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HYBRID WAVE/MODE ACTIVE VIBRATION CONTROL

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A hybrid approach to active vibration control is described in this paper. It combines elements of both wave and mode approaches to active control and is an attempt to improve on the performance of these approaches individually. In the proposed hybrid approach, wave control is first applied at one or more points in the structure. It is designed on the basis of the local behaviour of the structure and is intended to absorb vibrational energy, especially at higher frequencies. Then modal control is applied, being designed on the basis of the modified global equations of motion of the structure-plus-wave controller. These are now normally non-self-adjoint. Since the higher order modes are relatively well damped, hybrid control improves the model accuracy and the robustness of the system and gives better broadband vibration attenuation performance. Hybrid wave/mode active vibration control is described with specific reference to the control of a cantilever beam. The particular case considered is that of collocated, point force/sensor feedback wave control combined with modal control designed using pole placement. Numerical and experimental results are presented.

1. INTRODUCTION

There has been increasing interest in active vibration control in recent years. In part, this is due to demands for mechanical structures to be lighter and faster, and hence more prone to vibration, while passive methods, such as adding damping, etc., are often inappropriate. In active vibration control, desirable performance characteristics are achieved through the application of control forces to a structure. Usually, the structural response is measured and used to determine the appropriate control forces.

Vibrations can be described in a number of ways, with the most common descriptions being in terms of modes and in terms of wave motion. Active vibration control can be

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designed in terms of either modal or wave behaviour, with each method having advantages and disadvantages. In broad terms, modal control aims to control the global behaviour (i.e., the modes of vibration) of the structure while wave control aims to control the flow of vibrational energy through the structure. In this paper, a hybrid approach consisting of complementary wave- and mode-based control is described, which attempts to exploit the advantages of both methods.

In the modal approach, the response is described in terms of the undamped modes of vibration of the structure. A finite number of these modes are retained and the equation of motion written in matrix form. The equations of motion are then cast in state-space form, and the control is applied so as to modify the eigenstructure of the state-space equations in some way. In modal active vibration control, the aim is to control the characteristics of the modes of vibration [1–10], i.e., their damping factors, natural frequencies or mode shapes. There are a number of modal control design methods, two of the most widely used approaches being pole placement and optimal control [11]. Since the modal properties depend on the global properties of the structure, i.e., the material and geometric properties and the boundary conditions of the whole structure, this is a global approach. From the control perspective, modal control has been termed "high authority" [12], since it is designed on the basis of the global behavior of the whole structure.

In a continuous structure, vibrations can alternatively be regarded as being the superposition of waves travelling through the structure. These waves are reflected and transmitted at structural discontinuities. Active wave control aims to control the distribution of energy in the structure by either reducing the transmission of waves from one part of the structure to another (i.e., isolating one part of the structure from another) or absorbing the energy carried by the waves (i.e., adding damping). This is particularly useful for one-dimensional components in a structure, when a finite number of waves with given directions of propagation exist. Wave-based active vibration control is designed purely on the basis of the differential equation of motion and the local properties at and around the control region and is thus a "low authority" approach [12]. Wave control is normally designed in the frequency domain. Most implementations are feedforward [13–20]. Here the disturbance is detected, and a control force applied somewhere downstream to produce a destructive signal to cancel the incoming wave or to absorb the energy associated with it. Wave control can also be feedback [21–23]. In most previous cases, it has been applied to control wave motion in one-dimensional waveguides such as bending waves in beams, axial waves in rods, etc. Optimal controllers are usually non-causal. This reflects the implicit time delays involved in the propagation of waves from one point in a structure to another. Time domain implementations are, therefore, causal approximations to the optimums and are usually implemented digitally using FIR (or IIR) filters.

Both modal and wave active control have advantages and disadvantages. The advantages of modal methods include the generality of the approach and its global nature; the disadvantages include complexity and robustness problems. Since a continuous structure has an infinite number of (often lightly damped) vibrational modes, the control of the entire infinity of modes requires in principle an infinite number of actuators and sensors. In practice, the control is designed by considering only a finite number of modes. This degrades the system's performance. Firstly, control spillover occurs, in which those modes that are excluded from the designed control output are nevertheless excited. Secondly, modes that are excluded from the system model still contribute to the sensor measurements, causing observation spillover. Including more modes improves the performance but increases the size and complexity of the model, unavoidably increasing computational cost. Furthermore, the higher modes are inevitably uncertain: there is a trade-off between robustness and model accuracy with accurate modelling of the structure being essential for successful control system design. The modal properties may even depend on operation conditions, orientation of the structure, etc. Wave designs are based on the local properties of the structure and are inherently much less sensitive to system properties and, therefore, more robust than global models of structures, especially at higher frequencies. However, it does not consider global motion: the global behaviour can adversely affect the amount of control achieved.

A hybrid approach to active vibration control is described in this paper. It is an attempt to improve on the performance of modal and wave control approaches individually. It is not only a hybrid wave/mode approach, but also a hybrid low authority/high authority method. In the proposed hybrid approach, wave control is first designed and is targeted at higher frequencies. This wave control is intended to absorb vibrational energy and modifies the equations of motion of the structure, coupling the modes of the uncontrolled structure. Modal control is then designed for the lower modes of the structure based on the modified equations of motion of the structure-plus-wave-controller. Since the higher order modes are now well damped, the hybrid control improves the model accuracy and the robustness of the system, and hence gives better broadband vibration attenuation performance.

In principle, there is no restriction on which wave and mode control design approaches are used. In this paper, however, the particular case considered is that of collocated, point force/sensor feedback wave control combined with modal control designed using pole placement. Numerical and experimental results are presented for the case of hybrid control of a cantilever.

2. HYBRID ACTIVE VIBRATION CONTROL

Hybrid active vibration control is described in this section with particular reference to the case of the cantilever beam shown in Figure 1. The active control is applied in two stages. First, localized wave-based control is applied at a number of points—in this paper collocated point feedback control is adopted, although in principle any method could be used. This control is designed in the frequency domain and is based on a local model of wave propagation through the structure at and around the control location. It is intended to absorb energy, primarily at higher frequencies (above the frequency range for which modal control is designed). Secondly, active control of the global behaviour of the wave-controlled structure is then applied using a state-space approach. This is designed to control the lowest few modes of the wave-controlled structure. Here a pole-placement approach is used, although, once again, in principle any approach may be utilized. Before considering active control, the modal behaviour of the uncontrolled structure is described, since the mode shapes are later used as basis functions to determine the modes of the wave-controller structure.

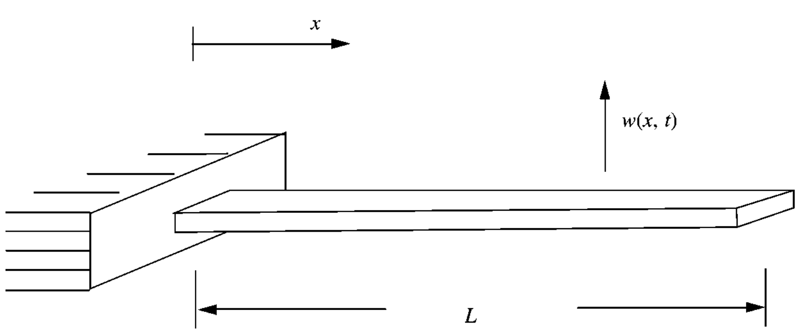

Figure 1. The cantilever beam.

2.1. THE UNCONTROLLED STRUCTURE

In the absence of damping, the equation of motion of a system can be written as [11]

$$L(x)\,w(x,\,t) + m(x)\,\ddot{w}(x,\,t) = f\,(x,\,t), \tag{1}$$

where $w(x,\,t)$ is the displacement, $m(x)$ the mass density, $f\,(x,\,t)$ the external force (the sum of the external disturbance and any control forces) and $L(x)$ is a stiffness operator, whose exact nature depends on the structure. For example, for bending waves in a thin beam of uniform section $L(x) = EI\,\partial^4/\partial x^4$, where EI is the bending stiffness. In the general case x may be a one-, two- or three-dimensional position vector.

In the absence of any excitation (i.e., $f = 0$), the eigensolution to this equation yields the natural frequencies ω_i and mode shapes $\phi_i(x)$ of the uncontrolled structure. While analytical solutions are available for some simple systems, in general these modal properties would be found numerically using, for example, a finite element model. The response can then be expressed as a sum of an infinite number of modal components as

$$w(x,\,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \tag{2}$$

where $q_i(t)$ ($i = 1, 2, \ldots, \infty$) are the modal co-ordinates. (Of course in any numerical model only a finite number n of modes are retained.) Here, the mode shapes are assumed to be mass-normalized such that

$$\int m(x)\phi_i(x)\phi_j(x)\,\mathrm{d}x = \delta_{ij}, \quad i, j = 1, 2, \ldots, \tag{3}$$

where the integral is taken over the whole structure. The equations of motion can then be written in terms of the modal co-ordinates by substituting equation (2) into equation (1), multiplying by $\phi_i(x)$ and integrating over the structure, giving

$$\ddot{q}_i(t) + \omega_i^2\, q_i(t) = f_i(t), \quad f_i(t) = \int f\,(x,\,t)\,\phi_i(x)\,\mathrm{d}x, \quad i = 1, 2, \ldots, \tag{4}$$

where f_i is the ith modal force.

2.2. WAVE CONTROL

First, wave control is applied at one or more points where the structure is uniform, so that the stiffness operator becomes a differential operator with constant coefficients and the mass density $m(x)$ is (locally) constant. For a one-dimensional structural member the motion can then be decomposed into a number of different waves (e.g., bending, axial or torsional waves) which propagate in both directions along the member.

Suppose a control force is applied to a structural component which vibrates solely as a thin beam in bending. Around the control location the equation of motion reduces to

$$EI\,\frac{\partial^4 w(x,\,t)}{\partial^4 x} + m\,\frac{\partial^2 w(x,\,t)}{\partial t^2} = 0. \tag{5}$$

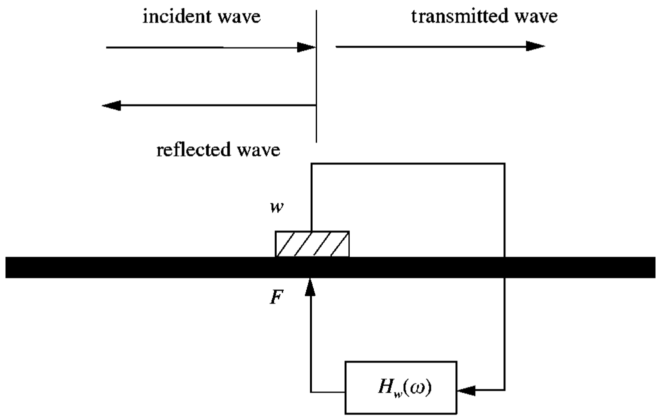

Figure 2. Collocated feedback control.

where EI and m constants because the beam is uniform. Assuming time harmonic motion at frequency ω, the response can be written in terms of four wave components as

$$w(x) = a^{+}\mathrm{e}^{-\mathrm{i}kx} + a^{-}\,\mathrm{e}^{\mathrm{i}kx} + a_N^{+}\,\mathrm{e}^{-kx} + a_N^{-}\,\mathrm{e}^{kx}, \qquad (6)$$

where the wavenumber $k = \sqrt[4]{m\omega^2/EI}$. The first two wave components represent propagating, energy-carrying waves, while the last two are nearfields, which decay exponentially with distance and normally carry no energy over significant distances. The aim of the wave control is to absorb the energy associated with the propagating waves.

In this paper, collocated point force/sensor negative feedback control is assumed to be applied at some point as shown in Figure 2. In the frequency domain, the wave control force is given by

$$F(\omega) = -\,H_w(\omega)\,W(\omega). \qquad (7)$$

The control is dynamically identical to an attached spring with a dynamic translational stiffness $H_w(\omega)$, which is normally frequency-dependent and complex [24, 25]. This situation has been the subject of a number of studies [21–24, 26]. A propagating wave is incident on the control location and gives rise to reflected and transmitted propagating and nearfield waves. It is shown in reference [21] that, if the objective of the control is to absorb as much of the energy carried by the incident wave as possible, then the frequency response of the optimal controller is given by

$$H_w^{(o)}(\omega) = 2\,\sqrt[4]{m^3 EI}\,(1+\mathrm{i})\,\omega^{3/2}. \qquad (8)$$

The control in effect involves a spring and damper with frequency-dependent parameters. The analysis in reference [21] assumes that the amplitudes of any incident near fields are negligible. If these exist then the performance of the control can deteriorate.

2.2.1. *Real-time implementation*

The optimal controller of equation (8) is non-causal. Hence, a real-time implementation must be some approximation to this ideal. There are many possible approaches to the implementation, and two are described here. In the first, the control is tuned to be optimum at some design frequency. The second approach involves the implementation of a digital FIR controller. This gives improved control from the wave perspective but complicates the design somewhat because of the requirement for a low-frequency approximation to the wave control as described below.

In tuned wave control proportional-plus-derivative (PD) feedback control is implemented, with the controller tuned so that it is equal to the optimal controller at some specific frequency ω_d. The controller then has the frequency response

$$H_w(\omega) = c_1 + c_2\,(\mathrm{i}\omega), \tag{9}$$

where

$$c_1 = 2\sqrt[4]{m^3\,EI}\omega_d^{3/2}, \quad c_2 = 2\sqrt[4]{m^3\,EI}\,\omega_d^{1/2}. \tag{10}$$

In the time domain, this corresponds to a tuned spring–damper combination. PD control is of course a classical control methodology, the control gains here being calculated from a wave perspective to be optimal at some desired tuned frequency ω_d.

The second approach involves digital implementation using an FIR filter. This offers superior performance. A causal approximation can be found by fitting a causal FIR filter to the optimum controller of equation (8) in the least-squares sense in the frequency domain [26]. This is in effect the same as truncating the impulse response of the optimal, non-causal, infinite-length FIR filter. The choice of the FIR filter length is a compromise between the accuracy of the controller and the calculation time for the control output.

2.2.2. *Time domain representation and approximation*

The wave control is designed in the frequency domain as described above. However, a time domain representation is required for the subsequent design of the modal control. If the force is applied at a point $x = x_w$, then the wave control force is $f_w(w, x, t) = f_w(w(x_w), t)\,\delta(x - x_w)$. For tuned PD control this becomes

$$f_w(x, t) = -\left[c_1 w(x, t) + c_2\,\dot{w}(x, t)\right]\delta(x - x_w). \tag{11}$$

For the case of digital control using a causal FIR controller, however, the time domain representation is not so straightforward.

Suppose that the implemented wave controller has some frequency response $H_w(\omega)$. It is convenient to approximate the controller by a polynomial in ω in the low-frequency region, where modal control is to be most effective. The real and imaginary parts are approximated separately over this frequency range using a least-squares procedure as [26]

$$\hat{H}_w = \hat{H}_{real} + \mathrm{i}\,\hat{H}_{imaginary}, \quad \hat{H}_{real} = a + b\omega^2, \quad \hat{H}_{imaginary} = c\omega. \tag{12}$$

The polynomials are chosen to be second order or lower so that the order of the system equations is not changed. The wave control force is thus approximated in this low-frequency region by

$$f_w(x, t) = -\left[a w(x, t) - b\,\ddot{w}(x, t) + c\,\dot{w}(x, t)\right]\delta(x - x_w). \tag{13}$$

2.3. MODAL CONTROL

Modal control is now implemented by applying state-space control to the wave-controlled structure. The equation of motion of the structure, equation (1), now includes the wave-control force $f_w(x, t)$ of equation (13) (or equation (11) if PD wave control is used). The motion is expressed in terms of the modes of the uncontrolled structure as in equation (2). The modal forces of equation (4) now become

$$f_i = \int f(x, t)\,\phi_i(x)\,\mathrm{d}x + \int f_w\,(w, x, t)\,\phi_i(x)\,\mathrm{d}x. \tag{14}$$

Only the first n modes of the uncontrolled system are now retained. For collocated wave control, and with the control force approximated by equation (13), the equations of motion can thus be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}, \tag{15}$$

where $\mathbf{f} = [f_1\ f_2\ \cdots\ f_n]^{\mathrm{T}}$ is the vector of external modal forces and where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping and stiffness matrices (written in terms of the modes of the uncontrolled structure) given by

$$M_{ij} = \delta_{ij} - b\phi_i(x_w)\phi_j(x_w), \quad C_{ij} = c\phi_i(x_w)\phi_j(x_w), \quad K_{ij} = \omega_i^2\delta_{ij} + a\phi_i(x_w)\phi_j(x_w). \tag{16}$$

In the absence of the wave-control force, the mass matrix $\mathbf{M}$ is an $n \times n$ identity matrix, $\mathbf{C}$ an $n \times n$ zero matrix and $\mathbf{K}$ is a diagonal matrix of natural frequencies squared. However, the wave control force couples the modes of the uncontrolled structure and the modes of the wave-controlled system are therefore changed. Furthermore, equation (15) is normally non-self-adjoint, so that the new modes are complex.

Upon introducing the state vector $\mathbf{X}(t) = [\mathbf{q}^{\mathrm{T}}(t) : \dot{\mathbf{q}}^{\mathrm{T}}(t)]^{\mathrm{T}}$, equation (15) is rewritten in state-space form as

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{f}, \tag{17}$$

where the coefficient matrices are

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}. \tag{18}$$

For modal control design, the state-space equation is transformed from the state space to the so-called *modal space* by using the eigenproperties of the characteristic matrix $\mathbf{A}$. Denote the eigenvalues of matrix $\mathbf{A}$ as $\Lambda = \mathrm{diag}(\lambda_i)$ $(i = 1, 2, \ldots, 2n)$, and the matrices of left and right eigenvectors of matrix $\mathbf{A}$ as $\mathbf{V}$ and $\mathbf{U}$ respectively (two eigenvector matrices are involved since $\mathbf{A}$ is asymmetric). By using the biorthonormality relations of the eigenvectors, i.e., $\mathbf{V}^{\mathrm{T}}\mathbf{U} = \mathbf{I}$ and $\mathbf{V}^{\mathrm{T}}\mathbf{A}\mathbf{U} = \Lambda$, and a linear transformation

$$\mathbf{X}(t) = \mathbf{U}\mathbf{Z}(t) \tag{19}$$

the state-space equation is transformed into the modal space as [27, 11]

$$\dot{\mathbf{Z}}(t) = \Lambda\mathbf{Z}(t) + \mathbf{V}^{\mathrm{T}}\mathbf{B}\mathbf{f}(t), \tag{20}$$

where $\mathbf{Z}$ is called the vector of *eigenmode states*. The modal control design is then performed in the modal space.

In distributed control, the modal control force $f(x, t)$ is assumed to be (continuously) spatially distributed across the structure. This raises profound implications for practical implementations. If that distributed control is not realizable, the control task is to be carried out by means of r discrete actuators instead. Then the control force $f(x, t)$ is written as

$$f(x, t) = \sum_{j=1}^{r} F_j(t)\, \delta(x - x_j), \tag{21}$$

where $F_j(t)$ is the amplitude of the control force at x_j and $\delta(x - x_j)$ the Dirac delta function. The modal control force is then given by

$$\mathbf{f}(t) = \Phi_F\, \mathbf{F}(t), \tag{22}$$

where $\mathbf{F}(t) = [F_1(t)\, F_2(t) \cdots F_r(t)]^{\mathrm{T}}$ and $\Phi_F = [\phi_i(x_j)]$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, r$.

Substituting equation (22) into equation (20) gives

$$\dot{\mathbf{Z}}(t) = \Lambda \mathbf{Z}(t) + \mathbf{PF}(t), \tag{23}$$

where $\mathbf{P} = \mathbf{V}^{\mathrm{T}}\mathbf{B}\Phi_F$.

Since the control is designed in the modal space first, the control force $\mathbf{F}(t)$ is written as

$$\mathbf{F}(t) = -\mathbf{G}^{\mathrm{T}}\mathbf{Z}(t), \tag{24}$$

where $\mathbf{G}^T = [g_1, g_2, \ldots, g_{2n}]$ is the control gain vector. Substituting equation (24) into equation (23) gives

$$\dot{\mathbf{Z}}(t) = \Lambda \mathbf{Z}(t) - \mathbf{PG}^{\mathrm{T}}\mathbf{Z}(t) = (\Lambda - \mathbf{PG}^{\mathrm{T}})\, \mathbf{Z}(t). \tag{25}$$

It can be seen from equation (25) that $(\Lambda - \mathbf{PG}^{\mathrm{T}})$ is the characteristic matrix of the system after control. By suitably designing $\mathbf{G}$, one can design the eigenproperties of the controlled system. One widely used method is pole allocation, in which the closed-loop poles of the controlled modes are selected in advance and the control gains then computed so as to match the desired closed-loop poles. The elements of the control gain matrix can then be determined as described in references [27, 11]. For the particular case of a single control force, which is considered in the experimental implementation below, Φ and $\mathbf{G}$ become vectors, with the elements of $\mathbf{G}$ being [27, 11]

$$g_j = -\frac{\prod_{k=1}^{2n} (\rho_k - \lambda_j)}{p_j \prod_{\substack{k=1 \\ k \neq j}}^{2n} (\lambda_k - \lambda_j)}, \tag{26}$$

where p_j are the elements of vector $\mathbf{P}$ and ρ_k the desired closed-loop eigenvalues (j, $k = 1, 2, \ldots, 2n$). The modal control force is then obtained by substituting the control gains into equation (24).

Note that the above control design is based on the eigenmode state vector $\mathbf{Z}$ instead of the actual sensor measurements. However, the relationship between the eigenmode state vector and the actual sensor measurements can be easily established. First, the eigenmode state vector $\mathbf{Z}$ and the modal state vector $\mathbf{X}$ are linearly related through the left eigenvector of the characteristic matrix $\mathbf{A}$ as

$$\mathbf{Z}(t) = \mathbf{V}^{\mathrm{T}}\mathbf{X}(t). \tag{27}$$

The modal state vector $\mathbf{X}$ can be found from the actual sensor measurements using modal filters [28]. Assume that the modal control design is to control the first n modes and the sensor measurements are taken at n discrete points $x_j, j = 1, 2, \dots, n$. From equation (2) and the definition of the modal state vector $\mathbf{X}$, one has

$$\mathbf{X}(t) = \begin{bmatrix} [\Phi_w^{\mathrm{T}}]^{-1} & 0 \\ 0 & [\Phi_w^{\mathrm{T}}]^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \dot{\mathbf{w}} \end{bmatrix}, \tag{28}$$

where $\mathbf{w} = [w(x_1)\ w(x_2)\ \cdots\ w(x_n)]$ and $\Phi_w = [\phi_i(x_j)]$, $i = 1, 2, \dots, n$; $j = 1, 2, \dots, n$. In the case when the number of sensor measurements is greater than the number of modes to be controlled, the pseudo-inverse can be used for the modal filtering purpose [26].

From equations (27) and (28), the relationship between the eigenmode state vector and the actual sensor measurements is established as

$$\mathbf{Z}(t) = \mathbf{V}^{\mathrm{T}} \begin{bmatrix} [\Phi_w^{\mathrm{T}}]^{-1} & 0 \\ 0 & [\Phi_w^{\mathrm{T}}]^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \dot{\mathbf{w}} \end{bmatrix}. \tag{29}$$

Substituting equation (29) into equation (24) gives the matrix of control gains in terms of the actual sensor outputs

$$\mathbf{R} = -\mathbf{G}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} \begin{bmatrix} [\Phi_w^{\mathrm{T}}]^{-1} & 0 \\ 0 & [\Phi_w^{\mathrm{T}}]^{-1} \end{bmatrix}. \tag{30}$$

In the numerical examples below the modal properties of the cantilever are known, while in the experimental implementation they were first measured.

3. NUMERICAL EXAMPLES

In this section, some numerical results will be presented for a cantilever whose geometrical and material properties are listed in Table 1. The lowest six natural frequencies are given in Table 2. A disturbance is applied at one point, wave control is applied at a second point and modal control is applied at two further points as given in Table 3. The positions of these points are chosen so as to avoid the nodes of the first six modes.

TABLE 1

Physical and geometric properties of the beam

Young's modulus (GN/m^2) E	Density (N/m^3) ρ	Width (mm) b	Depth (mm) h	Length (mm) L
190	7800	40	2	600

TABLE 2

The first six natural frequencies of the system

Mode number	1	2	3	4	5	6
Frequency (Hz)	4·46	27·97	78·33	153·49	253·73	379·03

TABLE 3

The locations of the disturbance, the controllers and the response point

	Disturbance	Wave controller	Modal controllers		Response point
Location	0·1L	0·2L	(1/6)L	(5/12)L	0·7L

Numerical results show the response at a position $x = 0{\cdot}7L$ per unit disturbance force. The controlled and uncontrolled frequency responses are compared for the cases of wave control alone, modal control alone and hybrid wave/mode control.

3.1. WAVE CONTROLLERS

Implemented wave controllers approximate the optimal control of equation (8). In the approximation involving tuned PD control (equation (9)), the controller is tuned to be optimal at 152 Hz. This is close to the fourth natural frequency of the structure; i.e., somewhat above the frequency range where modal control is to be most effective. In the causal FIR approximated implementation there are 20 terms in the FIR filter.

The frequency responses of the wave controllers are shown in Figure 3, while Figure 4 shows the incident energy absorbed for the case where only one wave is incident on the control location. In the ideal situation, half the incident energy is absorbed. The causal FIR controller is seen to be a better approximation than the tuned controller except in a narrow band around the tuned frequency.

3.2. RESPONSE AFTER CONTROL

Figures 5 and 6 show the frequency responses of the structure after application of the wave controllers alone. Without control, clear, sharp resonances can be observed. The different wave controllers are seen to add damping to the structure, and the amount of damping varies from mode to mode. They also change the natural frequencies somewhat.

It is seen that causal FIR control gives somewhat better performance. In both cases clear resonant behaviour is still apparent around the lowest natural frequencies: wave control approximates the optimum poorly in this frequency range, since it is designed to operate over a relatively wide frequency band.

Relatively poor performance can also be seen at high frequencies. The degradation of the performance at higher frequencies is due to the fact that the point of application of the wave controller lies close to a node of the 6th mode. In wave terms, it lies close to the node of a standing wave that exists, being reflected from the built-in end of the cantilever. Such effects depend on the specific form and location of the wave control. They can be minimized by applying wave control at a boundary, by implementing wave controllers which sense both displacement and rotation or by the suitable application of two or more wave controllers.

Modal control can be designed to control any modes provided the knowledge of the responses of those modes is adequate. Here the controller is designed to increase the damping factors of the first two modes of the structure to 0·5. The pole placement approach is adopted.

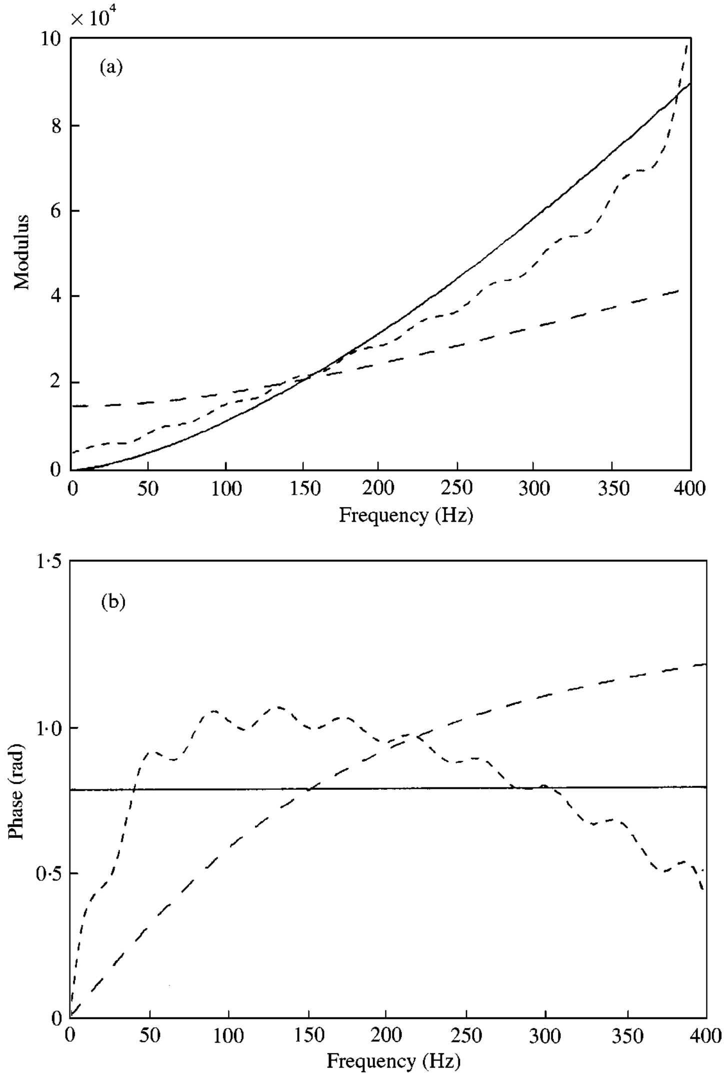

Figure 3. Frequency responses of wave controllers: (a) modulus; (b) phase; —— optimal, - - - - causal FIR and – – – – tuned PD controllers.

Figure 7 shows the frequency responses before and after modal control. The lowest two modes are clearly well controlled, while the remainder are virtually unaffected. Sharp resonances associated with the uncontrolled modes, therefore, still exist. Including more modes in the control design can alleviate this problem, but only at the cost of increased model complexity.

Figures 8 and 9 show the frequency responses after application of hybrid control using the tuned and causal FIR approximated wave controllers. The wave and modal controllers are designed as described above. Hybrid control clearly adds substantial damping, giving improvement over modal control alone. It also gives broadband control due to the energy-absorbing nature of the wave control, reducing the effects of higher resonances and spillover. Once again, causal FIR wave control gives somewhat better performance than PD control.

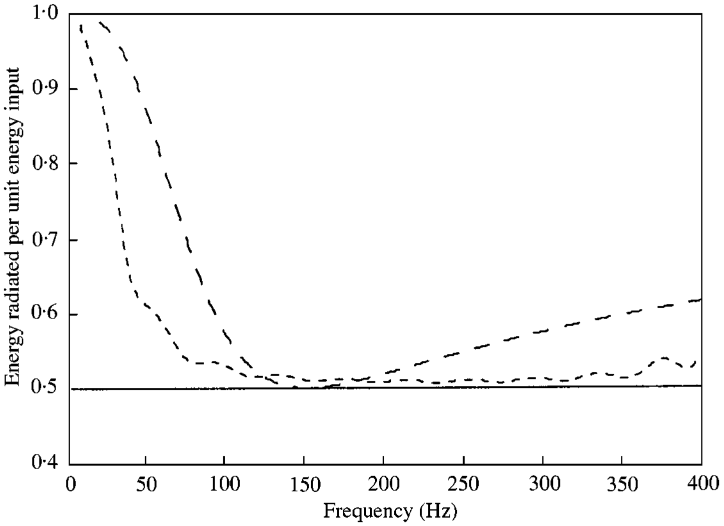

Figure 4. Energy radiated after wave control, with various wave controllers: —— optimal, - - - - causal FIR and – – – – tuned PD controllers.

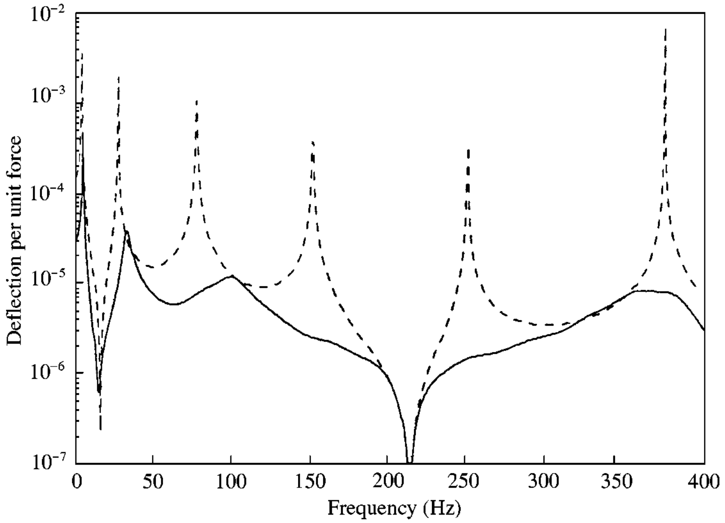

Figure 5. Frequency responses, - - - - before and —— after causal FIR wave control.

4. EXPERIMENTAL RESULTS

4.1. EXPERIMENTAL SET-UP

The experimental set-up is shown in Figure 10. The properties of the cantilever beam are given in Table 4. The modal behaviour of the beam was modelled in terms of the first five uncontrolled modes. Thus five sensors were used and their outputs combined by passing them through "modal filters" to obtain the modal states.

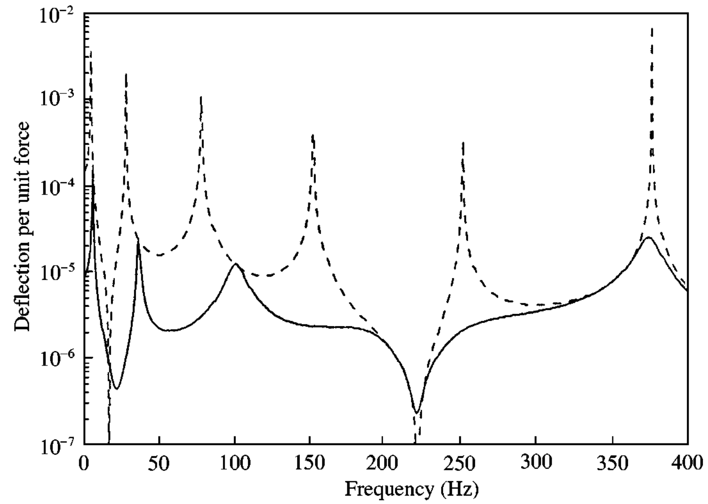

Figure 6. Frequency responses - - - - before and —— after tuned PD wave control.

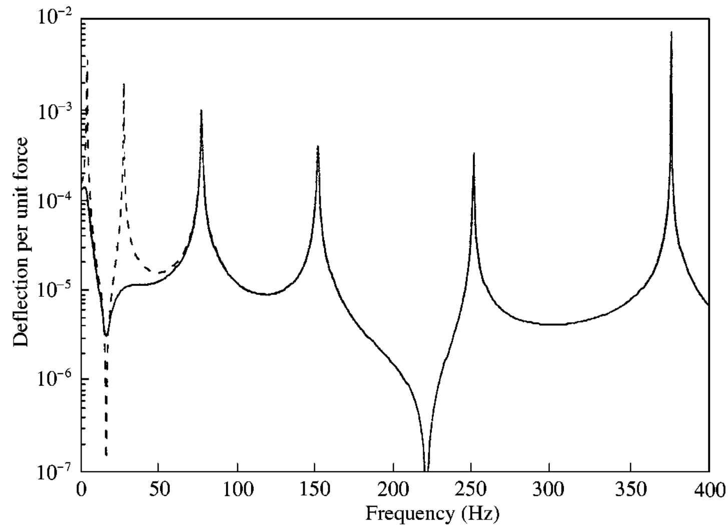

Figure 7. Frequency responses - - - - before and —— after modal control.

Accelerometers were used to measure vibrations and three electromagnetic shakers used to excite vibrations, one to create a disturbance signal, and the other two to create control signals. The locations of the sensors and actuators are given in Tables 5 and 6 respectively. The test signal used was a 'burst chirp" within the frequency range of interest. A Hewlett-Packard HP3565B 8-channel analyser was used to both provide the test signal and measure the frequency response of the system. Control was provided by a 100 MHZ IBM-PC Pentinum computer with a Metrabyte A/D-Dash-16F expansion board. Low-pass anti-aliasing and reconstruction filters and power amplifiers completed the arrangement. More details can be found in reference [26].

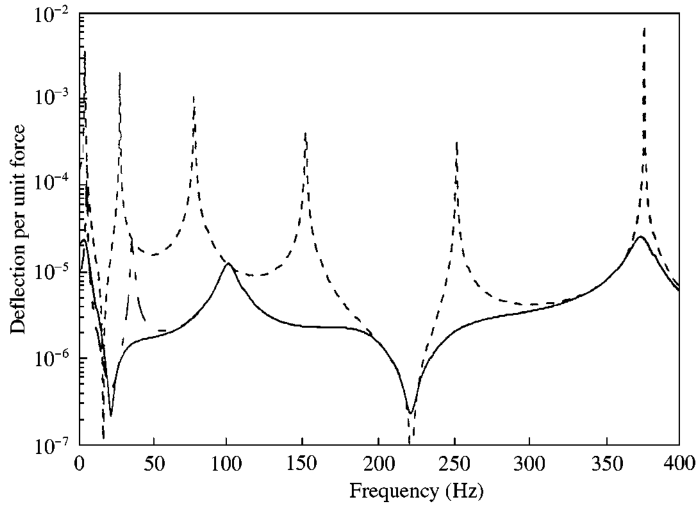

Figure 8. Hybrid control with tuned PD wave controller; frequency responses ----- before control, – – – – – – – after wave and —— hybrid control.

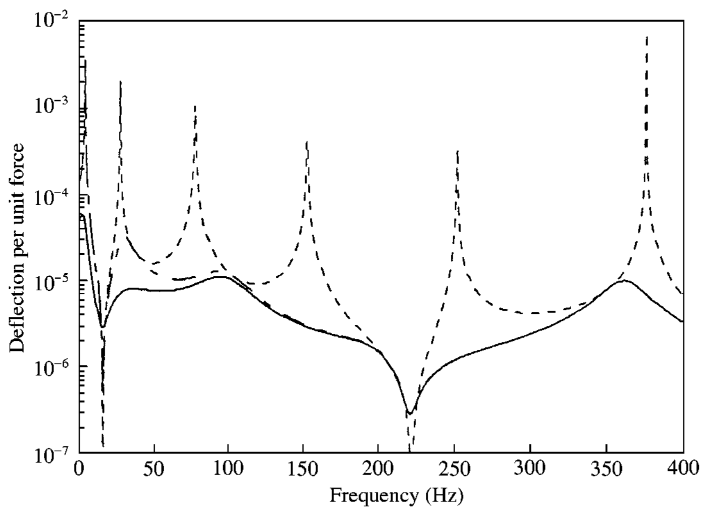

Figure 9. Hybrid control with causal FIR wave controller; frequency responses ----- before control, – – – – – – – after wave and —— hybrid control.

4.1.1. *Modal analysis of the uncontrolled system*

The accelerometers and actuators modify the dynamics of the system, whose modes consequently differ from those of a cantilever without their attachment. In particular, they add damping and change the natural frequencies.

The modal properties of the system were measured experimentally prior to the control design since they are required for the state-space design. The experimental modal analysis was performed for the lowest five modes using the peak value method [29]. Proportional damping was assumed. Figure 11 shows the measured frequency response and that

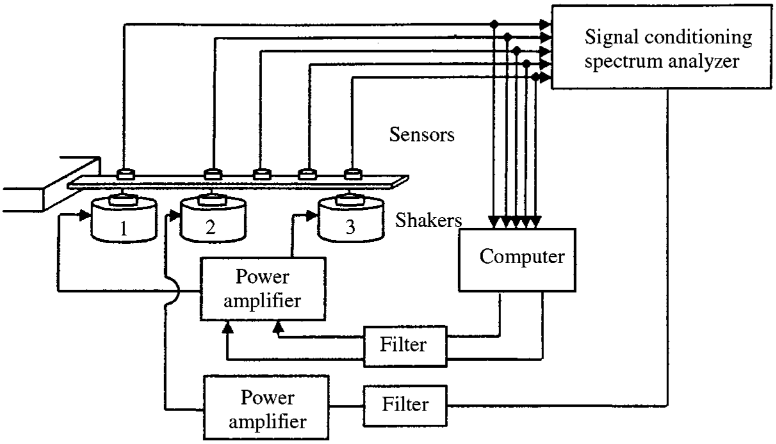

Figure 10. The experimental set-up. Shakers: 1. Wave control force, 2. Disturbance force, 3. Modal control force.

TABLE 4

Physical parameters of the experimental steel beam

Young's modulus (GN/m^2) E	Density (mg/m^3) ρ	Width (mm) b	Depth (mm) h	Length (mm) L
180	8600	40	2	600

TABLE 5

The locations of the sensors

Sensor	1	2	3	4	5
Location	100 mm	245 mm	400 mm	500 mm	580 mm

TABLE 6

The locations of the actuators

Actuator	Disturbance actuators	Wave control actuator	Modal control actuator
Location	245 mm	100 mm	400 mm

predicted using the identified modes. The response is shown at a point 100 mm from the clamped end. The agreement between simulation and experiment is good.

4.2. RESULTS

Wave, mode and hybrid controls were implemented. For modal control (see Figure 12) the controller was designed to increase the damping factors of the second and third modes

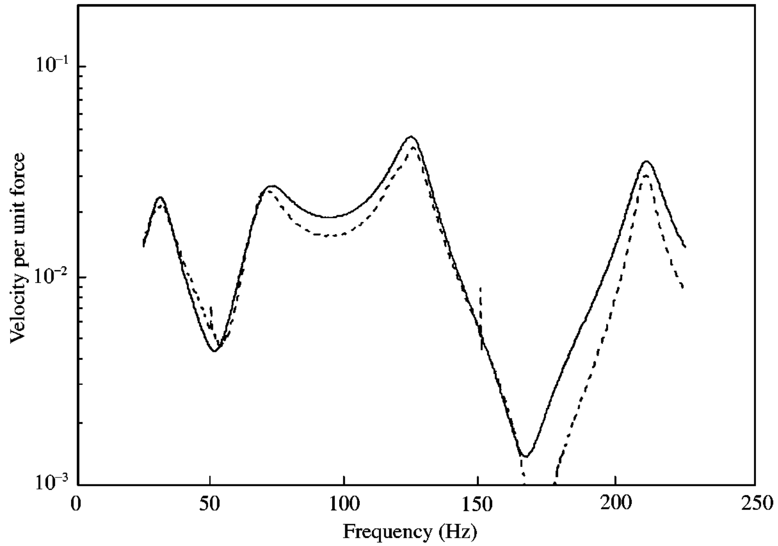

Figure 11. Frequency response before control: —— predicted and ---- measured.

to 0·7, with multi-input–single-output coupled modal control. The control gains were obtained in the discrete time domain based on the experimentally measured or estimated modal properties. Since accelerometers were used as sensors, the first mode responded very weakly and consequently the control design was aimed at a frequency range somewhat above this first natural frequency. Details are given in reference [26]. Wave and hybrid control were applied using tuned PD and FIR wave controllers (see Figures 13 and 14 respectively). The tuned PD controller was tuned to 70 Hz, with the FIR controller being a filter with 20 terms. The results show measured frequency responses at a location 100 mm from the clamped end. Generally, good agreement is obtained between measured and predicted behaviours.

For the case of modal control, it can be seen from Figure 12 that significant control is added to the second and third modes. Control spillover is observed in the uncontrolled residual modes. Wave control alone typically adds damping, but still leaves distinct resonant behaviour for the lower modes. Hybrid control alleviates this resonant behaviour. For both the predicted and the measured behaviour, the performance of the hybrid approach is consistently better than wave or modal approaches applied alone.

5. CONCLUDING REMARKS

In this paper, a hybrid approach to active vibration control was described. In the proposed approach, local, low-authority wave feedback control, designed on the basis of local wave propagation, is implemented first, the aim being to absorb broadband vibrational energy. After the implementation of the wave control, the equation of motion of the system is modified. Global, high-authority modal control is then designed, based on this modified equation, which aims to control the lowest few modes of the system. The specific case of collocated point force/sensor wave control and pole placement modal control was considered. Wave controllers were designed using either FIR filters or tuned PD control. Numerical and experimental results were presented for the active vibration control of a cantilever beam.

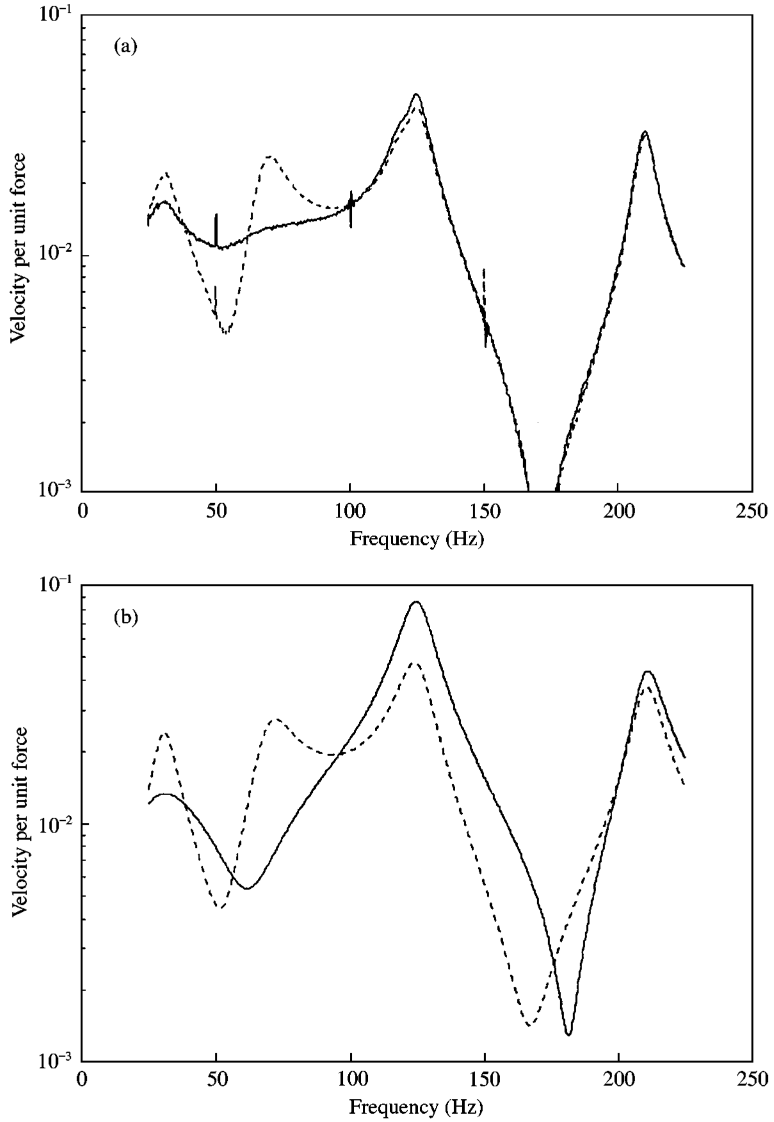

Figure 12. Frequency response ---- before and —— after modal control: (a) measured; (b) predicted.

The hybrid approach exhibits better broadband active vibration control performance than the cases with either modal or wave control alone. There is a reduction in the order of the models required without there being a significant increase in the system representation associated uncertainty. Finally, the effects of the unmodelled modes are reduced and the robustness is improved.

Alternative forms of wave and mode control could of course be used and would consequently alter the detail of the implementation. For example, in the design of the wave control, near fields were ignored, although these can deteriorate the performance. In a similar way, if propagating waves are incident from both sides of the control location then they can interfere and the performance may again deteriorate. The wave controllers that were implemented, however, were guaranteed to be stable, so there are no consequences for

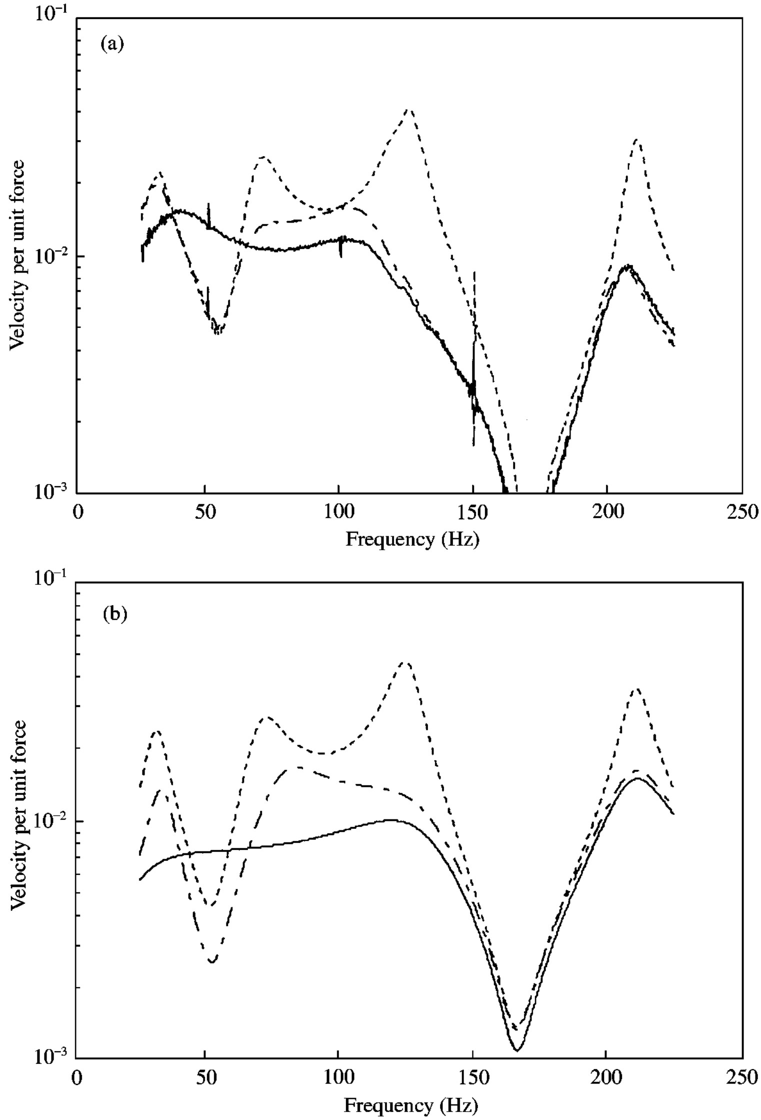

Figure 13. Hybrid control with tuned PD wave controller, - - - - before control, – – – – – – – after wave control and —— after hybrid control: (a) measured; (b) predicted.

closed-loop stability. These issues could be avoided in a number of ways, for example by implementing wave controllers which sense both displacement and rotation or by the suitable application of two or more wave controllers.

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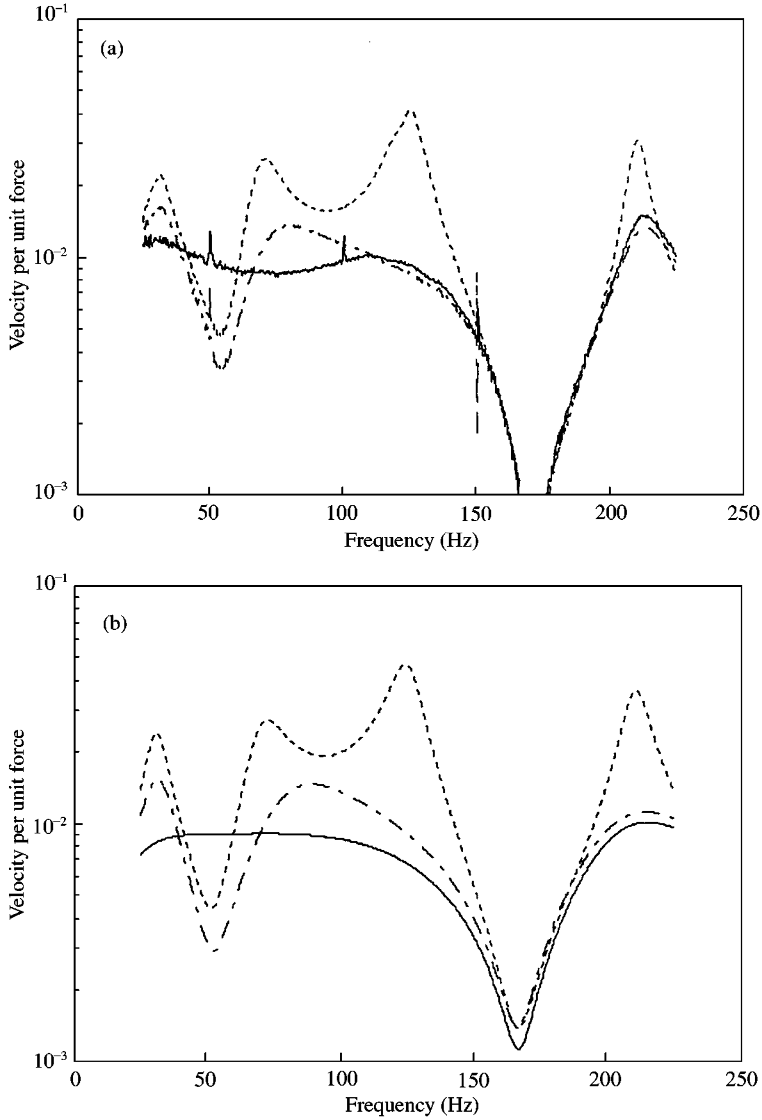

Figure 14. Hybrid control with causal FIR wave controller, ----- before control, ------- after wave control and —— after hybrid control: (a) measured; (b) predicted.

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