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A METHOD TO PREDICT ACOUSTIC RADIATION FROM AN ENCLOSED MULTICAVITY STRUCTURE

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The acoustical reciprocity theorem can be used to solve the problem of vibroacoustic coupling. However, the theorem can be used only on the presupposition that the scattered sound field of the elastic surface concerned is known. This is the key point and the most difficult point for many complicated surfaces, such as a multicavity structure. A new method, covering-domain method, which transforms the calculation of scattered sound field of an arbitrary-shaped closed shell into that of a series of simply closed spherical shells, is applied in this paper to calculate the scattered sound field of a multicavity structure with elastic surfaces. So the radiated sound pressure of an elastic multicavity structure excited by an external force can be predicted by using the acoustical reciprocity theorem. It is verified to be correct by a corresponding test in this paper.

1. INTRODUCTION

In practical machinery or most aerospace vehicle structures, there are some multicavities that consist of cambered structures, such as car, aeroplane, steamship, etc. The study of vibroacoustic coupling of these multicavity structures is extremely important. The finite element method and the statistics energy analysis method (SEA) may be used to study the coupled multicavity structures. The finite element method is a better method, but calculation work is very tedious for some complicated structures [1]. In general, the SEA method is suitable for light-coupled subsystems. When it is used for a vibroacoustic coupling system, modal density is a very important parameter. For a non-ideal subsystem, theoretical estimation for modal density is difficult and experiment technology has to be used in this case [2]. Besides, the SEA method is unsuitable for analyzing low-frequency vibroacoustic problems.

The concept of covering-domain has been used to express the strains–stresses relation of an arbitrary-shaped elastomer excited by an external force in reference [3]. This concept has been used for reference and the covering-domain method has been put forward in acoustics first [4]. The covering-domain method has been applied to calculate the radiated sound field of a complex-shaped single cavity in reference [4] and the sound radiation from a plate-ended cylindrical shell in reference [5]. Compared with the existing methods for solving coupled vibroacoustic problems, the covering-domain method has the following apparent advantages [6]: (1) it is more accurate and suitable for broadband frequency vibroacoustic problems; (2) it is able to distinguish as to which part of the structure provides the greatest contribution to the sound pressure at a certain interior point; (3) it is able to calculate the interior sound field of an arbitrary-shaped closed thin shell, whose thickness is either equal or unequal. In reference [6], the boundary element method and the covering-domain method are used, respectively, to calculate the radiated sound field from a single-cavity rectangular chest, and the calculation efficiency and the calculation precision are compared between them. It is indicated that the covering-domain method is more accurate and more efficient. Therefore, the covering-domain method is presented to study the vibroacoustic coupling of multicavity structures in this paper.

In section 2, the basic principle of the covering-domain method is described. In section 3, the formula for calculating the radiated sound pressure of an elastic multicavity structure due to an external exciting force is presented. In section 4, the comparison between the calculation results and the experiment results of the interior sound field of a two-cavity rectangular structure is given. The conclusions are presented in section 5.

2. THE BASIC PRINCIPLE OF THE COVERING-DOMAIN METHOD

Supposing the elastomers A and B are, respectively, fixed in two separate co-ordinate systems, when the two co-ordinate systems are overlapped, we can say that B covers A if point $M \in A$, then $M \in B$.

In general, the curved boundary surface C of an arbitrary-shaped closed shell A can always be fitted by n pieces of spherical surfaces C_1, $C_2, \ldots,$ and C_n. To calculate the interior sound field of the closed shell A, a series of closed spherical shells A_k $(k = 1, 2, \ldots, n)$ can be used to cover A. The spherical shell A_k has only a part of its surface to coincide with C_k and has the same thickness as the original spherical surface C_k. It is obvious that the common domain of all of $A_k (k = 1, 2, \ldots, n)$ is the domain occupied by the closed shell A.

Although it is difficult to calculate the interior sound field of a closed shell with a complicated shape directly, it is easy to calculate the interior sound field of these closed spherical shells. We can make use of the concept of covering-domain to transform the problem of the interior sound field of a complicated shell into a simple problem of a series of closed spherical shells. Therefore, the interior scattered sound field of an arbitrary-shaped closed shell can be expressed as follows:

$$P_s(\mathbf{r}) = \sum_{k=1}^{n} P_s^{(k)}(\mathbf{r}) \tag{1}$$

where $P_s^{(k)}(\mathbf{r})$ is the scattered sound field of the kth covering spherical shell at an interior point $\mathbf{r}$ of the arbitrary-shaped closed shell.

3. THE CALCULATION OF RADIATING PRESSURE OF MULTICAVITY STRUCTURE DUE TO AN EXTERNAL EXCITING FORCE

3.1. THE EXPRESSION FOR THE SCATTERED SOUND FIELD OF A CLOSED SPHERICAL SHELL

(1) *Calculation of the interior scattered sound field of a closed spherical shell*

Considering a system with spherical co-ordinates (r, θ, φ), we can make the center of the co-ordinate system coincide with the center of a close thin-walled elastic spherical shell. It is supposed that there is a point sound source q of unit strength at the interior point $\mathbf{r}_0$ $(r_0, \theta_0, \varphi_0)$ of the spherical shell. Due to the action of q, the spherical shell will bring about vibration, generate interior sound field $P_1(\mathbf{r})$, and external sound field $P_2(\mathbf{r})$ respectively. $P_1(\mathbf{r})$ consists of two parts, i.e., free sound field $P_0(\mathbf{r})$ generated by q, and interior scattered sound field $P_{s1}(\mathbf{r})$.

It is easy to obtain the $P_0(\mathbf{r})$ as follows [4]:

$$P_0(r,\theta,\varphi) = -\frac{\mathrm{i}\omega\rho}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR_1}}{R_1}\mathrm{e}^{-\mathrm{i}\omega t}$$

$$= \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{\mathrm{i}\omega\rho}{4\pi}(2n+1)\,k\,\frac{(n-m)!}{(n+m)!}\mathrm{P}_n^m(\cos\theta)\,\mathrm{P}_n^m(\cos\theta_0)\,\mathrm{e}^{\mathrm{i}m(\varphi-\varphi_0)}\,\mathrm{j}_n(kr_0)\mathrm{h}_n^{(1)}(kr)\mathrm{e}^{-\mathrm{i}\omega t}. \tag{2}$$

The external sound field and interior scattered sound field, which are related to the vibration of the spherical shell, can be given as follows:

$$P_2(r,\theta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} B_n\,\mathrm{h}_n^{(1)}(kr)\,\mathrm{P}_n^m(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{3}$$

$$P_{s1}(r,\theta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} C_n\,\mathrm{j}_n(kr)\,\mathrm{P}_n^m(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{4}$$

where $R_1 = \sqrt{r^2 + r_0^2 - 2r\,r_0\cos\beta}$, β is the angle between vector $\mathbf{r}$ and $\mathbf{r}_0$, $\mathrm{i} = \sqrt{-1}$, ω is circular frequency, ρ is media density, $k = \omega/c$ is the wave number, in which c is sound velocity, B_n and C_n are unknown coefficients, $\mathrm{h}_n^{(1)}(\cdot)$ is the first kind of the spherical Hankel function, $\mathrm{j}_n(\cdot)$ is the spherical Bessel function, and $\mathrm{P}_n^m(\cdot)$ is the first kind of the associated Legendre function.

According to reference [7], the radial displacement w of the spherical shell should meet the following equation:

$$\varepsilon\nabla^6 w + r_1\nabla^4 w + r_2\nabla^2 w + r_3 w + W = 0, \tag{5}$$

where $\varepsilon = h^2/12R^2$, $k_t = 1 + \varepsilon$, $k_r = 1 + 3h^2/(20R^2)$, $K_s = 2k_s/(1-\mu)$,

$$r_1 = \varepsilon[3 - \mu - 2(1+\mu)k_s] + \varepsilon[k_t + k_r + k_t K_s](kR)^2,$$

$$r_2 = 1 - \mu^2 - k_t(kR)^2 + 2\varepsilon[1 - \mu - (3 + 2\mu - \mu^2)k_s]$$
$$+ \varepsilon[(1-\mu)k_t + 2k_r - 2(1+\mu)k_r k_s - 4\mu k_t K_s](kR)^2$$
$$+ \varepsilon k_t[k_r + (k_t + k_r)K_s](kR)^2\omega^2,$$

$$r_3 = [2(1-\mu^2) + (1+3\mu)k_t(kR)^2 - k_t^2(kR)^2\omega^2] - 4\varepsilon(1-\mu^2)k_s$$
$$- 2\varepsilon k_s[(1+3\mu)k_t + 2(1+\mu)k_r](kR)^2$$
$$+ \varepsilon k_t[2k_t k_s - (1+3\mu)k_r K_s](kR)^2\omega^2 + \varepsilon k_t^2 k_r K_s(kR)^2\omega^4,$$

$$W = -[1 - \varepsilon K_s(\nabla^2 + 1 - \mu + k_r(kR)^2)]H,$$

$$H = \frac{(1-\mu^2)R^2}{Eh}(\nabla^2 + 1 - \mu + k_t(kR)^2)(P_1 - P_2),$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial\theta^2} + \mathrm{ctg}\,\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2},$$

where h is the thickness of the spherical shell, R is the nominal radius of the closed spherical shell, E is Young's modulus, μ is the Poisson ratio of the shell material, and k_s is an averaging coefficient of the shear.

Here suppose

$$w = \sum_{m=0}^{\infty} \sum_{m=-n}^{n} A_n \mathrm{P}_n^m(\cos\theta)\, \mathrm{e}^{\mathrm{i}m\varphi} \mathrm{e}^{-\mathrm{i}\omega t}. \tag{6}$$

Because the spherical harmonic function $S(\theta, \varphi) = \mathrm{P}_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\varphi}$ $(m = 0, \pm 1, \ldots, \pm n)$ satisfies the following differential equation when $0 \leqslant \theta \leqslant \pi$ and $0 \leqslant \varphi \leqslant 2\pi$ [8]:

$$\frac{\partial^2 S}{\partial \theta^2} + \operatorname{ctg}\theta \frac{\partial S}{\partial \theta} + \frac{1}{\sin^2\theta} \frac{\partial^2 S}{\partial \varphi^2} + n(n+1)S = 0, \tag{7}$$

we can simplify the six order equation (5) as follows according to expression (6) and equation (7):

$$[-\varepsilon n^3(n+1)^3 + r_1 n^2(n+1)^2 - r_2 n(n+1) + r_3]w + W = 0. \tag{8}$$

Besides, the following boundary condition should be met on the interfaces of the shell and acoustic medium:

$$\left.\frac{1}{\mathrm{i}\omega\rho} \frac{\partial P_1}{\partial r}\right|_{r=R-h/2} = \left.\frac{1}{\mathrm{i}\omega\rho} \frac{\partial P_2}{\partial r}\right|_{r=R+h/2} = \frac{\partial w}{\partial t}. \tag{9a}$$

Because the shell wall considered here is thin ($h \ll \lambda_0$, in which λ_0 is the wavelength of sound wave in the material of the shell), the medium vibration velocity on the interior and external surface of the shell ($r = R \pm h/2$) can be replaced by the vibration velocity on the middle surface of the shell $r = R$. In this case, the boundary conditions may be written as

$$\left.\frac{1}{\mathrm{i}\omega\rho} \frac{\partial P_1}{\partial r}\right|_{r=R} = \left.\frac{1}{\mathrm{i}\omega\rho} \frac{\partial P_2}{\partial r}\right|_{r=R} = \frac{\partial w}{\partial t}. \tag{9b}$$

Then, the following result can be obtained by equations (8) and (9b):

$$C_n = \frac{b_{n1}}{a_{n2}} \frac{\mathrm{i}\omega}{4\pi c}(2n+1)\frac{(n-m)!}{(n+m)!} \mathrm{P}_n^m(\cos\theta_0)\, \mathrm{e}^{-\mathrm{i}m\varphi_0} \mathrm{j}_n(kr_0), \tag{10}$$

where

$$a_{n2} = -a_{n1} \frac{\mathrm{j}_n'(kR)}{\rho c \omega} + b_n \mathrm{j}_n(kR),$$

$$b_{n1} = \frac{a_{n1}}{c} \mathrm{h}_n^{(1)\prime}(kR) - b_n \omega\rho \mathrm{h}_n^{(1)}(kR),$$

$$a_{n1} = a_n + b_n \frac{\mathrm{h}_n^{(1)}(kR)}{\mathrm{h}_n^{(1)\prime}(kR)} \omega\rho c,$$

$$a_n = -\varepsilon n^3(n+1)^3 + r_1 n^2(n+1)^2 - r_2 n(n+1) + r_3,$$

$$b_n = \frac{(1-\mu^2)R^2}{Eh}[-n(n+1) + (1-\mu) + k_t(kR)^2]$$

$$[1 - \varepsilon K_s(-n(n+1) + (1-\mu) + k_r(kR)^2].$$

Therefore, if there is a point sound source of unit strength in the closed thin spherical shell, its interior scattered sound field can be obtained by expression (4).

According to the similarly derived process, we can obtain the following results.

(2) When a point sound source of unit strength is at the external point $\mathbf{r}_1(r_1, \theta_1, \varphi_1)$ of a closed thin spherical shell with radius R, the expression for its scattered sound pressure at the external point $\mathbf{r}(r, \theta, \varphi)$ is

$$P_{s2}(r, \theta, \varphi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} D_n \mathrm{h}_n^{(1)}(kr) \mathrm{P}_n^m(\cos\theta) \mathrm{e}^{im\varphi} \mathrm{e}^{-\mathrm{i}\omega t}, \tag{11}$$

where

$$D_n = \frac{d_n \mathrm{h}_n^{(1)}(kr_1) \mathrm{j}_n'(kR)(b_n \mathrm{j}_n(kR) - d_{n1})}{\mathrm{h}_n^{(1)\prime}(kR) d_{n1} - \mathrm{b}_n \mathrm{h}_n^{(1)}(kR) \mathrm{j}_n'(kR)},$$

$$d_n = \frac{\omega\rho}{4\pi} \mathrm{i}(2n+1)k \frac{(n-m)!}{(n+m)!} \mathrm{P}_n^m(\cos\theta_1) \mathrm{e}^{-\mathrm{i}m\varphi_1},$$

$$d_{n1} = b_n \mathrm{j}_n(kR) - \frac{a_n}{\omega^2\rho} \mathrm{j}_n'(kR) k.$$

(3) When a point sound source of unit strength is at the interior point $\mathbf{r}_0$ $(r_0, \theta_0, \varphi_0)$ of a closed thin spherical shell with radius R, the expression for its scattered sound pressure at the external point $\mathbf{r}(r, \theta, \varphi)$ is

$$P_{s3} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} C_n \frac{\mathrm{j}_n'(kR)}{\mathrm{h}_n^{(1)\prime}(kR)} \mathrm{h}_n^{(1)}(kr) \mathrm{P}_n^m(\cos\theta) \mathrm{e}^{im\varphi} \mathrm{e}^{-\mathrm{i}\omega t}, \tag{12}$$

where C_n is the same as expression (10).

(4) When a point sound source of unit strength is at the external point $\mathbf{r}_1(r_1, \theta_1, \varphi_1)$ of a closed thin spherical shell with radius R, the expression for its scattered sound pressure at the interior point $\mathbf{r}(r, \theta, \varphi)$ is

$$P_{s4} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} D_n \frac{\mathrm{h}_n^{(1)\prime}(kR)}{\mathrm{j}_n'(kR)} \mathrm{j}_n(kr) \mathrm{P}_n^m(\cos\theta) \mathrm{e}^{im\varphi} \mathrm{e}^{-\mathrm{i}\omega t}, \tag{13}$$

where D_n is the same as expression (11).

3.2. THE EXTERNAL SCATTERED SOUND FIELD OF AN ARBITRARY-SHAPED CLOSED THIN SHELL

As shown in Figure 1, supposing that the boundary curved surface L of an arbitrary-shaped closed thin shell M can be fit by m pieces of spherical surfaces $L_1, L_2, \dots,$ and L_m, thus the m pieces of corresponding covering spherical shells cover the cavity M. Now our problem is to calculate the scattered sound pressure at an external point **X**, when there is a point sound source of unit strength at the external point **Y**.

According to the above description, when the positions of the point sound source and the considered spatial point relative to the closed spherical shell are different, the expression of scattered sound field at the spatial point is different. There are four probable situations: (1) there are m_1 pieces of covering spherical shells which cover the points **X** and **Y**, (2) there are m_2 pieces of covering spherical shells which cover neither the point **X** nor the point **Y**,

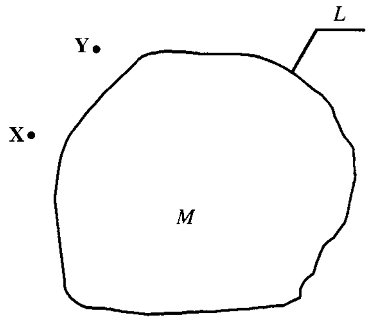

Figure 1. Calculation of the external scattered field of closed thin shell.

(3) there are m_3 pieces of covering spherical shells covering the point **Y**, but not **X**, and (4) there are m_4 pieces of covering spherical shells covering the point **X**, but not **Y**. Of course, $m = m_1 + m_2 + m_3 + m_4$. Since the covering spherical shell produces radiation sound field not only to its interior, but also to its exterior, the external scattered sound field of M at the point **X** can be regarded as the corresponding superposition of scattered sound fields of the above four situations. Therefore when a point sound source of unit intensity is at the external point **Y** of M, the scattered sound pressure at **X** can be given by the expression:

$$P_{SO}^{(M)}(\mathbf{X},\mathbf{Y}) = \sum_{i=1}^{m_1} P_{S1}^{(i)}(\mathbf{X},\mathbf{Y}) + \sum_{j=1}^{m_2} P_{S2}^{(j)}(\mathbf{X},\mathbf{Y}) + \sum_{s=1}^{m_3} P_{S3}^{(s)}(\mathbf{X},\mathbf{Y}) + \sum_{t=1}^{m_4} P_{S4}^{(t)}(\mathbf{X},\mathbf{Y}), \quad (14)$$

where $P_{S1}^{(i)}(\mathbf{X},\mathbf{Y})$ denotes the scattered sound pressure of the ith covering spherical shell in case (1), $P_{S2}^{(j)}(\mathbf{X},\mathbf{Y})$ denotes the scattered sound pressure of the jth covering spherical shell in case (2), $P_{S3}^{(s)}(\mathbf{X},\mathbf{Y})$ denotes the scattered sound pressure of the sth covering spherical shell in case (3), and $P_{S4}^{(t)}(\mathbf{X},\mathbf{Y})$ denotes the scattered sound pressure of the tth covering spherical shell in case (4).

3.3. CALCULATION OF THE INTERIOR RADIATED SOUND FIELD OF AN ELASTIC MULTICAVITY STRUCTURE DUE TO AN EXTERNAL EXCITING FORCE

Let us consider a two-cavity structure first. As shown in Figure 2, the elastic two-cavity structure $\bar{\Omega}$ consists of two cavities $\bar{\Omega}_1$ and $\bar{\Omega}_2$, and the common boundary of the cavities $\bar{\Omega}_1$ and $\bar{\Omega}_2$ is Γ. The interior acoustic space of $\bar{\Omega}_i$ is Ω_i $(i = 1, 2)$. S_1 is the boundary of $\bar{\Omega}_1$ excluding Γ and S_2 is the boundary of $\bar{\Omega}_2$ excluding Γ.

Here we can use many spherical surfaces to fit the curved boundary surface of $\bar{\Omega}$. Supposing that S_1 can be fit by n_1 pieces of spherical surfaces $A_1, A_2, \ldots,$ and A_{n_1}, the n_1 pieces of corresponding covering spherical shells cover the cavity $\bar{\Omega}_1$. S_2 can be fit by n_2 pieces of spherical surfaces $B_1, B_2, \ldots,$ and B_{n_2} and n_2 pieces of corresponding covering spherical shells cover the cavity $\bar{\Omega}_2$. Γ can be fit by n_3 pieces of spherical surfaces $F_1, F_2, \ldots, F_{n_3}$, and n_3 pieces of corresponding covering spherical shells also cover the cavity $\bar{\Omega}_2$.

Now the problem is to calculate the scattered sound pressure at an arbitrary interior point $\mathbf{r}$ of $\bar{\Omega}_1$ when a point sound source of unit strength is at an interior point $A(\mathbf{r}_A)$ of $\bar{\Omega}_1$.

We have described the covering-domain method for the single cavity above. But it can not be used for a multicavity structure directly. Therefore, we use the following transformation to deal with a two-cavity structure as a single-cavity structure.

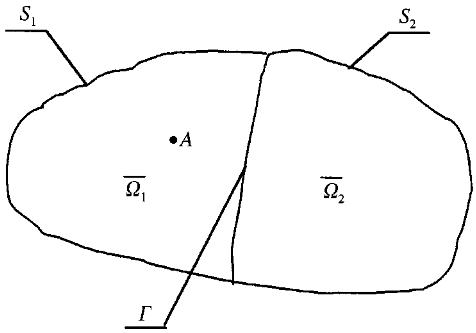

Figure 2. A sketch of an elastic two-cavity coupling.

For the elastic two-cavity structure, the scattered sound pressure at the interior point $\mathbf{r}$ of $\bar{\Omega}_1$ by the coupled cavity $\bar{\Omega}_2$ can be calculated by expression (14). If there exists a special shell that produces the same scattered sound pressure as the cavity $\bar{\Omega}_2$ at $\mathbf{r}$, we may call it the equivalent shell. That means that scattered sound pressure at the interior point $\mathbf{r}$ of $\bar{\Omega}_1$ from the equivalent shell equals to the scattered sound pressure from the cavity $\bar{\Omega}_2$. Thus, we obtain a single-cavity structure enclosed by the curved surface S_1 and the equivalent shell. Now the scattered sound pressure at the point $\mathbf{r}$ by the two-cavity structure can be expressed as

$$P_S(\mathbf{r}, \mathbf{r}_A) = \sum_{i=1}^{n_1} P_{S1}^{(i)}(\mathbf{r}, \mathbf{r}_A) + P_{SO}^{(\bar{\Omega}_2)}(\mathbf{r}, \mathbf{r}_A). \tag{15}$$

Here we need to note that the equivalent shell does not exist in all cases. But for a simple and convenient reason, we suppose that there always exists the equivalent shell in all cases.

According to the reciprocity theorem, the radiation sound pressure of an elastic body at a point $\mathbf{r}_0$ due to the action of the external force can be calculated. Supposing that there is a point sound source q of unit strength at $\mathbf{r}_0$, the scattered sound field $P_S(\mathbf{r}, \mathbf{r}_0)$ of the elastic body at an arbitrary point $\mathbf{r}$ due to the action of q is known. Then the radiation sound pressure of the elastic body at the point $\mathbf{r}_0$ excited by external force at the point $\mathbf{r}$ can be calculated by the following expression [9]:

$$P(\mathbf{r}_0) = -\frac{1}{\mathrm{i}4\pi\omega\rho}\iint_S \frac{\partial P_s(\mathbf{r}, \mathbf{r}_0)}{\partial \mathbf{n}} f(\mathbf{r})\,\mathrm{d}S, \tag{16}$$

where $f(\mathbf{r})$ is the external force acting on the elastomer at $\mathbf{r}$, S is the elastic surface, $\mathbf{n}$ is the normal of the elastic surface, which directs toward outside, $P(\mathbf{r}_0)$ is radiation pressure at $\mathbf{r}_0$, and $P_S(\mathbf{r}, \mathbf{r}_0)$ is the scattered sound field of the elastic surface.

Thus, we can use the above method to calculate the interior radiated sound pressure of an elastic two-cavity structure excited by an external force. For a structure coupled by three or more cavities, the same principle can be used.

From the above description about the covering-domain methods, it seems that the method can handle only the closed thin convex shells. In fact, for the closed thin shells with concave surfaces, if some “imaginary surfaces” are used, they can be divided into several convex “single-cavity” structures, which form a “multicavity” structure. Of course, these “imaginary surfaces” do not radiate sound pressure. Therefore, the covering-domain method can theoretically be used to calculate the radiation sound field of an arbitrary-shaped closed thin shell.

4. CALCULATION AND EXPERIMENT OF THE INTERIOR SOUND FIELD OF A TWO-CAVITY RECTANGULAR STRUCTURE

In order to verify the validity of the above theory, a rectangular chest with two cavities are studied experimentally and theoretically in this paper. As shown in Figure 3, the rectangular structure consists of two rectangular cavities $\bar{G}_1$ and $\bar{G}_2$, which is welded by steel plates 2·5 mm in thickness. $\bar{G}_i (i = 1, 2)$ includes the internal sound space G_i and its boundary. The common boundary of $\bar{G}_1$ and $\bar{G}_2$ is the baffle Γ. There is a window of $0{\cdot}15 \times 0{\cdot}15\ \mathrm{m}^2$ on the left-hand side of the chest, which is covered by a covering steel plate 2·5 mm in thickness in the experiment. To prevent sound from leaking, a rubber ring is padded between the covering plate and the end surface. Let the origin of the co-ordinate system be located at C on the left of the chest; the directions of the co-ordinates are shown in Figure 3.

In order to reduce the environmental influence, all the experiments are carried out in an anechoic room. As shown in Figure 4, the essential testing system consists of two parts, i.e., the excitation part and the corresponding testing part. The signal analyzer B&K2035 is used in all the experiments.

In the experiment, the two-cavity structure is suspended at its four corners by soft ropes. To measure the inside sound pressure of the two-cavity structure, we design an adjustable measuring device, which consists of a longitudinal steel tube, vertical regulating stem and microphone. The steel tube with a diameter of 1·4 cm and a length of 3·1 m goes through the longitudinal symmetry axis of the chest. The regulating stem, whose radial size is adjustable, can be moved along the longitudinal steel tube. The microphone is installed on the regulating stem. The position of the microphone can be read by a scale and regulated by the corresponding movement and rotation.

In the experiment, the excitation point is located at the point (0·155, 0·247, 1·0) (by meter, the same as the following) on the top. When the harmonic forces with corresponding frequencies of 55, 75, and 120 Hz are used, respectively, the radiated sound pressures are measured at different internal points that are designated as $1, 2, \ldots,$ and 14, whose co-ordinates are shown in Table 1. Here the reason that these measurement points are selected on the center vertical surface is only to make the experiment simple and convenient and other points inside the two-cavity rectangular structure can also be selected as measurement points without doubt. The amplitudes of the bestirred forces are 3 N.

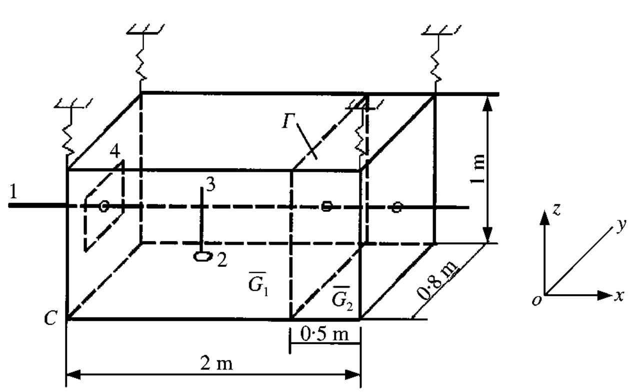

Figure 3. Experiment model of an elastic thin-walled two-cavity rectangular structure: 1, steel tube; 2, microphone; 3, regulating stem; 4, covering plate.

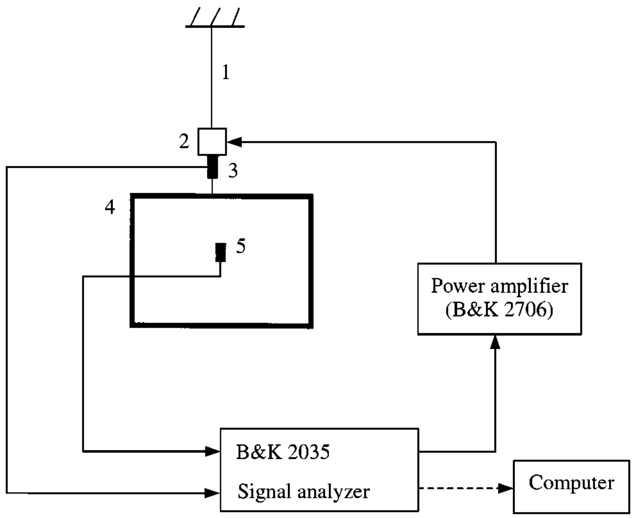

Figure 4. Sketch of an essential testing system for the two-cavity rectangular structure: 1, rubber sling; 2, generator (B&K 4809); 3, force sensor (B&K8001); 4, two-cavity rectangular structure; 5, microphone (B&K4155).

TABLE 1

The Cartesian co-ordinates of the measured points (unit: m)

Order of measured points	Co-ordinates of measured points	Order of measured point	Co-ordinates of measured points
1	(1·3, 0·4, 0·615)	8	(1·0, 0·4, 0·385)
2	(1·3, 0·4, 0·385)	9	(0·9, 0·4, 0·615)
3	(1·2, 0·4, 0·615)	10	(0·9, 0·4, 0·385)
4	(1·2, 0·4, 0·385)	11	(0·8, 0·4, 0·615)
5	(1·1, 0·4, 0·615)	12	(0·8, 0·4, 0·385)
6	(1·1, 0·4, 0·385)	13	(0·7, 0·4, 0·615)
7	(1·0, 0·4, 0·615)	14	(0·7, 0·4, 0·385)

Although we only consider three frequencies of 55, 75 and 120 Hz in low-frequency band, it is necessary to point out that the covering-domain method is suitable for broadband frequency, which can be understood from the theoretical derivation. The only limit to the covering-domain method is $h \ll \lambda_0$ in equation (9), in which λ_0 is the wavelength of sound wave in the material of the shell. Thus, for a thin-walled elastic structure, the calculating frequency can reach 20 kHz theoretically.

In the following, we will use the covering-domain method to calculate the radiated sound field from the closed two-cavity rectangular structure. According to the method, the boundary surfaces of the two-cavity structure should be fit by some spherical surfaces and the corresponding covering spherical shells should cover the interior sound space. Theoretically speaking, when a planar wall of the two-cavity rectangular structure is fit by a spherical surface with large radius, the large the radius the more precise is the approximation. But here, a problem of numerical calculation exists. Supposing the radius of the spherical surface is r and the wave number is k, when kr is very big, the progressions in expressions (4) and (11)–(13) have slow convergence, and we have to sum more terms of the series in order to obtain a satisfactory result. The more terms are summed up, the longer is the calculating time required. Besides, the number of summed terms is also limited by computer capability. Therefore, it is impossible to take the order number of the series to be very big and the radius of the spherical surface as infinity. According to our experience with calculations, it is effective to take r as 10–15 times of the biggest dimension of the cavity structure. Here, to simplify the theoretical calculation, every wall of the two-cavity structure can be fit by a corresponding spherical surface with radius $r = 20$ m.

Based on the above theoretical process in section 3, we adopt the following three steps to calculate the radiated sound pressures from the two-cavity structure excited by the external harmonic forces. The first step is to calculate the external scattered sound pressures from the rectangular cavity $\bar{G}_2$ at the measurement points. For the rectangular cavity $\bar{G}_2$, the planar walls can be fitted by six spherical surfaces respectively. It is very important that the corresponding covering spherical shells should cover the sound space G_2 of the cavity $\bar{G}_2$. The second step is to calculate the scattered sound pressures from the two-cavity rectangular structure at the measured points. Here the planar boundaries of the rectangular cavity $\bar{G}_1$ excluding the baffle Γ can be fitted by five spherical surfaces, respectively, and the corresponding covering spherical shells cover the sound space G_1 of the cavity $\bar{G}_1$. The last step is to calculate the radiated sound pressures at every measured point under the same condition as the experiment by applying the acoustic reciprocity theorem. In the theoretical calculation, because the center of each covering spherical shell is different from one another, a transformation of the co-ordinates has to be done.

In the calculation, we truncate the progression according to the rule that the difference between two contiguous terms of the progression is less than 0·000005. In this case, the calculation time on computer Pentium 586 for every measurement point under a considered frequency is about 20 min by using the MATLAB language. From expressions (4) and (11)–(13), we cannot conclude that the case of higher frequency needs more calculation time. In fact, the calculation time of higher frequency is almost the same as that of lower frequency.

The comparisons between the measured results and the computed results are shown in Figure 5 in which the horizontal coordinate denotes the order of measured points and the

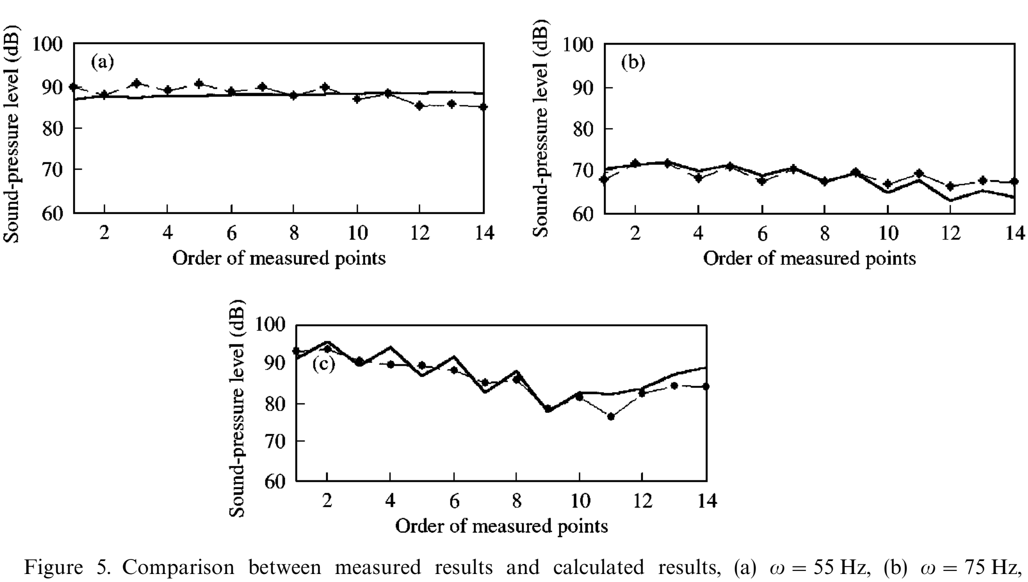

Figure 5. Comparison between measured results and calculated results, (a) $\omega = 55$ Hz, (b) $\omega = 75$ Hz, (c) $\omega = 120$ Hz: —, calculated results; —♦—, measured results.

vertical co-ordinate denotes the sound-pressure level (dB). It can be seen that there is a good agreement between them at most points. But there is a little discrepancy at several points, which may be caused by theoretical calculation error in the progression truncation and sound leaking and random measuring error in the experiment. As a whole, there is a credible agreement between them. So the method presented in this paper is verified to be valid.

5. CONCLUSIONS

In this paper, the covering-domain method is put forward to calculate sound radiation from an enclosed multicavity structure. It is verified to be correct by corresponding experiments for a coupled two-cavity rectangular structure. In addition, although we calculate only the interior sound field of the elastic two-cavity structure in the paper, the external radiation sound field of the elastic multicavity structure can also be calculated that will largely expand practical applications of the acoustic reciprocity theorem.

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REFERENCES

1. H. KARDESTUNCER 1996 *Finite Element Handbook* (Chinese version). Beijing: Science Press, 1145.
2. P. R. KESWICK and M. P. NORTON 1987 *Applied Acoustics* **20**, 137. A comparison of modal density measurement techniques.
3. L.-G. WANG and X. LIN 1987 *Acta Mechanica Sinica* **19**, 323–331. A general expression and its discussing for solutions of a set of Fredholm integration equation in the elastic mechanics.
4. J. H. WU, H. L. CHEN and X. L. HU 2000 *Acta Acustica* **25**, 468–471. Method to calculate interior sound field of arbitrary-shaped closed thin shell.
5. J. H. WU, H. L. CHEN and W. B. AN 2000 *Journal of Sound and Vibration* **237**, 793–803. A method to predict sound radiation from a plate-ended cylindrical shell excited by an external force.
6. J. H. WU 2001 *Ph.D. Thesis*, *Xi'an Jiaotong University of China*. The covering-domain method for vibroacoustic studies inside vehicle.
7. C. PRASAD 1964 *Journal of the Acoustical Society of America* **36**, 489–494. On vibration of spherical shells.
8. Z.-X. WANG and D.-R. GUO 1965 *Generality of Special Function*. Beijing: Science Press of China.
9. E. Л. Шендеров 1983 *The wave problems of Underwater Sound* (Chinese version). Beijing. National Defense Industry Press, 95.