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THE NON-LINEAR DYNAMIC BEHAVIOR OF AN ELASTIC LINKAGE MECHANISM WITH CLEARANCES

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In terms of Newton two-state model, by choosing two sets of generalized co-ordinates, this paper develops a unified dynamic model between the separation and collision process for the elastic linkage mechanism. This model incorporates the effects of rigidity and elasticity coupling and the angular velocity of crank is assumed to be variable in the operation. In addition, this paper provides a more simple and practical numerical solution method for convenient analysis. Through an example, the dynamic responses of the elastic linkage mechanism with clearances are analyzed, both the effects of elasticity and clearance on the dynamic behaviors of the mechanism are analyzed simultaneously and the non-linear behaviors caused by the clearance joints are analyzed by the dynamic model of rigid mechanism.

1. INTRODUCTION

The presence of clearance in the joints of mechanisms is inevitable. Small suitable clearance is necessary to move a mechanism smoothly. However, when the amount of clearance is excessive, not only does the occurrence of motion decrease but also, the vibrations and noises due to impulsive acceleration will become greater and greater. Therefore, more interest has already been drawn to modelling study of the mechanisms with clearance connections [1–5]. Moreover, in recent years, the high productivity, high-technology system demanded by the new type of mechanical industry also require very high speeds, light weights and high-precision machinery. So, it may be inaccurate to treat certain links in such mechanisms as rigid links. The dynamic effect of elastic deformations in the mechanisms has been investigated by some literatures [4–9].

Although the clearance effects and link elasticity have been analyzed individually, researches considering both the effects have been carried out widely only in recent years. Treatment of the complete problem is very difficult because modelling elastic mechanisms with clearances involves three types of coupling motions. First is the angular motion of rigid links, which is a large displacement, slowly variable process. Second is the impact motion caused by joint clearance, which is instantaneous, suddenly changing and a very complicated process. The last is elastic deformation motion, which is a smaller displacement, quickly variable process. Moreover, the importance of including both the effects of elastic deformation and joint clearance in dynamic modelling has been recognized for some time. In 1973, Winfrey [10] first analyzed the motion of a cam system with clearances by structural dynamics (finite-element) FE method. The modelling analysis was very simple for the constant mass and stiffness matrices and superposition was used by assuming small deformation. Dubowskey [11] developed a more general dynamic model for an elastic linkage mechanism with clearance connections by Lagrangian approach and then advanced this research to the spatial mechanism [12]. This model is either comprehensive or complicated and needs to be simplified for a possible numerical solution.

In terms of complicity of the problem, this study is established in developing a dynamic model which is quite simple for an easy solution and more accurate than most for the elastic linkage mechanism with bearing clearances. Based on a combination of Newton's second law and step function, by choosing two sets of generalized co-ordinates, this paper develops a unified dynamic model which incorporates the effects of rigidity and elasticity coupling where the angular velocity of crank is assumed to be non-constant in the operation.

Owing to the inclusion of more factors in dynamic modelling, the model must be more complicated than previous models. The residual problem for modelling is how to solve the complex dynamic equations. The key point of the solution is how to treat the above-mentioned coupling components of several motions. Literatures [12, 13] apply perturbation co-ordinate method which treats the angular motion caused by clearances and elastic deformation as very small variables and neglects the high order and high-time components of the small variables. This method simplifies the dynamic equations but is not suitable for the conditions of large elastic deformation or large clearances. In addition, some literatures [14, 15] regard the clearance as a link without mass, apply continuous contact assumption at the clearance joint and adopt KED method to solve the model. This solution method is very simple and suitable for a fairly simple analysis of the mechanisms with more number of links and clearances but does not show the detailed characteristic of the collision process in the clearance joints. From the viewpoint of dynamic equations themselves, this study treats the coupling component of rigid angular motion with clearance and elastic deformation as excitation components and provides a more concise iterative solution method which combines the numerical integration and model analysis methods. With the help of an example, the dynamic responses of the elastic linkage mechanism with clearances are analyzed. The effects of both elasticity and clearance on the dynamic behaviors of the mechanism are analyzed at the same time.

Although the dynamic modelling work for the elastic linkage mechanism with clearance connections has attracted more and more attention and gained some achievements, the non-linear behavior analysis caused by joint clearances has only been analyzed initially in the literature [16] and has not been carried out widely. The alternative change of contact, impact and loss of contact in the elastic linkage mechanism with clearances is equivalent to a piecewise linear system belonging to non-linear dynamic systems. Theoretically, under certain conditions, this system can show some non-linear phenomenon such as chaos and bifurcation. In general, the non-linear phenomenon of elastic linkage mechanism is mainly caused by the joint clearances. Whether the non-periodic motion arising in the system is chaotic behavior or not should be studied practically. So in this paper, the non-linear behavior of a rigid linkage mechanism with clearances is analyzed through the phase portraits and Poincare maps. The analysis result is also fit for the elastic linkage mechanism with clearance connections.

2. DEVELOPMENT OF EQUATIONS OF MOTION

For a convenient analysis, the dynamic equations of four linkage elastic mechanisms whose pair B has a clearance as described in Figure 1 are derived theoretically to show the dynamic modelling process of the elastic mechanism with clearance joint. The clearance has

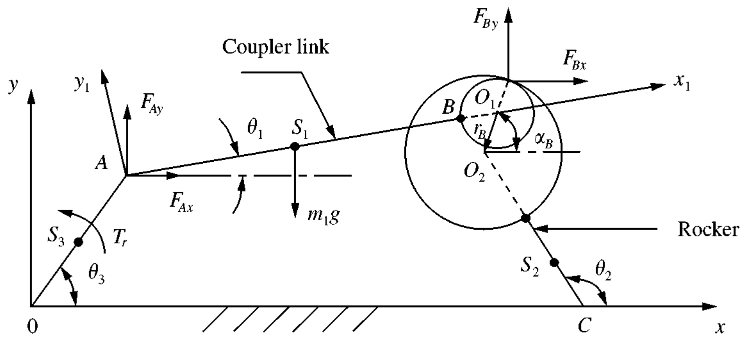

Figure 1. Four linkages mechanism with elastic linkage and clearance connections.

been greatly exaggerated for illustration. x_1Ay_1 represents the local reference frame. The model of the mechanism with multiple elastic links or multiple clearances can be derived analogously. In order to analyze the main point of the problem, several basic assumptions have been used in this analysis.

(1) Link deformation is sufficiently limited to allow a deflection approximation as embodied in Euler beam theory.
(2) The link is rigid in the axial direction and the deformation between the pairing elements in the clearance joint is limited to linear spring model.
(3) The impact force and operation loads have no effect on the axial deformation of the elastic link.

2.1. THE CHOICE OF GENERALIZED CO-ORDINATES

Regarding the motion of clearance elastic mechanism with clearance as the synthesis of the deformation of elastic links and the angular motion of rigid mechanism with clearance, we choose two sets of generalized co-ordinates as follows: the first set includes $(\theta_1, \theta_2, \theta_3, r_B, \alpha_B)$ which represent the angular motion of rigid mechanism with clearance pair B. θ_j $(j = 1, 2, 3)$ expresses the angular position of the links, r_B and α_B express the relative radial displacement and the contact angle between pairing elements at clearance joint B. The second set includes $\{q\}_j$ $(j = 1, 2, 3)$, which represent the deformation motion of elastic links. This process of choosing generalized co-ordinates makes the dynamic analysis very simple and clear, and improves the solution efficiency.

2.2. DYNAMIC MODEL FOR THE FOUR LINKAGE ELASTIC MECHANISM WITH CLEARANCE

Using the above-mentioned generalized co-ordinates, we can successfully develop the dynamic model of the mechanism shown in Figure 1. The relative motion variables between the elements at clearance pairs are independent of each other because their kinematics position constraints are relaxed. The joint constraint force that can be computed from the rigid model with clearance is equal to the external force acting at the end of clearance connection. For ease of demonstration, the dynamic behaviors of the coupler link are studied first as a case and the dynamic equations of other links can be derived similar to that of the coupler link.

In Figure 1, we assume the input torque T_r imposed on the crank OA which rotates at a non-constant speed to be

$$T_r(\dot{\theta}_3) = a - b\dot{\theta}_3, \tag{1}$$

$$a = i\mu(9{\cdot}8 \times n_C M_b)/(n_C - n_B), \qquad b = i \times i \times (60 \times 9{\cdot}8 \times M_b)/(2\pi(n_C - n_B)), \tag{2}$$

where $\dot{\theta}_3$ is the angular velocity of crank OA, a and b are the constants related to the characteristic of drive electromotor whose expressions are derived in the literature [17], i and μ denote the transmission ratio and mechanical efficiency of the retarder, respectively, and n_C, n_B, M_b represent synchronous rotate speed, rated rotate speed and rated torque of the electromotor respectively.

The dynamic equations of motion for coupler link AB are derived from Newton's second law as follows:

$$m_1\ddot{y}_{s1} = F_{Ay} + F_{By} - m_1 g - G_1\cos\theta_1, \quad m_1\ddot{x}_{s1} = F_{Ax} + F_{Bx} + G_1\sin\theta_1, \tag{3}$$

$$J_A\ddot{\theta}_1 = M_1 - m_1 a_A^t l_{s1} - M_{G1}, \tag{4}$$

where

$$\ddot{x}_{s1} = -(\dot{\theta}_3^2 l_3\cos\theta_3 + \ddot{\theta}_3 l_3\sin\theta_3 + \dot{\theta}_1^2 l_{s1}\cos\theta_1 + \ddot{\theta}_1 l_{s1}\sin\theta_1), \tag{5}$$

$$\ddot{y}_{s1} = -(\dot{\theta}_3^2 l_3\sin\theta_3 - \ddot{\theta}_3 l_3\cos\theta_3 + \dot{\theta}_1^2 l_{s1}\sin\theta_1 - \ddot{\theta}_1 l_{s1}\cos\theta_1), \tag{6}$$

$$a_A^t = \ddot{\theta}_3 l_3\cos(\theta_3 - \theta_1) - \dot{\theta}_3^2 l_3\sin(\theta_3 - \theta_1), \tag{7}$$

$$M_1 = (F_{Bx}\sin\theta_1 - F_{By}\cos\theta_1)l_1 - m_1 g l_{s1}\cos\theta_1 - (F_{By}\cos\alpha_B - F_{Bx}\sin\alpha_B)R, \tag{8}$$

l_1, l_3 are the lengths of coupler link and crank, m_1 is the mass of coupler link, s_1 is the center of mass of coupler link, l_{s1} is the length of coupler link between center of mass and joint, J_A is the inertial moment of coupler link with respect to joint A, g is the gravitational acceleration (9·8 m/s^2), $\ddot{x}_{s1}$ and $\ddot{y}_{s1}$ are accelerations of the coupler center of mass in the axial directions of x and y, and a_A^t is the absolute acceleration normal to link AB at joint A. G_1 and M_{G1} are the inertial force and inertial moment caused by the effects of elastic coupler deformation, such that

$$G_1 = \int_0^{l_1} \rho\ddot{y}_1(x_1, t)\,\mathrm{d}x_1, \qquad M_{G1} = \int_0^{l_1} \rho\ddot{y}_1(x_1, t)x_1\,\mathrm{d}x_1. \tag{9}$$

The deflection equation of coupler link is derived by using FE method as follows:

$$y_1(x_1, t) = \sum_{i=1}^{3} N_{1i}(x_1)q_{1i}(t), \quad 0 \leqslant x_1 \leqslant \frac{l_1}{2},$$

$$y_1(x_1, t) = \sum_{i=4}^{6} N_{1i}(x_1)q_{1(i-2)}(t), \quad \frac{l_1}{2} \leqslant x_1 \leqslant l_1. \tag{10}$$

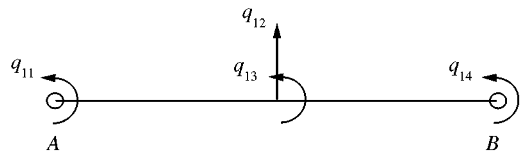

Figure 2. Finite-element model of the coupler.

Their two order derivatives is

$$\ddot{y}_1(x_1, t) = \sum_{i=1}^{3} N_{1i}(x_1)\ddot{q}_{1i}(t), \quad 0 \leqslant x_1 \leqslant \frac{l_1}{2},$$

$$\ddot{y}_1(x_1, t) = \sum_{i=4}^{6} N_{1i}(x_1)\ddot{q}_{1(i-2)}(t), \quad \frac{l_1}{2} \leqslant x_1 \leqslant l_1, \tag{11}$$

where $N_{1i}(x_1)$ is the element shape function, $q_{1i}(t)$ is the node displacement vector as shown in Figure 2.

Experiences have shown that using the three-time Hermite function as shape function to simulate the deflection curve of the elastic coupler is sufficient to give satisfactory results. According to some previous studies [11–13], the condition of no deflection at the ends of the link is usually assumed in the analysis. So, we regulate the boundary condition of Hermite functions as no deflection at the ends and every link is divided into two elements for an easy analysis. Thus, the coupler link corresponds to a simple-supported beam. This simplification can greatly reduce the analysis process but does not lose generalization.

In Figure 2 the elastic dynamic equation of coupler link is

$$[M]_1\{\ddot{q}\}_1 + [K]_1\{q\}_1 = \{F\}_1, \tag{12}$$

where $\{q\}_1 = [q_{11}\ \ q_{12}\ \ q_{13}\ \ q_{14}]^{\mathrm{T}}$, $[M]_1$ and $[K]_1$ are the mass and stiffness matrices in the elastic reference frame respectively. $[F]_1$ is the generalized force including the external forces and external moments caused by the impact motion and equivalent node forces applied to the ends of the coupler link in the elastic reference frame, for example, $M_B = F_B^t R$ is the external frictional moment at joint B, where F_B^t is the tangential component of impact force at clearance pair B. The expression of equivalent nodal force is $\{Q\}^e = \int [N]^{\mathrm{T}}\{f\}\,\mathrm{d}x$, where $[N]$ is the shape function matrix and $\{f\}$ is the transverse distributed excitation force matrix including distributed external forces, inertial forces and Coriolis inertial forces caused by rigid angular motion of clearance pairing elements.

The dynamic behaviors of clearance joint B in the motion process are analyzed as follows: The geometric constraint provides the following two relations:

$$r_{Bx} = l_3\cos\theta_3 - l_4 + l_1\cos\theta_1 - l_2\cos\theta_2, \qquad r_{By} = l_3\sin\theta_3 + l_1\sin\theta_1 - l_2\sin\theta_2, \tag{13}$$

$$r_B = \sqrt{r_{Bx}^2 + r_{By}^2}, \tag{14}$$

where r_{Bx} and r_{By} are the radial relative displacement between pin and brush at clearance joint B in the x- and y-axis directions.

In the constraint equations, the longitudinal deformations are neglected and the transverse deformations are assumed small, so the link AB is assumed to be constant and the deformations have no effect on the constraint equations. Here, a step function is induced whose expression is

$$S(u) = \begin{cases} 1, & u \geqslant 0, \\ 0, & u < 0, \end{cases} \tag{15}$$

where u is the contact deformation whose expression is $u = r_B - e$, e is the radius of normal clearance.

The relative velocity components (v_1 and v_n) in the normal and tangential directions are

$$v_t = \dot{r}_{By}\cos\alpha_B - \dot{r}_{Bx}\sin\alpha_B + (\dot{\theta}_1 - \dot{\theta}_2)R - \dot{\theta}_2\sqrt{r_{Bx}^2 + r_{By}^2}, \qquad v_n = \dot{r}_{Bx}\cos\alpha_B + \dot{r}_{By}\sin\alpha_B. \tag{16}$$

Thus, we can derive the impact force components (F_n and F_t) in the normal and tangential directions as follows:

$$F_n = Ku + C_n v_n, \qquad F_t = -f\,\mathrm{sign}(v_t)F_n - C_t v_t, \tag{17}$$

where C_n, C_t are viscous damping coefficients in the normal and tangential directions, respectively, K is the contact stiffness in the normal direction, f is the coefficient of Coulomb's friction, and R is the radius of the journal.

Therefore, the impact force components in the directions of x- and y-axis are

$$F_{Bx} = S(u)(F_t\sin\alpha_B + F_n\cos\alpha_B), \qquad F_{By} = S(u)(-F_t\cos\alpha_B + F_n\sin\alpha_B). \tag{18}$$

When the magnitude of the relative displacement r_B is less than the radial joint clearance e, the contact forces and moments are zero, that is, $S(u) = 0$. If r_B is equal to or greater than e, there is contact, and a contact force and moment, that is $S(u) = 1$. The clearance bearing contact angle expression is

$$\alpha_B = \mathrm{arctg}(r_{By}/r_{Bx}). \tag{19}$$

To this point, we can obtain the whole dynamic equations for coupler link by unifying rigid motion equations (2) and (3) with elastic deformation equations (12) directly.

In the light of this dynamic analysis process of the coupler link, the dynamic equations of other links can be derived similar to the equations (2), (3) and (12). Therefore, a set of coupled non-linear dynamic equations is written in a simple form as follows:

$$[M_e]\{\ddot{\eta}\} + [K_e]\{\eta\} = \{F_e\}, \tag{20}$$

where η is the system total co-ordinate matrices, $[M_e]$, $[K_e]$ and $[F_e]$ are the total mass and stiffness matrices and the total generalized force matrix of the mechanism respectively.

In order to include the structural damping effect, the equations are uncoupled by using modal analysis method and the damping matrix $[C]$ is introduced to the uncoupled equations. Here, the damping matrix is assumed to be diagonal matrices whose diagonal element expression is $C_{ii} = 2\xi(K_{ii}/M_{ii})^{1/2}$, where ξ is the structural damping ratio, K_{ii} and M_{ii} is the diagonal element of stiffness matrix and mass matrices in the uncoupled equations respectively.

Finally, the total dynamic equations in matrix form for the clearance elastic mechanism can be written as

$$[M]\{\ddot{\eta}_r\} + [C]\{\dot{\eta}_r\} + [K]\{\eta_r\} = \{F_r\}, \tag{21}$$

where $\{\eta_r\}$, F_r, $[M]$ and $[K]$ are mode co-ordinate, mode force, mode mass and stiffness matrices respectively.

3. NUMERICAL SOLUTION OF DYNAMIC EQUATIONS

Since both the effects of link elasticity and connection clearances on the dynamic behavior of planar linkage mechanisms are considered, the dynamic model is a set of strong coupled, strong non-linear, variable coefficient differential equations. The model involves three types of motions and the evolvement of dynamic parameters in every type of motions is different from each other. Therefore, direct solution of the equations analytically is very difficult or even impossible. In addition, using only one numerical solution method is not suitable and not accurate because of different parameters changing process in the different motion parts. In terms of the small deformation assumption of the elastic links, this study provides a more simple and more accurate solution method for the elastic linkage mechanism with clearance connections. This method is based on two kinds of methods, which are numerical integral such as Runge–Kutta method used to solve the stiff differential equations and mode analysis method used to solve the elastic deformation equations. This solution process applied the iterative approaching idea and its detailed procedures are given as follows.

(1) The initial conditions for the method are regarded as the results of kinematics and dynamic analysis of the mechanism in which all the joints are assumed to be zero clearance. Under the initial conditions, the angular position, angular velocity and angular acceleration of every link in the mechanism are derived in one circle of crank rotation by using fourth order Runge–Kutta numerical integral method without considering the effects of elastic link deformation. Then, the inertial forces, Coriolis inertial forces, inertial moments and impact forces can be obtained. So, the excitation forces and torques for elastic vibration can be achieved in the same time.
(2) By incorporating the excitation forces and torques derived from step (1), the elastic link deformations are developed through modal analysis method or mode superposition method. Thus, the inertial forces and inertial moments caused by elastic link can be obtained at the same time.
(3) By considering the effects of inertial forces and inertial moments, the parameters in step (1) are derived in the same rotation circle again.
(4) Once again solve the more accurate elastic link deformations by using the results of step (3).
(5) Eventually, the subsequent initial conditions for each circle in the simulation are obtained from the final conditions of the previous circle. The iterative process can be terminated until the kinematics conditions of previous circle are basically consistent with the next circle.

4. NUMERICAL RESULTS AND DISCUSSION

As an illustrative example of this dynamic analysis process, an elastic four linkages mechanism with a radial clearance in pair B as shown in Figure 1 was studied and the numerical results were discussed.

Table 1

Parameters used in the simulations

Constant	Description	Value
l_1	Length of the coupler link	0·181 m
l_2	Length of the rocker	0·1175 m
l_3	Length of the crank	0·03 m
l_4	Length of the ground link	0·2 m
m_1	Mass of the coupler link	0·1743 kg
m_2	Mass of the rocker	0·2916 kg
m_3	Mass of the crank	0·4489 kg
R	Radius of the journal	0·015 m
E	Modulus of elasticity	21 mn/cm^2
e	Normal radial clearance	0·000029 m
J_O	Moment of inertia of the crank	$1{\cdot}5 \times 10^{-4}$ kg m^2
J_A	Moment of inertia of the coupler link	$2{\cdot}7 \times 10^{-3}$ kg m^2
J_C	Moment of inertia of the rocker	$1{\cdot}34 \times 10^{-3}$ kg m^2
K	Contact stiffness in the normal direction	$7{\cdot}15 \times 10^6$N/m
C_n	Viscous damping coefficient in the normal direction	228 N s/m
C_t	Viscous damping coefficient in the tangential direction	0
f	Coefficient of Coulomb's friction	0·01
n_C	Synchronous rotate speed	3000 rpm
n_B	Rated rotate speed	2825 rpm
M_b	Rated torque of the electromotor	2·2 kg m
μ	Mechanical efficiency of the retarder	0·8
ξ	Damping ratio	0·01

4.1. DYNAMIC RESPONSE ANALYSIS FOR THE RIGID MOTION WITH CLEARANCES

A comprehensive dynamical response illustration for the mechanism with bearing clearances are the following.

In Figure 3(a) and 3(b), the angular acceleration response of coupler link shows very significant differences between the mechanism with and without radial clearance. In Figure 4, the crank angular acceleration variation shows a severe jump when the collision occurs at the clearance joint. In addition, from the trend of these curves, the peak value of dynamic response of the mechanism with bearing clearance are far greater than that of the case without bearing clearance and there are a series of fluctuations in the process of collision motion. These results clearly demonstrate that the mechanism stability with radial clearances is far lower than the mechanism without radial clearances.

4.2. IMPACT FORCE RESPONSES ANALYSIS

Figure 5 shows typical bearing force behavior. The bearing force displays a marked increase when the collision motions take place. For a long time, there is a constant bearing contact, which does not differ from the nominal or zero clearance behavior. At some point in Figure 5(b), the radial force at one of bearings passes through zero and the link end loses contact. By the time recontact occurs, there is a large relative velocity and impact results. This is followed by loss of contact and impact at the end of the link and then by a series of more and more weak impacts at the end.

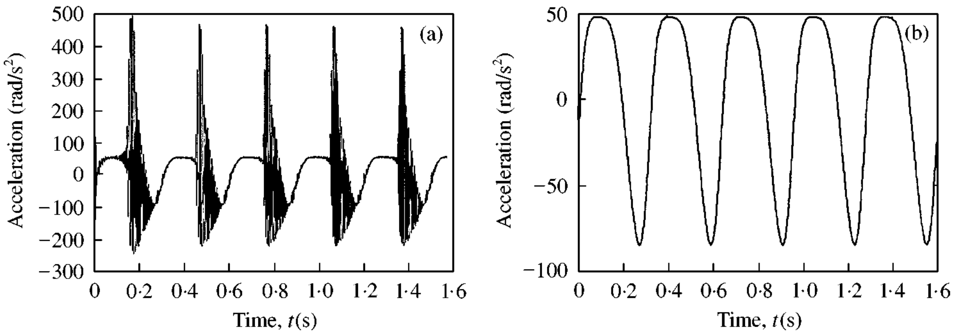

Figure 3. Acceleration response of coupler. (a) clearance joint; (b) idealized joint.

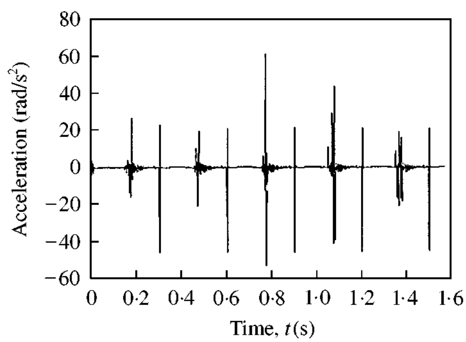

Figure 4. Acceleration response of crank.

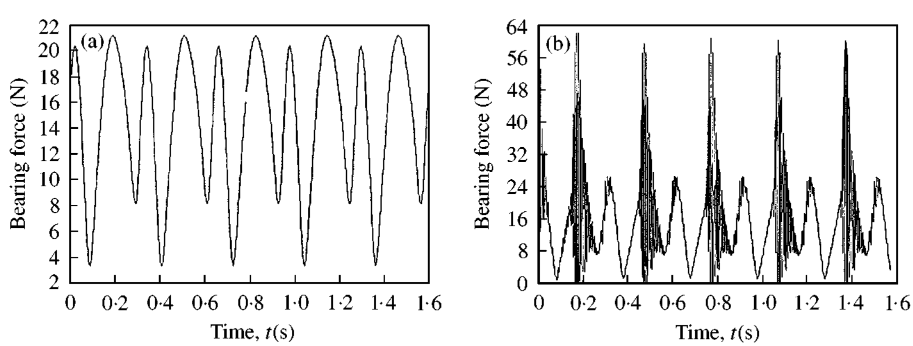

Figure 5. Contact force response without elasticity. (a) idealized joint; (b) clearance joint.

These results can only be obtained when the numerical integral time-step is little to the most but limited to the integral error boundaries. Both Figures 5(b) and 6(a) show that the contact forces increase when clearance and elasticity exist individually, especially even more severe effects of bearing clearance. The inclusion of both elasticity and clearance in the analysis cause no more essential change to the contact force than the clearance existence only except that the peaks of impact force are reduced by some degree as compared to that of no elasticity mechanism as described in Figure 6(b). The direction change of contact force is reflected in Figure 7. It is shown that the contact force directions change very drastically

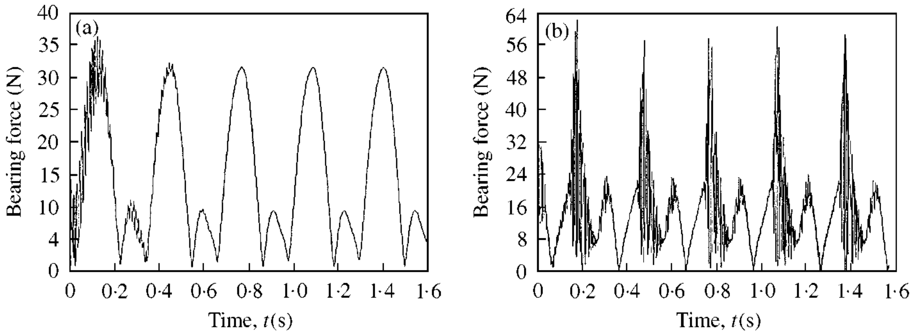

Figure 6. Contact force response with elasticity. (a) idealized joint; (b) clearance joint.

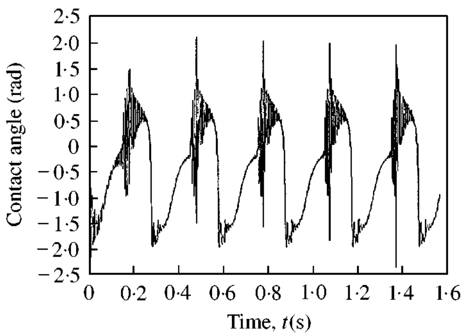

Figure 7. Contact angle response.

when collision motions occur and the change trend is basically identical in every running period.

4.3. ELASTIC DYNAMIC RESPONSE ANALYSIS

The presence of connection clearances greatly increases the elastic link deformation and reduces the motion stability and regularity of the elastic link as described in Figure 8. Figure 9(a) shows that the inclusion of suitable link structural damping consistently tends to reduce these excessive elastic deformations and make the elastic vibration more regular, and this result is the same with the mechanism with clearances shown in Figure 9(b). Otherwise, Figure 9 also shows that the suitable elastic deformation can change the contact position and collision frequency, that is, reduce the impact degree between the clearance pairing elements. Therefore, techniques of incorporating passive viscoelastic damper and active vibration control technologies can be used to reduce or eliminate excessive deformation caused by the bearing clearances theoretically if the damping is optimal and suitable [18]. This passive or active control method is adopted to indirectly control contact force by mainly reducing elastic deformation. Since the clearance effects are more important, more accurate and more practical control techniques to eliminate loss of contact are necessary to solve this problem. This aspect of theoretical and experimental work is scarce and should be taken up now.

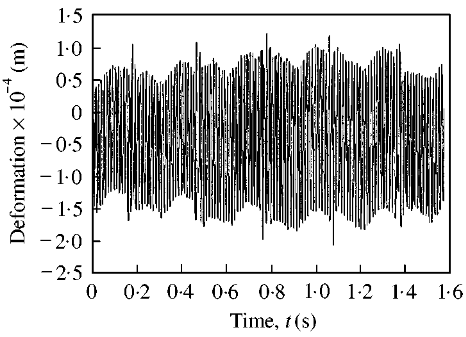

Figure 8. Mid-point transverse deformation response with clearance joint.

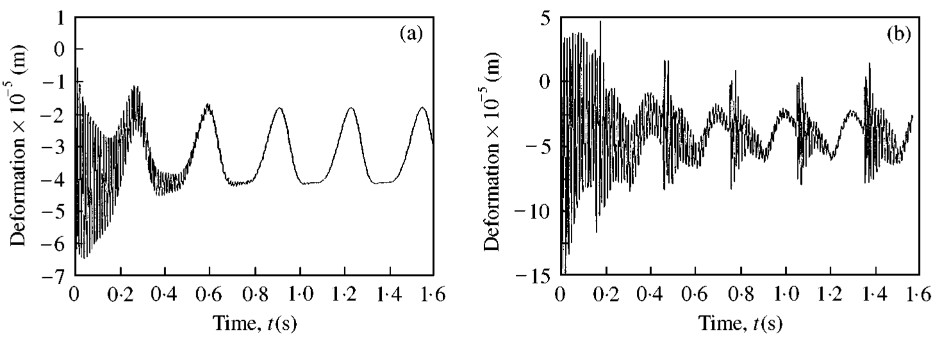

Figure 9. Mid-point transverse deformation response with structural damping: (a) idealized joint; (b) clearance joint.

4.4. NON-LINEAR DYNAMIC BEHAVIORS ANALYSIS

Under certain conditions, the elastic linkage mechanism can show some non-linear phenomenon which is mainly caused by the joint clearances. So, the non-linear behavior research for the rigid mechanism with clearance is enough to demonstrate the problem. We can assume that the second set of generalized co-ordinates are all equal to zero. Thus, the dynamic model is translated into a rigid mechanism model. It can be used to investigate the non-linear phenomenon arising in the mechanism. The result is suitable for the elastic mechanism with bearing clearances. Figure 10 describes the relative radial displacement time histories in five circles of the crank rotation. It indicates that the relative motion between journal and bearing at clearance pair B is steady and convergent. The phase portraits of relative displacement and velocity between journal and bearing at clearance joint B are described in Figure 11(a) and 11(b) respectively. They show that the relative motion trajectory between journal and bearing is irregularly non-repeatable. Poincare maps of different initial conditions are plotted in Figure 12(a–c). It is shown that the motion of the mechanism with different clearances is both periodic in the contact process and non-periodic in the separation process. In the periodic, contact maintained region the clearance response curves are made of isolated points and not dependent on the choice of the initial conditions. Otherwise, the non-periodic motion response curves made of numerous points are sensitive to the initial conditions and accord with typical characters of chaotic motions. So, it is found that clearance is the main factor that causes chaotic behavior and the system motion with clearances is a kind of complicated non-linear motion.

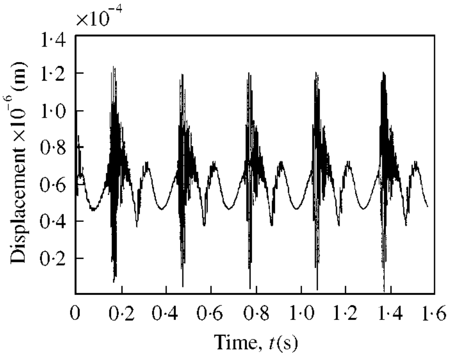

Figure 10. Radial relative displacement response.

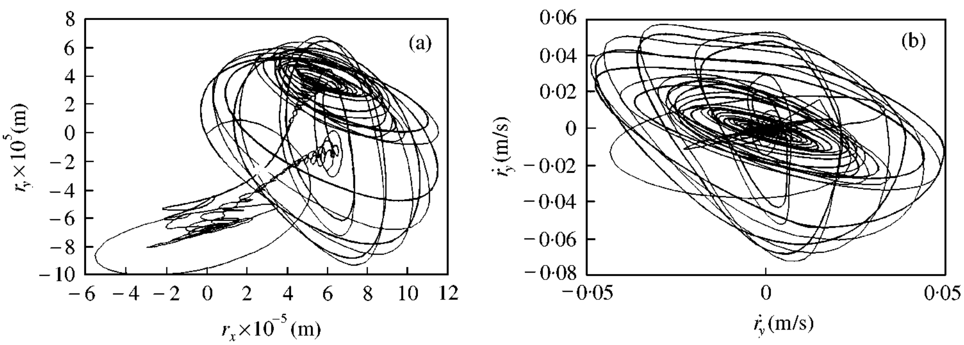

Figure 11. Phase portrait. (a) Displacement, (b) velocity.

5. CONCLUSIONS

This paper develops a unified dynamic model considering both the effects of rigidity and elasticity for the elastic linkage mechanism with clearance connections. In addition, this paper provides a more simple iterative solution method and an illustrative example has been investigated. From the analytical results, the following conclusions can be obtained:

(1) The mechanism stability with excessive radial clearances is far lower than the mechanism without radial clearances.
(2) The contact force increases when clearance and elasticity exist individually. Especially, the contact force increases even faster and changes very drastically with bearing clearance existence only. The inclusion of elasticity in the mechanism can reduce the peaks of impact force to some degree than that of no elasticity mechanism.
(3) The inclusion of suitable link structural damping tends to reduce elastic deformations and make the elastic vibration more regular.
(4) The system motion with clearances is a kind of complicated non-linear motion which is both periodic and non-periodic motion. The non-periodic motion that can be interpreted as chaotic motion is non-repeatable and sensitive to the initial conditions.

In short, both the effects of elasticity and bearing clearance should be considered when a dynamic investigation of the mechanism is carried out. In addition, this practice can also be beneficial for optimal mechanical design.

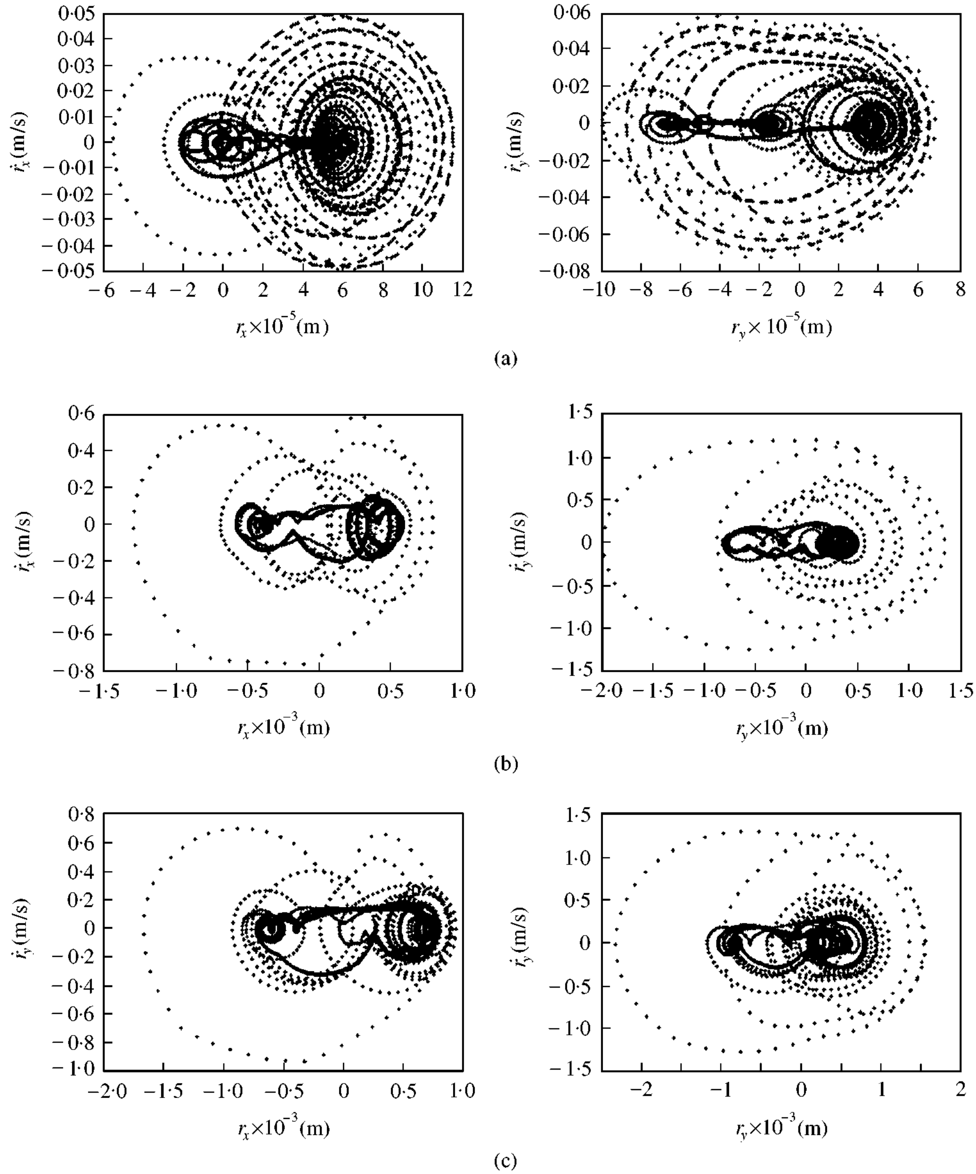

Figure 12. Poincare maps: (a) $e = 0{\cdot}029$ mm, $i = 15$; (b) $e = 0{\cdot}029$ mm, $i = 3$; (c) $e = 0{\cdot}29$ mm, $i = 3$.

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