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LETTERS TO THE EDITOR

TORSIONAL RESONANCE ANALYSIS IN AIR HANDLING UNITS

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1. INTRODUCTION

Many of the vibration problems and blower failures in small furnaces and air handlers can be eliminated by a more thorough analysis during the design stage. These problems and failures are caused by incompatibilities of the dynamic properties of the blower assemblies. One of the vibration problems that causes fatigue failure results from the torsional resonance of the wheel–shaft–rotor assembly. This torsional resonance frequency depends on both the rotor and wheel inertia moments, and the shaft stiffness that relies on shaft material property, shaft size, and the shaft length between the setscrew and rotor. The size of the shaft flat and the tightness of the setscrew are also part of the equation. A theoretical prediction of this torsional resonance, including the effect of shaft flat, is developed. The effect of setscrew tightness is also discussed based on the test results.

Vibration problems also involve the forces exciting the system. Usually the driving force of this torsional mode is the primary 120 Hz excitation of the motor. But it is quite possible that the 240 Hz excitation may also exist in some applications. A safe margin of the system response to these forces is discussed based on the life cycle test results.

The resonance induced by the wheel–shaft–rotor assembly could result in hub failure, setscrew loosening, and shaft gouging, or in some extreme cases wheel or motor mount failure. Typical fatigue failures related to setscrew, shaft, and hub are discussed. Based on the dynamic properties of the wheel–shaft–rotor assembly, three test methods, including both operating and non-operating vibration tests, have been developed to identify this resonance frequency. They are winding excitation test, strain gage test, and impact test. To minimize the potential vibration problems and related failures in units due to the wheel–shaft–rotor torsional resonance, several techniques can be used to shift this torsional resonance frequency.

2. WHEEL–SHAFT–ROTOR TORSIONAL FREQUENCY

A single-blower system mainly consists of a wheel, a shaft, and a motor rotor. This is a semidefinite vibration system [1]. The model is given in Figure 1. The natural frequency of this model is given by

$$f = \frac{1}{2\pi}\sqrt{k\left(\frac{1}{J_1} + \frac{1}{J_2}\right)}, \tag{1}$$

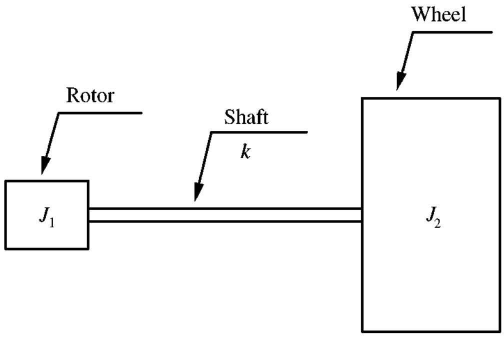

Figure 1. A wheel–shaft–rotor system.

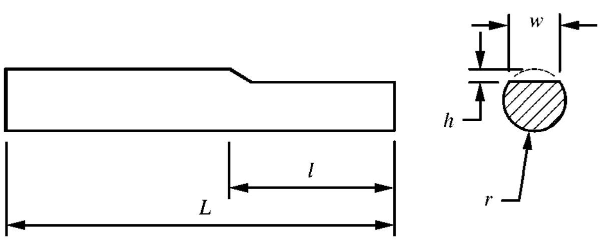

Figure 2. A shaft with flat.

where J_1 = inertia moment of rotor, in lb s^2, J_2 = inertia moment of wheel, in lb s^2, and k = stiffness of shaft, lb/in.

In the case where there is a flat on the shaft as shown in Figure 2, the shaft stiffness consists of two parts (k_1 and k_2) given by equation (2). k_1 is the stiffness of the shaft from the section without flat, while k_2 is for the section with the flat. Both of them are given by equations (3) and (4), respectively:

$$k = \frac{k_1 k_2}{k_1 + k_2}, \tag{2}$$

where

$$k_1 = \frac{\pi r^4 G}{2(L-l)}, \qquad k_2 = \frac{JG}{l}, \tag{3, 4}$$

$$J = \frac{\pi r^4}{2} - \frac{r^4}{2} \arccos\left(\frac{r-h}{r}\right) + \frac{r-h}{3}\sqrt{2hr - h^2}\left(\frac{3r^2}{2} + h^2 - 2hr\right) \tag{5}$$

where r = shaft radius, in and G = shear modulus (lb/in^2). In equation (5), J is the inertia moment of the flat section.

For a typical steel shaft with 0·5 in diameter and 4 in length, the percentage decrease of the shaft stiffness k versus the percentage increase of the flat height h is given in Figure 3, provided l is equal to L. Usually, the flat width w is 0·25 in and its corresponding height h is 0·0335 in, which is 13·4% of the shaft radius. This will cause a 5·12% decrease in the stiffness of k, and a 2·6% decrease in the wheel–shaft–rotor torsional natural frequency.

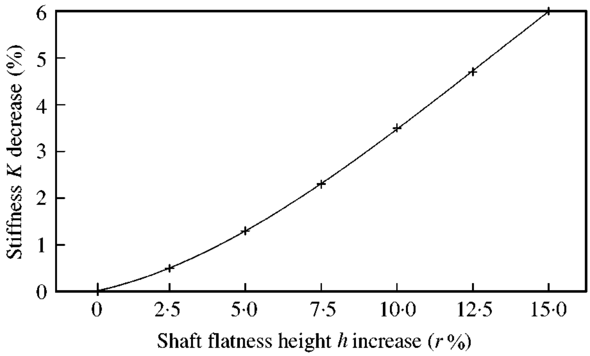

Figure 3. The percentage change of k versus h.

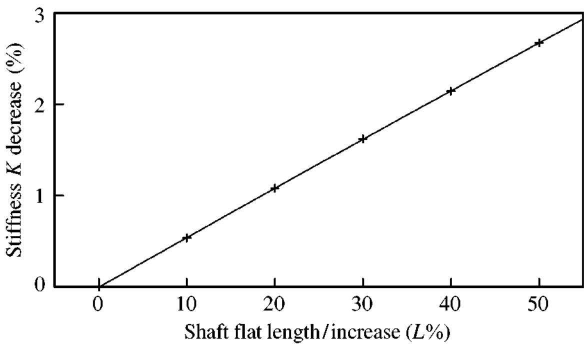

Figure 4. The percentage change of k versus l.

With a flat width at 0·25 in, the shaft stiffness is a function of the flat length l. The shaft stiffness k versus the percentage increase in the flat length l is given in Figure 4. Figure 4 indicates that if the flat length l equals half the shaft length L, the stiffness k will be reduced by 2·67%. This will cause the torsional natural frequency to decrease by 1·34%.

3. SETSCREW TIGHTNESS AND TORSIONAL FREQUENCY

A wheel–shaft–rotor assembly, which has a torsional frequency of 118 Hz was tested with different setscrew tightnesses [2]. The setscrew was torqued to 130 in lb and tested 10 times. Each time the wheel–shaft–rotor was reassembled and tightened with the same torque. The final result was averaged by dropping both the highest and lowest frequencies. Then the above tests were repeated eight more times. The torque was reduced 13 in lb each time. The test results are listed in Table 1 and plotted in Figure 5. The torsional frequency could be reduced by 10% if the setscrew is not tight.

4. WHEEL–SHAFT–ROTOR TORSIONAL RESONANCE

The wheel–shaft–rotor torsional resonance usually causes setscrew loosening, shaft gouging, and noise problems. In some extreme cases this mode may also cause other

TABLE 1

Torsional frequency versus setscrew tightness

T	130	117	104	91	78	65	52	39	26
f	118	118	118	118	116	115	114	106	102

Note: T: torque (in lb); f: frequency (Hz).

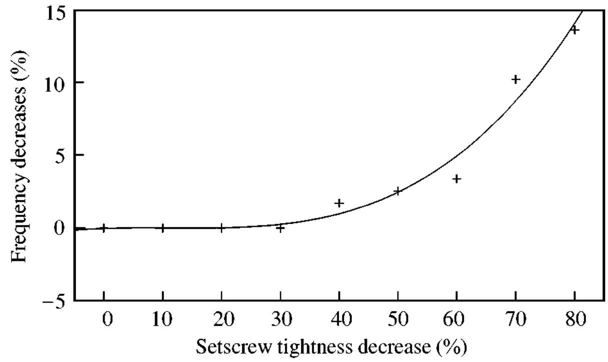

Figure 5. Tosional frequency decrease versus torque change.

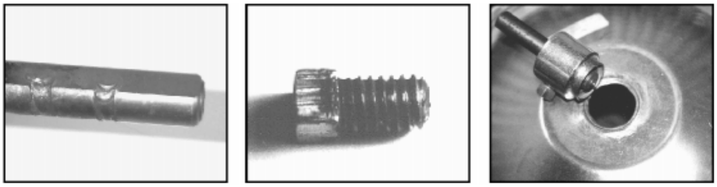

Figure 6. A gouged shaft, a damaged setscrew and wheel.

failures. The excitation force of this mode is usually at a primary torque pulsation of 120 Hz. It could be 240 Hz in some applications [3, 4]. This excitation force and the inertial moment forces from wheel and rotor will force the setscrew to move in one direction and the shaft in the opposite direction. Both of them shift directions at the same frequency of the torque pulsations. This change causes the setscrew to impact the shaft and induces a reaction pulsation force acting on the hub thread. As the process continues, the impact and fatigue damage of the thread result in setscrew loosening and shaft gouging. At the same time, excessive noise may be present. Another type of failure related to this torsional resonance mode is that this torsional resonance induces an excessive force to break the hub loose from the center plate and wear away the hub rollover. Then the center plate moves around on the rotating shaft and cuts off the shaft. In Figure 6 are shown a gouged shaft, a damaged setscrew, and wheel. The tip of the damaged setscrew was worn away and some fretting corrosion is also visible. The shaft was cut off by the center plate.

The primary torque pulsation excitation in a single-phase PSC motor is 120 Hz. Theoretically speaking, the fourth harmonic order of the power line frequency (240 Hz) will not occur unless the supply power wave forms (either voltage or current) are distorted or truncated. When a speed controller is used to adjust the operating speed it may be possible that the power wave forms are distorted or truncated. According to Fourier series theory, this will generate the fourth harmonic order line frequency torque pulsation excitation at 240 Hz. Usually, the amplitude of this torque pulsation excitation is small and will not do any harm to the blower assembly. If the wheel–shaft–rotor system has a torsional response frequency near the excitation frequency this torque pulsation excitation will be amplified significantly and will make excessive noise. An oscilloscope can be used to check if the power wave forms are distorted or truncated.

5. TEST METHODS

There are three methods that can be used to measure the wheel–shaft–rotor torsional natural frequency. Each of them has its advantage and disadvantage. Different applications may require different test methods.

The winding excitation test [5] is a non-operating test. This test requires a variable frequency/voltage power source to drive the motor using a pair of diodes, which will prevent rotation, but will provide torsional excitation. For a typical single-phase PSC motor, connect one of the motor capacitor wires to one of the diodes, while the other motor capacitor wire is idle. No capacitor is required. For motors above $\frac{3}{4}$ hp, 10–20 V provides the necessary excitation. Increase the excitation voltage accordingly for lower horsepower motors. The bottom line is to use enough voltage to excite the system without overheating the test motor. The increment of the excitation frequency should be set the same as the revolution of the analyzer. A winding excitation test is a relatively quick and easy test. There is no need to take the test blower apart. Therefore, the original system assembly conditions are well preserved, and the result is usually clear and repeatable. The frequency sweep range is unlimited. Besides the wheel–shaft–rotor torsional frequency, the motor mount torsional frequency can also be tested at the same time.

The rotor impact test is another non-operating test method to find the wheel–shaft–rotor torsional natural frequency. This method does not require a signal generator and power amplifier, but it does require both the test blower and the test motor to be taken apart.

A strain gage test is required if the rotor and shaft or the wheel hub and shaft have a resilient connection. A resilient connection may create a non-linear torsional system. The resonance is not only a function of the system parameters, but is also related to the excitation forces. Therefore, this resonance must be measured at the operating condition. In this method, the motor torque pulsation serves as an excitation force. For a 60 Hz motor, the power line frequency can go as high as 75 Hz without overheating the motor. If the resonance is close to 240 Hz, a truncated sine wave power source may be used to induce fourth harmonic order of the line frequency torque pulsation. This test can be conducted with a variable frequency/voltage power source and by mounting two torque strain gages on the shaft between the setscrew and rotor parallel to the shaft. The strain gage test requires experience and skill to get satisfactory results.

A wheel–shaft–rotor system was tested by using the above three methods. The results are plotted in Figures 7–9.

6. SAFETY RANGES OF THE RESPONSE FREQUENCY

Most direct drive blowers use a single-phase PSC motor with 120 Hz torque pulsation. In some applications, side band excitations at 120 Hz $\pm$ r.p.m./60 may also cause torsional

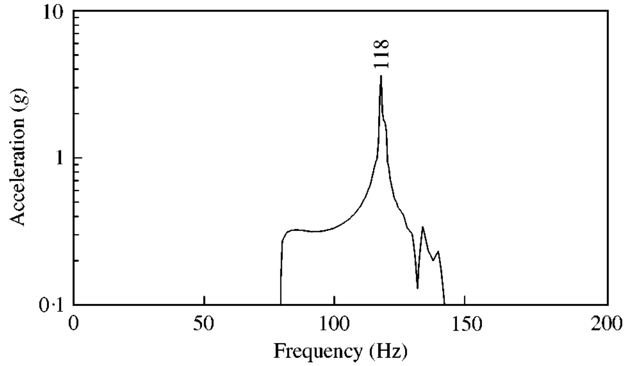

Figure 7. Winding excitation test result.

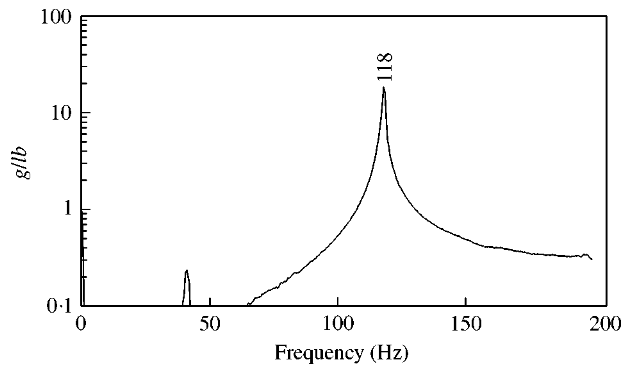

Figure 8. Rotor impact test result.

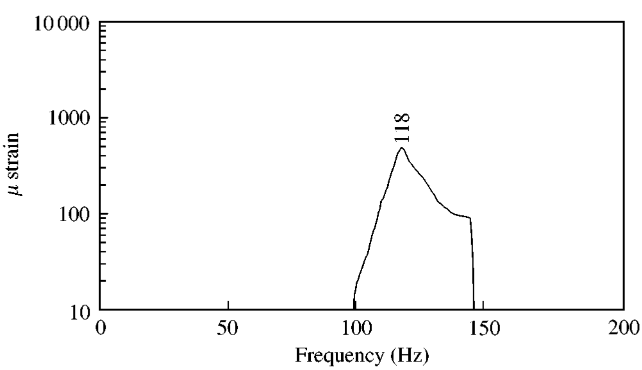

Figure 9. Strain gage test result.

vibration and noise problems if the system response frequency is close to one of these excitation frequencies. Based on experience and life cycle test results, the torsional natural frequency (f_t) of a wheel–shaft–rotor system will be acceptable if the following inequality is satisfied:

$$x_{120} \times (120 - (\text{r.p.m.}/60)) > f_t > 1{\cdot}14 \times (120 + (\text{r.p.m.}/60)), \quad (6)$$

TABLE 2

x values and f_t limits

Motor poles	4	6	8	10	12
r.p.m.	1800	1200	900	720	600
x_{120}	0·767	0·804	0·82	0·829	0·834
x_{240}	0·871	0·882	0·887	0·89	0·892
f_{b120}	69	80	86	90	92
f_{t120}	171	160	154	151	148
f_{b240}	183	194	200	203	205
f_{t240}	297	286	281	277	275

Note: f_{b120}: bottom limit of f_t at 120 Hz ($f_{b120} > f_t$); f_{t120}: top limit of f_t at 120 Hz ($f_t > f_{t120}$); f_{b240}: bottom limit of f_t at 240 Hz ($f_{b240} > f_t$); f_{t240}: top limit of f_t at 240 Hz ($f_t > f_{t240}$).

where

$$x_{120} = \frac{240 - 1{\cdot}14 \times (120 + (\text{r.p.m./60}))}{120 - (\text{r.p.m./60})}. \quad (7)$$

x_{120} is calculated based on the same percentage increase and decrease at both side bands.

In the case when a 240 Hz torque pulsation also exists, both inequalities (6) and (8) should be satisfied:

$$x_{240} \times (240 - (\text{r.p.m./60})) > f_t > 1{\cdot}1 \times (240 + (\text{r.p.m./60})), \quad (8)$$

where

$$x_{240} = \frac{480 - 1{\cdot}1 \times (240 + (\text{r.p.m./60}))}{240 - (\text{r.p.m./60})}. \quad (9)$$

The values of x_{120}, x_{240}, and the f_t limits at 120 and 240 Hz are listed in Table 2.

For a six-pole motor, as an example, the torsional natural frequency (f_t) of the wheel–shaft–rotor system should satisfy both of the following inequalities:

$$f_t \neq 80 \text{ to } 160 \text{ Hz}, \qquad f_t \neq 194 \text{ to } 286 \text{ Hz}. \quad (10, 11)$$

The safety factors in inequalities (6) (14%) and (8) (10%) are chosen to avoid side band excitation forces that may be a problem in some applications. For those applications without side band concern these safety factors may be a little conservative.

7. SOLUTIONS

Resonance problems involve the forces exciting the system and the response of the system to these forces. Different systems respond to the same exciting force differently. If a force acts on the wheel in a blower assembly, the vibration of the wheel may be present. If the same force is applied to a very solid concrete wall, nothing will happen. A system responds to a cyclic excitation force like a force amplifier. When the frequency of the excitation force coincides with the system's natural frequency, this force will be greatly amplified. The factor of the force amplified by the system is called response function. Thus, one could in principle get rid of any objectionable vibration, stress, or noise either by reducing the exciting force or by reducing the response function of the system. Experience tells us that a reduction of the exciting force tends to be much more costly than a reduction of the system response function [6]. Therefore, the following discussions are focused on the wheel–shaft–rotor torsional response.

The wheel–shaft–rotor torsional natural frequency depends on the wheel, rotor, and shaft length between the setscrew and rotor. It is necessary to find out how these components affect the system response. Then different solutions may be available for different applications. Let us look at a typical wheel–shaft–rotor assembly with the following parameters: wheel diameter = 11 in, wheel width = 10 in, wheel I = 0·472 in lb s^2, rotor I = 0·0142 in lb s^2, shaft diameter = 0·5 in, shaft length = 4 in, shaft shear modulus = 115×10^5 lb/in^2.

The torsional natural frequency of this system is 180 Hz calculated by equation (1). If the wheel inertia moment J_2 in equation (1) is reduced the torsional frequency will increase. Figure 10 shows a 50% decrease in the wheel moment of inertia which provides only a 1·4% increase in the torsional natural frequency, which indicates that the wheel does not affect the system response very much. This is because the wheel inertia moment is much higher than that of the rotor.

When the shaft length between the setscrew and rotor is reduced, this torsional natural frequency will dramatically increase as shown in Figure 11. Replacing a convex wheel with a concave wheel, or moving motor closer to the setscrew will shorten the shaft length. This is a very handy technique to solve the torsional resonance problems. If the shaft length decreases 50%, the frequency increases 42%.

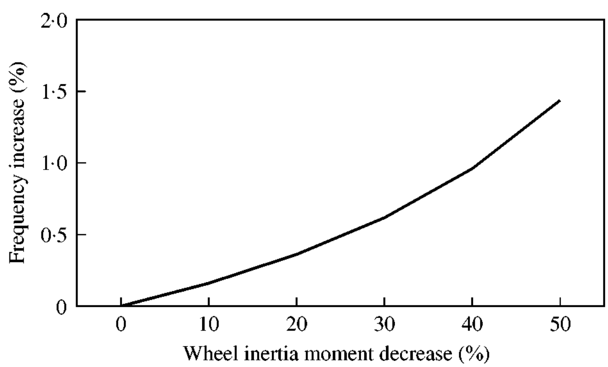

Figure 10. Frequency increase versus wheel inertia moment decrease.

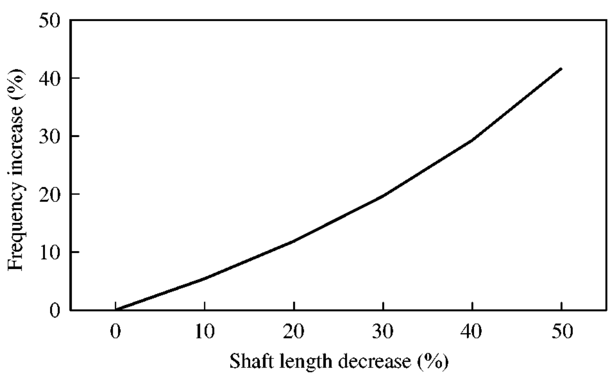

Figure 11. Frequency increase versus shaft length decrease.

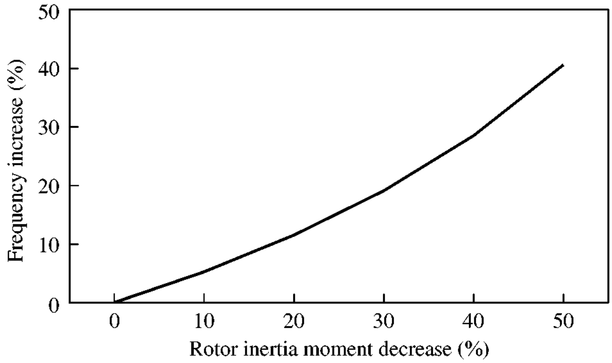

Figure 12. Frequency increase versus rotor inertia moment decrease.

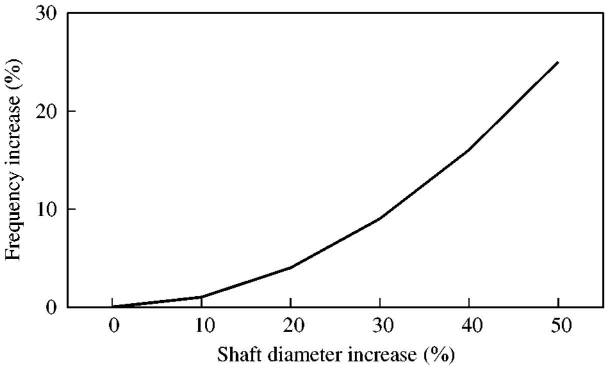

Figure 13. Frequency increase versus shaft diameter increase.

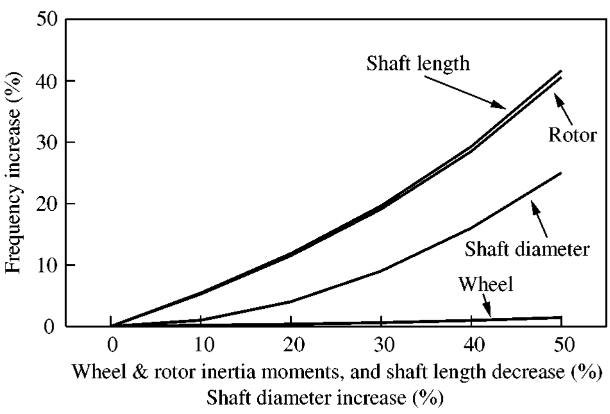

Figure 14. Frequency increase versus blower component change.

Figure 12 shows the comparison of frequency versus the rotor inertia moment. If the rotor inertia decreases 50% the frequency increases 41%. This means that this torsional natural frequency is also very sensitive to the change in the rotor inertia moment. This is why sometimes one motor is reliable while another one may not be. Increasing the shaft diameter will also affect this torsional natural frequency. If the shaft diameter is increased 50% the frequency will increase 25%, which is plotted in Figure 13. Each system has its own characteristic curve (see Figure 14). In this assembly, the shaft length between the setscrew and rotor affects the torsional resonance most, while changes in the wheel contribute very little to the response. In other words the torsional resonance problems result from the system dynamics, instead of the wheel itself.

8. CONCLUSIONS

The Wheel–shaft–rotor torsional natural frequency mainly depends on the shaft length between setscrew and rotor, rotor inertia moment, and shaft diameter. The changes in the wheel inertia moment do not affect this frequency very much. This is because the wheel inertia moment is much higher than that of the rotor. Changing the shaft length between the setscrew and rotor is one of the most effective ways to shift the wheel–shaft–rotor torsional frequency. Switching to an inverted centerplate wheel, or moving the motor closer to or away from the setscrew will change this shaft length. It is possible that using a different type of motor may cause this wheel–shaft–rotor torsional frequency to dramatically shift even though the shaft diameter, shaft length, and the wheel are the same. For a typical 0·5 in diameter shaft with a 0·25 in flat width, the shaft stiffness k reduces by 5·12% and the wheel–shaft–rotor torsional frequency decreases by 2·6%. Setscrew tightness affects the wheel–shaft–rotor torsional frequency. It could cause this frequency to decrease as much as 10%.

The winding excitation test is a relatively quick and easy test. The torsional frequency of the motor mounting system can also be tested at the same time. For a non-linear system a strain gage test is the best choice.

REFERENCES

1. D. D. Reynolds 1985 *Engineering Principles of Acoustics—Noise and Vibration Control*. Las Vegas: D.D.R. Inc.
2. M. A. Corbo 1996 *Proceeding of the 25th Turbomachinery Symposium, Texas A&M University* 189–222. Practical design against torsional vibration.
3. G. L. Godwin and E. F. Merrill 1970 *IEEE Transactions on Industry and General Applications* 258–265. Oscillatory torques during synchronous motor starting.
4. D. J. Sheppard 1988 *IEEE Transactions on Industry Applications* 812–817. Tosional vibration resulting from adjustable frequency AC drives.
5. T. M. Holdsworth and R. D. Morris 1976 *Sound and Vibration* 24–28. The use of induction motors as vibration generators.
6. P. K. Baade 1998 *Sound and Vibration*, July, 16–26. Vibration control of propeller fans.