Journal of Sound and Vibration (2001) **248**(4), 585–596
doi:10.1006/jsvi.2001.3558, available online at http://www.idealibrary.com on IDEAL®

SUPPRESSION OF RANDOM VIBRATION IN PLATES USING VIBRATION ABSORBERS

R. G. JACQUOT

Department of Electrical Engineering, University of Wyoming, Box 3295 *University Station, Laramie, WY* 82071-3295, *USA*

(*Received* 14 *August* 2000, *and in final form* 8 *January* 2001)

In the work presented here, a method to predict the effectiveness of application of damped dynamic vibration absorbers to suppress stationary random vibration of rectangular simply supported plates is given. Numerical examples of two different spatial distributions of the random-in-time forcing function are explored and optimal absorber parameters are presented.

1. INTRODUCTION

Plate and plate-like structures occur in many common applications. A few of these are bridge and building structures, naval structures and pressure vessels. One application that initially motivated this work was the vibration of edge-supported printed wiring boards for electronic systems. These above-cited structures are often excited by random acoustic fields or random accelerations.

There is a wealth of literature dealing with the free and deterministically forced vibration of systems that are composed of combinations of simpler assemblies such as beams, plates, sprung masses, dashpots, etc. [1–10]. There is a very limited body of literature dealing with the random vibration of such composite systems. The effect of elastic constraints on the random vibration of damped linear structures was considered by Howell [11]. More recently, the random vibration of combined linear systems has been considered by Bergman and Nicholson [12] in which the normal mode method and the Green functions are employed to express the cross-correlation functions and cross-spectral density functions of the beam response. Still more recently, Kareem and Sun [13] have considered the problem of random vibration of a structure carrying a tank of sloshing fluid which is modelled as a series of parallel attached linear oscillators. The theory given adds an additional number of degrees-of-freedom to handle the additional attached oscillators. The solution to the resulting problem is then given as the usual lumped parameter eigen solution and does not build on the knowledge of the problem prior to the addition of the tank of fluid. The random vibration of a damped tapered beam carrying masses is treated in the work of Yadav *et al.* [14] wherein the first and second order statistics of the responses are calculated for a cantilever beam with a base excitation. The random vibration of damped, modified beam systems has recently been published and it is shown that random vibration of a cantilever beam can be optimally suppressed by employing a dynamic vibration absorber [15].

In this work the suppression of random vibrations of a rectangular plate by a dynamic vibration absorber will be considered. Absorber location, mass, stiffness and damping will be the variables investigated. At the outset, a number of simplifying assumptions will be

0022-460X/01/490585 + 12 $35.00/0

made. The driven system will be assumed to be a thin, uniform, rectangular, elastic plate with simple support along the edges. The forcing function will be the product of a deterministic spatial distribution and a stationary random function in time. This assumption obviates many convective turbulent pressure fields such as those present in boundary layer flows.

2. THEORY

Consider a simply supported, thin rectangular plate on the rectangular domain $0 < x < a$ and $0 < y < b$ which carries a dynamic vibration absorber at point (x_0, y_0) as illustrated in Figure 1. The plate is under the action of two forcing functions, the first being the externally applied spatially distributed force $w(t)g(x, y)$ and the second being $p(t)$, the point force transmitted to the plate at a point by the attached dynamic absorber. The appropriate equation of motion for the plate is

$$\nabla^4 u + \frac{\rho h}{D}\frac{\partial^2 u}{\partial t^2} = \frac{w(t)g(x, y)}{D} + \frac{p(t)}{D}\delta(x - x_0)\delta(y - y_0) \tag{1}$$

where the flexural rigidity of the plate D is defined as

$$D = \frac{Eh^3}{12(1 - \nu^2)} \tag{2}$$

and where E is the modulus of elasticity, ν is the Poisson ratio, h is the plate thickness and ρ is the material density.

Here it has been assumed that the externally applied forcing function is $w(t)g(x, y)$, where $g(x, y)$ is a deterministic function of x and y and $w(t)$ is a stationary random function of time. The boundary conditions on equation (1) are that the displacement and bending moment are zero at the edges of the plate. The solution to this problem can be expanded in

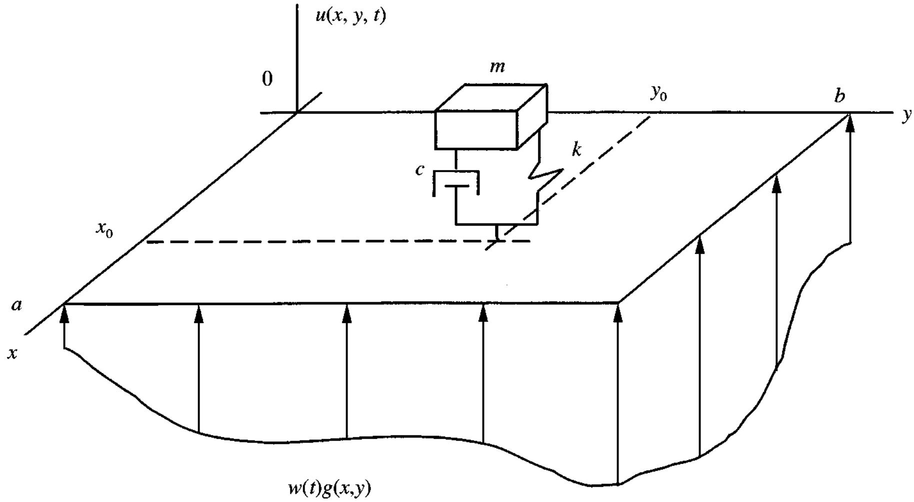

Figure 1. Simply supported plate carrying a dynamic absorber.

a two-dimensional Fourier series in the plate co-ordinates or

$$u(x, y, t) = \sum_{i,j=1,2,3,\ldots}^{\infty} q_{ij}(t) \sin\left(\frac{\mathrm{i}\pi x}{a}\right) \sin\left(\frac{\mathrm{j}\pi y}{b}\right). \tag{3}$$

Similarly, the spatial part of the forcing function can be expanded as

$$g(x, y) = \sum_{i,j=1,2,\ldots} a_{ij} \sin\left(\frac{\mathrm{i}\pi x}{a}\right) \sin\left(\frac{\mathrm{j}\pi y}{b}\right) \tag{4}$$

and the Dirac delta functions can also be expanded as

$$\delta(x - x_0)\delta(y - y_0) = \sum_{i,j=1,2,\ldots} b_{ij} \sin\left(\frac{\mathrm{i}\pi x}{a}\right) \sin\left(\frac{\mathrm{j}\pi y}{b}\right), \tag{5}$$

where the Fourier coefficients a_{ij} and b_{ij} are respectively

$$a_{ij} = \frac{4}{ab} \int_0^a \int_0^b g(x, y) \sin\left(\frac{\mathrm{i}\pi x}{a}\right) \sin\left(\frac{\mathrm{j}\pi y}{b}\right) \mathrm{d}y \, \mathrm{d}x, \tag{6}$$

$$b_{ij} = \frac{4}{ab} \sin\left(\frac{\mathrm{i}\pi x_0}{a}\right) \sin\left(\frac{\mathrm{j}\pi y_0}{b}\right). \tag{7}$$

Here the a_{ij} depend only on the spatial distribution of the random forcing function $g(x, y)$. If the expansions of relations (3), (4) and (5) are substituted into relation (1) and the Laplace transform is taken with respect to time, the result is a set of algebraic equations

$$\left[\left(\frac{\mathrm{i}\pi}{a}\right)^2 + \left(\frac{\mathrm{j}\pi}{b}\right)^2\right]^2 Q_{ij}(s) + \frac{\rho h}{D} s^2 Q_{ij}(s) = \frac{a_{ij}}{D} W(s) + \frac{b_{ij}}{D} P(s), \quad i, j = 1, 2, 3, \ldots. \tag{8}$$

If this is solved for the generalized co-ordinates $Q_{ij}(s)$ the result is

$$Q_{ij}(s) = \frac{1}{D} \frac{a_{ij} W(s) + b_{ij} P(s)}{[(\mathrm{i}\pi/a)^2 + (\mathrm{j}\pi/b)^2]^2 + (\rho h/D)s^2}, \tag{9}$$

then if both $W(s)$ and $P(s)$ were known then the s-domain motion of any point on the plate could be given as

$$U(x, y, s) = \frac{1}{D} \sum_{i,j=1}^{\infty} \frac{a_{ij} W(s) + b_{ij} P(s)}{[(\mathrm{i}\pi/a)^2 + (\mathrm{j}\pi/b)^2]^2 + (\rho h/D)s^2} \sin(\mathrm{i}\pi x/a) \sin(\mathrm{j}\pi y/b), \tag{10}$$

where $U(x, y, s)$ is the Laplace transform of $u(x, y, t)$ with respect to time. For a damped dynamic vibration absorber the relation between the motion of the point of attachment and the force transmitted to the plate at the point of attachment is

$$P(s) = -\,U(x_0, y_0, s) \frac{ms^2(cs + k)}{ms^2 + cs + k}. \tag{11}$$

The forcing function $P(s)$ in relation (10) can be eliminated by employing relation (11) to give

$$U(x, y, s) = \frac{1}{D} \sum_{i,j=1}^{\infty} \frac{\left(a_{ij} W(s) + b_{ij} \left[\frac{-ms^2(cs+k)}{ms^2+cs+k}\right] U(x_0, y_0, s)\right)}{[(i\pi/a)^2 + (j\pi/b)^2]^2 + (\rho h/D)s^2} \sin(i\pi x/a)\sin(i\pi y/b). \tag{12}$$

This gives the plate deflection at an arbitrary point (x, y) in terms of the forcing function $W(s)$ and the motion at the point of attachment (x_0, y_0). This relation must hold at the point of attachment (x_0, y_0) which gives

$$U(x_0, y_0, s)\left[1 + \frac{ms^2(cs+k)}{ms^2+cs+k} \frac{1}{D} \sum_{i,j=1}^{\infty} \frac{b_{ij} \sin(i\pi x_0/a)\sin(j\pi y_0/b)}{[(i\pi/a)^2 + (j\pi/b)^2]^2 + (\rho h/D)s^2}\right]$$

$$= \frac{W(s)}{D} \sum_{i,j=1}^{\infty} \frac{a_{ij} \sin(i\pi x_0/a)\sin(j\pi y_0/b)}{[(i\pi/a)^2 + (j\pi/b)^2]^2 + (\rho h/D)s^2}. \tag{13}$$

This can be solved for a transfer function between $W(s)$ and $U(x_0, y_0, s)$ to yield

$$\frac{U(x_0, y_0, s)}{W(s)} = \frac{\frac{1}{D} \sum_{i,j=1}^{\infty} \frac{a_{ij}}{[(i\pi/a)^2 + (j\pi/b)^2]^2 + (\rho h/D)s^2} \sin(i\pi x_0/a)\sin(j\pi y_0/b)}{1 + \frac{ms^2(cs+k)}{ms^2+cs+k}\left(\frac{1}{D}\right) \sum_{i,j=1}^{\infty} \frac{b_{ij}}{[(i\pi/a)^2 + (j\pi/b)^2]^2 + (\rho h/D)s^2} \sin(i\pi x_0/a)\sin(j\pi y_0/b)}, \tag{14}$$

where a_{ij} and b_{ij} are given by relations (6) and (7). The ijth radian natural frequency will be denoted as ω_{ij} which is

$$\omega_{ij} = \left[\left(\frac{i\pi}{a}\right)^2 + \left(\frac{j\pi}{b}\right)^2\right]\sqrt{\frac{D}{\rho h}}. \tag{15}$$

Expression (14) may be solved for $U(x_0, y_0, s)$ and then substituted into relation (12) to give the transform of the motion at any point which is $U(x, y, s)$. The absorber natural frequency is defined as

$$\omega_a = \sqrt{\frac{k}{m}}. \tag{16}$$

Now it is appropriate to define some non-dimensional parameters starting with the natural frequency ratio

$$\gamma_{ij} = \frac{\omega_{ij}}{\omega_{11}}. \tag{17}$$

The mass ratio μ is defined as the ratio of the absorber mass to that of the plate or

$$\mu = \frac{m}{\rho hab}. \tag{18}$$

The tuning ratio T is the ratio of the absorber frequency to the first natural frequency of the plate or

$$T = \frac{\omega_a}{\omega_{11}} \tag{19}$$

and the absorber damping ratio is defined as

$$\zeta = \frac{c}{(2\sqrt{km})}, \tag{20}$$

The plate aspect ratio R is defined as

$$R = \frac{b}{a}. \tag{21}$$

A new normalized Laplace transform p variable can now be defined as

$$p = \frac{s}{\omega_{11}}. \tag{22}$$

Using all these non-dimensional variables relation (14) may be written as

$$\frac{U(x_0, y_0, p)}{W(p)} = \frac{\dfrac{a^4R^4}{\pi^4(1+R^2)^2D}\displaystyle\sum_{i,j=1}^{\infty}\frac{a_{ij}\sin(i\pi x_0/a)\sin(j\pi y_0/Ra)}{\gamma_{ij}^2+p^2}}{1+4\mu\left(\dfrac{p^2(2\zeta Tp+T^2)}{p^2+2\zeta Tp+T^2}\right)\displaystyle\sum_{i,j=1}^{\infty}\frac{\sin^2(i\pi x_0/a)\sin^2(j\pi y_0/Ra)}{\gamma_{ij}^2+p^2}}, \tag{23}$$

Since the point of interest here is the stationary random motion of the plate, it is appropriate to calculate the frequency response function by letting $s = j\omega$ or in this case where the variable s has been normalized, it is appropriate to let $p = jf$ where the normalized frequency is defined as $f = \omega/\omega_{11}$ to give

$$\frac{U(x_0, y_0, jf)}{W(jf)} = \frac{\dfrac{a^4R^4}{\pi^4(1+R^2)^2D}\displaystyle\sum_{i,j=1}^{\infty}\frac{a_{ij}\sin(i\pi x_0/a)\sin(j\pi y_0/Ra)}{\gamma_{ij}^2-f^2}}{1-4\mu\left(\dfrac{f^2(j2\zeta Tf+T^2)}{T^2-f^2+j2\zeta Tf}\right)\displaystyle\sum_{i,j=1}^{\infty}\frac{\sin^2(i\pi x_0/a)\sin^2(j\pi y_0/Ra)}{\gamma_{ij}^2-f^2}}. \tag{24}$$

If this frequency response is termed $M(jf)$ and the forcing function $w(t)$ has power spectral density $S_w(f)$ the spectral density of the point of attachment is then

$$S_u(x_0, y_0, f) = |M(jf)|^2 S_w(f). \tag{25}$$

The mean square motion at point (x_0, y_0) can then be given by integrating the power spectrum of relation (25) to give

$$\sigma_u^2(x_0, y_0) = \frac{\omega_{11}}{2\pi}\int_{-\infty}^{\infty} |M(jf)|^2 S_w(f)\,\mathrm{d}f. \tag{26}$$

3. EXAMPLE 1

Consider the case where $g(x, y) = 1$ or that the forcing function is uniform in space and stationary and random in time. Assume that the plate has an aspect ratio of unity. In this case the a_{ij} Fourier coefficient are given by relation (6) to be

$$a_{ij} = \frac{16}{ij\pi^2}, \quad i,j = 1, 3, 5, \ldots \tag{27}$$

and zero otherwise. With this, relation (24) becomes

$$\frac{U(x_0, y_0, jf)}{W(jf)} = \frac{\dfrac{16a^4R^4}{\pi^6(1+R^2)^2D}\displaystyle\sum_{i,j=1}^{\infty}\frac{\sin(i\pi x_0/a)\sin(j\pi y_0/Ra)}{ij(\gamma_{ij}^2 - f^2)}}{1 - 4\mu\left(\dfrac{f^2(\mathrm{j}2\zeta Tf + T^2)}{T^2 - f^2 + \mathrm{j}2\zeta Tf}\right)\displaystyle\sum_{i,j=1}^{\infty}\frac{\sin^2(i\pi x_0/a)\sin^2(j\pi y_0/Ra)}{\gamma_{ij}^2 - f^2}}. \tag{28}$$

Assume that the absorber has been attached at the center of the plate so that $x_0/a = y_0/b = \frac{1}{2}$. For a tuning ratio $T = 1$, an aspect ratio of unity, a mass ratio $\mu = 0{\cdot}2$ and for a $w(t)$ with a constant power spectral density S_w the motion power spectral density $S_u(f)$ has been evaluated and is illustrated in Figure 2 in non-dimensional form for several values

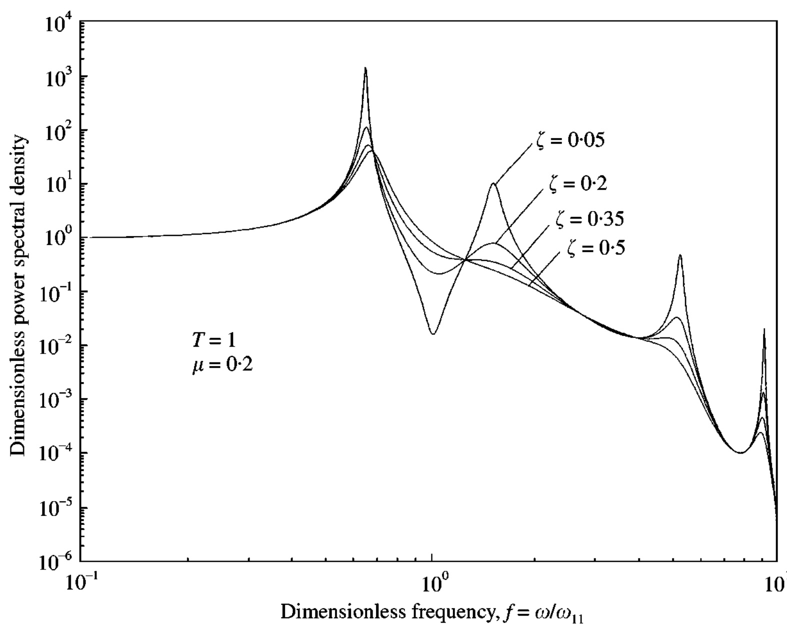

Figure 2. Dimensionless motion power spectral density for $g(x, y) = 1$, $R = T = 1$, $\mu = 0{\cdot}2$ and $x_0 = y_0 = a/2$ for several values of damping ratio.

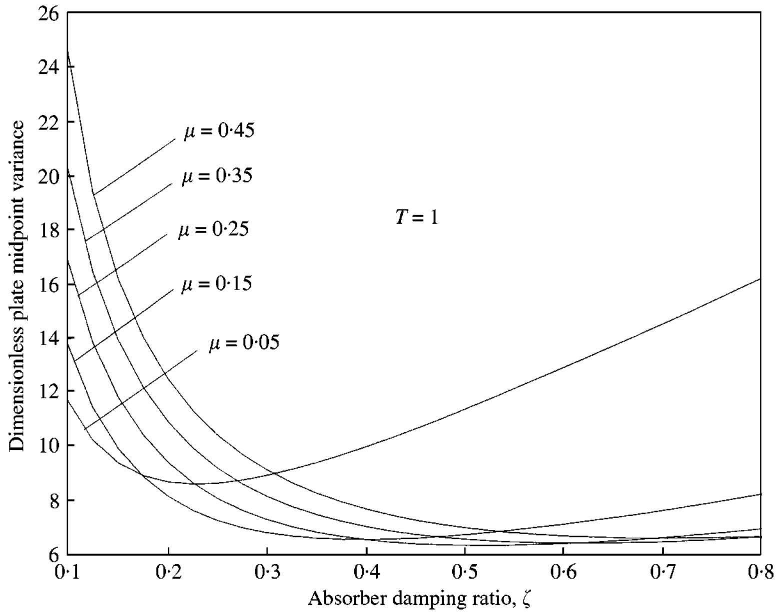

Figure 3. Dimensionless plate midpoint mean square motion for various mass ratios as a function of absorber damping ratio for the attachment of the absorber to the plate midpoint for $R = T = 1$.

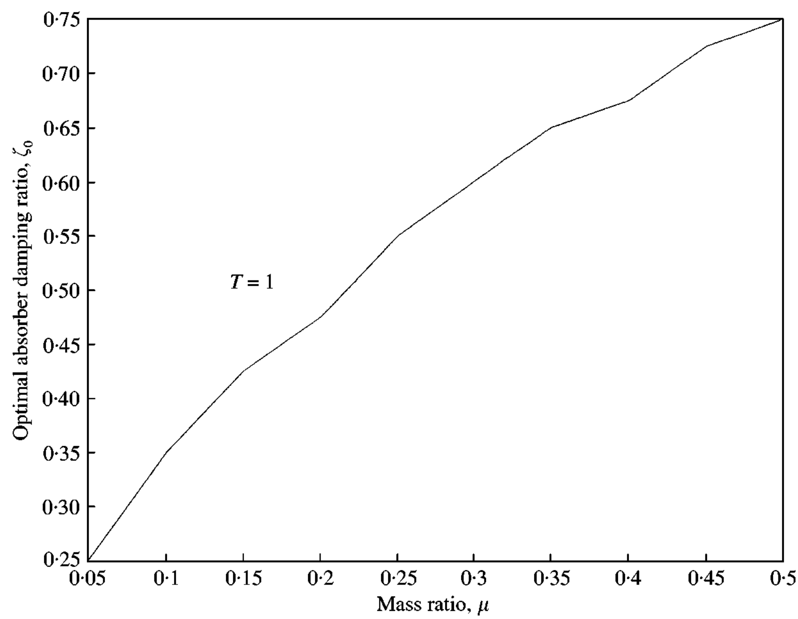

Figure 4. Optimal absorber damping ratio as a function of mass ratio for $g(x, y) = 1$, $R = T = 1$, $\mu = 0{\cdot}2$ and $x_0 = y_0 = a/2$.

of the absorber damping ratio ζ. If the integral of equation (26) is evaluated for various damping ratios and mass ratios the mean square motions of Figure 3 result. It is clear that for each particular mass ratio there is a value of damping ratio ζ that yields minimum mean square motion. The values of damping ratio which minimize the mean square motion for

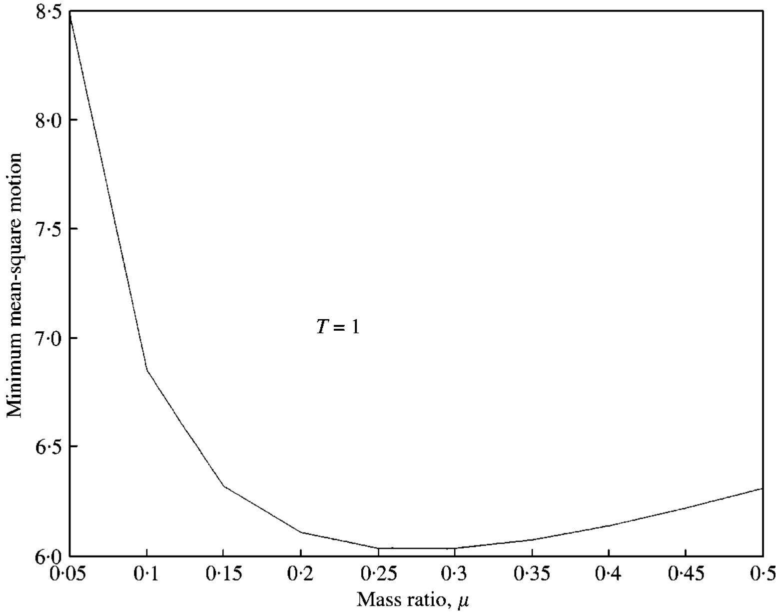

Figure 5. Mean-square midpoint motion as a function of mass ratio for optimal values of absorber damping ratio for $g(x, y) = 1$, $R = T = 1$ and $x_0 = y_0 = a/2$.

the various mass ratios in Figure 3 have been evaluated numerically and are shown in Figure 4 as a function of the mass ratio. The irregularity of the curve results from the inaccuracy of numerically finding the very shallow minima of the curves of Figure 3. The minimum mean square motion for a tuning ratio of $T = 1$ as a function of mass ratio is illustrated in Figure 5 and it is apparent that for a given tuning ratio there is a best mass ratio which in a variety of cases is approximately between 25% and 30% of the plate mass. Figure 4 then reveals that the best absorber damping ratio for this mass ratio is about $\zeta = 0{\cdot}45$.

It should now be noted that this example is naive from two points of view. The first is that excitation is never exactly symmetric, thus the modes with one or both of the indices, being even, will participate in the motion. The second is that the installation location of the absorber, (x_0, y_0), can never be established exactly at the center of the plate and this non-exact placement will cause coupling of energy into the modes with one or both of the indices being even. Although these modes are not expressly excited by the random forcing function, they can be excited by the absorber when its installation is not on a nodal line associated with a particular mode. These phenomena, called spillover in the distributed parameter automatic control literature [16], reflect the flow of energy from the odd modes driven by the external forcing function into the even modes by virtue of the intrinsic feedback loop created by the dynamic absorber.

In light of these results it would be interesting to examine the case, where the absorber is attached at a point not on nodal lines of the first 81 modes such as $x_0 = y_0 = 0{\cdot}55a$ for an aspect ratio of unity. The motion power spectral density for the point of absorber attachment is illustrated in Figure 6. From the resonant peaks present, for light damping it can be noted that the modes for even values of the modal indices are now participating due to the change of location of the absorber. This is due to the flow of energy from the forcing function through the odd modes which excites the absorber which due to its location excites

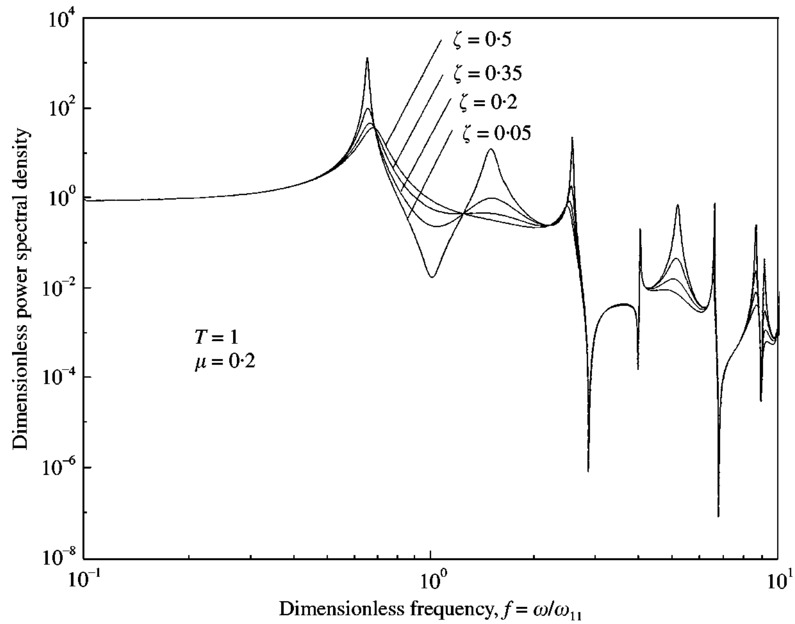

Figure 6. Dimensionless motion power spectral density for $g(x, y) = 1$, $R = T = 1$, $\mu = 0{\cdot}2$ and $x_0 = y_0 = 0{\cdot}55a$ for several values of damping ratio.

the even modes. For this new attachment point and fixed tuning and mass ratios, there are optimal values of the absorber damping parameter similar to those calculated for Figures 3–5.

4. EXAMPLE 2

Consider now the case where the spatial distribution of the forcing function is such that all the modes of the plate are excited by $w(t)g(x, y)$. As a simple example, let the spatial distribution be

$$g(x, y) = xy. \tag{29}$$

The Fourier coefficients for this distribution are given by relation (6) to be

$$a_{ij} = \frac{4Ra^2}{ij\pi^2}\cos(\mathrm{i}\pi)\cos(\mathrm{j}\pi), \quad \text{for all } i, j. \tag{30}$$

With these Fourier coefficients the frequency response function between the forcing function and the response at the point of attachment is

$$\frac{U(x_0, y_0, jf)}{W(jf)} = \frac{\dfrac{4a^4R^5}{\pi^6(1+R^2)^2D}\displaystyle\sum_{i,j=1}^{\infty}\frac{\cos(\mathrm{i}\pi)\cos(\mathrm{j}\pi)\sin(\mathrm{i}\pi x_0/a)\sin(\mathrm{j}\pi y_0/Ra)}{ij(\gamma_{ij}^2 - f^2)}}{1 - 4\mu\left(\dfrac{f^2(\mathrm{j}2\zeta Tf + T^2)}{T^2 - f^2 + \mathrm{j}2\zeta Tf}\right)\displaystyle\sum_{i,j=1}^{\infty}\frac{\sin^2(\mathrm{i}\pi x_0/a)\sin^2(\mathrm{j}\pi y_0/Ra)}{\gamma_{ij}^2 - f^2}}. \tag{31}$$

If the aspect ratio $R = 1$, the mass ratio $\mu = 0{\cdot}2$ and the tuning ratio $T = 1$, all the modes are excited by the forcing function. A good choice for the location of the dynamic absorber

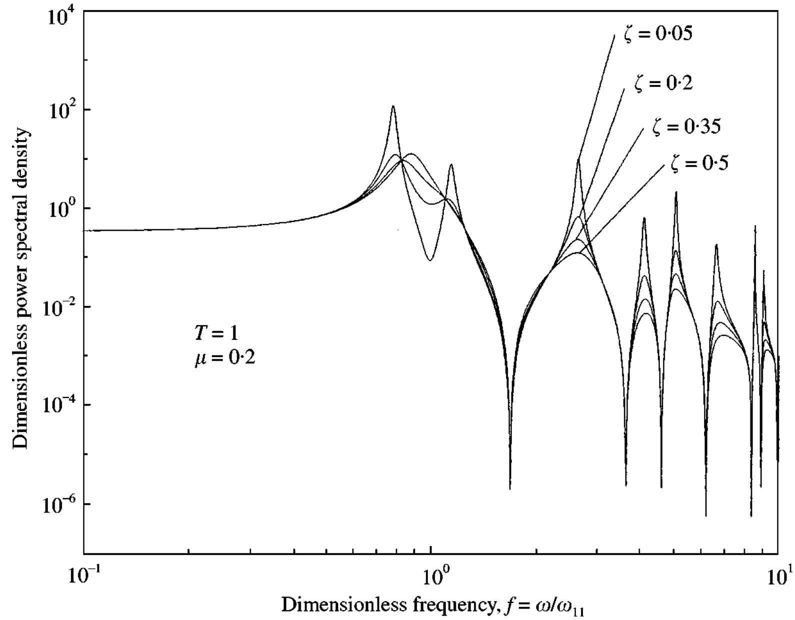

Figure 7. Dimensionless motion power spectral density for $g(x, y) = xy$, $R = T = 1$, $\mu = 0{\cdot}2$ and $x_0 = y_0 = 0{\cdot}77a$ for several values of damping ratio at the point of attachment.

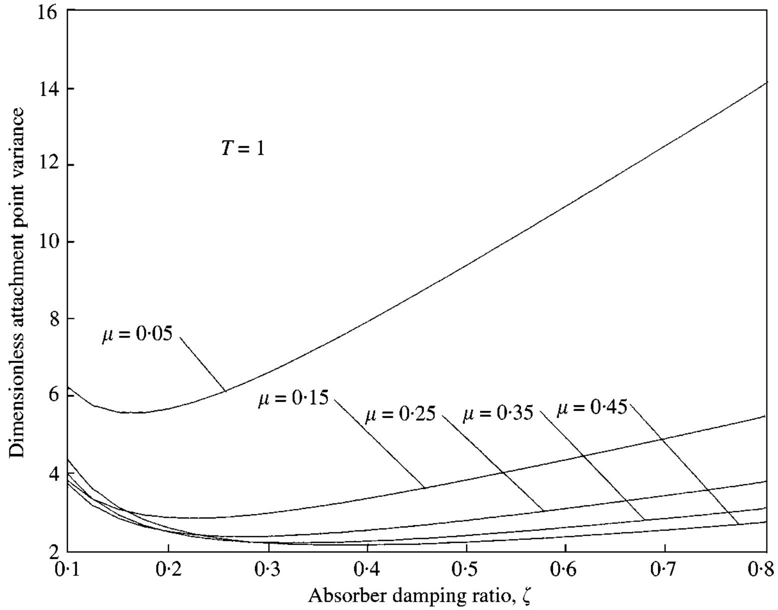

Figure 8. Dimensionless mean square motion for various mass ratios as a function of absorber damping ratio for the attachment of the absorber to the point $x_0 = y_0 = 0{\cdot}77a$ and $R = T = 1$.

is somewhere near where maximum motion will take place. Given that the spatial force distribution is $g(x, y) = xy$ and that the plate edges are restrained, a good location would be $x_0 = y_0 = 0{\cdot}77a$ such that it is driven by all the modes and in turn can dissipate the energy

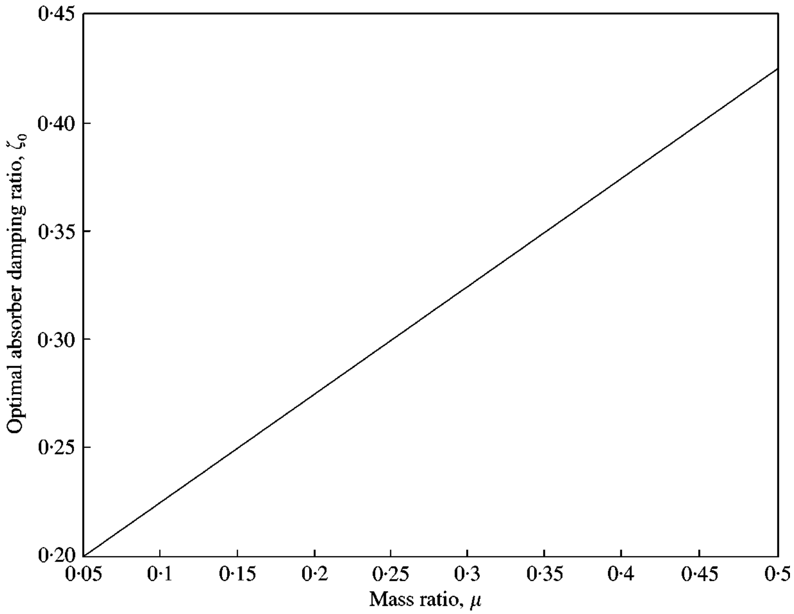

Figure 9. Optimal absorber damping ratio as a function of mass ratio for $g(x, y) = xy$, $R = T = 1$, $\mu = 0{\cdot}2$ and $x_0 = y_0 = 0{\cdot}77a$.

from all the modes. For this case the dimensionless power spectral density is illustrated in Figure 7 and it is clear that the modes are all participating in the motion. As in the previous example, once the absorber location is chosen the parameters may be chosen optimally to minimize the motion of the point of attachment. The mean-square motion of the point of attachment for a number of mass ratios as a function of the damping ratio is illustrated in Figure 8 and the optimal damping ratio as a function of mass ratio is illustrated in Figure 9. The linear function is somewhat surprising but is the result of a numerical minimum finding procedure.

5. CONCLUSION

In the work presented here dynamic vibration absorbers have been shown to suppress randomly forced vibrations significantly and can do so in a way which minimizes the mean-square motion at the point of attachment of the absorber. It is also possible to choose the absorber parameters so as to minimize the mean-square motion at some other point on the plate but the computations involved will be lengthy but can be accomplished.

Several interesting problems for future study are suggested by this investigation and by the reviewers of this paper. The first is that of minimization of the average mean-square motion over the whole domain of the plate, a problem which should be first explored in the one-dimensional context of a string or a beam. Another interesting problem for future work would be the optimal location(s) of one or more dynamic vibration absorbers on the domain of a distributed parameter system. This also would initially best be carried out on systems with only one spatial variable such as a string or beam.

ACKNOWLEDGMENT

The author would like to thank the reviewers for their comments that led to a considerable improvement in the manuscript.

REFERENCES

1. W. J. DUNCAN 1941 *Philosophical Magazine* **32**, 401–409. The admittance method for obtaining the natural frequencies of systems.
2. R. E. D. BISHOP and D. C. JOHNSON 1960 *The Mechanics of Vibration*. London: Cambridge University Press.
3. D. YOUNG 1948 *Transactions of the American Society of Mechanical Engineers*: *Journal of Applied Mechanics* **15**, 65–72. Vibration of a beam with a concentrated spring, mass and dashpot.
4. D. YOUNG 1952 *Proceedings of the First US National Congress of Applied Mechanics*, New York: American Society of Mechanical Engineers, 91–96. Theory of dynamic vibration absorbers for beams.
5. J. T. WISSENBERGER 1968 *Journal of Applied Mechanics* **35**, 327–332. Effect of local modifications on vibration characteristics of linear systems.
6. R. G JACQUOT and W. SOEDEL 1970 *Journal of the Acoustical Society of America* **47**, 1354–1358. Vibration of elastic surface systems carrying dynamic elements.
7. R. J. POMAZAL and V. W. SNYDER 1971 *American Institute of Aeronautics and Astronautics Journal* **9**, 2216–2221. Local modifications of damped linear systems.
8. E. H. DOWELL 1972 *American Society of Mechanical Engineers Journal of Applied Mechanics* **39**, 727–732. Free vibration of an arbitrary structure in terms of component modes.
9. J. HALLQUIST and V. W. SNYDER 1973 *Journal of Applied Mechanics* **40**, 312–313. Linear damped vibratory structures with arbitrary support conditions.
10. R. G. JACQUOT 1976 *Journal of Sound and Vibration* **49**, 345–351. The response of a system when modified by the attachment of an additional sub-system.
11. L. J. HOWELL 1975 *American Institute of Aeronautics and Astronautics Journal* **13**, 685–687. Effects of constraint modification on random vibration of damped linear structures.
12. L. A. BERGMAN and J. W. NICHOLSON 1985 *Journal of Sound and Vibration* **103**, 225–236. Stationary response of combined linear dynamic systems to stationary excitation.
13. A. KAREEM and W.-J. SUN 1987 *Journal of Sound and Vibration* **119**, 389–408. Stochastic response of structures with fluid-containing appendages.
14. D. YADAV, S. KAMLE and S. TALUKDAR 1996 *Journal of Sound and Vibration* **207**, 151–173. Transverse vibration of multimass loaded variable section beams under random excitation.
15. R. G. JACQUOT 2000 *Journal of Sound and Vibration* **234**, 441–454, Random vibration of damped modified beam systems.
16. R. G. JACQUOT 1986 *Journal of the Astronautical Sciences* **34**, 133–146. Spillover Minimization in the control of self-adjoint distributed systems.