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TRANSVERSE VIBRATIONS OF A SIMPLY SUPPORTED ORTHOTROPIC PLATE WITH AN OBLIQUE CUTOUT

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1. INTRODUCTION

Plates with cutouts are commonly encountered in many technological situations: aeronautical, civil, mechanical and naval engineering. Cutouts are used in most of the situations due to operational conditions: passage of ducts and conduits, cables, etc. Several studies have appeared in the case of circular, square and rectangular cutouts with free edges [1–3].

In the case of oblique cutouts (Figure 1) some studies have also been published and rough results for the fundamental frequency coefficients have been determined using simple approximations [4, 5].

The present investigation tackles two situations:

- The oblique edge is free.
- A simply supported oblique edge.

For the first problem an approximate analytical solution is obtained expressing the displacement amplitude in terms of a double Fourier series which identically satisfies the boundary conditions of the original, rectangular plate. The frequency determinant is generated using the classical Rayleigh–Ritz method by deducting the subsidiary functional corresponding to the hole from the energy functional corresponding to the virgin plate. An independent solution was obtained using a well-known finite element algorithm [6].

When the oblique edge is simply supported the frequency eigenvalues were only determined by means of the finite element procedure.

2. APPROXIMATE ANALYTICAL SOLUTION

Following previous studies [1–3] and using Lekhnitskii's well-established notation [7] one expresses the governing functional in the form

$$J[W] = \frac{1}{2}\iint\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_1\nu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y - \frac{\rho\omega^2 h}{2}\iint W^2\,\mathrm{d}x\,\mathrm{d}y. \tag{1}$$

where the integration is performed over the plate domain.

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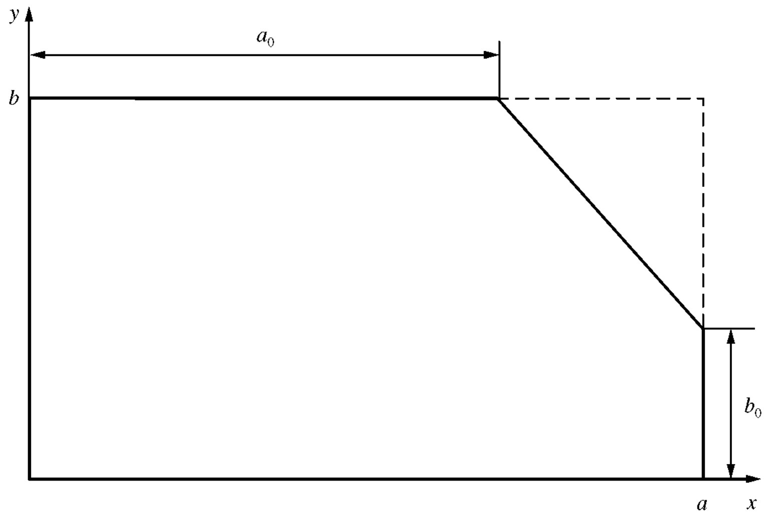

Figure 1. Vibrating structural system under study.

The displacement amplitude is approximated using the truncated double Fourier series

$$W \cong W_a = \sum_1^N \sum_1^M b_{nm} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}. \qquad (2)$$

Applying the classical Rayleigh–Ritz method one generates the frequency determinant whose roots are the frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}\,\omega_i a^2$ in the case of orthotropic plates. When dealing with isotropic plates one replaces D_1 by D in the dimensionless frequency coefficients.

3. FINITE ELEMENT SOLUTION

Calculations were performed only in the case of the square plate. The entire domain was divided into a mesh of (40 × 40) elements. In order to account for the presence of the hole, elements were suppressed until reaching the situation where $a_0/a = b_0/b = 0{\cdot}5$. Consequently, the original mesh of 1600 elements was finally reduced to 1410 elements taking into account the triangular elements neighboring the edge (see Figure 2). Due to the architecture of the ALGOR algorithm the triangular elements do possess the principal axes of orthotropy shown in Figure 2 which do not coincide with the axes of orthotropy of the original plate. However, it is felt that this constitutive-edge effect has minor influence on the global behavior of the structure and the lower natural frequencies. Obviously, this effect is not present when the plate is isotropic.

4. NUMERICAL RESULTS

Calculations were performed for $\nu = 0{\cdot}3$ in the case of isotropic plates and $D_2/D_1 = \frac{1}{2}$, $D_k/D_1 = \frac{1}{2}$ and $\nu_2 = 0{\cdot}3$ when dealing with orthotropic plates. The analytical determinations were performed for $N = M = 20$.

Table 1 depicts values of Ω_i in the case of isotropic, square plates with a free oblique edge. The results obtained by means of both methodologies are in good engineering agreement

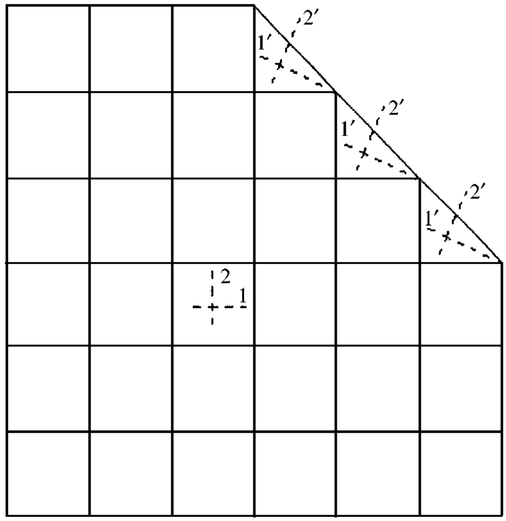

Figure 2. Finite element mesh. *Note*: 1 and 2, principal axes of orthotropy of the plate; 1′ and 2′, principal axes of orthotropy of the triangular, edge elements.

TABLE 1

Frequency coefficients of an isotropic square plate with a free, oblique edge: (*a*) *analytical solution*, (*b*) *finite element results*

	$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
(*a*)	1·00	19·738	49·348	49·348	78·956
	0·95	19·722	49·314	49·327	78·890
	0·90	19·670	49·235	49·250	78·721
	0·85	19·576	49·030	49·212	78·544
	0·80	19·438	48·805	49·213	78·438
	0·75	19·285	48·709	49·240	78·378
	0·70	19·142	48·680	49·279	78·388
(*b*)	1·00	19·744	49·368	49·368	79·038
	0·95	19·721	49·296	49·368	78·950
	0·90	19·658	49·112	49·368	78·741
	0·85	19·559	48·872	49·366	78·484
	0·80	19·450	48·605	49·356	78·098
	0·75	19·331	48·213	49·327	77·237
	0·70	19·203	47·499	49·259	75·615
	0·65	19·049	46·300	49·126	73·743
	0·60	18·841	44·727	48·896	72·646
	0·55	18·542	43·186	48·528	72·730
	0·50	18·128	42·098	47·969	73·791

for $a_0/a = b_0/b \geqslant 0{\cdot}7$. For $a_0/a = b_0/b < 0{\cdot}7$ the values determined by the analytical approach were rather high upper bounds and they are not included in the table. Similar considerations apply in the case of Table 2 which presents values of Ω_i for the orthotropic situation. Table 3 depicts eigenvalues for the isotropic rectangular plate ($b/a = \frac{2}{3}$ and $\frac{3}{2}$) determined analytically while Table 4 deals with the orthotropic plate. The situations

TABLE 2

Frequency coefficients of an orthotropic square plate with a free, oblique edge: (a) analytical solution, (b) finite element results

	$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
	1·00	19·984	43·471	51·188	79·509
	0·95	19·960	43·426	51·150	79·318
	0·90	19·882	43·267	51·019	79·207
(*a*)	0·85	19·733	43·021	50·801	78·901
	0·80	19·586	42·879	50·753	78·705
	0·75	19·382	42·691	50·757	78·327
	0·70	19·192	42·570	50·812	77·718
	1·00	19·990	43·495	51·209	79·568
	0·95	19·942	43·409	51·138	79·422
	0·90	19·834	43·224	50·992	78·985
(*b*)	0·85	19·664	42·980	50·817	78·454
	0·80	19·491	42·753	50·657	77·852
	0·75	19·311	42·491	50·458	76·670
	0·70	19·143	42·073	50·124	74·583
	0·65	18·976	41·336	49·587	72·366
	0·60	18·785	40·240	48·916	71·226
	0·55	18·535	39·012	48·241	71·542
	0·50	18·194	38·016	47·604	73·006

TABLE 3

Frequency coefficients of an isotropic, rectangular plate with a free, oblique edge (analytical results)

b/a	$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
	1·00	14·256	27·415	43·864	49·348
	0·95	14·245	27·394	43·851	49·320
	0·90	14·213	27·331	43·812	49·248
2/3	0·85	14·149	27·229	43·750	49·160
	0·80	14·048	27·110	43·724	49·093
	0·75	13·929	27·021	43·800	49·039
	0·70	13·823	26·967	43·909	48·973
	1·00	32·076	61·685	98·695	111·03
	0·95	32·052	61·636	98·665	110·97
	0·90	31·979	61·494	98·580	110·81
3/2	0·85	31·848	61·273	98·455	110·63
	0·80	31·664	61·080	98·342	110·74
	0·75	31·469	61·045	98·258	111·08
	0·70	31·277	61·079	98·160	111·15

of simply supported oblique edges are dealt with in Table 5 (isotropic plate) and Table 6 (orthotropic case).

The analysis of Tables 1 and 2 reveals the fact that when the free, oblique edge is introduced into the plate the fundamental frequency coefficient decreases slightly for $a_0/a = b_0/b \geqslant 0{\cdot}5$ while Ω_2 and Ω_4 decrease considerably, in the case of square isotropic and

TABLE 4

Frequency coefficients of an orthotropic, rectangular plate with a free oblique edge

b/a	$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
	1·00	30·229	63·909	79·509	115·17
	0·95	30·192	63·841	79·455	115·08
	0·90	30·078	63·594	79·270	115·54
2/3	0·85	29·867	63·321	78·864	113·11
	0·80	29·655	63·080	78·824	114·61
	0·75	29·376	62·974	78·626	114·84
	0·70	29·106	62·953	78·477	114·67
	1·00	14·818	26·487	43·471	44·926
	0·95	14·804	26·455	43·421	44·858
	0·90	14·755	26·348	43·342	44·817
3/2	0·85	14·650	26·115	43·109	44·709
	0·80	14·555	26·053	42·991	44·776
	0·75	14·408	25·918	42·915	44·824
	0·70	14·277	25·830	42·851	44·906

TABLE 5

Frequency coefficients of an isotropic, square plate with a simply supported oblique edge

$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
1·00	19·744	49·368	49·368	79·038
0·95	19·740	49·358	49·368	79·029
0·90	19·739	49·368	49·395	79·121
0·85	19·766	49·368	49·627	79·539
0·80	19·848	49·369	50·178	80·393
0·75	20·016	49·379	51·123	81·656
0·70	20·297	49·415	52·482	83·227
0·65	20·715	49·518	54·247	84·975
0·60	21·292	49·752	56·393	86·783
0·55	22·049	50·204	58·898	88·589
0·50	23·008	50·976	61·743	90·438

TABLE 6

Frequency coefficients of an orthotropic, square plate with a simply supported oblique edge

$a_0/a = b_0/b$	Ω_1	Ω_2	Ω_3	Ω_4
1·00	19·990	43·495	51·209	79·568
0·95	19·970	43·460	51·180	79·523
0·90	19·927	43·410	51·143	79·507
0·85	19·892	43·449	51·189	79·659
0·80	19·900	43·658	51·400	79·700
0·75	19·985	44·056	51·849	79·707
0·70	20·179	44·592	52·623	79·769
0·65	20·507	45·185	53·822	79·980
0·60	20·988	45·793	55·535	80·422
0·55	21·639	46·452	57·795	81·134
0·50	22·479	47·267	60·592	82·130

orthotropic plates. In the case of the rectangular plates investigated in the present study the variation of the values of Ω_i is very minor for $a_0/a = b_0/b \geqslant 0{\cdot}7$; see Tables 3 and 4.

As expected, the frequency coefficients experience considerable increments when the oblique edge is simply supported; see Tables 5 and 6. This is specially noticeable in the case of Ω_1, Ω_3 and Ω_4.

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