Journal of Sound and Vibration (2002) **249**(2), 251–261
doi:10.1006/jsvi.2001.3792, available online at http://www.idealibrary.com on IDEAL®

MODELLING DYNAMICS OF A CONTINUOUS STRUCTURE WITH A PIEZOELECTRIC SENSOR/ACTUATOR FOR PASSIVE STRUCTURAL CONTROL

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(*Received* 15 *September* 2000, *and in final form* 26 *March* 2001)

This paper presents the concept of a vibration control system in which motions of a continuous structure with piezoelectric sensors/actuators can be suppressed (or activated) through transforming mechanical energy to electrical one and *vice versa*. The study is focused on distributed parameter structures, in which electromechanical variables are spatially dependent, and therefore traditional methods of design of piezoelectric transformers do not apply. In this case, a different approach is necessary to account for the spatial dependency of the variables. To examine the feasibility of the proposed vibration control system, we have performed the vibration suppression analysis of the cantilevered beam with piezoelectric sensors/actuators subjected to an exciting force/moment(s). The experimental results indicate that the damping of the composite system increases by 8–10 times in comparison with the mechanical system.

As a result, the paper significantly expands the concept of passive damping mechanism for structural systems to take into account the dynamics of a continuous elastic structure piezoelectrically coupled to electrical network.

1. INTRODUCTION

Piezoelectric sensors/actuators have been widely used in structural vibration control. The original research in this area involved studies of the control of cantilevered beams using rectangular piezoelectric layers [1–6]. Recently, a new concept of modal sensors and actuators has been proposed: piezoelectric elements (PES) are trimmed in shapes according to modal functions [7, 8]. These modal sensors and actuators can sense and control only the response of structural vibration of one mode when the shape profile matches the mode of free vibration. As a result, modal sensors/actuators can independently control each corresponding vibration mode without influencing other modes of the structure. However, recent studies illuminated difficulties associated with the design of distributed modal sensors and explored practical limitations associated with positioning sensors on structures in general [9].

As a result, recent studies have been devoted to optimizing the position of rectangular sensors [10, 11]. Since proper selection of number and location of the piezoelectric sensors is critical to control structural vibration efficiently, determining the optimum placement of piezoelectric sensors for adaptive vibration control is one of the key issues to address [12–14]. Multi-level genetic algorithm was developed and used to solve the simultaneous optimal design problems of the number and positions of actuators in an actively controlled structure [15–17].

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To some extent the problem of proper selection of sensor/actuator locations can be avoided by passive vibration control using piezoelectric transformers [18].

To select suitable parameters of piezoelectric transformers to control flexible structural systems, it is necessary to account for the spatial dependency of electromechanical variables. The paper addresses modelling of dynamics of an elastic structure and electrical network through the piezoelectric effect. As a result, the paper significantly expands the concept of passive damping mechanism for structural systems to take into account the dynamics of a continuous elastic structure piezoelectrically coupled to electrical network. The electromechanical model is developed for cantilevered beams with surface mounted PEs.

2. ANALYSIS

First, the governing dynamic equations for a cantilevered beam and a piezoelectric element shunted by an electrical circuit (Figure 1) will be derived. Stress σ_a induced in PE is $\sigma_a = E_a(\varepsilon_a - \Delta_a + h_a\dot{\varepsilon}_a)$, where E_a is Young's modulus of PE, ε_a is the strain, h_a the damping coefficient of PE, $\dot{\varepsilon}_a$ the strain rate, $\Delta_a = d_{31}V_a(t)/t_a$ the strain induced by voltage $V_a(t)$, d_{31} the piezoelectric constant, and t_a is the thickness of PE (Figure 2).

Strain due to an applied voltage produces a bending moment

$$M_a = \int_{t_b/2}^{t_b/2+t_a} \sigma_a(t) w_a z \, \mathrm{d}z, \tag{1}$$

where w_a is the width of PE, $\sigma_a(t) = E_a(z\partial^2 w/\partial x^2 - d_{31}V_a(t)/t_a)$ the stress induced in the piezoelectric patch (the damping stress is not included). Therefore, bending moment M_a is given by

$$\int_{t_b/2}^{t_b/2+t_a} E_a\left(z\frac{\partial^2 w}{\partial x^2} - \frac{d_{31}V_a(t)}{t_a}\right) z w_a \, \mathrm{d}z = E_a w_a \frac{\partial^2 w}{\partial x^2}\left(\frac{t_b^2 t_a}{4} + \frac{t_a^2 t_b}{2} + \frac{t_a^3}{3}\right) - E_a d_{31} V_a(t) w_a \frac{(t_a + t_b)}{2}. \tag{2}$$

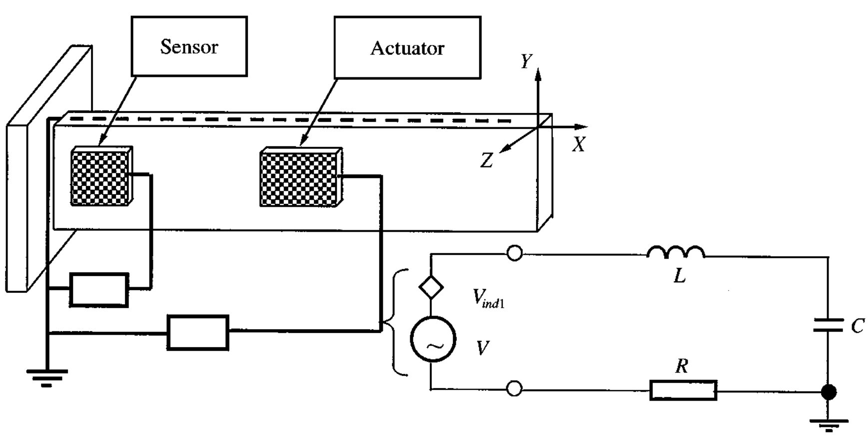

Figure 1. Schematic view of cantilevered beam and electromechanical system: $V_{ind\,1}$ is the induced potential difference ($V_{ind\,1} = h_1 r_1$, where h_1 is the piezoelectric constant and r_1 the displacement).

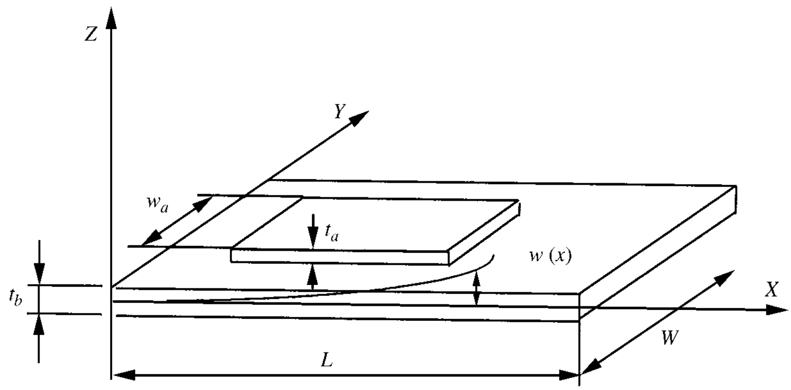

Figure 2. Cantilevered beam with co-ordinate system.

Taking into account that constant C_a depending on the geometry of the composite system is

$$C_a = \tfrac{1}{2} E_a d_{31} w_a (t_a + t_b), \tag{3}$$

the bending moment is

$$M_a = E_a w_a \frac{\partial^2 w}{\partial x^2}\left(\frac{t_b^2 t_a}{4} + \frac{t_a^2 t_b}{2} + \frac{t_a^3}{3}\right) - C_a V_a(t). \tag{4}$$

Consequently, the deflection of the beam is governed by the following differential equation:

$$\frac{\partial^2}{\partial x^2}\left[EI \frac{\partial^2 w(t,x)}{\partial x^2} + E_a w_a \frac{\partial^2 w(t,x)}{\partial x^2}\left(\frac{t_b^2 t_a}{4} + \frac{t_a^2 t_b}{2} + \frac{t_a^3}{3}\right) - C_a V_a(t,x)\right] + \rho A \frac{\partial^2 w(t,x)}{\partial t^2} = 0, \tag{5}$$

where ρA is the mass per unit length. Taking into account that

$$EI_{eq} = EI + E_a w_a \left(\frac{t_b^2 t_a}{4} + \frac{t_a^2 t_b}{2} + \frac{t_a^3}{3}\right),$$

equation (5) can be rewritten as follows:

$$\frac{\partial^2}{\partial x^2}\left[EI_{eq} \frac{\partial^2 w(t,x)}{\partial x^2} - C_a V_a(t,x)\right] + \rho A \frac{\partial^2 w(t,x)}{\partial t^2} = 0. \tag{6}$$

It is also necessary to take into account the structural damping coefficient of the beam [19, 20]. The stress arising in the beam due to damping is given by $\sigma_a = \mathrm{E}_a h \dot{\varepsilon}_a$, where

$$\varepsilon_a = -z \frac{\partial^2 w}{\partial x^2}, \qquad \dot{\varepsilon}_a = -z \frac{\partial^3 w}{\partial t \partial x^2}.$$

Bending moment of the body of the beam

$$M_{beam\ body} = 2\int_0^{t_{b/2}} zh\frac{\partial^3 w}{\partial t\partial x^2}Ezw_a\,dz = 2E\frac{\partial^3 w}{\partial t\partial x^2}hw_a\int_0^{t_{b/2}} z^2\,dz$$

$$= 2E\frac{\partial^3 w}{\partial t\partial x^2}hw_a\frac{z^3}{3}\bigg|_0^{t_{b/2}} = \frac{2}{3}\left(\frac{t_b}{2}\right)^3 hE\frac{\partial^3 W}{\partial t\partial x^2}w_a. \tag{7}$$

Bending moment of the actuator is

$$M_a = \int_{t_{b/2}}^{t_{b/2}+t_a} E_a z^2 h_a\frac{\partial^3 w}{\partial t\partial x^2}w_a\,dz = E_a w_a\frac{\partial^3 w}{\partial t\partial x^2}h_a\frac{z^3}{3}\bigg|_{t_{b/2}}^{t_{b/2}+t_a}$$

$$= E_a w_a h_a\frac{\partial^3 w}{\partial t\partial x^2}\left(\frac{t_b^2 t_a}{4}+\frac{t_a^2 t_b}{2}+\frac{t_a^3}{3}\right). \tag{8}$$

Total bending moment caused by damping is

$$M = M_a + M_{body\ beam} = \frac{\partial^3 w}{\partial t\partial x^2}C_{damp}, \tag{9}$$

where

$$C_{damp} = \frac{2}{3}\left(\frac{t_b}{2}\right)^3 Ehw_a + E_a h_a w_a\left(\frac{t_b^2 t_a}{4}+\frac{t_a^2 t_b}{2}+\frac{t_b^3}{3}\right).$$

Consequently, the equation of motion for the cantilevered beam with PEs is given by

$$\frac{\partial^2}{\partial x^2}\left[EI_{eq}\frac{\partial^2 w(t,x)}{\partial x^2}+\frac{\partial^3 w}{\partial t\partial x^2}C_{damp} - C_a V_a(t,x)\right] + \rho A\frac{\partial^2 w(t,x)}{\partial t^2} = P, \tag{10}$$

where

$$C_a = \tfrac{1}{2}E_a d_{31} w_a(t_a+t_b),$$

$$C_{damp} = \frac{2}{3}\left(\frac{t_b}{2}\right)^3 Ehw_a + E_a h_a w_a\left(\frac{t_b^2 t_a}{4}+\frac{t_a^2 t_b}{2}+\frac{t_a^3}{3}\right),$$

$$EI_{eq} = EI + E_a w_a\left(\frac{t_b^2 t_a}{4}+\frac{t_a^2 t_b}{2}+\frac{t_a^3}{3}\right),$$

where E is Young's modulus of the beam (B), h the damping coefficient of B, ρA the mass per unit length of B, P the external force, and $w(x,t)$ is the displacement of the beam under vibration.

The total charge accumulated by the PE is expressed as follows:

$$Q(t) = \int_{x_1}^{x_2} R(x)w_{sen}q(t,x)\,dx = \int_{x_1}^{x_2} R(x)w_{sen}\frac{k_{31}^2}{g_{31}}z_m\frac{\partial^2 w}{\partial x^2}\,dx, \tag{11}$$

where $R(x)$ is the function representing the shape of PE, w_{sen} the width of PE, $(x_2; x_1)$ the location of PE, k_{31} the coupling factor, g_{31} the voltage constant and $z_m = (t_b + t_a)/2$.

For transferring energy between mechanical and electrical systems it is necessary to introduce resistors, inductors and capacitors into the circuit. Consequently, one obtains

$$L\ddot{q} + R\dot{q} + \frac{q(t)}{C_c} - \frac{q_m(t)}{C_{ap}(x_2 - x_1)} = V, \tag{12}$$

where V is the external voltage. In the case of passive damping external voltage is equal to 0. Consequently,

$$L\ddot{q} + R\dot{q} + \frac{q(t)}{C_c} - \frac{q_m(t)}{C_e} = 0, \tag{13}$$

where $C_e = C_{ap}(x_2 - x_1)$ and C_{ap} is the capacitance of PE.

Consequently, one obtains the following governing differential equations:

$$\frac{\partial^2}{\partial x^2}\left[EI_{eq}\frac{\partial^2 w(t,x)}{\partial x^2} + \frac{\partial^3 w}{\partial t \partial x^2}C_{damp} - \frac{C_a Q(t)}{C_{ap}(x_2 - x_1)}\right] + \rho A\frac{\partial^2 w(t,x)}{\partial t^2} = P,$$

$$L\ddot{q} + R\dot{q} + \frac{q(t)}{C_c} - \frac{q_m(t)}{C_e} = 0,$$

$$q_m(t) = \int_{x_1}^{x_2} R(x)\frac{k_{31}^2}{g_{31}} z_m w_{sen}\frac{\partial^2 w}{\partial x^2}\,\mathrm{d}x. \tag{14}$$

The solution $w(t,x)$ can be sought in terms of normal modes $\phi_S(x)$:

$$w(t,x) = \sum_S \frac{P\phi_S(a)\phi_S(x)\sin(\omega t - \alpha_S)}{m[(\omega_S^2 - \omega^2)^2 + (c\omega_S^2\omega)^2]^{1/2}}, \qquad \tan\alpha_S = \frac{c\omega_S^2\omega}{\omega_S^2 - \omega^2}, \tag{15}$$

where $c = 2\zeta/\omega_S$, a is the co-ordinate x of external force P, ζ the modal damping coefficient and $\zeta = \omega_S\, C_{damp}/(2EI_{eq})$.

Taking into account that the voltage V is constant in the interval $(x_2; x_1)$, but undergoes a step change at each of the boundaries of this interval, the second derivative of the actuating voltage yields

$$\frac{\partial^2}{\partial x^2}\left(\frac{C_a}{C_e}q(t)\right) = [\delta'(x - x_1) - \delta'(x - x_2)]q(t)\frac{C_a}{C_e}, \tag{16}$$

where $\delta'(r)$ is the derivative of the Dirac delta function. Consequently, the structure with PEs can be described as follows:

$$\ddot{\varphi}_S + c\omega_S^2\dot{\varphi}_S + \omega_S^2\varphi_S = \frac{P\phi_S(a)\sin\omega t}{m} + \frac{C_a}{m}\frac{[\phi'_S(x_1) - \phi'_S(x_2)]q(t)}{C_e},$$

$$L\ddot{q} + R\dot{q} + \frac{q(t)}{C_c} - \frac{\int_{x_1}^{x_2} R(k_{31}^2/g_{31})z_m w_{sen}\left(\sum_S \varphi_S(t)(\partial^2\phi_S(x)/\partial x^2\right)\mathrm{d}x}{C_e} = 0. \tag{17}$$

3. APPLICATION

To estimate the influence of the circuit on the vibration of the beam, it is necessary to find $w(t, x)$. The solution can be sought in terms of normal modes $\phi_s(x)$:

$$w(t, x) = \sum_s \phi_s(x)\varphi_s(t), \tag{18}$$

where

$$\ddot{\varphi}_s + c\omega_s^2\dot{\varphi}_s + \omega_s^2\varphi_s = \begin{cases} \dfrac{P\phi_s(a)\sin\omega t}{m} & \text{(point force) or} \\ \dfrac{M\phi_s'(a)\sin\omega t}{m} & \text{(point moment).} \end{cases} \tag{19}$$

The solution is

$$\varphi_s = \frac{P\phi_s(a)\sin(\omega t - \alpha_s)}{m[(\omega_s^2 - \omega^2)^2 + (c\omega_s^2\omega)^2]^{1/2}}, \tag{20}$$

$$\tan\alpha_s = \frac{c\omega_s^2\omega}{\omega_s^2 - \omega^2}. \tag{21}$$

Consequently, one obtains

$$w(t, x) = \sum_s \frac{P\phi_s(a)\phi_s(x)\sin(\omega t - \alpha_s)}{m[(\omega_s^2 - \omega^2)^2 + (c\omega_s^2\omega)^2]^{1/2}}. \tag{22}$$

For mechanical system without piezoelectric damping,

$$\frac{\partial^2 w}{\partial x^2} = \sum_s \frac{P\phi_s(a)(\partial^2\phi_s(x)/\partial x^2)\sin(\omega t - \alpha_s)}{m[(\omega_s^2 - \omega^2)^2 + (c\omega_s^2\omega)^2]^{1/2}}. \tag{23}$$

Depending on the electrode profile $R(x)$, the charge produced by the mechanical system will be different (11). If $R(x)$ has the form of the second derivative of the modal function, i.e., $\partial^2\phi_s/\partial x^2$, then because of orthogonality (for modes $r \neq s$: $(\mathrm{d}^2\phi_s/\mathrm{d}x^2)(\partial^2 w/\partial x^2) \equiv 0$) the charge will depend on one of the modes only:

$$q_m(t) = \frac{k_{31}^2}{g_{31}} z_m w_{sen} \int_{x_1}^{x_2} \frac{\mathrm{d}^2\phi_s}{\mathrm{d}x^2}\frac{\partial^2 w}{\partial x^2}\,\mathrm{d}x. \tag{24}$$

If the form of the actuator does not correspond to modal shape, then one assumes the solution in the form (22). The charge produced by the mechanical system is

$$q_m(t) = \int_{x_1}^{x_2} R\frac{k_{31}^2}{g_{31}} z_m w_{sen}\left(\sum_s \varphi_s(t)\frac{\partial^2\phi_s(x)}{\partial x^2}\right)\mathrm{d}x. \tag{25}$$

Taking into account that the voltage V is constant in the interval $r_1 < r < r_2$, but undergoes a step change at each of the boundaries of this interval (16), the differential equation corresponding to each mode is

$$\ddot{\varphi}_s + cw_S^2\dot{\varphi}_s + \omega_S^2\varphi_s = \frac{P\phi_s(a)\sin\omega t}{m} + \frac{C_a}{m}\frac{[\phi_s'(x_1) - \phi_s'(x_2)]q(t)}{C_e},$$

$$L\ddot{q} + R\dot{q} + \frac{q(t)}{C_c} - \frac{\int_{x_1}^{x_2} R(k_{31}^2/g_{31})z_m w_{sen}\left(\sum_s \varphi_s(t)(\partial^2\phi_s(x)/\partial x^2\right)\mathrm{d}x}{C_e} = 0. \tag{26}$$

Consequently, the structure with piezoelectric devices can be described as two sets of coupled equations of motion and Maxwell's equation:

$$\begin{aligned} \mathbf{M}\ddot{\delta} + \mathbf{D}\dot{\delta} + \mathbf{K}\delta - \mathbf{H}q = \mathbf{P}, \\ \mathbf{L}\ddot{q} + \mathbf{R}\dot{q} + \mathbf{C}^{-1}q - \mathbf{H}^T\delta = 0, \end{aligned} \tag{27}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ the damping matrix, $\mathbf{K}$ the stiffness matrix, $\mathbf{H}$ the piezoelectric constant matrix; $\mathbf{P}$ the applied force; $\mathbf{L}$ the inductance matrix; $\mathbf{R}$ the resistance matrix, $\mathbf{C}$ the capacitance matrix, δ the displacement and q is the electrical charge.

The parameters of the electrical circuit can be tuned to a structural mode so as to minimize the maximum response of the mode. For passive control, mechanical energy may be transferred into electrical energy through the piezoelectric transformer and then dissipated by resistance in the electrical circuit [18]. To match the dynamic stiffness of the electrical system, one matches $\mathbf{M}s^2 + \mathbf{D}s + \mathbf{K}$ to $\mathbf{L}s^2 + \mathbf{R}s + \mathbf{C}^{-1}$ [18].

4. EXPERIMENTAL RESULTS

Experiments were conducted to test the validity of the analytical model. The cantilevered beam was 77 cm long, 5 cm wide and 6 mm thick as shown in Figure 3. The material of the beam is aluminium with the following properties: Young's modulus—76×10^9 N/m^2;

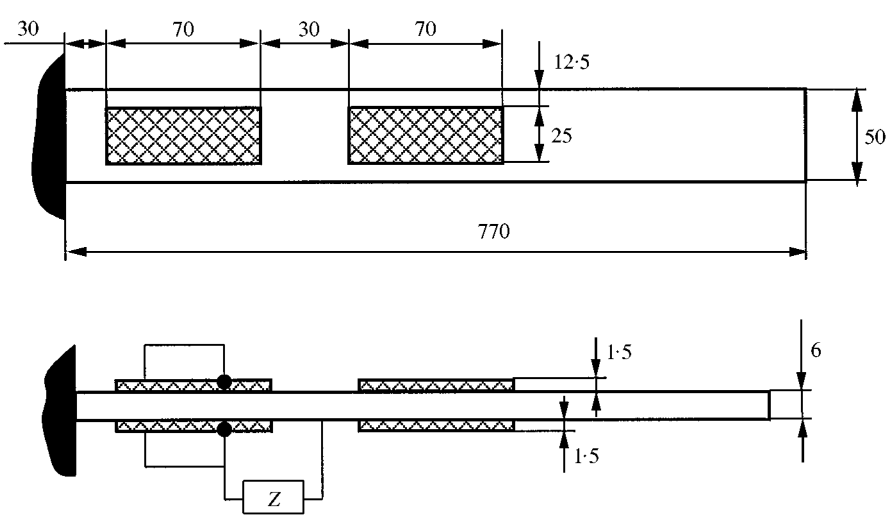

Figure 3. Cantilevered beam with two sets of piezoelectric patches.

TABLE 1

Material properties of the piezoelectric patches (PIC 151)

Density (g/cm^3)	7.80
Curie temperature (°C)	250
Dielectric constant	
$\varepsilon_{33}^{T}/\varepsilon_0$	2100
$\varepsilon_{11}^{T}/\varepsilon_0$	1980
Dielectric loss $\tan\delta$ [$\times 10^{-3}$]	15
Resistivity	10^{11}
Coupling factors	
k_p	0·62
k_{33}	0·69
k_{31}	0·34
Mechanical Q	120
Frequency constants (Hz m)	
N_p	2100
N_1	1500
N_3	1680
N_t	1950
Charge constants [$\times 10^{-12}$ m/V]	
d_{31}	−210
d_{33}	450
d_{15}	580
Voltage constants [$\times 10^{-12}$ V m/N]	
g_{31}	−11.5
g_{33}	22.8
Elastic constant [$\times 10^{-12}$ m^2/N]	
S_{11}^{E}	15
S_{33}^{E}	19
Elastic modulus	$E = 63$ GPA

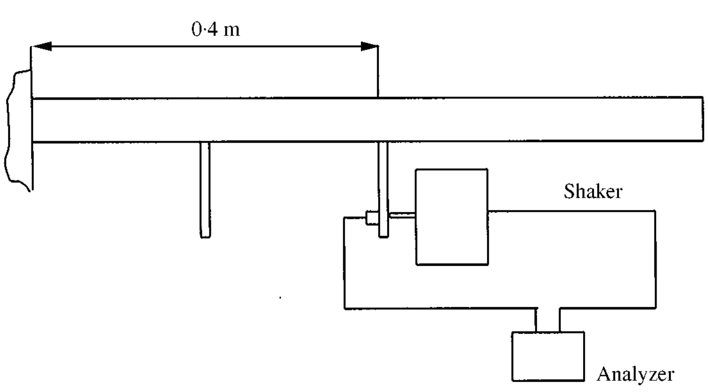

Figure 4. Experimental set-up of the beam with Mini-Shaker.

Table 2

Experimental parameters of the electrical circuit

Optimal inductance, *H*	2965	80	9·6
Optimal resistance, *k*	158	105	50
Capacitance, *F*	0·156	0·156	0·156
Frequency of the beam, Hz	7·8	50	140

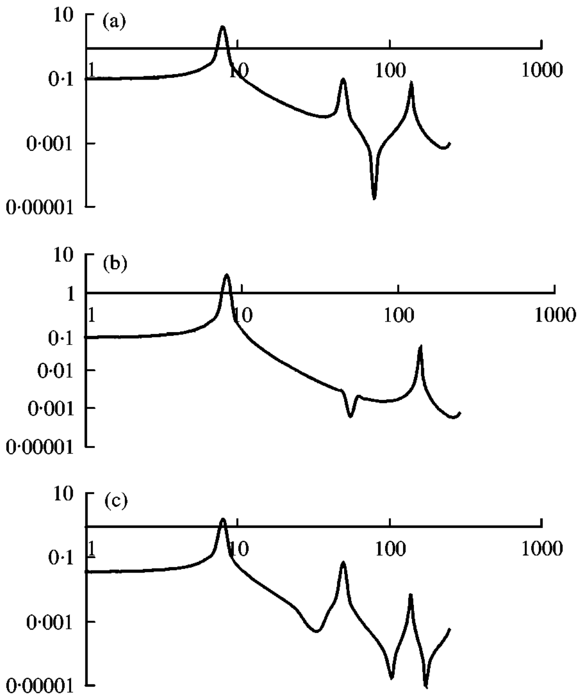

Figure 5. The frequency response of the beam (axis Y—absolute displacement, mm; axis X—frequency, Hz): (a) $x = 0{\cdot}77$ m; (b) $x = 0{\cdot}6$ m; (c) $x = 0{\cdot}4$ m.

density—2845 kg/m^3. Two sets of surface mounted piezoelectric patches PIC 151 ($70 \times 25 \times 1{\cdot}5$ mm) were bonded to the beam. The material properties and frequency constants of the piezoelectric patches are given in Table 1. The first pair was shunted while the second pair served to drive the beam. The shunted pair was located 30 mm from the base and extended 70 mm. The piezoelectric pairs were separated by 30 mm.

Mini Shaker type 4810 Bruel & Kjaer was used to excite the beam in the frequency range up to 150 Hz and at the location that is 40 cm away from the cantilevered end, see Figure 4. HP 35670A signal analyzer was used to generate the excitation signals, and to record and analyze the forced vibrations of the beam with and without the electromechanical system. The parameters of the electrical circuit that are given in Table 2 were tuned to structural modes so as to minimize the responses of the modes. Three frequency response curves of the beam at three different locations, namely $x = 0{\cdot}4$, $0{\cdot}6$ and $0{\cdot}77$ m were obtained respectively. Figure 5 shows the obtained three frequency response functions of the cantilevered beam without the electromechanical system and Figure 6 shows those obtained when the beam was coupled with the composite electromechanical system.

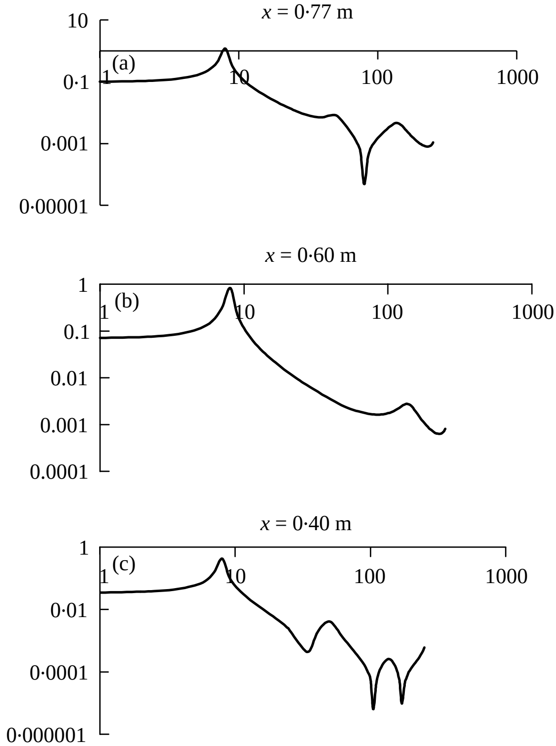

Figure 6. The frequency response of the beam with passive network (axis Y—absolute displacement, mm; axis X—frequency, Hz): (a) $x = 0{\cdot}77$ m; (b) $x = 0{\cdot}6$ m; (c) $x = 0{\cdot}4$ m.

It is found that because of the interaction between the electrical and mechanical systems, the damping of the combine beam–piezoelectric patches system increases by 8–10 times in comparison to that of the mechanical system. In particular, the vibration suspension of the beam when the piezoelectric patches are used is much more effective for higher modes, for instance, the second and third modes in this example. In consideration of practical applications, as applied loads may change from time to time, a self-adaptive electromechanical system, the parameters of electric circuits of which are self-adjusted according to the main frequency components of the beam response, needs to be developed to maximize the damping effect.

5. CONCLUSIONS

In this paper, research is focused on distributed parameter structures, in which electromechanical variables are spatially dependent, and therefore, traditional methods of the design of piezoelectric transformers do not apply.

In this case, a different approach is necessary to account for the spatial dependency of the variables. A new type of structural damping mechanism has been presented based on resonant shunting of piezoelectric materials by passive electrical circuits.

The analytical model of the electromechanical system, as well as the experimental validation of this model, provides a solid basis for future applications of piezoelectric transformers.

ACKNOWLEDGMENTS

The authors are thankful to the reviewers for their useful comments and suggestions. Financial support provided by the University of Technology, Sydney for this study is gratefully acknowledged.

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