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SUPPRESSION OF EFFECTS OF NON-LINEARITIES IN A CLASS OF NON-LINEAR SYSTEMS BY DISTURBANCE OBSERVERS

S. M. SHAHRUZ

Berkeley Engineering Research Institute, P. O. Box 9984, *Berkeley, CA* 94709, *U.S.A.*
E-mail: *shahruz@robotics.eecs. berkeley.edu*

AND

C. CLOET AND M. TOMIZUKA

Department of Mechanical Engineering, University of California, Berkeley, CA 94720, *U.S.A.*

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1. INTRODUCTION

Perhaps all natural and physical systems are governed by non-linear laws of nature. The dynamics of most of such systems can be mathematically represented by non-linear differential or integral equations, which can be studied by analytical or numerical techniques. These techniques, in many instances, can successfully explain certain phenomena that are exclusive to non-linear systems. One such phenomenon is the limit cycle (periodic) behavior of systems. Limit cycles can be considered as both desirable and unwanted responses of systems. For instance, oscillators (see, e.g., reference [1]) or certain types of lasers, such as self-pulsatig lasers (see, e.g., reference [2]), are expected to exhibit limit cycle behavior. However, in a positioning system, limit cycles are certainly unwanted and should be suppressed (see, e.g., reference [3]). In the past decades, researchers have devised techniques to suppress limit cycles in non-linear systems: see, e.g., references [4–11] and the references therein.

In this note, it is shown that an effective means of suppressing effects of non-linearities, and consequently possible limit cycles in a class of non-linear systems, is the application of disturbance observers. Disturbance observers are useful tools that were originally proposed in references [12, 13] as means of estimating disturbances to linear systems and cancelling them subsequently. Later, the theory of disturbance observers was advanced in reference [14]. Presently, disturbance observers are successfully used in achieving robust stability and performance in motion control systems, for instance, robotic systems, high-speed machining systems, (micro) positioning systems, disk drives; see e.g., references [15–21] and the references therein. It appears that disturbance observers are mostly designed for linear systems. There are some works where the application of disturbance observers to non-linear systems is reported; see references [11, 22–27]. The present note illustrates that disturbance observers can make members of a certain class of non-linear systems behave linearly.

The organization of the note is as follows. In section 2, the class of non-linear systems to be studied is presented. A non-linear system in this class has the property that its output is equal to the summation of the output of a stable single-input–single-output (SISO) linear

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time-invariant system and a bounded function of time. In section 3, a disturbance observer is designed to estimate the effects of non-linearities in a system in the class under consideration and cancel them subsequently. Upon having the non-linear effects cancelled, the system behaves linearly. An example is given to show that limit cycles in a Van der Pol-type system caused by a non-linearity can be effectively suppressed by a disturbance observer. In section 4, a non-linear feedback system is considered. The system is expected to be free of non-linear responses, such as limit cycles, and achieve desired goals, such as tracking step inputs. By using a disturbance observer, the non-linear effects in the system are suppressed. Thus, the system can be treated as a linear system for which a linear controller can be designed to achieve the desired goals. An example is given to illustrate the design of a controller that makes a Van der Pol-type system track step inputs.

2. NON-LINEAR SYSTEMS

In this section, a class of SISO non-linear systems is introduced. A member of this class is represented by

$$\begin{cases}\dot{\xi}(t) = A\xi(t) + f(\xi(t), t), \qquad \xi(0) =: \xi_0, \\ \eta(t) = c\xi(t),\end{cases} \tag{1}$$

for all $t \geqslant 0$, where the state vector $\xi(t) \in \mathbb{R}^n$, the non-zero initial state vector $\xi_0 \in \mathbb{R}^n$, the output $\eta(t) \in \mathbb{R}$, the coefficient matrices $A \in \mathbb{R}^{n\times n}$, $c = [c_{11}\ c_{12}\ \cdots\ c_{1n}] \in \mathbb{R}^{1\times n}$, and the non-linear function $f: \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is given by

$$f(\xi(t), t) = [f_1(\xi(t),\ t)\quad f_2(\xi(t), t) \cdots f_n(\xi(t), t)]^{\mathrm{T}}. \tag{2}$$

It is assumed that:

(A1) The pair (A, c) is completely observable.

(A2) The non-linear function f, though not exactly known, is norm bounded. More precisely,

$$\|f\|_\infty := \max_{1\leqslant i\leqslant n}\ \sup_{x\in\mathbb{R}^n}\ \sup_{t\geqslant 0}\ |f_i(x, t)| \leqslant k_f < \infty, \tag{3}$$

where $k_f > 0$ is a constant real number.

Suppose that system (1) exhibits non-linear behavior, such as limit cycle behavior, which is considered as undesirable. Thus, a control law has to be designed to suppress the non-linear behavior.

Let a scalar control input $v(\cdot)$ be applied to system (1) via an influence (input) vector $b \in \mathbb{R}^n$, thereby the system is represented as

$$N: \begin{cases}\dot{x}(t) = Ax(t) + bv(t) + f(x(t), t), \qquad x(0) =: x_0 = \xi_0, \\ y(t) = cx(t),\end{cases} \tag{4}$$

for all $t \geqslant 0$, where the state vector $x(t) \in \mathbb{R}^n$, the input $v(t) \in \mathbb{R}$, and the output $y(t) \in \mathbb{R}$. The system N is shown in Figure 1. It is assumed that:

(A3) The vector b is chosen such that the pair (A, b) is completely controllable.

The control law $v(\cdot)$ is to be designed to suppress the effects of the vector-valued non-linear function f. Note that, in general, it is not possible to cancel the vector f algebraically by the vector $b\ v(\cdot)$. Such a cancellation, even when it is possible, is not recommended since f is not exactly known.

With this set-up, an equivalent representation of system (4) can be obtained as follows. By assumption (A3), it is noted that A in equations (4) can be assumed to be a Hurwitz matrix.

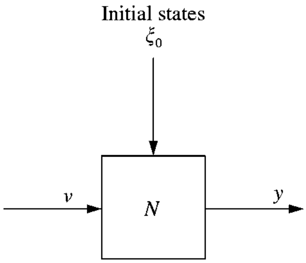

Figure 1. The non-linear system N represented by equation (4).

The reason is that by assumption (A3), there exists is linear state feedback control law that can make the coefficient matrix of system (4) Hurwitz. Having A a Hurwitz matrix, there exist constant real numbers $M > 0$ and $\sigma > 0$, such that

$$\| \exp(At) \|_{\infty} \leqslant M \exp(-\sigma t), \tag{5}$$

for all $t \geqslant 0$ (see, e.g., reference [28, p. 195]). This inequality is used to establish a useful result. From equations (4), it follows that the output of the non-linear system N is

$$y(t) = c \exp(At)\, x_0 + c \int_0^t \exp(A(t-\tau))\, b v(\tau)\, \mathrm{d}\tau + d(t), \tag{6}$$

for all $t \geqslant 0$, where

$$d(t) = c \int_0^t \exp(A(t-\tau))\, f(x(\tau), \tau)\, \mathrm{d}\tau \in \mathbb{R}. \tag{7}$$

Using inequalities (3) and (5) in equation (7), it is concluded that $t \mapsto d(t)$ is a bounded function of time. More precisely,

$$\| d \|_{\infty} := \sup_{t \geqslant 0} |d(t)| \leqslant \sum_{j=1}^{n} |c_{1j}| M k_f / \sigma < \infty. \tag{8}$$

From equations (6) and (7) and inequality (8), it is concluded that the output of the non-linear system N is equal to the summation of the output of the stable SISO linear time-invariant system

$$H\colon \begin{cases} \dot{\bar{x}}(t) = A\bar{x}(t) + bv(t), & \bar{x}(0) =: \bar{x}_0 = \xi_0, \\ \bar{y}(t) = c\,\bar{x}(t), \end{cases} \tag{9}$$

and the bounded function of time $d(t)$ for all $t \geqslant 0$, where the state vector $\bar{x}(t) \in \mathbb{R}^n$ and the output $\bar{y}(t) \in \mathbb{R}$. By assumptions (A1) and (A3), the representation of the system H in equations (9) is minimal. The transfer function of H is irreducible and is given by

$$H(s) = c(sI_n - A)^{-1} b, \tag{10}$$

where I_n denotes the $n \times n$ identity matrix.

A conclusion to be drawn from equations (6) to (10) is that the system N can be equivalently represented by the linear system in Figure 2. This system is denoted by H_{+d}. The transfer function from v to y in H_{+d} is $H(s)$. The representation in Figure 2 has a useful property to be exploited in the next section.

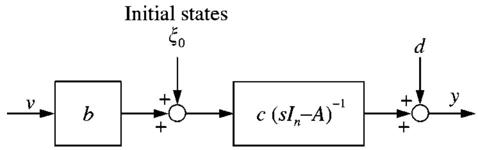

Figure 2. The system H_{+d}. This system is an equivalent representation of the system N.

3. LINEAR BEHAVIOR BY DISTURBANCE OBSERVERS

Representing the non-linear system N by the equivalent linear system H_{+d} in Figure 2 is of great advantage, because the effects of non-linearities in N appear as the bounded disturbance $d(\cdot)$ in H_{+d}. Therefore, if one seeks to suppress the effects of non-linearities in N, then one should design a control law that suppresses the effect of $d(\cdot)$ in H_{+d}. The latter can be achieved by a disturbance observer that estimates $d(\cdot)$ and cancels it subsequently. Therefore, the goal of this section is to design a disturbance observer to make N behave linearly and, for instance, be free of limit cycles.

A disturbance observer added to the system H_{+d} is shown in Figure 3. In this figure, $H_n(s)$ represents the nominal transfer function (mathematical model) corresponding to $H(s)$ in equation (10). Noting that the effect of the initial state vector ξ_0 asymptotically decays to zero by the stable transfer function $c\,(s\,I_n - A)^{-1}$, it follows that $\tilde{d}(t) := \; y(t) - y_n(t)$ is an estimate of the disturbance $d(t)$ as $t \to \infty$. In order to implement the disturbance observer, the filter $Q(s)$ is added to the system to make $Q(s)\,H_n^{-1}(s)$ a realizable (at least a proper) transfer function, because $H_s^{-1}(s)$ is often unrealizable. A successful design of a disturbance observer crucially depends on the design of $Q(s)$. Due to its important role, the design of $Q(s)$ has been extensively studied; see, e.g., references [12, 14, 15, 21]. It turns out that $Q(s)$ should be a low-pass filter of unity DC-gain. A typical form of $Q(s)$ is

$$Q(s) = \frac{\sum_{k=1}^{m-\rho} a_k\,(\tau s)^k + 1}{\sum_{k=1}^{m} a_k\,(\tau s)^k + 1}, \tag{11}$$

where ρ is at least equal to the relative degree of $H_n(s)$ and a_k and τ are positive real numbers. From Figure 3, it is concluded that the output of the system is

$$y(s) = [1 + b\,(1 - Q(s))^{-1}\,Q(s)\,H_n^{-1}(s)]^{-1}\,[c\,(sI_n - A)^{-1}\,\xi_0 + d(s)], \tag{12}$$

where $y(s)$ and $d(s)$ are the Laplace transforms of $y(\cdot)$ and $d(\cdot)$, respectively. Several comments regarding equation (12) should be made: (1) the filter $Q(s)$ should be designed such that the transfer function

$$[1 + b\,(1 - Q(s))^{-1}\,Q(s)\,H_n^{-1}(s)]^{-1}, \tag{13}$$

is stable; (2) since $Q(s)$ is a low-pass filter of unity DC-gain, the effect of the bounded disturbance $d(\cdot)$ in the system in Figure 3 is suppressed, and the output of the system converges to zero.

An implementation of the disturbance observer for the system N (equivalently H_{+d}) is shown in Figure 4. The system in this figure is denoted by N_{DOB} to indicate that a disturbance observer is added to N. The equivalence of N_{DOB} and the system in Figure 3 asserts that the effects of non-linearities in N_{DOB} are suppressed and the output of this system converges to zero.

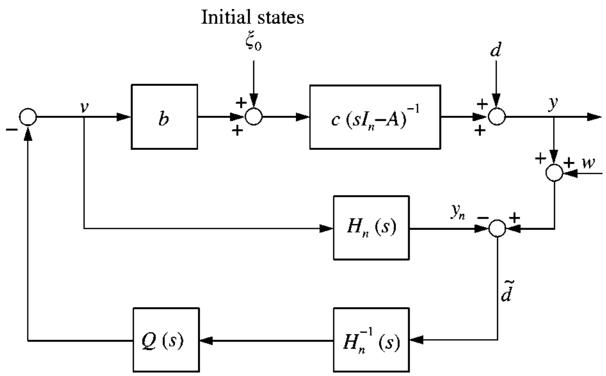

Figure 3. A disturbance observer added to the system H_{+d} (equivalently N) to estimate d which incorporates the effects of non-linearities in N. An estimate of d is $\tilde{d}$ which is cancelled subsequently.

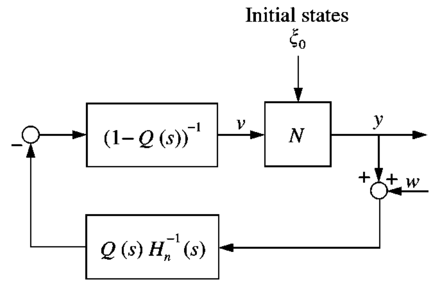

Figure 4. The system N_{DOB}. This system is N to which a disturbance observer is added.

Next, an example is presented to illustrate the efficacy of disturbance observers in suppressing the effects of non-linearities and limit cycles in a non-linear system in the class of systems considered in this note.

3.1. EXAMPLE: A VAN DER POL-TYPE SYSTEM

Consider the system

$$\begin{bmatrix} \dot{\xi}_1(1) \\ \dot{\xi}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} + \begin{bmatrix} \tanh(3\xi_1(t)) \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \xi_1(0) \\ \xi_2(0) \end{bmatrix} = \begin{bmatrix} \xi_{10} \\ \xi_{20} \end{bmatrix}, \tag{14a}$$

$$\eta(t) = [1 \;\; 0] \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix}, \tag{14b}$$

for all $t \geqslant 0$, where the states $\xi_1(t) \in \mathbb{R}$, $\xi_2(t) \in \mathbb{R}$, the non-zero initial state vector $[\xi_{10} \;\; \xi_{20}]^{\mathrm{T}} \in \mathbb{R}^2$, and the output $\eta(t) \in \mathbb{R}$. System (14) is a Van der Pol-type system (see reference [28, Chapter 2]). It is straightforward to verify that assumptions (A1) and (A2) hold for systems (14). Moreover, it can be shown that starting from any non-zero $[\xi_{10} \;\; \xi_{20}]^{\mathrm{T}}$, system (14) exhibits limit cycle behavior (see reference [28, Chapter 2]).

Let $[\xi_{10} \;\; \xi_{20}]^{\mathrm{T}} = [1 \;\; 0]^{\mathrm{T}}$. For these initial states, the output of system (14) is that depicted in Figure 5(a) and designated by η. It is evident that η is a periodic function of time, i.e., the

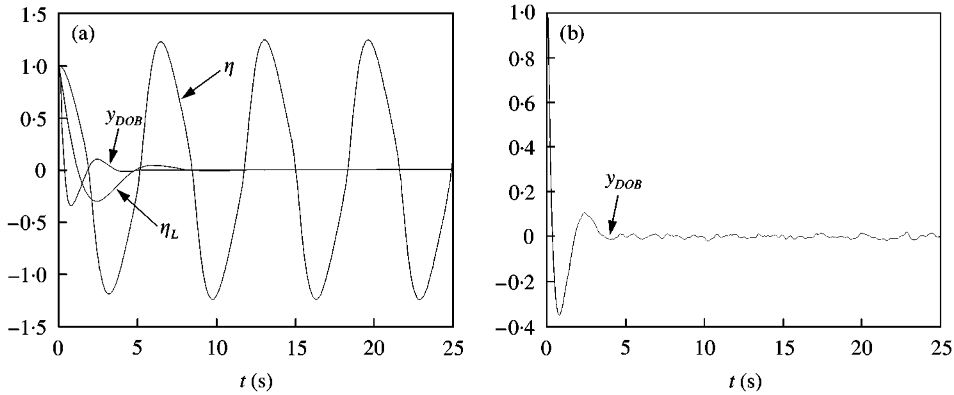

Figure 5. (a) Responses of the systems N, the non-linearity-free N, and N_{DOB}, designated by η, η_L, and y_{DOB}, respectively, in the absence of measurement noise w. It is evident that y_{DOB} is free of limit cycles. That is, the disturbance observer has suppressed the effects of the non-linearity. (b) Response of the system N_{DOB}, designated by y_{DOB}, in the presence of measurement noise w.

system has limit cycle behavior. If there were no non-linearity in the system, i.e., tanh $(3\xi_1(\cdot))$ in equation (14a) were absent, then the system output would decay to zero asymptotically, as shown in Figure 5(a) by η_L.

The difference between η and η_L is due to the non-linearity in system (14). It is now shown that the effect of this non-linearity, and consequently limit cycle behavior, can be effectively suppressed by a disturbance observer.

Let a scalar input $v(\cdot)$ be applied to system (14) via the influence vector $b = [0\ \ 1]^{\mathrm{T}}$, thereby the system is represented as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) + \begin{bmatrix} \tanh(3x_1(t)) \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{15a}$$

$$y(t) = [1\ \ 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \tag{15b}$$

for all $t \geqslant 0$.

It is straightforward to verify that assumption (A3) holds for system (15). The linear system H corresponding to system (15) can be obtained by neglecting the non-linear term in equation (15a). The transfer function of this system is

$$H(s) = \frac{1}{s^2 + s + 1}. \tag{16}$$

The implementation of the disturbance observer for system (15) is the same as the system N_{DOB} in Figure 4, where $H_n(s) = H(s)$, and

$$Q(s) = \frac{3000}{s^3 + 45\,s^2 + 650\,s + 3000}. \tag{17}$$

The output of N_{DOB} in the absence of measurement noise ($w \equiv 0$) is shown in Figure 5(a) and is designated by y_{DOB}. It is evident that y_{DOB} converges to zero. That is, the disturbance observer has successfully suppressed the effect of the non-linearity in the system.

The effect of measurement noise $w(\cdot)$ on the performance of the system N_{DOB} is studied next. Let $w(\cdot)$ be band-limited white noise. The output of the system in the presence of this source of noise is depicted in Figure 5(b) and is designated by y_{DOB}. It is evident that the system output decays to zero, however, it is noisy due to the measurement noise. Noisy output is the price to be paid to suppress limit cycles by feedback and a disturbance observer. Whenever there is a feedback, there is always measurement noise that affects the outputs of systems adversely.

4. CONTROLLERS FOR A CLASS OF NON-LINEAR FEEDBACK SYSTEMS

In this section, based on the results of section 3, a methodology for designing controllers for a class of non-linear feedback systems is presented. A member of this class is shown in Figure 6 and is denoted by $S(C, N)$. In this system, r, y, and w are, respectively, the reference input, the output, and the measurement noise. Furthermore, the system N is that represented in equations (4) for which assumptions (A1)–(A3) hold, and the controller C (either linear or non-linear) is to be designed to achieve desired goals, such as tracking step inputs. Since N is a non-linear system, there are no standard techniques for designing C. However, having assumption (A2) satisfied, the system $S(C, N)$ can be equivalently represented by the linear system in Figure 7. This system is denoted by $S(C, H_{+d})$. Due to the linearity of the system $S(C, H_{+d})$, the controller C can be chosen to be a linear system with transfer function $C(s)$. The design of $C(s)$ is now straightforward: set $d \equiv 0$ in $S(C, H_{+d})$ and use standard techniques from the theory of linear systems to design a $C(s)$ by which desired goals are achieved. Although the design of $C(s)$ is straightforward, the effects of non-linearities in the system which appear as the disturbance $d(\cdot)$, can still cause undesirable responses. In order to suppress such effects, a disturbance observer is added to the system $S(C, H_{+d})$. The resulting system is denoted by $S(C, N_{DOB})$ and is shown in Figure 8. Clearly, this system takes advantage of the linearizing effect of the disturbance observer and the control input provided by $C(s)$ to achieve the desired goals.

An example is now presented to examine how well the system $S(C, N_{DOB})$ can track step inputs.

4.1. EXAMPLE: A VAN DER POL-TYPE SYSTEM TRACKING STEP INPUTS

Consider the system $S(C, N)$ where N is that represented by equations (15) and let C be an integral (I) controller with the transfer function

$$C(s) = \frac{0{\cdot}36}{s}. \tag{18}$$

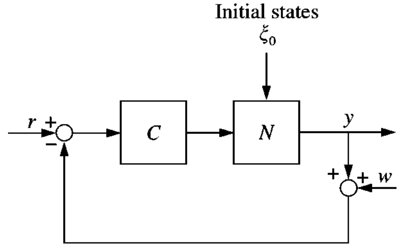

Figure 6. The non-linear feedback system $S(C, N)$.

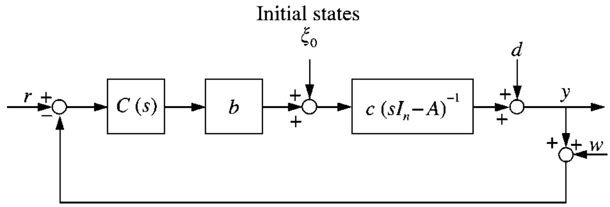

Figure 7. The linear feedback system $S(C, H_{+d})$. This system is an equivalent representation of $S(C, N)$.

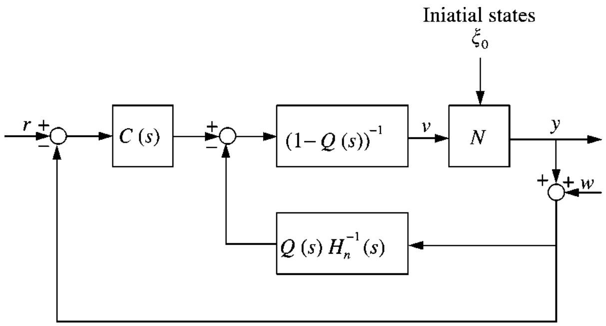

Figure 8. The non-linear feedback system $S(C, N_{DOB})$. This system is $S(C, N)$ to which a disturbance observer is added.

Let the initial states of N be $[0\ \ 0]^T$, $w \equiv 0$, and

$$r(t) = 0{\cdot}6, \tag{19}$$

for all $t \geqslant 0$. For this set-up, the output of the system $S(C, N)$ is that depicted in Figure 9(a) and designated by y. It is evident that $S(C, N)$ exhibits limit cycle behavior. If there were no non-linearity in the system, i.e., $\tanh(3x_1(\cdot))$ in equation (15a) were absent, then the system output would be that in Figure 9(b) designated by y_L. This output is the same as that of the system $S(C, H_{+d})$ when $d \equiv 0$. It should be remarked that the I controller is tuned to have the output of the disturbance-free ($d \equiv 0$) $S(C, H_{+d})$ track the step input $r(\cdot)$ with satisfactory transients.

The difference between y and y_L is due to the non-linearity in system (15). It is now shown that the effect of this non-linearity, and consequently limit cycle behavior, can be suppressed by a disturbance observer.

The implementation of the disturbance observer for the system $S(C, N)$ is the same as the system $S(C, N_{DOB})$ in Figure 8, where $H_n(s) = H(s)$, and $Q(s)$ is given in equation (17). The output of $S(C, N_{DOB})$ in the absence of measurement noise ($w \equiv 0$) is shown in Figure 9(a) and is designated by y_{DOB}. It is evident that y_{DOB} tracks the step input $r(\cdot)$. That is, the disturbance observer has successfully suppressed the effect of the non-linearity in the system.

The effect of measurement noise $w(\cdot)$ on the performance of the system $S(C, N_{DOB})$ is studied next. Let $w(\cdot)$ be band-limited white noise. Apply this noise together with the input in equation (19) to the systems $S(C, N)$, disturbance-free ($d \equiv 0$) $S(C, H_{+d})$, and $S(C, N_{DOB})$ to obtain, respectively, y_N, y_L, and y_{DOB} in Figure 9(b). It is evident that the output of $S(C, N_{DOB})$ due to the measurement noise is slightly noisy.

The input $v(\cdot)$ to the system N when the measurement noise is absent, is depicted in Figure 10(a). This input when the measurement noise is present, is depicted in Figure 10(b). It is evident that the magnitude of the input in this figure is not large.

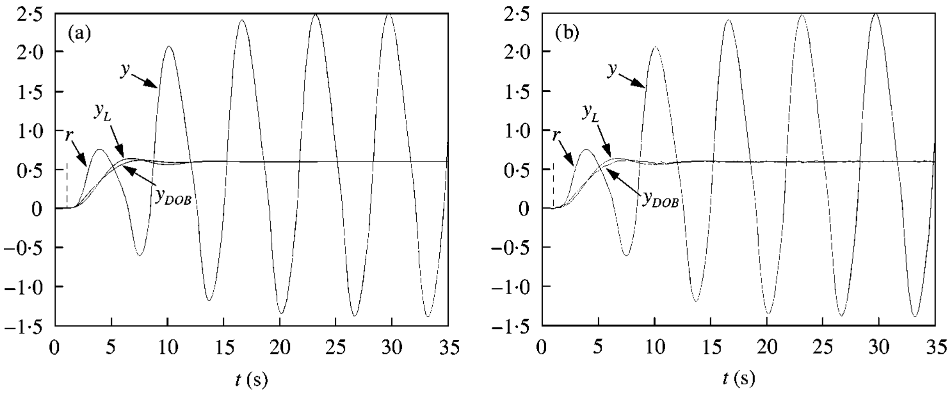

Figure 9. (a) Responses of the systems $S(C, N)$, the disturbance-free ($d \equiv 0$) $S(C, H_{+d})$, and $S(C, N_{DOB})$, designated by y, y_L, and y_{DOB}, respectively, in the absence of measurement noise w. It is evident that y_{DOB} tracks the step input of amplitude 0·6. That is, the disturbance observer has suppressed the adverse effects of the non-linearity. (b) Responses of the systems $S(C, N)$, the disturbance-free $S(C, H_{+d})$, and $S(C, N_{DOB})$, designated by y, y_L, and y_{DOB}, respectively, in the presence of measurement noise w.

An important remark regarding the design of disturbance observers and filters $Q(s)$ is as follows. It is possible to choose a very fast $Q(s)$ by placing its poles far to the left of the imaginary axis of the complex plane. By doing so, the effects of non-linearities in the system are completely concelled. For instance, $Q(s)$ in the example of section 4.1 has its poles at -10, -15, and -20. If the poles of $Q(s)$ were placed much farther to the left of the imaginary axis of the complex plane, then the outputs y_L and y_{DOB} in Figure 9(a) would overlap. A fast $Q(s)$ is the best choice, however, as long as there is no measurement noise. In the presence of noise, if a very fast $Q(s)$ is chosen, then the magnitude of the control input $v(\cdot)$ to the system N would be extremely large. Thus, there is a trade off between complete cancellation of the effects of non-linearities in a system and the magnitude of the required control. In section 4.1, the filter $Q(s)$ is only moderately fast. Therefore, the outputs y_L and y_{DOB} do not overlap. However, in the presence of measurement noise, the magnitude of the control $v(\cdot)$ is reasonably small as shown in Figure 10(b).

5. CONCLUSIONS

In this note, a class of single-input–single-output (SISO) non-linear systems is considered. A non-linear system in this class has the property that its output is equal to the summation of the output of a stable SISO linear time-invariant system and a bounded function of time. A disturbance observer is designed to estimate the effects of non-linearities in the system and cancel them subsequently. The disturbance observer is thus able to make the non-linear system behave linearly and, for instance, be free of limit cycle behavior. An example is given to show how limit cycles in a Van der Pol-type system caused by non-linearities can be effectively suppressed by a disturbance observer.

Furthermore, a methodology for designing controllers for a class of non-linear feedback systems is presented. The designed controllers consist of two parts: a disturbance observer by which the effects of non-linearities are suppressed and a linear controller to have the system achieve desired goals.

Two remarks are made regarding the design of disturbance observers and classes of non-linear systems: (1) there are some non-linear systems that satisfy assumption (A2) and

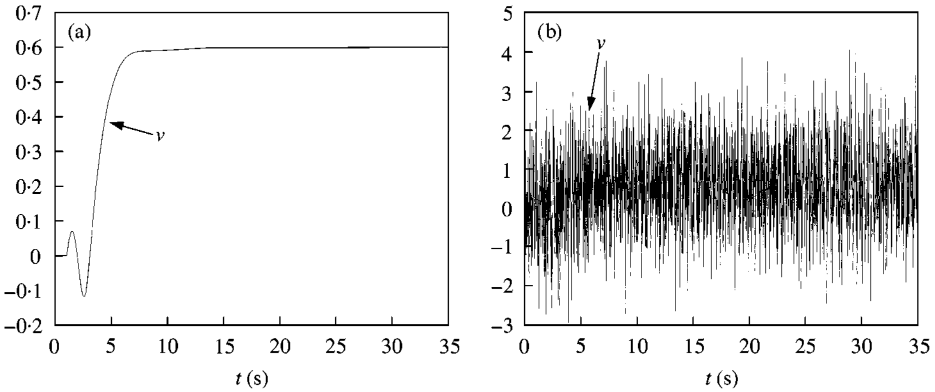

Figure 10. (a) Control input applied to the system N in the absence of measurement noise w. (b) Control input applied to the system N in the presence of measurement noise w. It is evident that the magnitude of the applied control input is reasonably small.

exhibit chaos. By designing disturbance observers, it is possible to suppress chaotic behavior in such systems; (2) there are some non-linear systems that do not satisfy assumption (A2), such as the standard Van der Pol oscillator (see, e.g., reference [29, Chapter 11]); however, the boundedness of their outputs can be inferred by some other means. By designing disturbance observers, it is possible to suppress the effects of non-linearities in such systems.

It is interesting to note that disturbance observers are linear systems, but yet they are able to suppress the effects of non-linearities.

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