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STOCHASTIC DYNAMIC SENSITIVITY OF UNCERTAIN STRUCTURES SUBJECTED TO RANDOM EARTHQUAKE LOADING

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The present study involves computation of stochastic sensitivity of structures with uncertain structural parameters subjected to random earthquake loading. The formulations are provided in frequency domain. A strong earthquake-induced ground motion is considered as a random process defined by respective power spectral density function. The uncertain structural parameters are modelled as homogeneous Gaussian stochastic field and discretized by the local averaging method. The discretized stochastic field is simulated by the Cholesky decomposition of respective co-variance matrix. By expanding the dynamic stiffness matrix about its reference value, the advantage of Neumann Expansion technique is explored within the framework of Monte Carlo simulation, to compute responses as well as sensitivity of response quantities. This approach involves only a single decomposition of the dynamic stiffness matrix for the entire simulated structure and the facility that several stochastic fields can be tackled simultaneously are basic advantages of the Neumann Expansion. The proposed algorithm is explained by an example problem.

1. INTRODUCTION

The nature and trend of the responses of structures is widely dependent on design parameters. Sensitivity is one of the ways to evaluate the performance of structures when they are under the influence of parametric changes. The study of sensitivity of structural systems is now acclaimed to be a very special area of interest by virtue of its utility in the field of computational structural mechanics. There has been a great interest in developing various methods for computing response sensitivity of the structure [1–5]. The sensitivity gradient is a major prerequisite for carrying out structural optimization, reliability study, parameter identification, etc. However, the conventional sensitivity analysis of structures is based on the assumptions of complete determinacy of structural parameters. But in reality, the occurrence of uncertainty due to variation of design variables is inevitable. In stochastic optimization and reliability-based design, the used performance function involves uncertain structural parameters. As a result, the gradients of the performance function are also uncertain in nature. Hence, the necessity to estimate the stochastic sensitivity gradient with respect to random design parameters arises.

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The recent development of the stochastic finite element methods (SFEM) [6–8] provides a rational framework for the analysis of complex uncertain systems subjected to stochastic excitation. The responses of a discrete system with uncertain parameters, when dynamic loads are considered are commonly determined by using perturbation methods [9, 10] or direct simulation technique [11, 12]. Perturbation and simulation-based SFEM have been applied to study the seismic response variability of soil sites [13, 14]. There are approaches [4, 15, 19] to exploit perturbation technique for sensitivity analyses of uncertain structures. But the method was found to be not so effective for large variability of design parameters. Nakagiri and Hisada [16] and Hisada and Nakgiri [9] concluded that the second order perturbation was too intractable to be of any practical interest in solving real physical problems. The Neumann Expansion method has been rarely used in the field of structural mechanics [7, 17]. As in perturbation, this method proved that implementation of higher order terms in the expansion is quite laborious. Interestingly, the Neumann Expansion is found to work efficiently when coupled with Monte Carlo simulation [17, 18, 20, 21].

In this study, it has been attempted to extend the Neumann Expansion method within the framework of Monte Carlo simulation for sensitivity analysis of stochastic dynamic systems. Here, seismically induced ground acceleration is treated as a stationary random process defined by Kanai–Tajimi model. A finite element formulation is used to discretize the structure and its dynamic response is formulated in the frequency domain. The spatial uncertainties of structural parameters are modelled as homogeneous Gaussian stochastic field and discretized by local averaging technique [22, 23]. By expanding the uncertain dynamic stiffness matrix about its reference value, the Neumann Expansion method is introduced within the framework of Monte Carlo simulation for computing response and its sensitivity. A representative example of a concrete wall with uncertain Young's modulus and mass density subjected to random earthquake loading is considered for illustration.

2. DETERMINISTIC SENSITIVITY

2.1. SENSITIVITY OF FREQUENCY RESPONSE FUNCTION

The dynamic equilibrium equation for a multi-degree-of-freedom system subjected to ground excitation can be written as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{1\}\{u_g\}, \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrix, respectively, $\{u\}$ is the total displacement of the system and $\{u_g\}$ is the displacement resulting directly from ground motion. To solve equation (1), the displacement of the finite element model subjected to unit amplitude ground motion $\ddot{u}_g = \mathrm{e}^{\mathrm{i}\omega t}$, can be assumed as $\{u\} = \{H_\omega \mathrm{e}^{\mathrm{i}\omega t}\}$ where H_ω is the complex frequency response function. Thus the velocity and acceleration response can be obtained by taking the time derivative and substituting it into equation (1): the equation of motion transforms to

$$([K] - \omega^2[M] + \mathrm{i}\omega[C])\{H_\omega\} = -[M]\{1\},$$

$$\text{i.e., } [D(h)]\{H_\omega(h)\} = \{F(h)\}, \tag{2}$$

where $[D(h)]$ is the dynamic stiffness matrix and $\{F(h)\}$ is the forcing vector. All these matrices and vectors are the functions of either any design variable "h", or in combination. In the present work, Young's modulus and mass density are considered as design variables.

Thus equation (2) is explicitly re-written as

$$[D(\{E\},\{m\})]\{H_\omega(\{E\},\{m\})\} = \{F(\{m\})\}. \tag{3}$$

Now if the sensitivity computation is performed with respect to modulus of elasticity as the only design variable, differentiation of equation (3) with respect to "E" and reorientation results in

$$\frac{\partial}{\partial\{E\}}\{H_\omega(\{E\},\{m\})\} = [D(\{E\},\{m\})]^{-1}\left[\frac{\partial}{\partial\{E\}}\{F(\{m\})\} - \{H_\omega(\{E\},\{m\})\} \times \frac{\partial}{\partial\{E\}}[D(\{E\},\{m\})]\right]. \tag{4}$$

For simplicity, equation (4) can be presented as

$$\{y\} = -[D(\{E\},\{m\})]^{-1}[\{H_\omega(\{E\},\{m\})\}[D']]$$

$$\text{or, } \{y\} = [D(\{E\},\{m\})]^{-1}\{F^*\}, \tag{5}$$

where sensitivity of frequency response function can be defined as

$$\{y\} = \frac{\partial}{\partial\{E\}}\{H_\omega(\{E\},\{m\})\} \quad \text{and} \quad [D]' = \frac{\partial}{\partial\{E\}}[D(\{E\},\{m\})],$$

thus $\{F^*\} = -[\{H_\omega(\{E\},\{m\})\}[D']]$.

2.2. SENSITIVITY OF SPECTRAL DENSITY FUNCTION AND MEAN SQUARE DISPLACEMENT

For a linear system with known transfer function of response, the spectral density function $\{S_\omega\}$ for any response variable $\{u\}$ can be readily obtained as

$$\{S_\omega\} = \{H_\omega(\{E\},\{m\})\}\{H_\omega(\{E\},\{m\})\}^{*\mathrm{T}} S_f. \tag{6}$$

Here, $\{H_\omega(\{E\},\{m\})\}^{*\mathrm{T}}$ is the complex conjugate of $\{H_\omega(\{E\},\{m\})\}$ and S_f is the power spectral density of the forcing function. The sensitivity of spectral density function can be easily obtained by differentiating equation (6) with respect to "E" as

$$\frac{\partial}{\partial\{E\}}\{S_\omega\} = \left[\{H_\omega(\{E\},\{m\})\}^{*\mathrm{T}}\frac{\partial}{\partial\{E\}}\{H_\omega(\{E\},\{m\})\} + \{H_\omega(\{E\},\{m\})\}\frac{\partial}{\partial\{E\}} \times \{H_\omega(\{E\},\{m\})\}^{*\mathrm{T}}\right] S_f$$

$$\text{or, } \{S'_\omega\} = \lfloor \{H_\omega(\{E\},\{m\})\}^{*\mathrm{T}}\{y\} + \{H_\omega(\{E\},\{m\})\}\{y\}^{*\mathrm{T}} \rfloor S_f \tag{7}$$

Here, $\{y\}^{*\mathrm{T}}$ stands for the complex conjugate of $\{y\}$.

Now the sensitivity of mean square displacement $\{U\} = \int_0^\infty \{S_\omega\}\,\mathrm{d}\omega$ can be computed as

$$\{U'\} = \frac{\partial}{\partial\{E\}}\{U\} = \frac{\partial}{\partial\{E\}}\int_0^\infty \{S_\omega\}\,\mathrm{d}\omega = \int_0^\infty \{S'_\omega\}\,\mathrm{d}\omega. \tag{8}$$

The approximate integration will have to be performed using Simpson's rule upto the cut-off frequency.

3. STOCHASTIC SENSITIVITY

It is evident from the above section that for deterministic analysis, design variables E_k and m_k $(k = 1, 2, \ldots, N)$ are deterministic quantities for all discretized N numbers of element of the structure. In stochastic analysis, E_k and m_k will be random from element to element. It is well-versed in SFEM analysis that if the design parameter E and/or m becomes uncertain, $[D\{\{E\}, \{m\}\}]$ as well as $\{H_\omega\{\{E\}, \{m\}\}\}$ will be random in nature, causing $\{F^*\}$ to be random. As a result, response sensitivity vector $\{y\}$ will no longer remain deterministic. Thus for kth element, equation (5) changes to

$$\{y_k\} = [D(\{E\}, \{m\})]^{-1}\{F_k^*\}. \tag{9}$$

Here

$$\{F_k^*\} = -[\{H_\omega(\{E\}, \{m\})\}[D'_k]] = -\{H_\omega(\{E\}, \{m\})\}\frac{\partial}{\partial E_k}[D(\{E\}, \{m\})]. \tag{10}$$

4. DISCRETIZATION AND SIMULATION OF STOCHASTIC FIELD

The purpose of the present section is to describe the procedure of stochastic discretization and generating sample function of the discretized Gaussian stochastic field necessary for subsequent simulation. Different researchers proposed several methods of discretization using continuous and discontinuous representation of stochastic field. Continuous representation involving Karhunen–Loeve expansion [7, 24] or general orthogonal series expansion [25] demands solution of the integral eigenvalue problem, which may not have a closed-form solution for realistic covariance function. But in discontinuous representation, the stochastic finite element models based on local averaging technique is found to converge more rapidly than the mid-point method [22]. Moreover, detailed knowledge about the correlation function of the random field is not essential. If it is available, the use of direct variance reduction function can be obtained. Hence, local averaging [7, 22] is used where the field variable over an element is approximated by spatial average.

A homogeneous random scalar field $\alpha(x, y)$ defined over the domain Ω is taken. This "α" may be any design variable. It is characterized by its mean $\bar{\alpha}(x, y)$, variance σ^2 and variance function $\rho(r_x, r_y)$ where $r_x = x - \bar{x}$ and $r_y = y - \bar{y}$. The local averages of the field over a rectangle A_i centered at (x_i, y_i) having sides L_{x_i} and L_{y_i} parallel to x- and y-axis, respectively, is defined as

$$\alpha(x_i, y_i) = \frac{1}{L_{x_i} L_{y_i}}\int_{x_i - L_{x_i}/2}^{x_i + L_{x_i}/2}\int_{y_i - L_{y_i}/2}^{y_i + L_{y_i}/2} \alpha(x, y)\,\mathrm{d}x\,\mathrm{d}y. \tag{11}$$

Assuming the correlation of the field to be quadrant symmetric, the mean vector, variance and covariance of the local averages can be written as [22]

$$E(\alpha) = \bar{\alpha}, \tag{12}$$

$$\mathrm{Var}(\alpha) = \sigma\gamma(L_x, L_y), \tag{13}$$

$$\mathrm{Cov}(\alpha_i, \alpha_j) = \sigma \frac{1}{4A_i A_j} \sum_{k,l=0}^{3} (-1)^{k+1} (L_{xk} L_{yl})^2 \gamma(L_{xk}, L_{yl}) \sigma, \tag{14}$$

where $\sigma = \mathrm{diag}[\sigma_1, \sigma_2, \ldots, \sigma_N]$ and

$$\gamma(L_{xk}, L_{yl}) = \frac{4}{L_{xk} L_{yl}} \int_0^{L_{xk}} \int_0^{L_{yl}} \left(1 - \frac{r_x}{L_{xk}}\right)\left(1 - \frac{r_y}{L_{yl}}\right) \rho(r_x, r_y)\, \mathrm{d}x\, \mathrm{d}y. \tag{15}$$

Here L_{xk} and L_{yl} are the distances characterizing the relative positions of any two discretized rectangles A_i and A_j as shown in Figure 1, and $\gamma(L_{xk}, L_{yl})$ is the normalized variance function of the local averages $\alpha(x, y)$ over the rectangle with sides L_{xk} and L_{yl}. If $\alpha(x, y)$ is separable, i.e., $\rho(r_x, r_y) = \rho(r_x)\rho(r_y)$, one-dimensional expression for variance reduction function can be easily used for two-dimensional purpose and becomes simply the product of two one-dimensional factors, $\gamma(L_{xk}, L_{yl}) = \gamma(L_{xk})\gamma(L_{yl})$.

Various available forms of analytic expressions, i.e., triangular, exponential, etc., characterized by a correlation parameter has been reported [24]. However, the suitability of any model can be justified by fitting actual experimental data. As no such data is available to ascertain the relative merits of alternative models, Gaussian models with zero mean and unit standard deviation having exponential correlation function have been selected for the purpose of illustrating the analytical procedure. Here, the one-dimensional variance function corresponding to $\rho(r) = \exp[-(r/b)^2]$ can be readily obtained as

$$\gamma(r) = \left(\frac{b}{r}\right)^2 \left[\frac{r}{b}\sqrt{\pi}\phi\left(\frac{b}{r}\right) + \mathrm{e}^{-(r/b)^2} - 1\right], \tag{16}$$

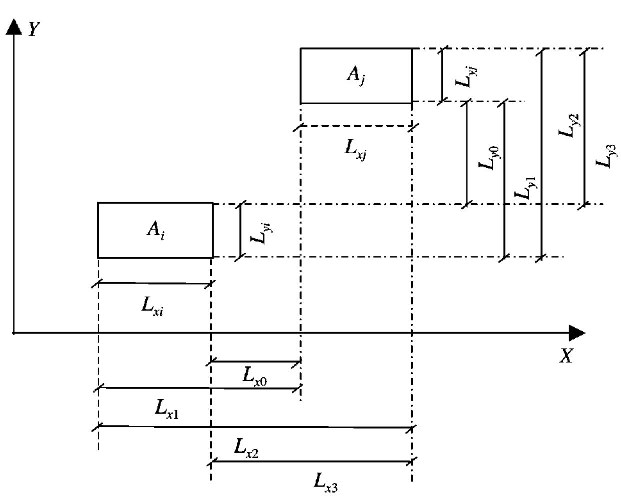

Figure 1. Definition of distances characterizing relative position of rectangles A_i and A_j.

where $\phi(\cdot)$ is the error function. Its value increases from zero to one as the argument of the function increases from zero to infinity and "b" is the correlation parameter. For the present 2-D problem, variance function of the following form is taken:

$$\rho(r_x, r_y) = \exp\left[-\left\{\left(\frac{r_x}{b_x}\right)^2 + \left(\frac{r_y}{b_y}\right)^2\right\}\right], \tag{17}$$

where b_x and b_y are the correlation parameters parallel to x- and y-axis respectively.

If there are "N" finite elements in the structure, auto-correlated random design vector $\{\alpha\} = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}^{\mathrm{T}}$ of the random field $\alpha(x, y)$ can be obtained as

$$\{\alpha\} = [L]\{Z\} \tag{18}$$

in which $\{Z\} = \{Z_1, Z_2, \ldots, Z_N\}^{\mathrm{T}}$ is a vector comprising "N" independent Gaussian random variates and $[L]$ is the lower triangular matrix derived through Cholesky decomposition of $\mathrm{Cov}(\alpha_i, \alpha_j)$. It can be shown that the generated random variables satisfy the original covariance property [17].

For the present study, Gaussian model has been selected for stochastic representation of modulus of elasticity, though non-Gaussian field models are also existent. The Gaussian models have limitations where design parameters experience large variations. Again the assumption of Gaussian distribution implies the possibility of generating unrealistic negative values of elastic properties. As truncated Gaussian distribution has been used for the generation of random sample functions of the uncertain parameter, these difficulties can be circumvented [17].

5. NEUMANN EXPANSION SOLUTION FOR RANDOM FREQUENCY RESPONSE

The direct Monte Carlo simulation is a technique to solve $\{H_\omega\{E\}, \{m\}\}$ directly for each simulated sample structure, by substituting elements of simulated random $\{E\}$ and $\{m\}$ at respective locations in $[D(\{E\}, \{m\})]$. Then by solving the associated deterministic problem, a population corresponding to desired response quantities can be obtained. It is worth mentioning that in the direct Monte Carlo simulation random stiffness matrix needs decomposition for each simulation. The Neumann Expansion technique can be utilized to avoid the repeated decomposition of random stiffness matrix. For kth finite element, the design variable "E_k" and "m_k" can be decomposed into its mean and fluctuating component as

$$E_k = E_{k0}(1 + \Delta E_k), \quad \text{where } \Delta E_k = E_{k0}\alpha_k^E$$

$$m_k = m_{k0}(1 + \Delta m_k), \quad \text{where } \Delta m_k = m_{k0}\alpha_k^m, \quad \text{where } k = 1, 2, 3, \ldots, N. \tag{19}$$

The random deviatoric part "α_k^E" or "α_k^m" is obtained through digital simulation as described in equation (18). Following this, the random global dynamic stiffness matrix is separated into mean and deviatoric parts.

$$[D\{\{E\}, \{m\}\}] = [D\{\{E_0\}, \{m_0\}\}] + [D\{\{\Delta E\}, \{\Delta m\}\}]. \tag{20}$$

Now applying the Neumann Expansion for the inversion of random stiffness matrix $[D\{\{E\}, \{m\}\}]$ and equation (20), the response vector which can be obtained from equation

(3) yields as

$$\{H_\omega\{\{E\},\{m\}\}\} = [D\{\{E\},\{m\}\}]^{-1}\{F\{m\}\} = \left(\sum_{n=0}^{\infty}[-P]^n\right)[D\{\{E_0\},\{m_0\}\}]^{-1}\{F\{m\}\}$$

$$= ([I]-[P]+[P]^2-[P]^3+\cdots)\{H_{\omega_0}\}$$

$$= \{H_{\omega_0}\} - \{H_{\omega_1}\} + \{H_{\omega_2}\} - \{H_{\omega_3}\} + \ldots, \tag{21}$$

where, $[P] = [D\{\{E_0\},\{m_0\}\}]^{-1}[D\{\{\Delta E\},\{\Delta m\}\}]$ and

$$\{H_{\omega_0}\} = [D\{\{E_0\},\{m_0\}\}]^{-1}\{F\{m\}\}. \tag{22}$$

The above series solution is equivalent to the solution to the following recursive equation:

$$[D\{\{E_0\},\{m_0\}\}]\{H^r_{\omega_i}\} = [D\{\{\Delta E\},\{\Delta m\}\}]\{H^r_{\omega_{i-1}}\}. \tag{23}$$

Once the decomposition of the deterministic part of the dynamic stiffness matrix i.e., $[D\{\{E\},\{m\}\}]$ is complete, $\{H_{\omega_0}\}$ can be computed for each simulated load vector and equation (23) can be used iteratively to obtain the random frequency response function for each simulated sample structure without further decomposition of the stiffness matrix.

6. NEUMANN EXPANSION SOLUTION FOR RANDOM RESPONSE SENSITIVITY

The Neumann Expansion can now be easily extended for each kth element to compute the random sensitivity of frequency response function $\{y_k\}$ and equation (9) yields

$$\{y_k\} = \left(\sum_{n=0}^{\infty}[-P]^n\right)[D\{\{E_0\},\{m_0\}\}]^{-1}\{F^*_k\}$$

$$= ([I]-[P]+[P]^2-[P]^3+\cdots)\{y_{k_0}\}$$

$$= \{y_{k_0}\} - \{y_{k_1}\} + \{y_{k_2}\} - \{y_{k_3}\} + \cdots, \tag{24}$$

where

$$\{y_{k_0}\} = [D\{\{E_0\},\{m_0\}\}]^{-1}\{F^*_k\}. \tag{25}$$

The solution of the above series can also be obtained by solving the following recursive equation.

$$[D\{\{E_0\},\{m_0\}\}]\{y^r_{k_i}\} = [D\{\{\Delta E\},\{\Delta m\}\}]\{y^r_{k_{i-1}}\}. \tag{26}$$

The expansion series may be terminated after a few terms depending on the required convergence and accuracy of the solution [17].

Once the ensemble of sensitivity of frequency response function is obtained, the statistical algorithms can be utilized to extract various statistical moments of the different sensitivity quantities. Hence the expected values and covariance of sensitivity of frequency response function are obtained for "N_s" number of simulation as

$$E[\{y\}] \cong \{\bar{y}\} = \frac{1}{N_s} \sum_{i=1}^{N_s} \{y\}_i$$

$$\text{and, } \mathrm{Cov}[\{y\}, \{y\}] = \frac{1}{N_s} \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} [\{y\} - \{\bar{y}\}]_i^{\mathrm{T}} [\{y\} - \{\bar{y}\}]_j. \tag{27}$$

Similarly, response statistics of spectral density function and mean square displacement are also obtained.

7. NUMERICAL EXAMPLE

To illustrate the efficiency of Neumann Expansion coupled with Monte Carlo simulation for sensitivity analysis of uncertain dynamic systems, as presented in the earlier section, an example of a concrete wall of 9·0 m × 9·0 m with 0·2 m thickness fixed at the base as shown in Figure 2 subjected to random earth earthquake loading is presented. Results are furnished at mid-point location where largest output is expected and are compared with those by direct simulation.

Mean values of modulus of elasticity (E) and mass density (m) are taken as $2{\cdot}0 \times 10^{10}$ N/m^2 and 2400·0 kg/m^3 respectively. The value of the Poisson ratio is considered as 0·15. Deviatoric part of elasticity and mass density are modelled as Gaussian process and

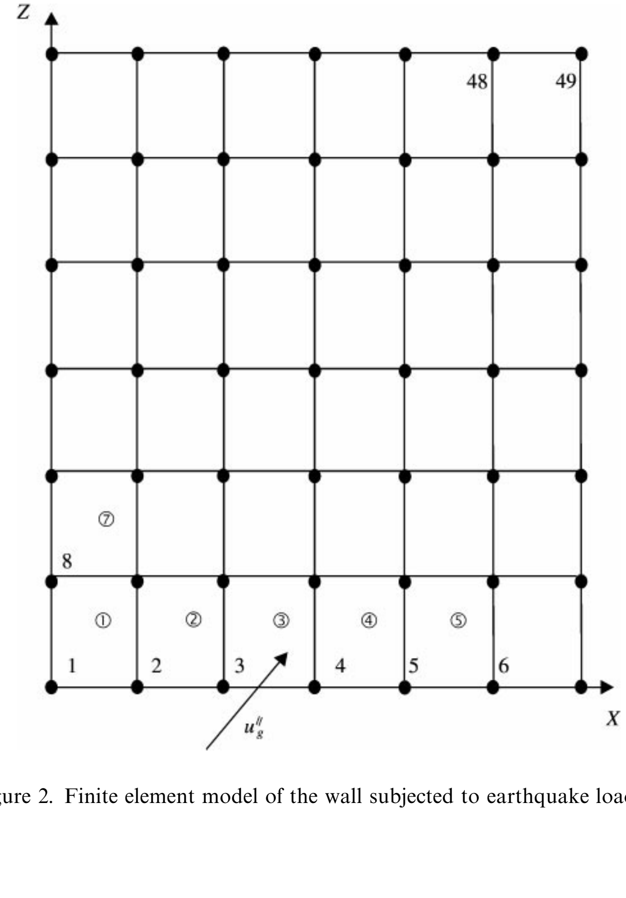

Figure 2. Finite element model of the wall subjected to earthquake loading.

generated as described in equation (18). In local averaging discretization, the correlation parameters are assumed as twice the element length.

In this study, for simplicity and ease of analysis, the ground motion is considered to be a stationary random process with constant frequency content truncated for a finite duration. The following power spectral density function as suggested by Kanai–Tajimi can be used to characterize a strong earthquake ground motion

$$S_f = \frac{[S_0[1 + 4\xi_g^2(\omega/\omega_g)^2]}{[1 - (\omega/\omega_g)^2]^2 + 4\xi_g^2(\omega/\omega_g)^2}. \tag{28}$$

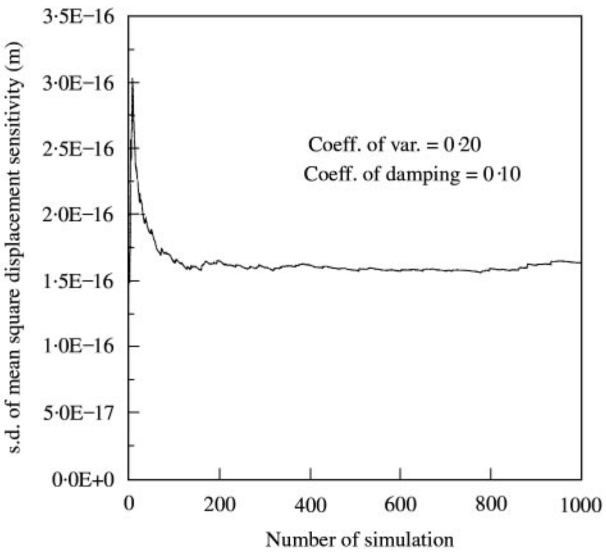

Figure 3. Fluctuation of s.d. of mean-square displacement sensitivity.

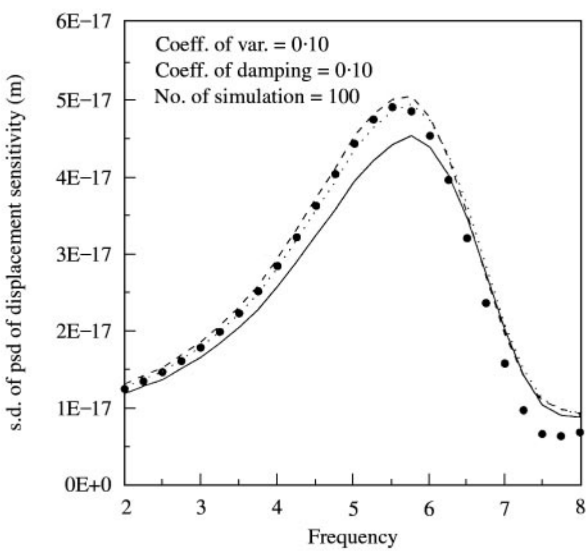

Figure 4. Comparison of s.d. of psd of displacement sensitivity: ——, neu. exp. (first term); ·······, neu. exp. (second term); - - - - -, neu. exp. (third term); ●, direct simulation.

In the above, "ω" is the component frequency, S_0 is the scale factor, "ω_g" and "ξ_k" are the equivalent natural frequency and damping ratio of the ground, respectively, characterized by a single-degree-of-freedom system. The different parameters of Kanai–Tajimi model of random earthquake loading are the following: scale factor $S_0 = 3{\cdot}73 \times 10^{-3}\,\mathrm{m^2/s^3}$, equivalent natural frequency $\omega_g = 15{\cdot}56\,\mathrm{rad/s^2}$ and equivalent damping ratio $\xi_g = 0{\cdot}65$. The sensitivity of responses are calculated at a frequency step of 0·25 and the cut-off frequency is taken as 17·0 rad/s.

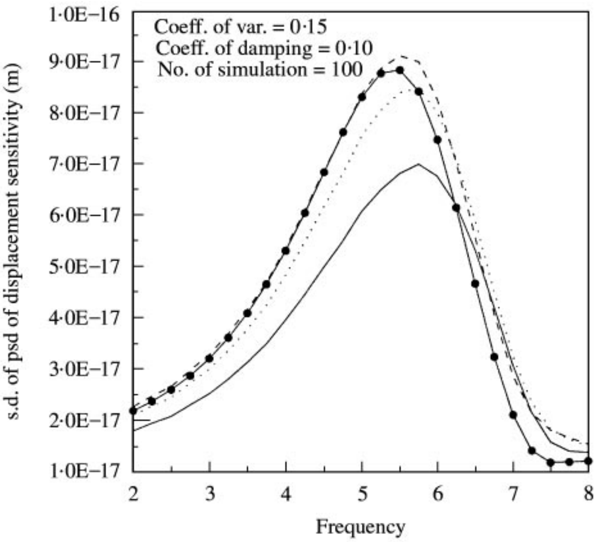

Figure 5. Comparison of s.d. of psd of displacement sensitivity: ——, neu. exp. (first term); ·······, neu. exp. (second term); - - - - -, neu. exp. (third term); —●—, direct simulation.

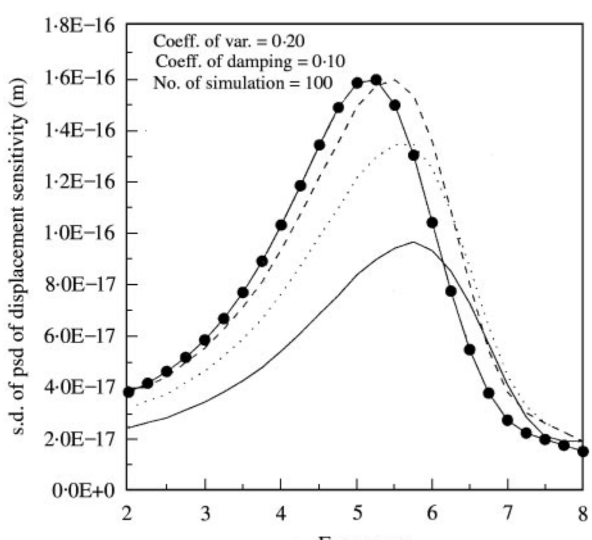

Figure 6. Comparison of s.d. of psd of displacement sensitivity: ——, neu. exp. (first term); ·······, neu. exp. (second term); - - - - -, neu. exp. (third term); —●—, direct simulation.

The size of the ensemble should be large enough to obtain statistically stable results. As it is observed from Figure 3 that after 100 simulations the fluctuation of sensitivity of standard deviation of mean-square displacement is negligibly small, the number of simulations has been fixed at 100. Figures 4–6 represent the comparison of power spectral density at different coefficient of variation (c.o.v). The plots infer that for a lower range of c.o.v. two terms of Neumann Expansion are well convergent, whereas for higher c.o.v. more number of terms need to be employed. However, the results obtained by the Neumann

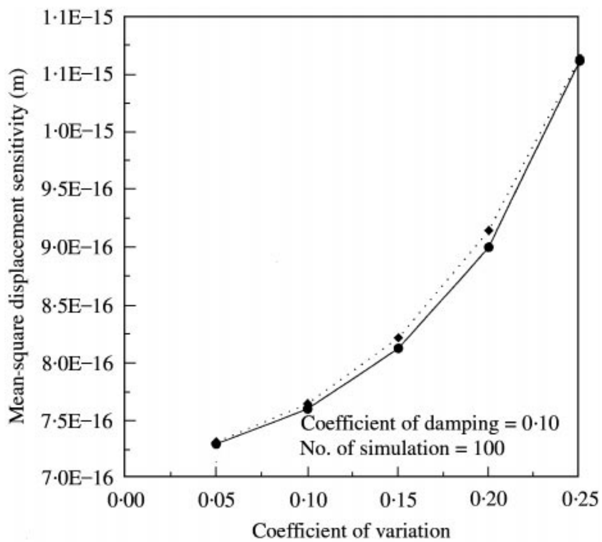

Figure 7. Comparison of mean-square displacement sensitivity versus coefficient of variation: ·····◆······, neu. exp. (third term); —●—, direct simulation.

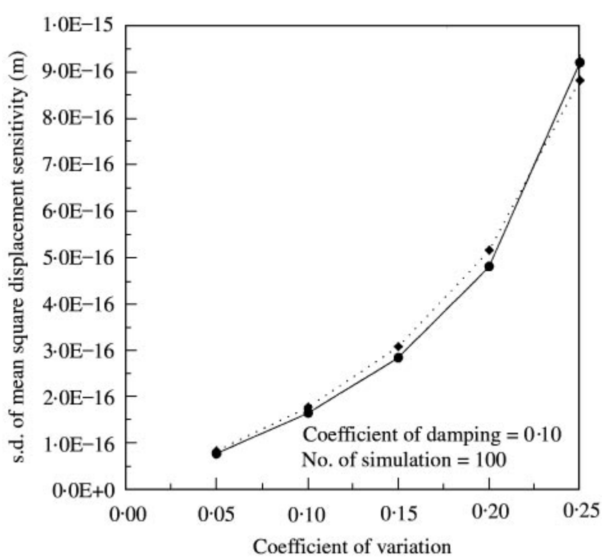

Figure 8. Comparison of s.d. of mean-square displacement sensitivity versus coefficient of variation: ·····◆······, neu. exp. (third term); —●—, direct simulation.

Expansion show some shift around the natural frequency and this shift increases at higher c.o.v. To show these shifts in a pronounced fashion, plots have been restricted within a frequency range of 2·0–8·0 rad/s. The comparison of sensitivity of mean-square displacement at different c.o.v. are furnished in Figures 7 and 8. The dependence of sensitivity of mean-square displacement on correlation parameter is shown in Figures 9 and 10. It is observed that as the correlation length increases, the efficiency of Neumann Expansion decreases. An increase in c.o.v. as well as correlation length causes the increase of

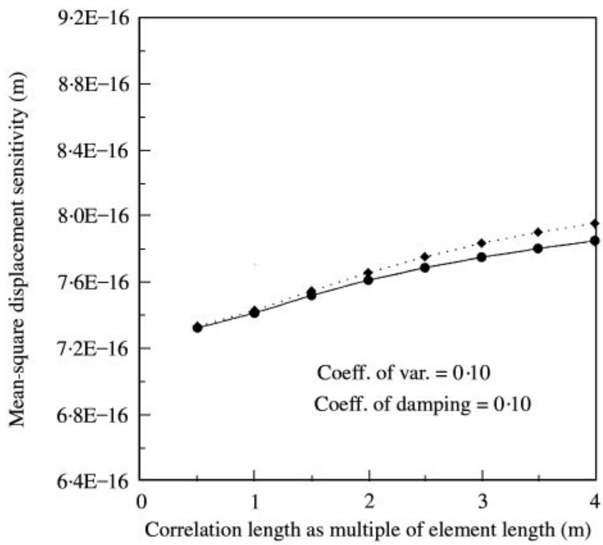

Figure 9. Comparison of mean-square displacement sensitivity versus correlation length: ·····◆······, neu. exp. (third term); —●—, direct simulation.

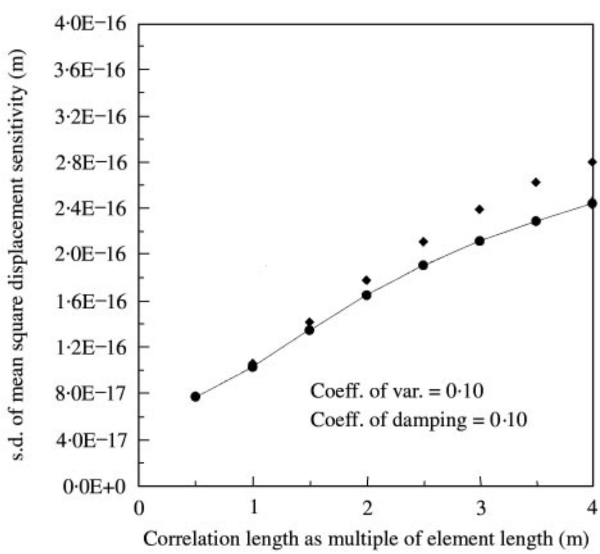

Figure 10. Comparison of s.d. of mean-square displacement sensitivity versus correlation length: ·····◆······, neu. exp. (third term); —●—, direct simulation.

stochasticity of the system and more number of terms are to be employed for convergence of results.

8. CONCLUSION

By decomposing dynamic stiffness matrix, the existing finite element program can be easily used, only incorporating random variable simulation subroutine, without involving further computational complexity. It is observed that for small c.o.v., two terms of Neumann Expansion are well convergent. But only three terms in the Neumann Expansion are not sufficient for good convergence at higher c.o.v. (more than 0·20). In that situation, more number of terms will be necessary but the advantage of the Neumann Expansion will be lost. It is evident that the Neumann Expansion is quite effective over direct simulation as it involves only single decomposition of dynamic stiffness matrix for the entire simulated structure. However, the accuracy and efficiency are largely dependent on the number of degrees of freedom, degree of accuracy required, models of uncertain parameters, etc. It is notable that more than one random parameter can be considered simultaneously for a single-step solution result. This is an important flexibility in this approach, which is difficult to obtain in perturbation technique.

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