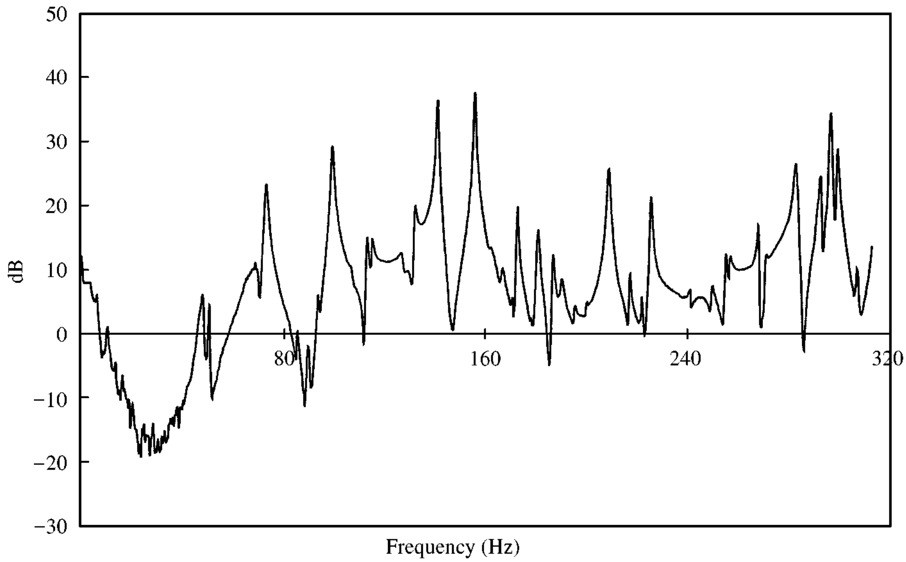

Figure 3. Typical bode magnitude plot from the response of the tank's base. Tank —— water test on 6 mm base (6.25%).

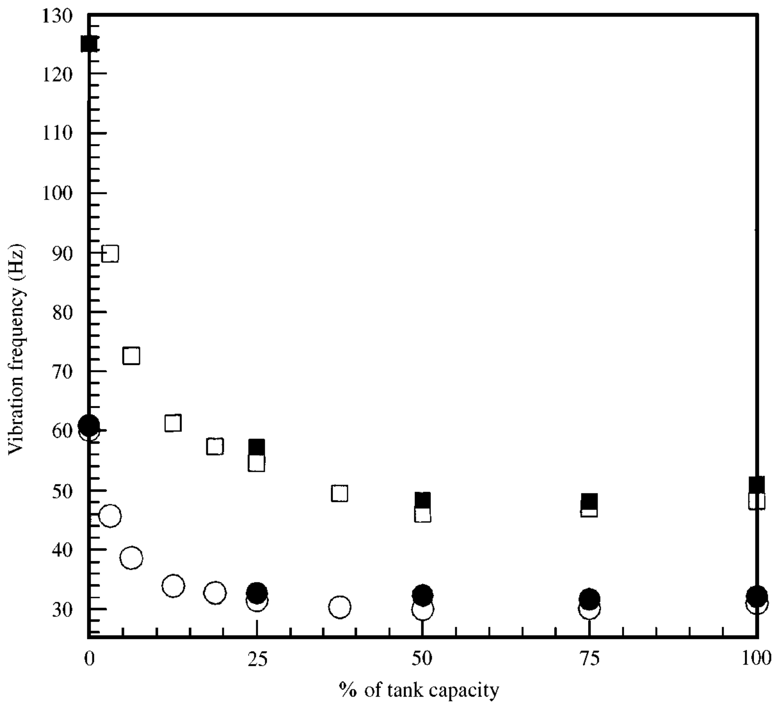

Figure 4. Fundamental vibration frequency of the base of the tank for both water and diesel: □, 6 mm plate—water; ○, 3 mm plate—water; ■, 6 mm plate—diesel; ●, 3 mm plate—diesel.

4. FINITE ELEMENT MODELLING

4.1. PROPERTIES OF STRUCTURAL ELEMENTS TO EMULATE WATER

The finite element modelling described in reference [3] was reproduced to "calibrate" the finite element work described here. This modelling was undertaken using STRAND7 finite element software. A half-model of the shell was created using 128 four-node thin-shell elements with fixed boundary on the base of the tank. The results of the analysis (Table 1) show similar if not slightly better correlation with the experimental results than the predictions of reference [3]. The percentage errors shown in the tables are $\varepsilon = 100\,(f_p - f_m)/f_m$. Our prediction of the $m = 1, n = 2$ mode was of similar error to that reported in reference [3]. As explained in reference [3], the lower order modes of vibration are most sensitive to the real quality of the boundary conditions that are not likely to be fully simulated in the FEA model.

The fluid in the half-model was represented using 128 eight-node hexahedral elements and 128 five-node pentahedral elements with its top surface unconstrained. The fluid–plate interface was modelled as continuous, that is fluid elements were directly connected to the plate elements. These boundary conditions were identical to those in reference [3]. The fluid properties were not defined in reference [3] so the properties of the isotropic solid elements from reference [4] were used; element density, $\rho = 1000\ \mathrm{kg/m^3}$, modulus of elasticity, $E = 666 \times 10^6\ \mathrm{N/m^2}$ and Poisson ratio, $\nu = 0{\cdot}45$. The results of this analysis are detailed in Table 2 and show extremely poor correlation between predictions and experimental results. Also the order of the modes changed, and coupled with the poor predictions suggests that the fluid properties used were inappropriate.

An iterative procedure was undertaken to determine the most appropriate property to model the fluid. Anisotropic properties for the solid elements were considered along with isotropic properties as the former could model the very low shear modulus of a fluid. For all cases, ν and ρ were fixed at 0·45 and $1000\ \mathrm{kg/m^3}$ respectively. It was found that the closest correlation between the FEA predictions and the experimental results of reference [3] over a range of modes was for $E = 2{\cdot}2 \times 10^6\ \mathrm{N/m^2}$; Figure 5 shows the FEA predictions for a range of element properties. Table 3 shows the FEA predictions using $E = 2{\cdot}2$ MPa and the theoretical results for all the modes and the respective frequencies reported in reference [3]. The agreement between experiment and predicted frequencies is excellent with lower

TABLE 1

STRAND7 FEA predictions compared with the results from reference [3] *for the empty cylinder*

Mode number		Experiment	[3] FEA predictions		STRAND7 FEA	
m	n	Frequency (Hz), f_m	Frequency (Hz), f_p	Error (%)	Frequency (Hz), f_p	Error (%)
1	3	616	634	2·9	633	2·8
1	2	708	815	15·1	811	14·5
1	4	945	948	0·3	948	0·3
1	5	1479	1481	0·1	1479	0·0
2	4	1628	1650	1·4	1635	0·4
1	1	—	1827	N/A	1820	N/A
2	3	1851	1842	0·5	1825	1·4
2	5	1969	2030	3·1	2010	2·1
1	6	2151	2158	0·3	2146	0·2

TABLE 2

FEA predictions using the properties from reference [4] *for the experiment of reference* [3] *for a full cylinder*

Mode number		Experiment	Strand FEA	
m	n	Frequency (Hz), f_m	Frequency (Hz), f_p	Error (%)
1	3	388	3239	735
1	2	421	2091	397
1	4	628	Not calculated	N/A
1	5	1027	Not calculated	N/A
2	4	1094	Not calculated	N/A
1	1	—	924	N/A
2	3	1245	3462	178
2	5	1299	Not calculated	N/A
1	6	1546	Not calculated	N/A

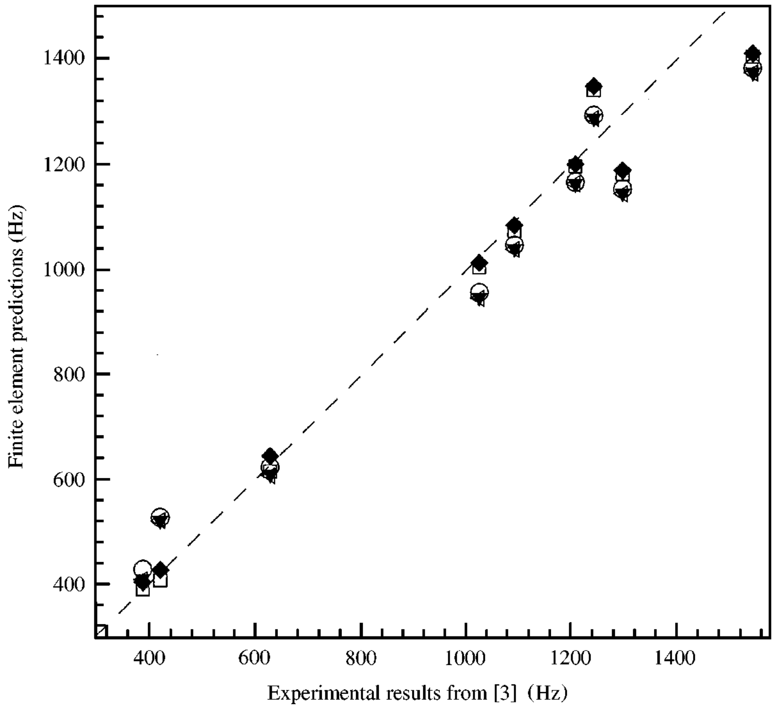

Figure 5. Correlation between the experimental results of reference [3] and the FEA predictions from STRAND7. Filled symbols are closest to the experimental results: □, isotropic, $E = 2$ MPa; ♦, isotropic $E = 2{\cdot}2$ MPa; ▼, anisotropic, $G = 0{\cdot}1$ MPa; ○, anisotropic, $G = 0{\cdot}5$ MPa; ◁, anisotropic, $G = 0{\cdot}05$ MPa; — — —, 1 : 1 correlation.

frequency modes being equal to or better than those reported in reference [3]. Interestingly, our predictions of the $m = 1$, $n = 2$ mode for the full tank were close to the experimental results unlike those reported in reference [3]. As shown in Table 3, the error increased for the higher frequency modes suggesting a limitation of the use of general solid elements to model fluid. For most engineering applications, however, the lower frequencies of vibration are the most important for design considerations.

TABLE 3

Predictions for a full cylinder from reference [3] using $E = 2{\cdot}2 \times 10^6$ N/m^2 for the solid elements

Mode number		Experiment	Strand7 FEA	
m	n	Frequency (Hz), f_m	Frequency (Hz), f_p	Error (%)
1	3	388	403	3·9
1	2	421	427	1·4
1	4	628	645	2·7
1	5	1027	1016	1·1
2	4	1094	1087	0·6
1	1	—	1203	N/A
2	3	1245	1351	8·5
2	5	1299	1191	8·3
1	6	1546	1414	8·5

4.2. FEA OF THE MODAL ANALYSIS EXPERIMENT

The finite element modelling for the test tanks were conducted using MSC Patran and Nastran. These packages were used in preference to STRAND7 as they were installed on a faster UNIX-based computer and also offered some useful post-processing functionality. Linear isotropic quadrilateral plate elements and triangular plate elements were used to model the test tanks. These elements were assigned the following properties: $E = 205 \times 10^9$ N/m^2, $\nu = 0{\cdot}29$ and $\rho = 7850$ kg/m^3. The fluid elements were modelled using isotropic hexahedrals and pentahedrals solid elements using the value of E derived in section 4.1 and $\nu = 0{\cdot}45$. The density of the elements was set to 1000 and 856 kg/m^3 for water and diesel respectively.

Searching for structural modes of vibration in this model was far from a trivial task. Modelling the fluid within the tanks introduced $3p$ extra modes of vibration, where p is the number of nodes connected to the solid elements only. These additional vibration modes of the fluid are irrelevant for this analysis with only the structural modes of the tank of interest. The easiest way found to determine whether the mode was structural or fluid was by viewing the results using a normalized displacement scale. When a structural vibration mode occurs, the fluid elements attached to the tank will undergo displacement with the tank as shown in Figure 6. If a fluid mode occurs, however, the tank will be displaced a minimal amount with a corresponding high displacement for the fluid as shown in Figure 7.

Figure 8 shows the predicted and measured fundamental frequency of vibration for the 3 and 6 mm thick base for both water and diesel. The agreement between predictions and measurements is extremely good for the 3 mm thick plate and overall quite good for the 6 mm plate. These results confirm the appropriateness of the properties for the solid elements.

5. ADDED MASS THEORIES

It is appropriate here to calculate A_p using equation (1) for the base of the tanks. The main assumption with the added mass proposal of reference [1] was that sufficient fluid was present to reduce the vibration frequency of the plate to its minimum value. That is, if this

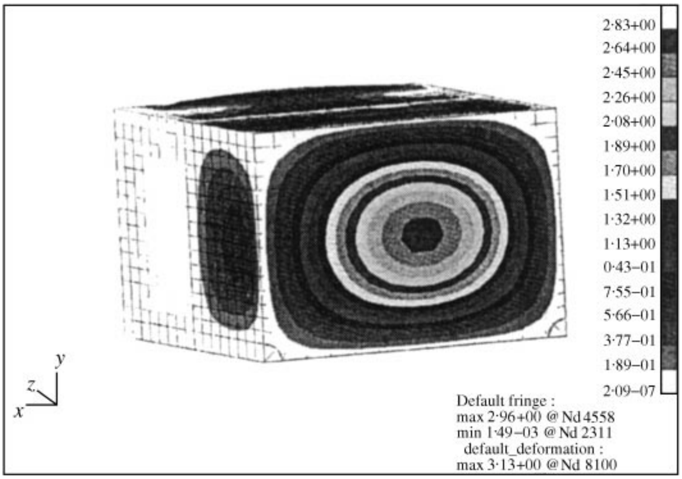

Figure 6. Fluid element normalized displacements for a structural mode.

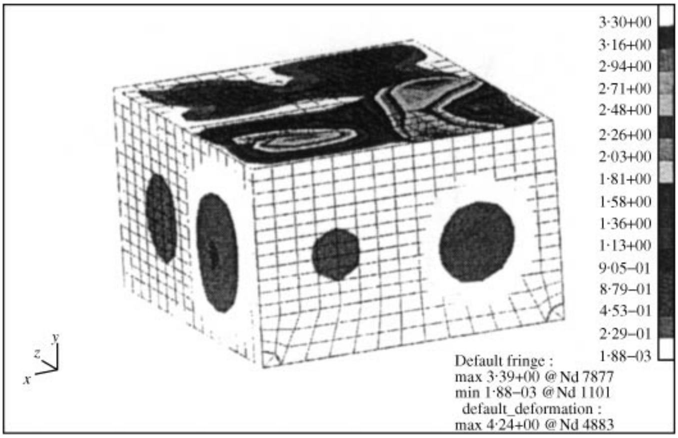

Figure 7. Normalized displacements for a fluid mode—only fluid elements are shown.

formulation were applicable to a vessel containing a finite volume of fluid, then the asymptote frequency for both the 3 and 6 mm base plate would be predicted.

The base was fully welded to the sides of the tank and as such was deemed to be better represented by the C–C–C–C assumption, completely clamped on all bounding edges, than the S–S–S–S assumption, simply supported on all bounding edges, in reference [2]. The form of $X_i(x)$ and $Y_j(y)$, the one-dimensional beam modes of equation (4) are given by [2]

$$X_i(x) = \cosh\left(\frac{\lambda_i x}{L}\right) - \cos\left(\frac{\lambda_i x}{L}\right) - \sigma_i\left[\sinh\left(\frac{\lambda_i x}{L}\right) - \sin\left(\frac{\lambda_i x}{L}\right)\right],$$

$$Y_j(y) = \cosh\left(\frac{\lambda_j y}{L}\right) - \cos\left(\frac{\lambda_j y}{L}\right) - \sigma_j\left[\sinh\left(\frac{\lambda_j y}{L}\right) - \sin\left(\frac{\lambda_j y}{L}\right)\right].$$

For the fundamental mode in both the X and Y directions, $\lambda_1 = \lambda_i = \lambda_j = 4{\cdot}73$ and $\sigma_1 = \sigma_i = \sigma_j = 0{\cdot}983$ [2]. The integrals in the numerator and denominator of equation (3)

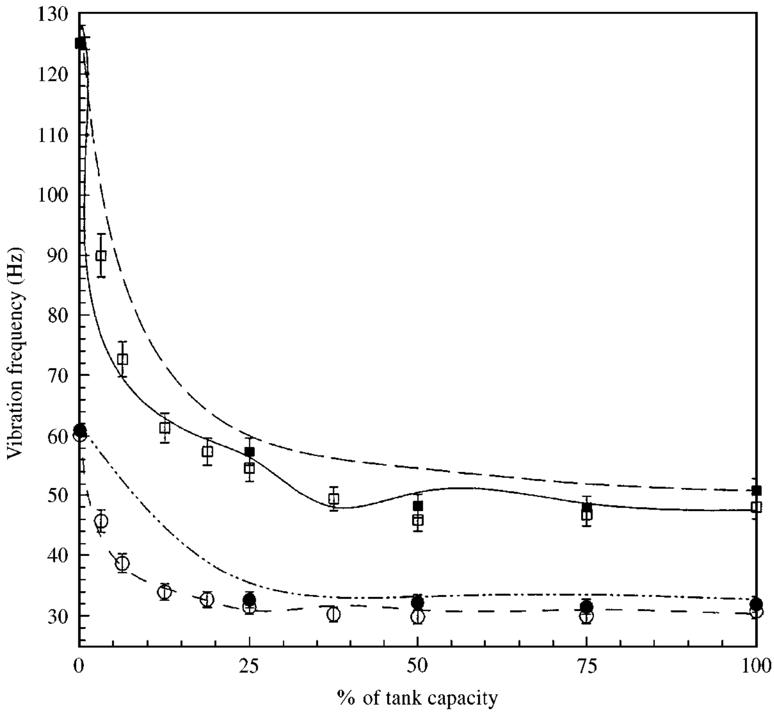

Figure 8. Measured and predicted fundamental frequency of vibration for the 3 and 6 mm base plate for both water and diesel. Error bars are shown on the measurements: □, water results 6 mm—experimental; ——, water results 6 mm—FEA; ○, water results 3 mm—experimental; – – -, water results 3 mm—FEA; ■, diesel results—6 mm experimental; — — -, diesel results—6 mm FEA; ●, diesel results—3 mm experimental; –·· – ··–, diesel results—3 mm FEA.

were evaluated with the aid of Mathcad and were found to be

$$\int_A z_{ij}\,\mathrm{d}A = \int_A X_i(x)\,Y_j(y)\,\mathrm{d}A = 0{\cdot}345 \quad \text{and} \quad \int_A z_{ij}^2\,\mathrm{d}A = \int_A X_i^2(x)\,Y_j^2(y)\,\mathrm{d}A = 0{\cdot}5.$$

The value of α_{11} is therefore given by $\alpha_{11} = (0{\cdot}345)^2/2 \times 1 \times 0{\cdot}5 \times 0{\cdot}5 = 0{\cdot}23805$. Unfortunately, the value of α_{11} listed in reference [2] was 0·3452, which appears to be incorrectly transferred from reference [1]. To determine A_p for the water tests, $\beta = 0{\cdot}72$ from reference [2] and as the base plate is $1\,\mathrm{m} \times 0{\cdot}5\,\mathrm{m}$, then A_p is given by $A_p = 0{\cdot}238 \times 0{\cdot}72 \times 1000 \times 0{\cdot}5 \times 1{\cdot}0^2 = 85{\cdot}7\,\mathrm{kg}$. The frequency ratio can then be simply found using equation (1).

Table 4 shows the frequency ratio predictions using Greenspon's [1] proposal and those of FEA predictions and modal analysis measurements for both the 3 and 6 mm thick base plates. There is good agreement between the FEA predictions and measurements further confirming the accuracy of the modelling. The added mass proposal of reference [1], however, not only shows poor correlation with measurements but also exhibited an incorrect trend; the predicted frequency increases as a function of the plate thickness, whereas experiments and finite element predictions suggest it should decrease. The direct application of Greenspon's [1] added mass principle appears inappropriate to the base of a rectangular vessel of finite volume.

TABLE 4

Frequency ratio calculated from equation (1) *and from FEA and modal analysis results*

Base plate	Added mass theory	FEA	Modal analysis	Added mass error (%)	FEA error (%)
3 mm	0·465	0·492	0·506	8·1	2·7
6 mm	0·599	0·369	0·384	– 56·0	4·9

6. MODIFIED ADDED MASS PROPOSAL

Two important observations can be drawn from the experimental results presented in this paper: the thickness of the base plate influences the amount of fluid that couples and the general shape of the frequency—liquid depth curve is invariant with plate thickness. Rearranging equation (1) with A_p as the subject yields

$$A_P = M_P\left[\left(\frac{f_{air}}{f_{fluid}}\right)^2 - 1\right]. \tag{5}$$

Figure 9 shows the added mass determined from experimental results as a function of fluid depth for both tank with water and diesel. It appears that the added mass is insensitive to fluid density, although the density of water and diesel are reasonably close, so this may not be true in general.

The general form of the added mass proposal of reference [1], equation (2), appears appropriate for rectangular plates. We propose a modification to this equation by introducing a variable, B, which accounts for the plate thickness and the depth of the fluid in the tank. That is

$$A_P = B\alpha_{ij}\beta\rho ab^2. \tag{6}$$

The measurements suggest the following form for B:

$$B = 0{\cdot}022\,\frac{t^2}{ab}(1 - \mathrm{e}^{-0{\cdot}045d/t}), \tag{7}$$

where d is the depth of the fluid in the tank.

The final form of the added mass equation from the experimental data presented here is

$$A_P = 0{\cdot}022\,\frac{t^2}{ab}\,\alpha_{ij}\beta\rho ab^2(1 - \mathrm{e}^{-0{\cdot}045d/t}). \tag{8}$$

This formulation is only valid for predicting the fundamental frequency on the base of a fully welded rectangular tank. Furthermore, it assumes that the fundamental frequency converges to an asymptote with increasing fluid depth. This assumption is consistent with the measurements, after accounting for experimental error, and with the predictions of Chiba [6]. Predictions from equation (8) are plotted in Figure 9 and show in general close agreement for most depths to the measurements for both the 3 and 6 mm base. The discrepancy between equation (8) and the measurements for 3 and 6 mm base are similar to those between the finite element prediction and measurements. This equation appears to adequately predict the exponential effect that the depth of fluid and the thickness of the plate have on the added mass over a uniformly mass-loaded plate. This equation, however,

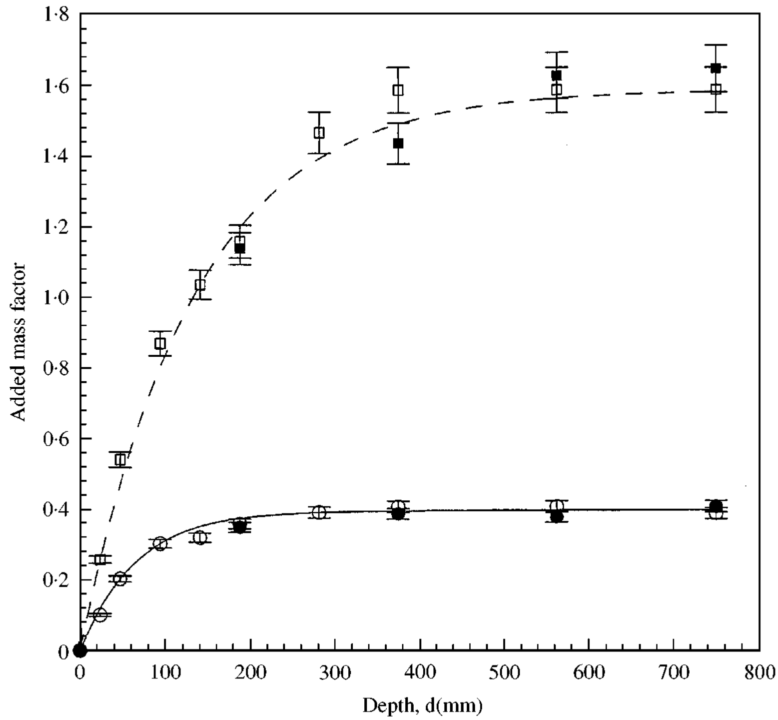

Figure 9. Added Mass factor for 3 mm and 6 mm thick base plates. Error bars are shown on the measurements: □, 6 mm plate—water; — — -, proposed equation; ■, 6 mm plate—diesel; ○, 3 mm plate—water; ——, proposed equation; ●, 3 mm plate—diesel.

has limitations, as it is only applicable to rectangular plates without any cutout holes or stiffeners. Further work is needed to extend this principle to plates with more complex structural geometry and to shapes other than rectangular.

7. CONCLUSIONS

The work described in this paper should assist in reducing the cost of undertaking a dynamic finite element analysis on vessels containing fluid. Fluid in a vessel is typically modelled using specialized solid fluid elements, which are generally restricted to advanced and expensive packages. Solid structural elements, which form part of the element library of most finite element packages, are generally not used for this purpose, as the freedom conditions are incorrect. Further there appears to be no published data suggesting the most appropriate property-set to use for these solid elements. We found that the first few modes of the dynamic response of a vessel filled with either water or diesel could be accurately modelled using solid finite elements with the following isotropic property: $E = 2{\cdot}2 \times 10^6\ \mathrm{N/m^2}$, Poisson ratio of 0·45 and density of $1000\ \mathrm{kg/m^3}$.

By far the most time efficient way to model fluid within a vessel for a dynamic analysis is to adjust the mass of the walls and base of the vessel to account for the presence of the fluid. This task is far from trivial, as the vibration mode shapes for the walls of a partially filled vessel are a function of the level of fluid within the tank. A formulation has been developed here to determine the amount of mass to be added to the base of a partially filled rectangular tank. Measurements suggest that the fundamental vibration frequency is a function of the level of fluid in the tank, the thickness of the base and to a lesser extent, the density of the fluid within the tank. A formulation has been devised based on the added mass principle of Greenspon [1] to determine the amount of mass to be added to the base plates in a finite element model of the tank containing fluid. This equation is currently limited to only rectangular base plates without cutout holes or stiffeners. Further work is needed to extend this principle to base plates of complex structural geometry and to shapes other than rectangular.

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