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BLIND SOURCE SEPARATION: A TOOL FOR ROTATING MACHINE MONITORING BY VIBRATIONS ANALYSIS?

G. GELLE AND M. COLAS

Laboratoire d'Automatique et de Microélectronique (LAM), Université de Reims Champagne-Ardenne, B.P. 1039 *F*-51687 *REIMS Cedex* 2, *France. E-mail: guillaume.gelle@univ-reims.fr*

AND

C. SERVIERE

Laboratoire des Images et des Signaux (LIS), ENSIEG, BP 46, 38402 *Saint Martin d'Hères cedex, France*

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Blind source separation (BSS) is a general signal processing method, which consists of recovering, from a finite set of observations recorded by sensors, the contributions of different physical sources independently from the propagation medium and without any *a priori* knowledge on the sources. Such methods are attractive for the monitoring or the diagnosis of mechanical systems. It is shown that BSS allows the vibratory information generated from a single rotating machine working in a noisy environment to be recovered by freeing the sensor signal from the contribution of other working machines. In that way, BSS can be used as a pre-processing step to rotating machine fault detection and diagnosis. In this paper, two possible approaches to solve the BSS problem of rotating machine signals are compared; that is, the temporal or frequential approach. The first method developed initially for temporally white signals is used in an experimental context and it is shown that the results are comparable to the frequential domain approach specially developed for rotating machine signals. These two approaches are tested on real signals from a mechanical testing bench, and the implementations of the different methods as well as their performances are discussed. An example to bearing fault detection is given in the final part, to illustrate the potential of this approach as a pre-processing step to improve the diagnosis.

1. INTRODUCTION

Blind source separation (BSS) is a general signal processing method, which consists of recovering, from a finite set of observations recorded by sensors, the contributions of different physical sources independently from the propagation medium and without any *a priori* knowledge on the sources. These methods have been successfully used in many fields of applied sciences and engineering including medicine, telecommunications, audio processing, noise reduction or data processing [1].

Until now, BSS methods have seldom been used for the monitoring and diagnosis of mechanical system, such as the signals from rotating machines. The existing contributions to this field include references [2–5] for a blind signal separation purpose, and also reference [6] for data analysis. However, BSS is a promising tool for non-destructive monitoring because the signals recorded by sensors in an industrial application are always disrupted by the environment. (For example, ambient noise, other mechanical systems.) Using BSS as

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a pre-processing step would enable the specific signature of each rotating system to be used for diagnosis, thus isolating them from interference from the environment.

The purpose of this paper is the application of BSS to rotating machine signals separation and the comparison between temporal [4] and frequential [3] approaches for convolutive mixtures. First, this comparison is made on two synthetic signals following a model of gearbox vibrations [7] artificially mixed using a known transfer system. This preliminary stage shows the potential of BSS for rotating machine signals when the assumption of linear mixing is verified. After that, BSS was applied to real signals from a test bench with two motors fixed to the same structure. The objective is to extract the signature of each machine from each sensor, which would allow an improvement in the fault detection method and system diagnosis. Finally, an example of bearing fault detection was used to illustrate the performance of BSS approaches and the capability, the advantages and the drawbacks of temporal and frequential BSS algorithms for this framework are also discussed.

2. PRINCIPLE OF BSS

2.1. GENERAL CONCEPT

Blind source separation is a class of signal processing methods by which unobserved signals, also called sources, are recovered from the observation of several mixtures of them. Typically, the observations are obtained as the output of a set of sensors (antenna), where each sensor receives a different combination of source signals. The adjective “blind” indicates that the source signals are not observed and also that no information is available about the mixture. This type of approach is potentially most useful when it is impossible to model the transfer from the sources to the sensors. The lack of knowledge about the mixture and the sources is compensated by assuming the mutual independence of the sources.

This assumption allows exploitation of the spatial diversity provided by many sensors and is the fundamental basis of BSS. The general model of BSS is shown in Figure 1, which assumes the existence of m statistically independent signals $X(n) = [x_1(n), \ldots, x_m(n)]$ and the observation of at least m mixtures $Y(n) = [y_1(n), \ldots, y_m(n)]$ such as

$$Y(n) = f(X(n), X(n-1), \ldots, X(0)) + B(n), \tag{1}$$

where $B(n) = [b_1(n), \ldots, b_m(n)]$ denotes an additive noise which can be Gaussian or not.

The solution consists of finding an estimate $S(n)$ of the sources $X(n)$ by adapting an unknown separating function which leads to independence of $S(n)$.

In the most general case these functions are non-linear with respect to the sources as well as to several time lags. Nevertheless, in many fields such as telecommunications or biomedicine the mixture is commonly assumed to be a linear, time-independent combination of the sources. This case of a linear memoryless mixing system is also called

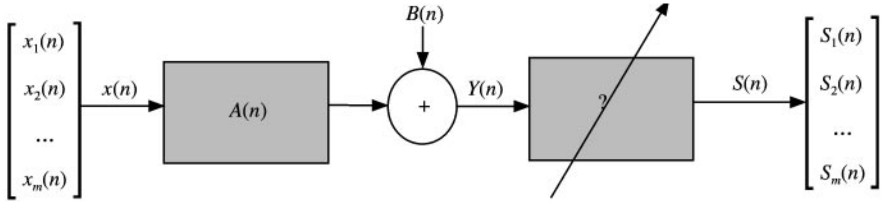

Figure 1. BSS general scheme.

instantaneous mixture. Mechanical systems are one example in which an instantaneous assumption does not always hold true due to the transfer delays of the vibrations through the structures. Usually, the model used for this application is a linear time-dependent mixture, called a convolutive mixture. However, an instantaneous model can hold when the structure under investigation has a high rigidity and a small size in order to consider the transmission delays in the mechanical structure to be negligible compared to the sampling period [5].

2.2. THE INSTANTANEOUS MIXING MODEL

For instantaneous mixtures and if a noise-free case is assumed, the general model (1) becomes

$$\mathbf{Y}(n) = \mathbf{A}\mathbf{X}(n), \tag{2}$$

where both $\mathbf{A}$ and $\mathbf{X}$ are unknown. This instantaneous mixing model is also called an ICA model (independent component analysis model) related to the aim of the problem; i.e., to find a linear transformation, which relies on independent components (sources) contributing to the observations of a mixture of them.

The aim of separation or ICA is to estimate a separating matrix $\mathbf{C}$ whose outputs are

$$\mathbf{S}(n) = \mathbf{C}\mathbf{Y}(n) = \mathbf{C}\mathbf{A}\mathbf{X}(n) \tag{3}$$

to give an estimate of the vector $\mathbf{X}(n)$. The ideal result is obtained with $\mathbf{C} = \mathbf{A}^{-1}$, but it is not possible to realize this without any assumptions. The assumption of independent components allows the estimation of the matrix $\mathbf{C}$, such that the product $\mathbf{CA}$ is equal to a diagonal matrix $\mathbf{D}$ up to a permutation matrix.

2.3. THE CONVOLUTIVE MIXING MODEL

The general model of a convolutive mixture can be represented as follows.

Each $A_{ij}(z)$ represents the linear transfer function from the ith source to the jth sensor and is given by $A_{ij}(z) = \sum_{i=0}^{\infty} A_{ij}(l) . z^{-1}$. The whole mixing system can be summed up as:

$$\mathbf{A}(z) = \begin{bmatrix} A_{11}(z) & \dots & A_{1m}(z) \\ \vdots & \ddots & \vdots \\ A_{k1}(z) & \dots & A_{km}(z) \end{bmatrix}.$$

Thus, the observations can be written as a convolution between the sources and the mixing process which is usually given as

$$\mathbf{Y}(n) = [\mathbf{A}(z)]\, \mathbf{X}(n) + \mathbf{B}(n), \tag{4}$$

where z^{-1} is both the backward-shift operator, i.e., $z^{-1}\mathbf{X}(n) = \mathbf{X}(n-1)$ as well as the complex variable in the z transform. So the aim of separation is to estimate a stable inverse system of $\mathbf{A}(z)$, i.e., a filter such as

$$\mathbf{S}(n) = [\mathbf{C}(z)\, \mathbf{A}(z)]\, \mathbf{Y}(n).$$

2.4. INDETERMINACIES AND UNIQUENESS

Intrinsically, the BSS problem is confronted with two inherent ambiguities.

First, it is impossible to know the original labelling of the sources, due to the insensitivity of the mathematical independence to the permutation of the sources. Assuming that $\mathbf{P}$ is a permutation matrix, the noise-free case ($\mathbf{B}(n) = 0$) can be written as

$$\mathbf{Y} = [\mathbf{AP}^{-1}]\,\mathbf{PX},$$

where the elements of $\mathbf{PX}$ are the permuted original sources and the mixing matrix $\mathbf{AP}^{-1}$ is a new mixing matrix estimated by the BSS algorithm.

The second indeterminacy is that it is impossible to identify uniquely the sources due to the insensitivity of the mathematical independence to a scaling factor applied on the sources. Hence,

$$\mathbf{Y}(n) = [\mathbf{AD}^{-1}]\,\mathbf{DX}(n).$$

So, for an instantaneous mixture, any scalar multiplier in one of the sources x_i can be cancelled by dividing the corresponding column of matrix $\mathbf{A}$. So a single solution does not exist but a class of solutions to which the independent sources belong. For a convolutive mixture, this scaling indeterminacy becomes a filtering indeterminacy and equation (1) does not define the filter $\mathbf{A}(z)$ uniquely (i.e., they are not identifiable) but to a linear filtering $\mathbf{D}(z)$.

The non-uniqueness of the BSS results can pose many problems to the user, especially in monitoring or diagnosis purpose. This difficulty will be tackled in the next part of this paper.

3. BSS OF INSTANTANEOUS MIXTURES

From the basic ICA model (2), it is necessary to recover X from Y by identifying a linear transformation such as

$$\mathbf{CA} = \mathbf{DP}. \tag{5}$$

3.1. SECOND ORDER INADEQUACY

The mixing matrix $\mathbf{A}$ can be expressed as a product of three matrices by singular value decomposition (SVD) as

$$\mathbf{A} = \mathbf{V}\boldsymbol{\Delta}^{1/2}\boldsymbol{\Pi}, \tag{6}$$

where $\mathbf{V}$ and $\boldsymbol{\Pi}$ are two unitary matrices, and $\boldsymbol{\Delta}$ is a diagonal matrix.

To understand the inadequacy of second order statistics for finding a transformation $\mathbf{C}$ for checking equation (5) independent sources of unit variance are considered. The covariance matrix of the observation Y is given by

$$\mathbf{R}_\mathbf{Y} = \mathrm{E}[\mathbf{YY}^\mathrm{H}] = \mathrm{E}[(\mathbf{AX})(\mathbf{AX})^\mathrm{H}] = \mathbf{AA}^\mathrm{H} = \mathbf{V}\boldsymbol{\Delta}\mathbf{V}^\mathrm{H}, \tag{7}$$

where the superscript denotes the Hermitian transposition.

Consequently, $\mathbf{R}_\mathbf{Y}$ does not depend on the rotation matrix $\boldsymbol{\Pi}$, which is needed to identify matrix $\mathbf{A}$.

In short, the BSS problem cannot be solved by using only second order statistics because independence is a stronger condition than uncorrelation. To estimate the missing unitary matrix $\mathbf{\Pi}$, it is necessary to resort to an independence criterion using the spatial diversity provided by the ICA model.

3.2. MEASURING INDEPENDENCE

A mathematical definition of independence is given by the probability theory. Random variables are said to be independent if their joint probability density function (p.d.f.) is equal to the product of their respective marginal densities, i.e., assume a $m \times 1$ random vector $X = [X_1, X_2, \ldots, X_m]$ with a multivariate probability density function $p(x)$, then independence allows $p(x)$ to be factorized as

$$p(x) = p_1(x_1) p_2(x_2) \ldots p_m(x_m). \tag{8}$$

In other words, the p.d.f. of one random variable x_i is unaffected by the observation of the other variables of the vector $\mathbf{X}$. This property allows the simplification of the problem because it makes the calculation of the statistics much simpler; hence, it is then possible to use a one-dimensional marginal p.d.f. instead of performing the calculation on the multi-dimensional joint p.d.f. Nevertheless, since definition (8) involves the p.d.f. of the random variable, measuring independence can be difficult. To circumvent this difficulty, it is possible to express independence in terms of random variables by the way of cumulants or generalized moments. So the following relations can be obtained for the two independent variables X_i and X_j.

R1: "two signals are statistically independent, if all their cross-cumulants (at any order k) are equal to zero". [8]

$$Cum_k(X_i, X_j) = 0, \tag{9}$$

where Cum_k denotes the cumulant operator at order k. This property makes possible the derivation of empirical contrasts as in section 3.1.

R2: for any function f and g (allowing the calculus of mathematical expectation),

$$\mathrm{E}\{f(X_i)g(X_j)\} - \mathrm{E}\{f(X_i)\}\,\mathrm{E}\{g(X_j)\} = 0 \quad \text{for } i \neq j. \tag{10}$$

This property will be used to approximate independence in section 3.1.

Once again, it can be seen that independence is a stronger condition than uncorrelation for which these are only

$$\mathrm{E}\{X_i, X_j\} - \mathrm{E}\{X_i\}\,\mathrm{E}\{X_j\} = 0 \quad \text{for } i \neq j. \tag{11}$$

There is a special case for which uncorrelation is equivalent to independence, which is for random variables with Gaussian joint p.d.f. This case explains why Gaussian variables are inadequate to solve ICA problem. In fact, the Darmois theorem [9] shows that separation can be achieved only if one more than source is Gaussian.

Although uncorrelation does not imply independence, it can be used as the first step to reduce the ICA problem. Actually, if the sources $\mathbf{X}$ have unit variance and are spatially white, then if a whitening matrix $\mathbf{W}$ is assumed such as

$$\mathbf{Z} = \mathbf{WY} = \mathbf{WAX} \tag{12}$$

then, the product **WA** is necessarily a rotation matrix because it connects two spatially white vectors.

Such a whitening matrix **W** can be obtained by Choleski decomposition or by principal component analysis (PCA), and allows a whitened (or sphered) version (Z) of the observation vector **Y** to be obtained.

After this step, equation (12) shows that the ICA problem is now reduced to the estimation of a rotation matrix leading to independent sources. Usually, this unknown rotation matrix is obtained by maximization or minimization of an objective function also called the contrast function which implements independence.

3.2.1. *Contrast functions*

Contrast functions for BSS were first introduced by Comon [10] in 1994. A contrast function applied to BSS is a function of a probability distribution of the separating system (i.e., **CY**) which has the property that it reaches its minimum (or maximum) value when the source separation is achieved. Of course such a function must take the indeterminacies inherent to the BSS into account.

Many different approaches exist to develop contrast functions according to the prior knowledge about the model or about the sources (for example, some knowledge about the distribution, some moments, etc.), but most of them can be derived from the maximum likelihood (ML) principle given in reference [11]:

$$\hat{\mathbf{A}} = \max_{\mathbf{A}} q(\mathbf{A}^{-1}\mathbf{Y}) \tag{13}$$

where $q(x)$ is the sources distribution which is supposed to be known.

In fact, it is possible to show that the ML principle coincides up to a constant term with Kullback divergence $\Psi(S, X)$ between the ideal solution obtained from observations **Y** (i.e., $\mathbf{A}^{-1}\mathbf{Y}$) and the real sources **X**. A sufficient condition to obtain independent components at the separator output is then given by the maximization of the contrast (called ML contrast):

$$\Phi_{ML}(S) = \Psi(S, X) = -\int p(u)\log\left(\frac{p(u)}{q(u)}\right)\mathrm{d}u \tag{14}$$

where $p(u)$ denotes the p.d.f. of the estimated sources S and $q(u)$ the p.d.f. of the true sources **X**.

The Kullback divergence of equation (14) can also be shown to be

$$\Psi(p(S), q(X)) = I(p(S)) + \sum_i \Psi(p_i(s_i), q_i(x_i)), \tag{15}$$

where $I(p(S))$ is called mutual information (MI) and is defined as

$$\Phi_{MI}(S) = I(p(S)) = \int p(s)\log\left(\frac{p(S)}{\prod_i p_i(s_i)}\right)\mathrm{d}S. \tag{16}$$

The first term provides a measure of the gap between the joint density of S and the product of the marginal densities, whereas the second term is a measure of marginal matching between the true and the estimated sources. Therefore, MI only measures the independence of the estimated sources without any assumption about the distribution of the true sources (which are generally unknown).

As the Kullback divergence is insensitive to any non-singular linear transformation applied on both densities $p(u)$ and $q(u)$, equation (14) can be written [12] as

$$\Phi_{ML}(S) = \Psi(\mathbf{g}(S), \mathbf{V}) = -H(\mathbf{g}(S)), \tag{17}$$

where $\mathbf{V}$ is an M-dimensional uniformly distributed random vector, $\mathbf{g}(X)$ is the cumulative density function associated with the p.d.f. $q(u)$ of the true sources X, and $H(\mathbf{g}(S))$ is the differential entropy associated with the random vector $\mathbf{g}(S)$ and defined for a random variable U as

$$H(U) = -\int p(u) \log(p(u))\, \mathrm{d}u. \tag{18}$$

The differential entropy evaluated at the output of the non-linear function $g(\cdot)$ of the estimated sources provides the well-known *Infomax* contrast (IM):

$$\Phi_{IM}(S) = -H(\mathbf{g}(S)). \tag{19}$$

This contrast function is very popular in the neural network community although it is equivalent to the ML one. As $\mathbf{g}(S)$ is restricted in $[0,1]^M$, $\Phi_{IM}(S)$ is maximized for $\mathbf{g}(S)$ uniformly distributed in $[0,1]^M$, that is for S distributed as $\mathbf{X}$. Thus, it can be seen that $\Phi_{IM}(S)$ and $\Phi_{MI}(S)$ are closely related and equivalent to $\Phi_{ML}(S)$.

The ML contrast can easily be optimized by a gradient descent algorithm [13]:

$$\nabla_S \Phi_{ML}(S^k) = \mathrm{E}\{\boldsymbol{\varphi}(S^k) S^{k^{\mathrm{T}}}\} - \mathbf{I},$$

$$S^{k+1} = S^k - \nabla_S \Phi_{ML}(S^k) S^k, \tag{20}$$

where $\boldsymbol{\varphi}(S)$ is the score function defined as $\boldsymbol{\varphi}(S) = [-\log(q_1)', \ldots, -\log(q_i)', \ldots, -\log(q_M)']$.

When the source densities are unknown, the choice of $\varphi(S)$ can be extended to more general non-linear functions as proposed by Herault and Jutten [14]. This approach will be presented in detail in the case of convolutive mixtures in section 4.1.

In the cases mentioned above, the use of p.d.f. is not easy to handle. One way to simplify the Kullback divergence-based contrast is to introduce cumulants through polynomial density expansion [15]. The first example is given by Gaeta and Lacoume [16] with an approximation of the likelihood by a Gram–Charlier expansion. Comon [10] suggests an approximation of the mutual information by an Edgeworth expansion. Hyvarinen makes some interesting remarks about this approach [17].

Another way to derive some empirical contrast is the approximation of independence using equations (9) and (10).

For computational reasons, the approximation of cumulants and moments is generally done up to the fourth order. The minimization of cross-cumulants leads to empirical contrast like kurtosis cancellation [18], some fourth order cumulants [19], or even a set of second and fourth order cross-cumulants [11].

Using some additional assumptions, it is also possible to restore independent components in equation (2) using only the second order statistics. These assumptions can be made on the mixing matrix or on the temporal structure of the sources. For the first assumption, the mixing matrix can be found using PCA only if A is a symmetrical matrix and also if $A_{ii} = 1$ [20]. If the sources are time correlated processes, they can be restored using frequency information. In this way it is possible to achieve separation even for Gaussian sources, under condition of different spectra. So the estimation of the missing unitary matrix is given by a joint diagonalization of a set of covariance matrixes with different time lags [21].

4. BSS OF CONVOLUTIVE MIXTURES

Many approaches can be found in the literature to realize BSS using a convolutive mixing model. In this paper two approaches of BSS used to monitor the vibrations of rotating machines [3, 4] are stressed. The first method is based on a temporal approach and implemented through an iterative algorithm. The second approach is based on frequential model of convolutive mixture discussed in section 4.2.

4.1. TIME DOMAIN BASED SEPARATION

It was emphasized previously that it is possible to restore the source signals up to a linear filtering. It is possible, however, to reduce the shape indeterminacy in model (4) by setting a constraint either on matrices $\mathbf{A}(z)$ (the diagonal terms of $\mathbf{A}(z)$ are usually supposed to be unity) or on $\mathbf{X}(n)$ (its components are generally assumed to have unit variance). The first constraint, (on $\mathbf{A}(z)$), permits the model shown in Figure 2 for two sources to be simplified. It is a realistic approximation if it is assumed that the sensors are as close to the sources as possible. Indeed, it is believed that the ability to detect fault decreases with the increase of distance between the sensor and the fault source. Thus, it is very important for diagnosis that the sensor is as close as possible to the engine being monitored. This assumption implies that the parameters A_{ij} are equal to 1 when $i = j$. This simplified model is usually assumed when the linear filters are estimated in the time-domain and the parameters A_{ij} $(i \neq j)$ are directly computed. After this assumption, the mixing matrix becomes

$$\mathbf{A}(z) = \begin{bmatrix} 1 & A_{12}(z) \\ A_{21}(z) & 1 \end{bmatrix}. \tag{21}$$

In this case A_{12} and A_{21} represent the cross coupling between the two processes. The second assumption to reduce the BSS problem is that $A_{ij}(z)$ are linear and causal filters with a finite impulse response (FIR). Then the coefficient $A_{ij}(z)$ can be written as

$$A_{ij}(z) = \sum_{k=0}^{L_{ij}-1} a_{ij}(k)\, z^{-k}, \tag{22}$$

where L_{ij} are the filter lengths which are assumed to be known.

This assumption is not necessary to solve our problem, but allows the permutation of the restored sources to be freed [22].

Now, the general idea of convolutive temporal BSS consists of identifying an inverse matrix of $[\mathbf{A}(z)]$ by

$$\mathbf{C}(z) = \frac{1}{D(z)} \begin{bmatrix} 1 & -C_{12}(z) \\ -C_{21}(z) & 1 \end{bmatrix} \tag{23}$$

with $D(z) = 1 - A_{12}(z) A_{21}(z) \neq 0$ to respect the condition of invertibility of $[\mathbf{A}(z)]$.

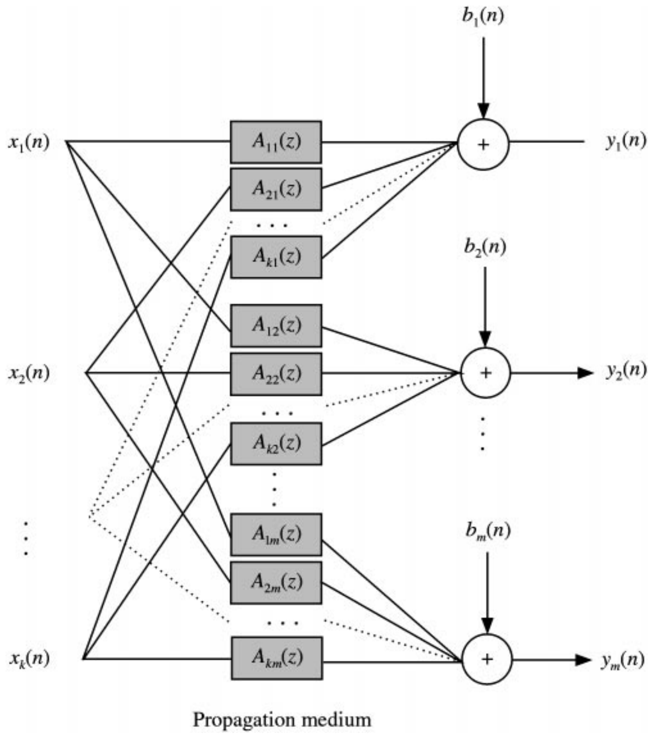

Figure 2. Convolutive mixing model.

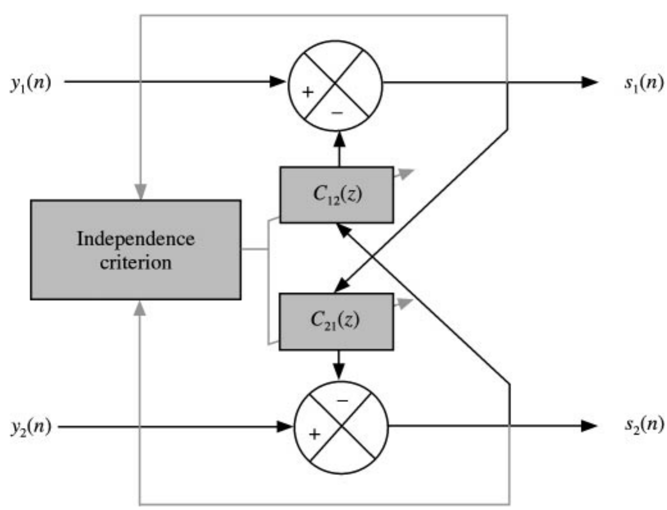

Figure 3. Recurrent procedure for BSS.

The sources will be reconstituted by filtering signals from the sensors through this transfer matrix. The two unknown filters can be identified by a back propagation or recurrent procedure as shown in Figure 3.

The solution of this system can be obtained from

$$s_i(n) = y_i(n) - C_{ij}(n) * s_j(n) = y_i(n) - \sum_{k=0}^{L_{ij}-1} C_{ij}(n,k)\, s_j(n-k) \tag{24}$$

with $i \neq j$ and $i, j \in [1,2]$ for two sources and L_{ij} is the length of the filter C_{ij}. This solution can be written in matrix formulation as

$$\mathbf{S}(n) = [\mathbf{C}(z)]\, Y(n).$$

That is

$$\mathbf{S}(n) = \frac{1}{D(z)} \begin{bmatrix} 1 & -C_{12}(z) \\ -C_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & -A_{12}(z) \\ -A_{21}(z) & 1 \end{bmatrix} \mathbf{X}(n), \tag{25}$$

$$\mathbf{S}(n) = \frac{1}{D(z)} \begin{bmatrix} 1 - C_{12}(z)A_{21}(z) & A_{12}(z) - C_{12}(z) \\ A_{21}(z) - C_{21}(z) & 1 - C_{21}(z)A_{12}(z) \end{bmatrix} \mathbf{X}(n). \tag{26}$$

System stability will be ensured if the zeros of the polynomial $D(z)$ remain within the unit circle, i.e., if the filter $1 - C_{12}(z)C_{21}(z)$ is minimum phase.

Separation is obtained when S_i equal to X_i up a permutation matrix and a linear filtering. Here equation (26) yields two solutions:

either $C_{12}(z) = A_{12}(z)$ and $C_{21}(z) = A_{21}(z)$ and the perfect theoretical solution, $S_1(z)$ equal to $X_1(z)$ and $S_2(z)$ equal to $X_2(z)$ is obtained,

or $C_{12}(z) = 1/A_{21}(z)$ and $C_{21}(z) = 1/A_{12}(z)$ are also possible leading to independent sources.

In this case, the solution $S_i(z)$ is equal to $-X_j(z)/A_{ji}(z)$. However, this solution implies that the filters are infinite impulse response (IIR), which is inconsistent with our hypothesis.

The aim of BSS is now to estimate the filters A_{12} and A_{21}, and then, x_1 and x_2 could be recovered from y_1 and y_2 by inverse filtering. The method used afterwards consists of finding estimates $C_{12}(z)$ and $C_{21}(z)$ of $A_{12}(z)$ and $A_{21}(z)$ by maximizing a criterion function of system outputs (24) through the stochastic iteration

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu_{ij}\,\Phi_{ij}(n,k), \tag{27}$$

where $\Phi_{ij}(n,k)$ is a function maximizing the independence between the components of $S(n)$ and leading to the convergence $\Phi_{ij}(n,k) = 0$, where μ_{ij} are small gain factors that govern stability and rate of convergence.

Many of the independence criteria presented in section 3.2 can be extended to convolutive mixtures. Three of them have been implemented for this application.

(a) Output decorrelation [23],

A1:
$$\Phi_{ij}(n,k) = \mathrm{E}\{S_i(n)S_j(n-k)\}. \tag{28}$$

(b) Output non-linear function cancellation [24],

A2:
$$\Phi_{ij}(n,k) = \mathrm{E}\{f(S_i(n))g(S_j(n-k))\}. \tag{29}$$

(c) Output cross-cumulants cancellation [24],

$$\text{A3:} \qquad \Phi_{ij}(n,k) = -\,sign\frac{\partial\, cum_{13}\{S_i(n), S_j(n-k)\}}{\partial\, c_{ij}(n,k)}, \quad cum_{13}\{S_i(n), S_j(n-k)\} \tag{30}$$

for $i \neq j$ and $i, j \in [1,2]$, and $0 \leqslant k < L$, $L = \max(L_{ij})$.

A recent study of A1 and A2 was presented in reference [25], including stability analysis and asymptotic behaviour, for a large class of separating functions. The authors showed that the stability conditions are related to the source statistics, the separating functions, and the mixing filters for independent identically distributed (*i.i.d.*) random sequences. Cruces and Castedo in reference [26] performed a similar study for the cumulant algorithm A3.

For a good implementation of these classes of algorithms, it is necessary to make some remarks. The independence between the output i at time n and the output j at a different time $(n-k)$ is tested to obtain as many equations as coefficients in filters ($2L$). However, the output independence test will be sufficient if these $2L$ equations are independent. Therefore,

$$\Phi_{ij}(n,k) \neq \Phi_{ij}(n,k-1). \tag{31}$$

This condition is verified if the signals have sufficiently broadband spectra and if the sampling frequency (F_s) is not too high, so that $s_j(n)$ and $s_j(n-1)$ can be actually independent. Otherwise, over-sampling would damage the algorithm because it would generate adaptation equations that are too similar.

4.2. FREQUENCY DOMAIN BASED SEPARATION

In the frequency domain, the convolutive mixture is reduced to an instantaneous complex mixture for each frequency bin and can be written as

$$\mathbf{Y}(n,f) = \mathbf{A}(f)\mathbf{X}(n,f) + \mathbf{B}(n,f), \quad f = 0, \ldots, N-1, \tag{32}$$

where $\mathbf{Y}(n,f)$ (respectively, $\mathbf{X}(n,f)$ and $\mathbf{B}(n,f)$) is the N-points discrete Fourier transform (DFT) of the nth data block of the data vector $\mathbf{Y}(n)$ (respectively $\mathbf{X}(n)$ and $\mathbf{B}(n)$).

The transfer matrix $\mathbf{A}(f)$ characterizes the linear propagation from sources to sensors and must be non-singular to recover the sources at the frequency bin f. The hypotheses of independence about sources and noise are assumed to be just the same as for the temporal model.

The sources separation is performed in each frequency band by a BSS algorithm for instantaneous complex mixed sources as described in Figure 4.

Setting a constraint of unit variance of the sources eliminates the scaling indeterminacy here. The mixing matrix $\mathbf{A}(f)$ is then expressed as the product of three matrices, after a singular value decomposition:

$$\mathbf{A}(f) = \mathbf{V}(f)\mathbf{\Delta}(f)^{1/2}\mathbf{\Pi}(f), \tag{33}$$

where $\mathbf{V}(f)$ and $\mathbf{\Pi}(f)$ are two unitary matrices. $\mathbf{\Delta}(f)$ is a diagonal matrix. The two matrices $\mathbf{V}(f)$ and $\mathbf{\Delta}(f)$ are identified from the second order statistics as in equation (7). They contain, respectively, the eigenvectors and the eigenvalues of the spectral matrix of the observation $\mathbf{Y}(n)$. After projection of the observation $\mathbf{Y}(n,f)$ in the subspace spanned by

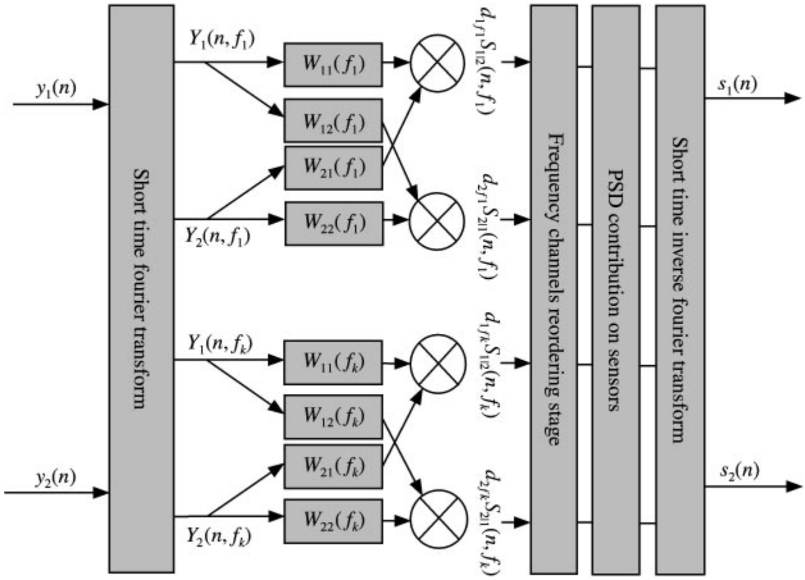

Figure 4. Frequential BSS separation.

the eigenvectors and normalization, the data $\mathbf{Z}(n,f)$ are whitened. They are linked to the components of the source vector $\mathbf{X}(n,f)$ by a remaining unitary transformation as in equation (12):

$$Z(n,f) = \mathbf{\Pi}(f)\mathbf{X}(n,f). \tag{34}$$

The matrix $\mathbf{\Pi}(f)$ can be identified by any of the approaches for BSS of instantaneous mixtures described in section 3.2, using higher order statistics or temporal correlation of the sources into each frequency bin.

Concerning the temporal correlation of the sources, it is proved in reference [27] that matrix $\mathbf{\Pi}(f)$ results on the diagonalization of delayed interspectral matrices. Concerning statistical independence, the additional information exists only with the hypothesis of non-Gaussian sources. Fourier transform is often thought to converge towards Gaussianity, but it was shown in reference [28] that this assertion is not valid for spectral lines signals such as those of rotating machines. More precisely, the spectral kurtosis defined as

$$K_N^X(f) = \frac{(1/L)\sum_{n=0}^{L-1} |X_N(n,f)|^4 - |(1/L)\sum_{n=0}^{L-1} X_N(n,f)^2|^2}{((1/L)\sum_{n=0}^{L-1} |X_N(n,f)|^2)^2} - 2$$

tends toward -1 at all the harmonic bins when the number of averages is large enough. Hence, for the latter, any higher order based source separation algorithm for instantaneous complex mixtures can be used in each frequency bin. Here, a maximum likelihood based method [11] is applied to estimate $\mathbf{\Pi}(f)$. In the case of two sources, $\mathbf{\Pi}(f)$ is a complex Givens rotation, parameterized with two angles. The maximum likelihood function is computed, using a Gram–Charlier expansion of the p.d.f. of the sources. The expansion is stopped at the fourth order and $\mathbf{\Pi}(f)$ is expressed with fourth order cumulants of the observations.

As the mixing matrix is recovered up to a permutation matrix $\mathbf{P}(f)$ and a diagonal complex matrix $\mathbf{D}(f)$, a matrix $\mathbf{W}(f)$ has been estimated such that:

$$\mathbf{W}(f)\mathbf{A}(f) = \mathbf{D}(f)\mathbf{P}(f). \quad (35)$$

To remove the permutation indeterminacy the re-ordering step described in reference [28] can be used. It aims to recover the statistical relationship between the estimated sources at one frequency bin and the temporal sources $X_i(n)$. It uses the redundant information between $S_i(n,f)$ and $S_i(n+1,f)$ on the temporal source. Indeed, the MA filter

$$F_f(z) = 1 - z\,\mathrm{e}^{-(2i\pi f/N)}$$

reconstitutes the temporal signal $(X_i(n) - X_i(n-N))$ when applied on $S_i(n,f)$ in each frequency band. The permutations are then detected and removed, comparing the coherence between the filtered estimated sources between different frequency bins f. Choosing f_{ref} as the reference channel and labelling S_{ref} the source lying in it, the re-organizing step is performed following the rule

$$S_q(n,f) \in S_{ref} \quad \text{if } q = \max_j CC(F_{f_{ref}}(S_{ref}(n,f_{ref})),\ F_f(S_j(n,f)))$$

with $CC(X,\ Y)$ defined as is a coherence function defined as

$$CC(X,\ Y) = \frac{\mathrm{E}[X\,Y^*]}{\sqrt{\mathrm{E}[|X|^2]\,\mathrm{E}[|Y|^2]}}.$$

A constraint is then applied to $\mathbf{X}(n)$ by an energy normalization of $S_i(n,f)$ in each frequency bin followed by a correlation measure between $S_i(n,f)$ and $Y_j(n,f)$ for $j = 1, 2$. In that way, filter indeterminacy can be partially circumvented by restoring the PSD contribution of each source on each sensor. This approach is equivalent to that described in section 4.1 when sensors are actually near the sources.

5. APPLICATIONS

As previously stated, the aim of BSS is to recover unknown sources without any prior knowledge about them or the mixing process. Different statistical criteria can be used to quantify the independence of signals such as those presented in the previous sections. This part focuses on the three algorithms in temporal domain described in section 4.1 and the frequential domain approach described in section 4.2.

5.1. SIMULATIONS

The purpose of this part is to provide an illustration of the capability of BSS algorithms to separate signals from rotating machine vibration. From this point of view, two synthetic signals are generated following the model below, which represents an example of vibrations from a gearbox [7].

$$x(n) = \sum_{i=1}^{a} A_i \sin(2\pi i f_m n + \phi_i)\left[1 + \sum_{j=1}^{b} B_j \sin(2\pi i f_p n + \phi_j)\right] + e(n),$$

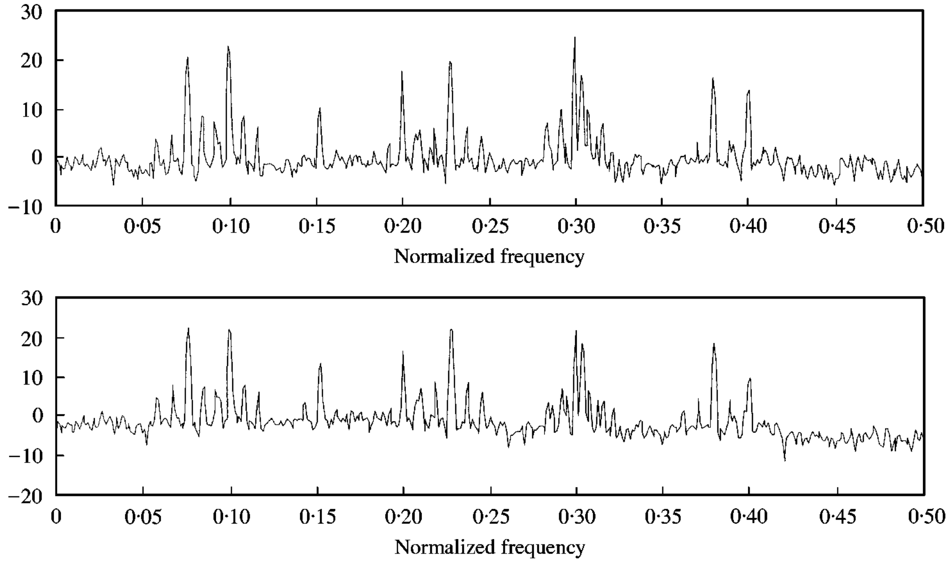

Figure 5. Spectra of the observed signals (mixtures).

where A_i and B_j represent the amplitude modulation law, $e(n)$ is a real Gaussian noise, and ϕ_i and ϕ_j are random phases uniformly distributed in $[0, 2\pi]$. The mixtures are obtained using a transfer matrix defined in equation (21). As the choice of mixing filters is not really important, for this experiment $A_{12} = [0,\ 0{\cdot}01, -0{\cdot}09,\ 0{\cdot}05, -0{\cdot}18,\ 0{\cdot}59,\ 0{\cdot}46]^{\mathrm{T}}$ and $A_{21} = [0,\ 0{\cdot}1,\ -0{\cdot}15,\ 0{\cdot}55,\ 0{\cdot}42, -0{\cdot}12,\ 0{\cdot}04]^{\mathrm{T}}$ and the two simulated observations are presented in Figure 5.

After separation step, the results are illustrated in Figure 6. The separation performances are presented in the Table 1 in terms of:

Residual cross-talking error (RCTE), defined as

$$RCTE(s_i, x_i) = 10 \log \left\{ \frac{E\,[(s_i - x_i)^2]}{E(x_i^2)} \right\}. \tag{36}$$

Mean square error (MSE) between the filter coefficients:

$$MSE_{ij} = \frac{1}{L} \sum_k (c_{ij}(k) - A_{ij}(k))^2, \tag{37}$$

where L is the number of coefficients for each source A_{ij}. This measure is generally used for white sources but this example shows that a harmonic signal with sufficiently high number of components allows a good estimation of the mixing filters.

Examining these performances, every algorithm provides satisfactory separation results (RCTE < -23 dB for each estimated source). However, A1 and A3 seem to be less sensitive to coloured sources, which are implicitly assumed for temporal methods to estimate filter coefficients. Nevertheless it is explained in reference [29] that filter coefficients are estimated in order to match an independence criterion and not to ensure the correct estimation of the filters. That is why MSE criterion is not really significant for non-temporally white signals.

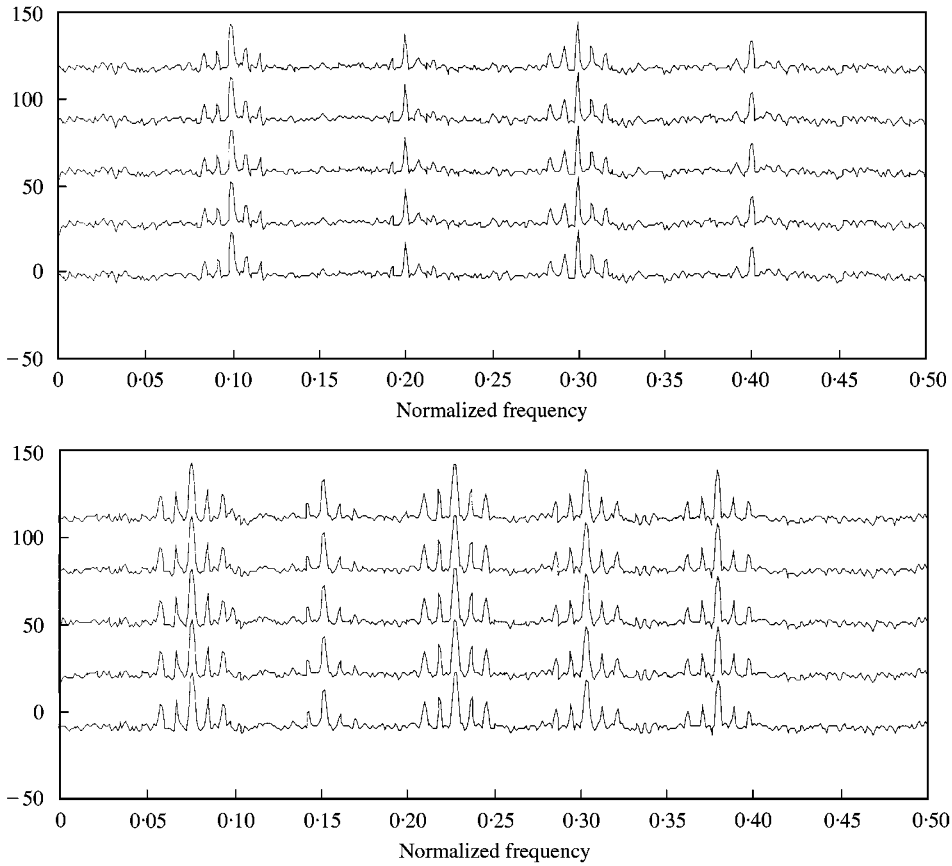

Figure 6. Spectra of the estimated and the real sources. Bottom to top: real source, A1, A2, A3 and frequency domain approaches (FDA).

TABLE 1

Performances comparison

	$RCTE_1$ (dB)	$RCTE_2$ (dB)	MSE_{12}	MSE_{21}
A1	− 31·76	− 31·09	0·0004	0·0003
A2	− 27·20	− 23·99	0·0018	0·0023
A3	− 34·92	− 32·79	0·0003	0·0005
FDA	− 29·25	− 27·23	0·0020	0·0019

5.2. EXPERIMENTAL RESULTS

5.2.1. *Experimental context*

The experiments were made on a test bench carrying two DC motors (1·4 and 1·1 kW) with different rotation speeds. The two motors were fixed to the same structure as in Figure 7. Two accelerometers were glued on each motor to measure vibrations.

The problem illustrated by this experiment is one of a factory in which two rotating machines operate simultaneously, but each machine must be diagnosed separately. Thus,

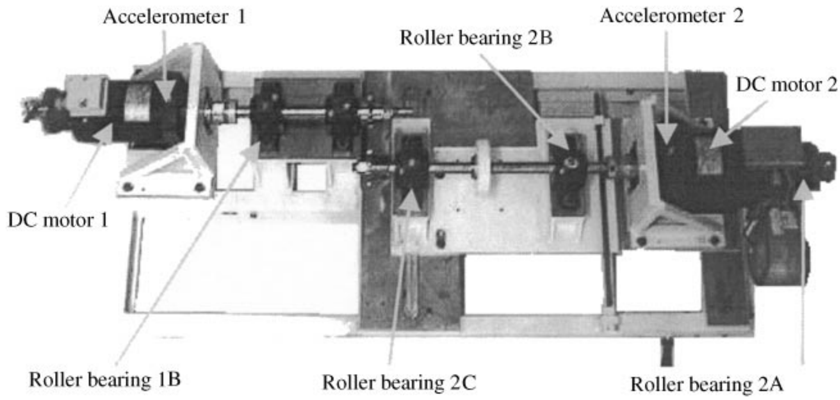

Figure 7. Test bench.

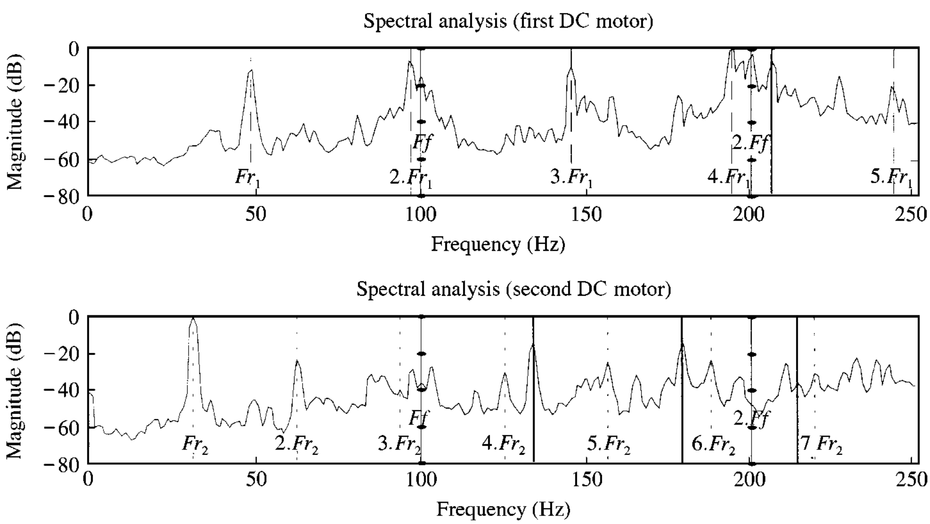

Figure 8. Spectra of the source references.

according to the superposition principle, signals from the other machine disrupt signals received by sensors placed on each machine. There is a great interest in the use of BSS methods as part of the diagnostic process because BSS should free us from noisy environment; that is restoring on each sensor the signature of its own machine without having to stop the machines which would be damaging to the production. For this purpose, BSS can be viewed as a pre-processing step (de-noising) that improves the diagnosis. Traditional methods of fault detection could then be applied to the specific signatures of the system to be diagnosed.

When treating real recordings, it is very difficult to measure the separation quality. Here, prior knowledge about the sources was used; that is, harmonic frequencies in relation to the mechanical components as well as the signals recorded on each source separately in the real environment (the reference).

The two reference signals are shown in Figure 8. The rotation speed of the two motors are set to 48·5 Hz for motor 1 (1·1 kW) and 31·5 Hz for motor 2 (1·4 kW). Motor 1 is fed by

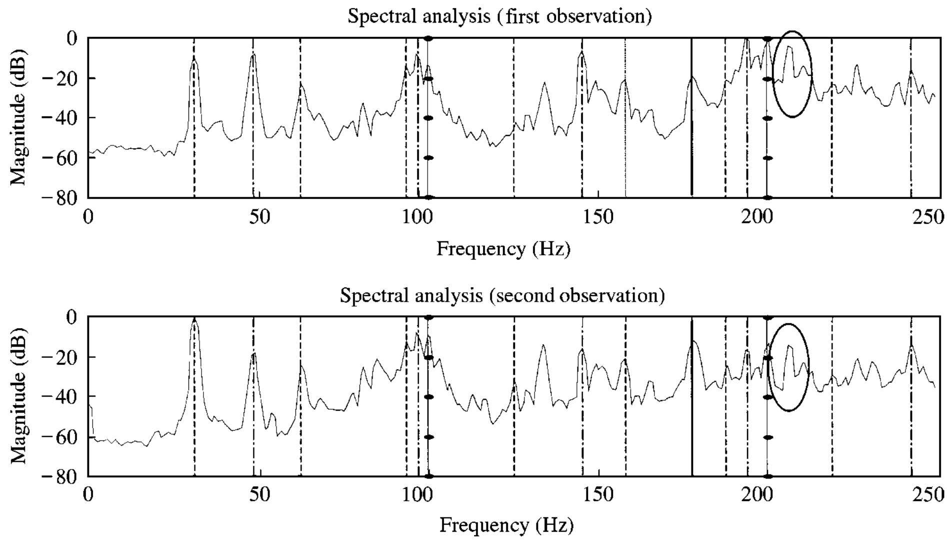

Figure 9. Spectra of the sensor signals (mixtures).

single-phase wiring (rectified) which provides 100 Hz for the fundamental frequency plus the harmonics. The motor 2 is fed by three-phase wiring (rectified) which presents 100, 200 and 300 Hz frequencies. These wiring frequencies are represented in the figures in dashed and dotted lines. The harmonics of the low rotating frequency (motor 2) are drawn in dotted lines and those for the high rotating frequency (motor 1) are drawn in dashed lines. Each motor is fitted out with two single-row roller bearings (6203 RS C3) and drives a main shaft filled with two self-aligning roller bearings (2207 KTV C3). Roller bearing 2A, 2B, 2C, and 1B were found to be faulty and to induce two defect frequencies at 134 Hz (outer race fault on 2C), 179 Hz (outer race fault on 2A), 207 Hz (outer race fault on 1B) and 210 Hz (inner race fault on 2B). These frequencies are drawn in Figures 8–10 in solid lines. Twenty thousand samples were recorded at F_s equal 2 kHz and resampled for a temporal approach to 500 Hz. Figure 7 presents the PSD estimated with the Welch averaged method of the two records obtained on each sensor. Each PSD was normalized by its maximal value.

5.2.2. *Settings*

With regard to the temporal methods, the number of filter coefficients L for the mixture was experimentally estimated by the impulse response method equal to 100 for a sampling frequency equal to 500 Hz. All temporal algorithms were implemented with a constant gain (for different n) but with multiple passes of the observations for different values of μ_{ij} in order to refine the results around the separating solution. Both the adapting steps μ_{ij} were set to [0·2, 0·1, 0·01, 0·001]. For the frequency domain method, the mixing matrix has been estimated for 256 frequency bins in the frequency band [0–250] Hz. Twenty thousand samples were considered for each frequency channel to perform separation.

5.2.3. *Rotating and feeding frequencies*

The results obtained with the four approaches are depicted in Figure 10. For legibility, each plot is shifted (20 dB) with respect to frequency domain approach.

The results indicate that all the methods except cross-cumulants cancellation give satisfactory results for the two motor rotating frequencies plus harmonics (position and

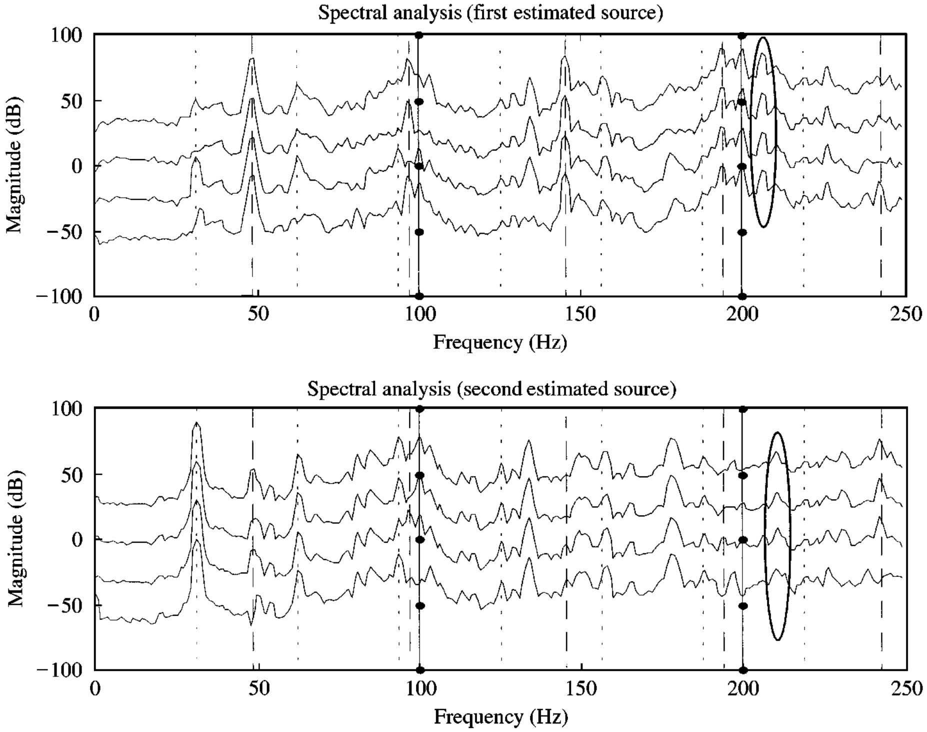

Figure 10. Spectra of the estimated sources. Bottom to top: frequency domain approach (FDA), A3 approach, A1 approach, A2 approach.

magnitude) except for the fifth harmonic of motor 1 for which temporal BSS was not efficient. The A1 criterion seems to provide a slightly better result for the fundamental frequency of motor 2 and is the more accurate temporal method to solve the BSS problem in this experiment. Another remark that favours this solution is that rotating machine signals are present with obviously highly temporal correlation and all the temporal methods used in this paper are dedicated to temporally white signals. Among the temporal methods, the second order criterion seems to be less sensitive to this assumption about the sources. The FDA approach is fitted to this hypothesis and gives better results for the low power harmonics (fifth harmonic).

With regard to the feeding frequencies (k.100 Hz) present in both sources but prevalent on motor 1, the frequency domain method also provides better separation whilst the, temporal approach attributes one feeding harmonic in each source.

5.2.4. *Bearing fault detection*

Bearings are one of the most widely used components in various kinds of processes as well as robots, manufacturing processes and rotating machines. In order to enhance productivity, product quality and reliability, a monitoring system is essential to check the status of the different components. The interest of BSS applied to monitoring appear in for example the case of bearing fault detection, where some fault related frequencies appear as a function of the axle rotating speed [30]. One of the most useful methods is in detecting and following these fault related frequencies and then to know which type of defect the bearing has as well as its importance. So an early detection of these frequencies required knowledge about axle bearing rotation speed and also a maximum local signal-to-noise ratio around the fault related frequency. In this experiment, many processes worked simultaneously and observe a mixture of them was observed. The mixing phenomenon complicates the calculus and the detection of the bearing fault related frequencies (especially because there are several rotating speed in the same mechanical system).

To illustrate the potential of BSS in bearing fault detection, it is stressed that this paper deals with only one clearly identified fault related to a bearing. This fault is positioned on the outer race of the axle driving roller bearing of the motor 1 (bearing 1B). The next relation gives the calculus of the corresponding defect frequency:

$$F_{ir} = \frac{n}{2}\frac{RPM}{60}\left(1 - \frac{Bd}{Pd}\cos\Phi\right) = 207\,\mathrm{H}z, \tag{38}$$

where Pd is the pitch diameter, n the number of elements, Bd the ball diameter, Φ the contact angle and RPM the rotation frequency.

Figure 10 clearly shows that this frequency (207 Hz) is associated with the 'good' source and so it is easier to calculate the origin of the mechanical failure (for this example, a defect on outer race related to the motor operating at 48·5 Hz). Here, BSS can be viewed as a pre-processing step, which make easier and enhance the detection and the monitoring of the mechanical system to be diagnosed.

Other bearing faults are present on the process (see section 5.2.1), and it can be verified that most of these faults are clearly re-associated with its driving shaft. However, the fault connected with the bearing 2C (outer race), which is farther from the sensor seems to be attributed to the 'good' source only with a frequential approach. For temporal methods, no BSS is performed at this frequency. A plausible hypothesis is that the faulty bearing corresponding to this frequency is separated from the principal vibratory sources (i.e., the motors). In this case, the faulty bearing acts like a third source towards the sensors and therefore the filtering indeterminacy (section 2.4) cannot be removed. The frequential domain approach is insensitive to this fact due to the presence of only one source in the frequency bin of interest and so the separation in this frequency bin tends to attribute the frequency component to the closest source in the maximum coherence sense.

5.2.5. *Practical point of view*

From a practical point of view, it must be noted that the frequency domain algorithm is extremely computationally difficult to implement. The good results obtained with the decorrelation approach (A1) and the attractive computational cost seems to be a good alternative to meet the objective. However, to obtain a very good separation quality in few frequency bins, the frequency domain method seems to be more accurate. A combination of these two methods could provide a good alternative which is neither too computationally costly nor too accurate.

6. CONCLUSION

This paper has described the basic principles of BSS, and has discussed its application to rotating machine vibrations, through simulations and experiment. In the most general case with linear assumptions, the mixing process generated by mechanical systems is assumed to be convolutive. In this framework, two approaches have been presented which are suited to solving this problem. The harmonic nature of the rotating machine signals complicates the separation procedure with temporal methods, which are initially developed for temporally white signals. In spite of this difficulty it is shown that both temporal and frequential BSS approaches give rise to similar results. So the results provided in this paper allow BSS to be considered as a promising tool to pre-process the data in mechanical fault diagnosis applications. However, the high computational cost of the frequential approach with respect to the temporal one is prohibitive for an implementation for all the frequency bins and the use of FDA is recommended only if the frequency channel of interest are *a priori* known and also if the gap from model hypothesis (linearity for example) is no longer negligible. Future work will consist of extending and studying the feasibility of BSS methods to more than two sources. Another way is to generalize BSS methods to non-linear mixtures in order to take account of more complicated physical models for generation and interactions of vibratory sources.

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