Journal of Sound and Vibration (2001) **246**(4), 737–740
doi:10.1006/jsvi.2001.3601, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF A THIN ELLIPTICAL PLATE WITH A CONCENTRIC, CIRCULAR FREE EDGE HOLE

P. A. A. LAURA, R. H. GUTIÉRREZ AND E. ROMANELLI

Departamento de Ingeniería, Universidad Nacional del Sur and CONICET 8000-Bahía Blanca, Argentina

(*Received* 8 *January* 2001)

1. INTRODUCTION

A survey of the literature reveals that the title problem has not been treated in the open literature [1]. Clamped and simply supported outer edges are considered in the present study which makes use of an approach previously developed by the senior author and coworkers [2, 3].

In the case of a clamped edge the boundary conditions at the outer edge are identically satisfied while for the situation where it is simply supported only the essential outer edge condition is considered. The natural boundary conditions at the hole edge are not taken into account. This is permissible since use is made of the classical Rayleigh–Ritz method using the polynomial co-ordinate functions applicable in the case of solid plates.

In the case of simply connected clamped and simply supported elliptical and circular plates the fundamental eigenvalues determined using the present approach are in excellent agreement with the results available in the literature [1, 2].

2. APPROXIMATE SOLUTION

In the case of normal modes of vibration the governing functional is given by

$$J(W) = D\iint_{\bar{P}} [(W_{\bar{x}^2} + W_{\bar{y}^2})^2 - 2(1-\nu)(W_{\bar{x}^2} W_{\bar{y}^2} - W^2_{\overline{xy}})]\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - \rho h\omega^2 \iint_{\bar{P}} W^2\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y}. \quad (1)$$

Introducing the dimensionless variables

$$\bar{x} = ax, \qquad \bar{y} = by \quad (2)$$

and the dimensionless parameters (see Figure 1)

$$\lambda = \frac{a}{b} \quad (a = 2a_1,\ b = 2b_1), \qquad \Omega^2 = \frac{\rho h a_1^4}{D}\,\omega^2,$$

0022-460X/01/390737 + 04 $35.00/0

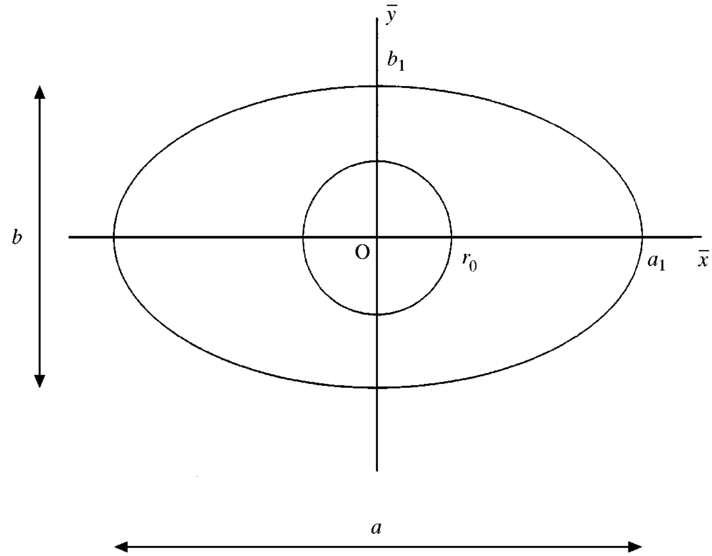

Figure 1. Vibrating structural element under study.

one obtains, by substituting into equation (1),

$$\frac{\lambda a^2}{D} J(W) = \iint_P [(W_{x^2} + \lambda^2 W_{y^2})^2 - 2(1-\nu)\lambda^2(W_{x^2} W_{y^2} - W_{xy}^2)]\,\mathrm{d}x\,\mathrm{d}y - 16\Omega^2 \iint_P W^2\,\mathrm{d}x\,\mathrm{d}y. \tag{3}$$

Using the approximation

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(x, y) \tag{4}$$

and applying the Rayleigh–Ritz method one obtains

$$\frac{\lambda a^2}{2D}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{N} \left\{ \iint_P [(\varphi_{jx^2} + \lambda_{iy^2}^2)(\varphi_{ix^2} + \lambda^2 \varphi_{iy^2}) - (1-\nu)\lambda^2(\varphi_{jy^2}\varphi_{ix^2} + \varphi_{jx^2}\varphi_{iy^2} - 2\varphi_{jxy}\varphi_{ixy})]\,\mathrm{d}x\,\mathrm{d}y - 16\Omega^2 \iint_P \varphi_j \varphi_i\,\mathrm{d}x\,\mathrm{d}y \right\} C_j = 0. \tag{5}$$

In the case of a clamped plate one takes

$$\varphi_1(x, y) = (x^2 + y^2 - 1/4)^2 \tag{6}$$

while for the simply supported plate it is convenient to use

$$\varphi_1(x, y) = x^2 + y^2 - 1/4. \tag{7}$$

TABLE 1

Fundamental frequency coefficient of a clamped elliptical plate with a free edge circular hole ($\lambda = a_1/b_1$; $\rho = r_0/a_1$)

λ/ρ	0·1	0·2	0·3	0·4
1·1	11·497	12·016	13·243	15·836
1·2	12·855	13·631	15·135	18·799
1·3	14·273	15·149	17·178	21·315
1·4	15·833	16·902	19·165	24·976

TABLE 2

Fundamental frequency coefficient of a simply supported elliptical plate with a free edge circular hole ($\lambda = a_1/b_1$; $\rho = r_0/a_1$)

λ/ρ	0·1	0·2	0·3	0·4
1·1	5·498	5·584	5·746	6·003
1·2	6·121	6·242	6·428	6·778
1·3	6·800	6·936	7·181	7·574
1·4	7·547	7·711	7·981	8·550

Accordingly, the remaining $\varphi_i(x, y)$ are

$$\varphi_2(x, y) = \varphi_1(x, y)x^2, \qquad \varphi_3(x, y) = \varphi_1(x, y)y^2, \qquad \varphi_4(x, y) = \varphi_1(x, y)x^2y^2. \tag{8a–c}$$

3. NUMERICAL RESULTS

All the numerical calculations were performed for $N = 4$ and $\nu = 0{\cdot}30$. Table 1 depicts values of Ω_1 for a doubly connected plate rigidly clamped along the outer boundary while Table 2 deals with the simply supported case. The fundamental frequency coefficient is tabulated as a function of $\lambda = a/b = a_1/b_1$ and $\rho = r_0/a_1$. One immediately concludes that for all the cases considered, Ω_1 increases with the size of the free–edge hole. This constitutes a clear example of the dynamic stiffening effect [4]. For moderate sized holes, say $r_0/a_1 \leqslant 0{\cdot}2$, the value of Ω_1 corresponding to the clamped case is approximately twice the value corresponding to the simply supported situation. This is the case when dealing with solid, circular plates (10·22 and 4·94† respectively).

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur (Project Director: Professor R. E. Rossi).

† $\Omega_1 = 4{\cdot}935$ for $\nu = 0{\cdot}30$.

REFERENCES

1. A. W. Leissa 1969 *NASA SP* 160. Vibration of plates.
2. P. A. A. Laura, J. C. Paloto and R. D. Santos 1975 *Journal of Sound and Vibration* **41**, 177–180. A note on the vibration and stability of a circular plate elastically restrained against rotation.
3. E. Romanelli and P. A. A. Laura 1977 *Computers and Structures* **62**, 795–797. An approximate method for analyzing transverse vibrations of circular, annular plates of non-uniform thickness and a free inner boundary.
4. P. A. A. Laura, R. E. Rossi and M. J. Maurizi 1991 *The Shock and Vibration Digest* **23,** 3–7. Dynamic stiffening of structural elements.