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A TRANSIENT LOCAL ENERGY APPROACH AS AN ALTERNATIVE TO TRANSIENT SEA: WAVE AND TELEGRAPH EQUATIONS

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This paper is concerned with the high-frequency dynamics of structures in the time domain. In this frequency range, a differential equation governing the vibrational energy density is proposed. This simplified equation leads to the prediction of large scale space and time evolution of the energy quantities averaged over time, frequency and space. For one-dimensional undamped structures, the energy equation is the classical wave equation whereas its generalization for damped structures leads to a telegraph-type equation. The discussion is focussed on the comparison with the diffusion equation sometimes encountered in the literature.

1. INTRODUCTION

Medium- and high-frequency dynamics is the subject of a fantastic devotion and the great number of works and publications is evidence of the interest of the scientific community in such a question. As a matter of fact, there is still a real need to have predictive tools well suited to medium and high frequencies and taking into account the numerous features of this domain. Those peculiarities make the use of the classical methods (finite element, boundary element, etc.), not always possible or of less suitability in the non-modal range.

Among possible alternatives, one can mention the well-known *Statistical Energy Analysis* [1] (SEA). SEA gives the vibrational energy of each subsystem of complex structures. Nevertheless, SEA requires some improvements. In this matter, the reader can consult references [2–4] for a review of SEA. An interesting survey and a number of critical comments are given. Beyond these studies, a number of works attempt to enhance the SEA robustness and predictivity. Among those tentatives, one can note the earlier work of Belov *et al.* [5] and the interesting investigations of Nefske and Sung [6], who proposed the use of the heat conduction analogy to obtain not only the total energy available in SEA, but also the space spread of energy density within the subsystems. This leads to an energy formulation of the dynamical equation of motion instead of the classical formulation based on the displacement field. Basically, this way of thinking can be viewed as a *local energy* formalism whereas the SEA formalism is based on global energies of finite subsystems. This model has been improved by Bernhard and his team [7–9] and also discussed in references [10, 11].

However, a number of questions and debates are addressed towards the validity of the heat conduction analogy, also called the vibrational conductivity approach. The reader can

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find more details in references [11–18]. *It should be noted that all of those contributions concern only the steady state dynamics.* In all these studies, it has been shown that the thermal analogy is a very particular case for the representation of the energy repartition in the steady state condition. In fact, the thermal analogy is restricted to very simple one-dimensional systems with one propagating wave. Further models have been proposed for complex one-dimensional systems [17, 18] and for multi-dimensional systems [11, 16, 19]. From "exact" consideration of energy flow, Xing and Price [20, 21] have treated the case of semi-infinite systems, and have remarked that for a semi-infinite vibrating rod in tension or in compression, the time-varying energy does not satisfy the heat conduction equation. But no equation concerning the finite rod is given. Finally, it should be noted that, a simplified displacement method called, the transfer function technique (TFT) was proposed by Colton *et al.* [22]. This method is based on a wave approach similar to the technique used in this paper for an energy differential equation proof. It is restricted to the analysis of a simple one-dimensional propagator. The differential energy equation studied in this paper differs from the TFT in the simplification hypothesis and tends to be a general tool for high-frequency modelling of one or multi-dimensional elastoacoustic phenomenon [11, 16, 17, 19].

The high-frequency dynamics in the time domain is not a frequent subject in the existing literature. Only a few papers deal with transient responses from an energetic point of view. An earlier work on the subject has been published by Manning [23] in the context of transient SEA. More recently, Lai and Soom [24, 25], and Pinnington and Lednik [26, 27] published additional transient SEA results. However, transient SEA requires more theoretical investigations in order to clarify several problems, among them the coupling loss factor definition versus time. Transient SEA seems to be relevant for the determination of structural reverberation time. In the particular context of energy flow methods, there exists an attempt by Nefske and Sung [6] to generalize the heat analogy in the time domain. As in transient SEA, the authors introduce the term $\partial W/\partial t$ where W is the total energy density to complete the heat conduction equation. Neither a rigorous demonstration nor a justification is given. However, it is to be acknowledged that the authors did not solve the heat conduction equation for transient problems. Only steady state results have been shown.

After this brief description of the state of the art, the main goal of this paper is to generalize and extend the discussion about the thermal analogy by considering both space and time variables. Actually, by using a wave approach very similar to the one in reference [17] in the steady state case, the time domain is tackled for both damped and undamped cases. It will be shown that in both cases, the thermal analogy fails in representing the energy spread for very simple systems. A *wave type* equation is obtained in the undamped case, whilst a *telegraph type* equation is derived for the energy in the time and space domain in the damped case. Comments and comparisons are finally given in order to point out the differences between this model and the vibrational conductivity approach. In particular, it is shown that the equivalence between vibrational energy flow and heat flow is physically a very restricted matter.

2. SOME DEFINITIONS AND ASSUMPTIONS

This section introduces the main definitions and concepts needed to derive the energy equations, to discuss its features and to appreciate its interest. In the subsequent presentation, the terms $f(s-ct)$ and $g(s+ct)$ will designate travelling waves in a one-dimensional medium, considered as an homogeneous waveguide. The space variable

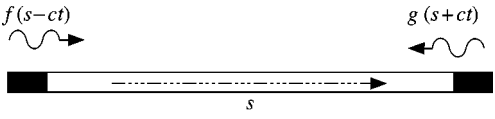

Figure 1. Wave designation for a simple waveguide.

along the system is sand the time is denoted by t. The first term is a wave propagating in the s-positive direction (right-travelling) whereas the second is propagating in the s-negative direction (left-travelling, see Figure 1). In the case where the disturbance is governed by the wave equation, d'Alembert's solution shows that, in general, two such waves propagate simultaneously in the system.

With the view of describing the energy transfer inside the medium, two continuous energy fields are introduced. The first energy quantity is the total energy density $W(s, t)$ defined as the sum of the potential energy density and the kinetic energy density. The second energy variable $I(s, t)$ is the energy flow defined as the energy per unit time flowing through a section of the waveguide. This quantity is attached to the propagating part of the wave. For pure evanescent waves it vanishes and these waves are not taken into account in the evaluation of the energy transfer process. These energy quantities are local, as opposed to the energies per subsystem involved in SEA.

Both propagating waves $f(s - ct)$ and $g(s + ct)$ involve partial energy quantities defined as the energy variables associated to those waves. Henceforth, $I^+(s, t)$, $I^-(s, t)$, $W^+(s, t)$ and $W^-(s, t)$ will represent, respectively, the energy flows and the energy densities attached to the right-travelling and the left-travelling waves.

In order to establish a relationship between the partial energies $W^\pm(s, t)$ and $I^\pm(s, t)$ and the total ones $W(s, t)$, $I(s, t)$, an additional assumption is introduced. This assumption postulates that interferences are not considered in the model. As a consequence, the superposition principle is valid for energies as is the case for the disturbance fields f and g in the linear situation:

$$W(s, t) = W^+(s, t) + W^-(s, t), \quad I(s, t) = I^+(s, t) + I^-(s, t). \tag{1}$$

In addition, it is well known, in the literature of wave dynamics, that a proportionality exists between the energy flow and the energy density associated with travelling waves [28, 29],

$$I^\pm(s, t) = \pm cW^\pm(s, t), \tag{2}$$

where c is the energy velocity of waves. The sign before the energy velocity on the right-hand side stems from the direction of propagation. This relationship must be considered as the definition of the energy velocity. It should be noted that the energy velocity and the group velocity match for a lossless medium [28]. However, for light damping (a condition that is met in nearly all practical structures) the energy velocity does not significantly differ from the group velocity. Equation (2) has been encountered in many papers concerning high-frequency literature. For instance, Nefske introduced this expression in his famous paper [6] giving the diffusion equation. Langley [30] used this expression in order to establish the *Wave Intensity Technique* equations. Many other verifications can be found in references [7, 17–19, 13].

The main step in deriving the energy equation is the local energy balance for a non-loaded region,

$$\frac{\partial I}{\partial s}(s, t) + \frac{\partial W}{\partial t}(s, t) + p_{diss}(s, t) = 0, \tag{3}$$

where p_{diss} is the power density being dissipated. For undamped systems, the local energy balance (3) becomes

$$\frac{\partial I}{\partial s}(s, t) + \frac{\partial W}{\partial t}(s, t) = 0. \tag{4}$$

The damping model adopted here is the same as in SEA: power density being dissipated is proportional to the energy density. Hence

$$p_{diss}(s, t) = \eta\omega W(s, t), \tag{5}$$

where η is the damping loss factor and ω the circular frequency. The validity of this relationship has been discussed in the literature [24, 26] about SEA. Additional comments and discussions concerning equation (5) are included in reference [1]. From the power balance (3) and the given dissipation model (5), an equation giving the time and space evolution of W can be written only if the energy flow I can be related to the energy density W with an appropriate intrinsic relationship. To seek such a relationship is the purpose of the next sections.

3. VIBRATIONAL CONDUCTIVITY APPROACH

The most natural of these intrinsic relationships stems from the remark that the problem of the repartition of vibrational energy along absorbing systems is similar to the heat conduction problem [5]. In reference [6], this analogy is invoked to postulate that Fourier's law should be valid. The energy flow is proportional to the gradient of the energy density. The involved coefficient is readily found on the basis of a wave analysis. It yields

$$I(s, t) = \frac{c^2}{\eta\omega}\frac{\partial W}{\partial x}(s, t). \tag{6}$$

Now, substituting relationship (6) into the power balance (3) gives

$$-\frac{c^2}{\eta\omega}\frac{\partial^2 W}{\partial s^2}(s, t) + \frac{\partial W}{\partial t}(s, t) + \eta\omega W(s, t) = 0. \tag{7}$$

This equation is termed *the heat conduction equation or the diffusion equation.* As a matter of fact, it is analogous to the heat conduction equation with a convective term $\eta\omega W$ [6]. This equation (7) predicts only the non-oscillating evolution of the energy density. The solution of this diffusion equation may be viewed as the frequency average or the local space average of the energy density deduced from classical governing equations [7]. Particular energy boundary and coupling conditions [31, 32] are finally introduced to solve the energy equation.

4. TRANSIENT ENERGETICS FOR UNDAMPED CASE

Consider an undamped waveguide with a right-travelling and a left-travelling wave. As the disturbance fields f and g attached to these waves are solutions of the equation of motion, the energy balance is satisfied separately for each wave,

$$\frac{\partial I^{\pm}}{\partial s}(s, t) + \frac{\partial W^{\pm}}{\partial t}(s, t) = 0, \tag{8}$$

where the dissipation term has been discarded. Introducing equations (2) into the power balance (4) leads to partial differential equations for only the partial energy densities:

$$\pm c \frac{\partial W^{\pm}}{\partial s}(s, t) - \frac{\partial W^{\pm}}{\partial t}(s, t) = 0. \tag{9}$$

This is exactly the energy equation obtained by Xing and Price [20, 21] for simple semi-infinite rods in tension or compression, derived from another point of view. In these papers, the authors start from the governing equations of small disturbances in a continuum elastic medium and derive equation (9) without any simplification. A further substitution of equation (2) into equation (9) gives

$$\frac{\partial I^{\pm}}{\partial t}(s, t) = - c^2 \frac{\partial W^{\pm}}{\partial s}(s, t). \tag{10}$$

By applying the linear superposition relationships (1), equation (10) yields

$$\frac{\partial I}{\partial t}(s, t) = - c^2 \frac{\partial W}{\partial s}(s, t). \tag{11}$$

At this stage, one can clearly confirm that the relationship relating the energy flux and the energy density is of a different kind when comparing it to the classical Fourier's law (6), even for very simple vibrating systems.

Finally, by introducing relationship (11) into the derivative respect to t of the energy balance equation (4), it becomes

$$- c^2 \frac{\partial^2 W}{\partial s^2}(s, t) + \frac{\partial^2 W}{\partial t^2}(s, t) = 0. \tag{12}$$

Equation (12) describes the energy spread within a one-dimensional system when interferences are not taken into account. This equation is a wave-type equation instead of the diffusion equation presented by Nefske [6].

5. TRANSIENT ENERGETICS FOR DAMPED CASE

Consider now the case of damped systems. The linear superposition (1) of energy previously used is still valid. The derivation of the energy equation comes, once again, from the local energy balance (3) for damped systems. For a particular travelling wave solution, the power balance (3) may still be applied, and so

$$\frac{\partial I^{\pm}}{\partial s}(s, t) + \frac{\partial W^{\pm}}{\partial t}(s, t) + p^{\pm}_{diss}(s, t) = 0. \tag{13}$$

The power density being dissipated (5) is obviously satisfied by the particular travelling wave solutions, so that

$$p^{\pm}_{diss}(s, t) = \eta \omega W^{\pm}(s, t). \tag{14}$$

Now, by substituting equations (14) and (2) into the power balance equations (13), one obtains:

$$c^2 \frac{\partial W^{\pm}}{\partial s}(s, t) + \frac{\partial I^{\pm}}{\partial t}(s, t) + \eta \omega \mathrm{I}^{\pm}(s, t) = 0. \tag{15}$$

By adding equation (15),

$$-c^2 \frac{\partial W}{\partial s}(s, t) = \frac{\partial I}{\partial t}(s, t) + \eta\omega I(s, t). \tag{16}$$

Once again, relationship (16) can be compared with Fourier's law (6). The former has a time-derivative term that is not present in the latter. In fact, relationship (16) generalizes Fourier's law (6) by adding the time-derivative term revealed by relationship (11). Deriving the local power balance equation (3) with respect to time leads to the expression

$$-c^2 \frac{\partial^2 W}{\partial s^2}(s, t) - \eta\omega \frac{\partial I}{\partial s}(s, t) + \frac{\partial^2 W}{\partial t^2}(s, t) + \eta\omega \frac{\partial W}{\partial t}(s, t) = 0, \tag{17}$$

where $\partial I/\partial t$ has been evaluated from equation (16). Finally, the energy equation is obtained from equation (17) by substituting the expression of $\partial I/\partial s$ from the energy balance (3):

$$-c^2 \frac{\partial^2 W}{\partial s^2}(s, t) + \frac{\partial^2 W}{\partial t^2}(s, t) + 2\eta\omega \frac{\partial W}{\partial t}(s, t) + (\eta\omega)^2 W(s, t) = 0. \tag{18}$$

Equation (18) is the energy equation representing the time and space evolution of the non-oscillating part of the energy density. This equation is a telegraph-type equation and it generalizes directly the wave equation (12) obtained for undamped systems.

6. BOUNDARY CONDITIONS

The simplest boundary conditions to be applied to a waveguide of length L are for non-dissipative extremities. Indeed, in this case

$$I(0, t) = 0, \quad I(L, t) = 0 \tag{19, 20}$$

should be valid at any time.

In the presence of absorbing boundaries, the absorption coefficient α is introduced. It is related to the reflection efficiency $R = 1 - \alpha$ defined as the ratio of reflected energy flow to incident energy flow. So, for the left boundary location at $s = 0$ (see Figure 2)

$$I^+(0, t) = -(1 - \alpha) I^-(0, t). \tag{21}$$

The net or total energy flow is then

$$I(0, t) = I^+ + I^- = \alpha I^-(0, t). \tag{22}$$

Now, consider the energy densities. Propagation condition (2) leads one to re-write equality (21) as

$$W^+(0, t) = (1 - \alpha) W^-(0, t) \tag{23}$$

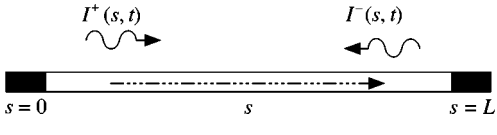

Figure 2. Waveguide boundaries.

and the following is obtained for the total energy density:

$$W(0,t) = W^+ + W^- = (2-\alpha)W^-(0,t). \tag{24}$$

By comparing equations (22) and (24), a local relationship involving only total energy quantities is derived;

$$I(0,t) = -c\frac{\alpha}{(2-\alpha)}W(0,t). \tag{25}$$

Applying a similar condition for the right boundary $s = L$ with a reverse sign, one obtains now two boundary conditions for absorbing extremities:

$$I(L,t) = c\frac{\alpha}{(2-\alpha)}W(L,t). \tag{26}$$

Indeed, the case $\alpha = 0$ matches conditions (19). More interesting is the case $\alpha = 1$ for which

$$I(0,t) = -cW(0,t), \quad I(L,t) = cW(L,t). \tag{27}$$

Boundary conditions (27) to be applied for totally absorbing ends are nothing more than propagation conditions (2).

7. COMMENTS

Many comments and remarks can be made owing to the formulations presented before. For the sake of clarity, one can first summarize the three equations for the energy obtained in previous sections:

$$-c^2\frac{\partial^2 W}{\partial s^2}(s,t) + \eta\omega\frac{\partial W}{\partial t}(s,t) + (\eta\omega)^2 W(s,t) = 0, \tag{28}$$

$$-c^2\frac{\partial^2 W}{\partial s^2}(s,t) + \frac{\partial^2 W}{\partial t^2}(s,t) = 0, \tag{29}$$

$$-c^2\frac{\partial^2 W}{\partial s^2}(s,t) + \frac{\partial^2 W}{\partial t^2}(s,t) + 2\eta\omega\frac{\partial W}{\partial t}(s,t) + (\eta\omega)^2 W(s,t) = 0. \tag{30}$$

First of all, it should be noted that under steady state conditions, no difference exists between the model of diffusion equation (28) and model (30) presented here. In this way, equation (30) proposed in this paper generalizes effectively the vibrational conductivity equation under steady state conditions widely studied in the literature [12, 8]. However, when both time and space domains are involved, many fundamental differences appear between equations (28) and (30).

From a purely mathematical point of view, it should be noted that energy equation (28) is of different nature than the others two (29, 30). In fact, energy equation (28) is a *parabolic* equation whilst equations (29, 30) are *hyperbolic* (if time t is considered as being the second co-ordinate). More precisely, equations (29, 30) admit two families of real characteristics. The d'Alembert solutions for wave equation (29) and telegraph equation (30) are of the form

$$W(s,t) = \mathrm{e}^{-\eta\omega t}F(s-ct) + \mathrm{e}^{-\eta\omega t}G(s+ct), \tag{31}$$

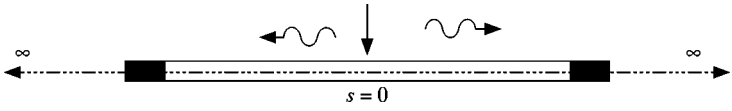

Figure 3. Waveguide under a unit energy source at $s = 0$ and $t = 0$.

where $F(s - ct)$ and $G(s + ct)$ are the right- and left-travelling energy densities $W^+(s - ct)$ and $W^-(s - ct)$. Finally, parabolic equation (28) has only one family of characteristics which leads to a drastically different solution behaviour. In particular, the oscillatory properties of their fundamental solutions are different. It should be noted that these oscillatory properties condition the numerical interest of energy-based models at high frequencies.

Among additional comments to be given, the energy derivative with respect to time is a fundamental matter. In particular, the first derivative of the energy density with respect to time, namely the term $\partial W/\partial t$ appears in both diffusion equation (28), and telegraph equation (30). This term traduces a non-conservation of energy and breaks the symmetry with respect to time. This means that a different kind of solution is obtained, if the sign of the time t is reversed. This non-conservation of energy is accompanied by a continuous increasing change in entropy as time goes on.

Equation (28) is a "perfect" diffusion equation with respect to both time and space. However, the diffusion equation predicts that the temperature will rise instantaneously everywhere, under the effect of an instantaneous heat source. Similarly, equation (28) means that the energy flow *propagates instantaneously* from a source to an observation point on the vibrating system. This energy flow is obviously physically impossible. On the other hand, the wave or the telegraph equation predicts a *finite velocity of propagation* of mechanical energy flow. Consequently, there are some lacks in the analogy between the vibrational energy flow and the thermal flow. This systematic analogy leads to misunderstanding of the physical phenomenon and to an energy equation of limited consistency.

In order to continue the discussion, Green functions corresponding to the system represented in Figure 3 for the different energy equations are summarized as follows:

$$G(s, t) = \frac{\sqrt{\eta\omega}}{2c\sqrt{\pi t}} \mathrm{e}^{(-\eta\omega/4c^2t)s^2} \mathrm{e}^{-\eta\omega t} \mathrm{H}(t) \tag{32}$$

for diffusion equation (28),

$$G(s, t) = \frac{1}{2c} \mathrm{H}(c^2t^2 - s^2)\mathrm{H}(t) \tag{33}$$

for wave equation (29),

$$G(s, t) = \frac{1}{2c} \mathrm{e}^{-\eta\omega t} \mathrm{H}(c^2t^2 - s^2)\mathrm{H}(t) \tag{34}$$

for telegraph equation (30) proposed in this paper. In these expressions, H denotes the Heaviside function.

First of all, it should be pointed out that the Green function (33) for the wave equation can be, as expected, obtained from the Green function (34) of the telegraph equation when the damping loss factor η goes to zero.

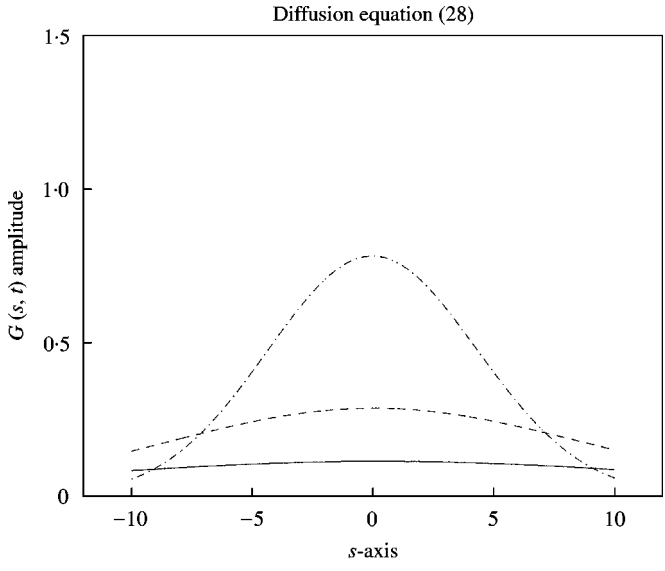

Figure 4. Green function versus s-axis for the diffusion equation at various time values; solid line $ct = 1$, – – – – line $ct = 4$ and – · – · – · – line $ct = 9$.

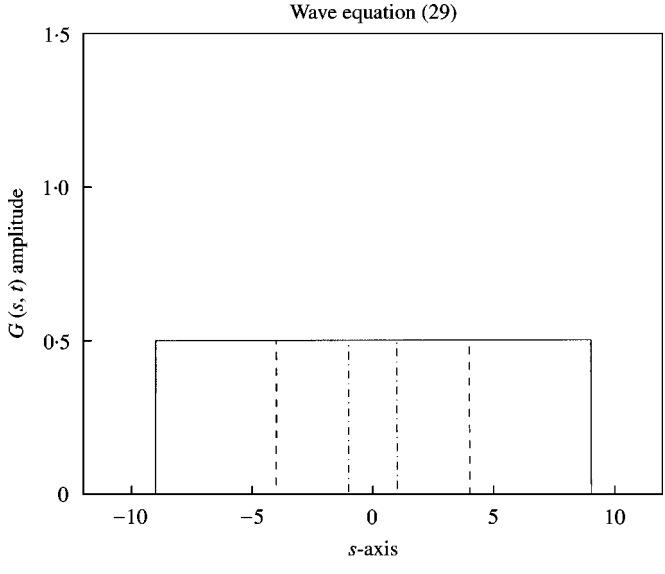

Figure 5. Green function versus s-axis for the wave equation at various time values; solid line $ct = 1$, – – – – line $ct = 4$ and – · – · – · – line $ct = 9$.

In Figures 4–6, Green the respective functions are plotted versus the s-axis for various time values. These figures confirm that the various Green functions have different properties and behaviour.

Ultimately, the last comment to be mentioned, in the analysis of these energy equations, concerns the manner in which diffusion equation (28) may be a possible approximation of telegraph equation (30). As stated in reference [33] for the heat flow analysis, *after*

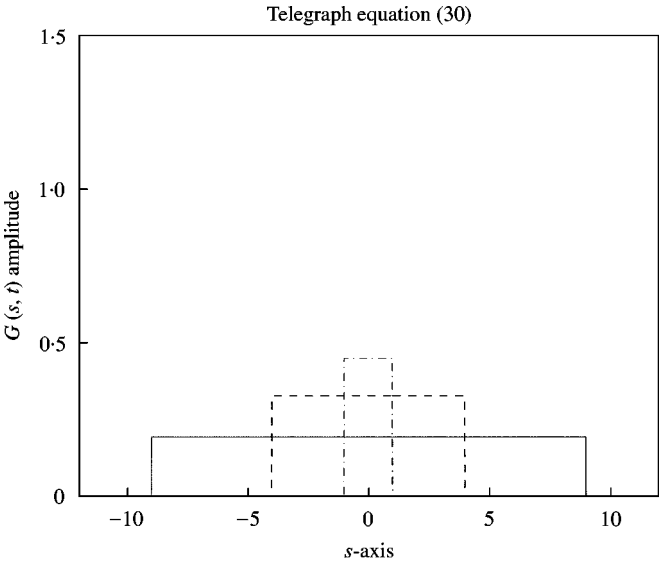

Figure 6. Green function versus s-axis for the telegraph equation at various time values; solid line $ct = 1$, – – – – line $ct = 4$ and – · – · – · – line $ct = 9$.

a sufficiently long time, instantaneous propagation of energy can be acceptable, and hence also the diffusion equation. In reference [33], a correction of the diffusion equation is proposed. As instantaneous propagation is impossible, an additional term $1/c^2(\partial^2 \cdot / \partial t^2)$ is introduced. A similar procedure cannot be introduced immediately when dealing with energy behaviour. In fact, the correction of the diffusion equation must give asymptotically the diffusion equation for large values of the energy velocity ($c \to \infty$) or equivalently, when ct is much greater than $|s|$ ($ct \gg |s|$). This approximation cannot be done here as the equivalent diffusion coefficient is energy velocity dependent, so that the behaviour when $c \to \infty$ is different from that of the diffusion equation. Hence, no obvious approximation can be given for the diffusion equation, confirming once again the difference between heat flow and mechanical energy flow.

Thus, telegraph equation (30) that has been derived in this paper from a propagative approach, is not a simple correction of diffusion equation (28) but is something completely different.

8. CONCLUSION

In this paper, some energy equations in the space-time domain have been studied within the context of SEA alternatives well suited for medium- and high-frequency dynamics. Actually, a propagative approach dealing with wave contents of studied systems is used, leading to transient energy-based equations. These equations remove the oscillating parts of quadratic fields and consequently predict only the incoherent quadratic levels. It is well known that such an approximation is, in return, significantly advantageous for the numerical cost [8, 9].

The transient local energy equations given here point out that Fourier's law (6) fails for the time-varying vibrational energy. It should be replaced by relationship (16) in which a time-derivative term has been included. The analogy between the vibrational energy flow and the thermal flow in the conduction problem, is no longer valid. Transient equation (30)

derived in this paper shows that the energy flow is governed mainly by a propagation process rather than a diffusion one. So that, a telegraph-type equation which reduces to a wave equation as a particular case, seems to be more adequate for energy transfer prediction.

It should be pointed out that the propagative approach proposed in this short paper can readily be generalized to multi-propagative modes in one-dimensional waveguides [17] or to multi-dimensional systems [11, 19].

Finally, in the context of some specific applications in technical engineering, such as impact noise, shock response problems and so on, the transient local energy approach proposed in this paper, could turn out to be an additional tool for transient SEA. So, in addition to the simulation of decay rate and energy envelopes [25] provided by transient SEA, the transient local energy equation permits the prediction of the repartition of vibrational energy inside subsystems.

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