



ON THE ORIGINS AND DEVELOPMENT OF MOBILITY AND IMPEDANCE METHODS IN STRUCTURAL DYNAMICS

P. GARDONIO AND M. J. BRENNAN

*Institute of Sound and Vibration Research, University of Southampton,
Southampton SO17 1BJ, England. E-mail: pg@isvr.soton.ac.uk*

(Received 6 December 2000, and in final form 26 June 2001)

An account of the conception and early development of *mobility* and *impedance* methods in structural dynamics is presented in this paper. As often happens in science, the concepts and ideas developed in one branch of science have inspired new approaches and theories in another branch, and indeed this was the case for mechanical *mobility* and *impedance*, which has its origin in the field of electricity. The conception and formulation of both direct and inverse electromechanical analogies are described in this paper together with the formulation of *ad hoc* impedance and mobility methods that were developed for mechanical systems. Finally the impedance, mobility and transmission matrix methods that evolved for flexible, distributed mechanical systems are discussed. This paper has been written with reference to a large number of published papers many of which have been listed in the reference section.

© 2002 Academic Press

1. INTRODUCTION

In this paper, the conception and evolution of *mobility* and *impedance* methods in structural dynamics is presented. The authors use these methods in most of their research work involving, for example, passive/active vibration control, structural acoustics problems, and rotor dynamics problems. They have found that there is only patchy treatment of the mobility and impedance approaches in many text books and in contemporary literature, and yet the approach is very powerful. It allows structures to be subdivided and does not have high-frequency limitations as with the finite element method, it formulates problems in terms of variables that can be easily measured, and it is particularly useful in the training of novice researchers who are forced to think of the physics of the problem in hand, rather than the mathematics. The motivation behind this paper is therefore (1) to give some exposure to the mobility and impedance approach and (2) to describe the way in which the approach has been developed, and in doing so provide a bibliography on the subject. It is also hoped that this will inspire some new research in the area where new techniques may be combined with an old approach. Examples where this is already happening are described later in the paper.

The task of reconstructing the origin of a methodology developed for the study of physical phenomena is challenging. It requires interpretation of documents written at the early stages of the formulation of a methodology, and thus it could be limited by the lack of material published in scientific journals. This is probably the case in this attempt to identify the milestones that led to the concepts of mechanical impedance and mechanical network

theory. However, the authors believe that they have included the key papers in the subject's development which turn out to be mainly from Western Europe and the USA.

The paper is organized as follows. Following this introduction, in section 2, the origin of the impedance method for mechanical systems is described. In section 3, the development of mechanical network theory is discussed together with its analogy to an electrical network. The way in which the analogy was developed into the mobility concept is the subject of section 4, and the extension of the approach to multi-degree-of-freedom systems is described in section 5. Finally, following a review of recent research on mobility and impedance techniques in section 6, the paper is brought to a close with some concluding remarks.

2. ORIGIN OF THE MECHANICAL IMPEDANCE CONCEPT

The early analysis of vibratory systems in terms of mechanical impedances was probably linked to the studies of electrical communication. Many documents indicate that a number of inventions such as the electromagnetic telegraph by Henry in 1830, the telephone by Bell in 1876, the phonograph by Edison in 1878 and the motion pictures by Edison in 1891 gave scientists a completely new set of problems to solve around the turn of the 20th century [1]. Although primitive versions of these instruments were able to convert sound into mechanical and electrical signals and *vice versa*, they were not able to replicate speech or music. Both the electromechanical transducers and the electrical circuits used to build these new instruments were characterized by low efficiency and sharp resonances. Thus, only a few octave bands of the acoustic signals were uniformly transmitted [2]. Scientists, therefore, faced the challenges of increasing the number of uniformly reproduced octave bands, and increasing the efficiency of acoustic to electrical (and *vice versa*) conversions [3].

Under pressure from commercial development of the electric telegraph and the telephone at the turn of the 20th century, the theory of electric networks or circuits, was developed. In 1884 Heaviside gave a definition of electric impedance [4]. He wrote on page 371 of Vol. I: "impedance is here and later substituted for apparent resistance. It is the ratio of the amplitude of the impressed force to that of the current when their variations are simply harmonic." This, together with theorems by Kirchhoff [5], Thévenin [6, 7] and Norton [7], enabled the development of a new method for the systematic study of electrical networks using the principles of superposition, reciprocity and compensation. The main advantage of this new approach was that linear circuits could be studied without using differential equations [8–10]. The first circuit studied with this new approach was the band-pass filter by Campbell [11, 12]. He also applied the network theory to obtain a system with uniform transmission over a wide frequency range. By matching the termination impedances of several resonant elements, he made a device which had high and uniform energy transmission efficiency over the widest possible frequency range.

Professor Arthur G. Webster was the first to realize the possibility of using the impedance concept in the study of vibrating mechanical systems. In 1914, during a meeting of the Physical Society in Philadelphia, Webster read a paper [13] where, as shown in Figure 1, he first defined the acoustic impedance for an oscillating system. He went on to define the impedance for a mass, spring and dash-pot mechanical oscillator system as the ratio of the periodic force applied on the mass $f(t) = F(\omega)\exp(j\omega t)$ over the equally periodic displacement of the mass $v(t) = V(\omega)\exp(j\omega t)$ so that $Z_m(\omega) = F(\omega)/V(\omega)$. Webster's decision to define the mechanical impedance as the ratio of a dynamic parameter over a kinematic quantity rather than the inverse quotient was probably because he wished to be consistent to the definition of electrical impedance [14]. Electrical impedance represents the ratio between the e.m.f. across an electrical element and the current flowing through it. This

ACOUSTICAL IMPEDANCE, AND THE THEORY OF HORNS AND OF THE PHONOGRAPH

BY ARTHUR GORDON WEBSTER

DEPARTMENT OF PHYSICS, CLARK UNIVERSITY

Communicated, May 8, 1919*

The introduction more than thirty years ago of the term 'impedance' by Mr. Oliver Heaviside has been productive of very great convenience in the theory of alternating currents of electricity. Unfortunately, engineers have not seemed to notice that the idea may be made as useful in mechanics and acoustics as in electricity. In fact, in such apparatus as the telephone one may combine the notions of electrical and mechanical impedance with great advantage. Whenever we have permanent vibrations of a single given frequency, which is here denoted, as usual, by $n/2\pi$, the notion of impedance is valuable in replacing all the quantities involved in the reactions of the system by a single complex number. If we follow the convenient practice of denoting an oscillating quantity by e^{int} and taking its real part (as introduced by Cauchy) all the derivatives of e^{int} are obtained by multiplication by powers of in , or graphically by advancing the representative vector by the proper number of right angles.

If we have any oscillating system into which a volume of air X periodically enters under an excess pressure p , I propose to define the impedance by the *complex* ratio $Z = p/X$. If we call $dX/dt = I$ the current as in electricity, if we followed electrical analogy we should write $Z = pI$ so that the definition as given above makes our impedance lead by a right angle the usual definition. I believe this to be more convenient for our purposes than the usual definition and it need cause no confusion.

If we have a vibrating piston of area S as in the phonometer, we shall refer its motion to the volume $S\xi$ it carries with it and the force acting on it to the pressure, so that $F = Sp$. The differential equation of the motion is

$$m \frac{d^2\xi}{dt^2} + \kappa \frac{d\xi}{dt} + f\xi = F = Sp, \quad X = S\xi, \quad (1)$$

we have

$$Z_1 = (f - mn^2 + ikn)/S^2, \quad (2)$$

where m is the mass, κ the damping, f the stiffness. The real part of S^2Z , $f - mn^2$, is the uncompensated stiffness, which is positive in a system tuned too high, when the displacement lags behind the force, by an angle between zero and one right angle, negative when the system is tuned too low, when the

*This article was read in December 1914 at the meeting of the American Physical Society at Philadelphia, and has been held back because of the continual development of the experimental apparatus described in a previous paper in these PROCEEDINGS.

Figure 1. Front page of the paper in which Professor Arthur Gordon Webster first introduced the concept of mechanical impedance (from reference [13]).

could be seen as the ratio between the cause (e.m.f.), and its effect (current). Webster's definition of mechanical impedance represents the ratio between the cause of motion (force), and its effect (displacement).

3. DIRECT AND INVERSE ELECTROMECHANICAL ANALOGIES

Once the similarity between mechanical and electrical systems was realized, many scientists began to analyze electromechanical transducers using network theory. Results from early studies appeared in patents, journal publications and books [15–24]. The first study of a purely mechanical system using network theory was probably carried out by Harrison [25] on improving the design of a phonograph (system on the right-hand side of Figure 2).

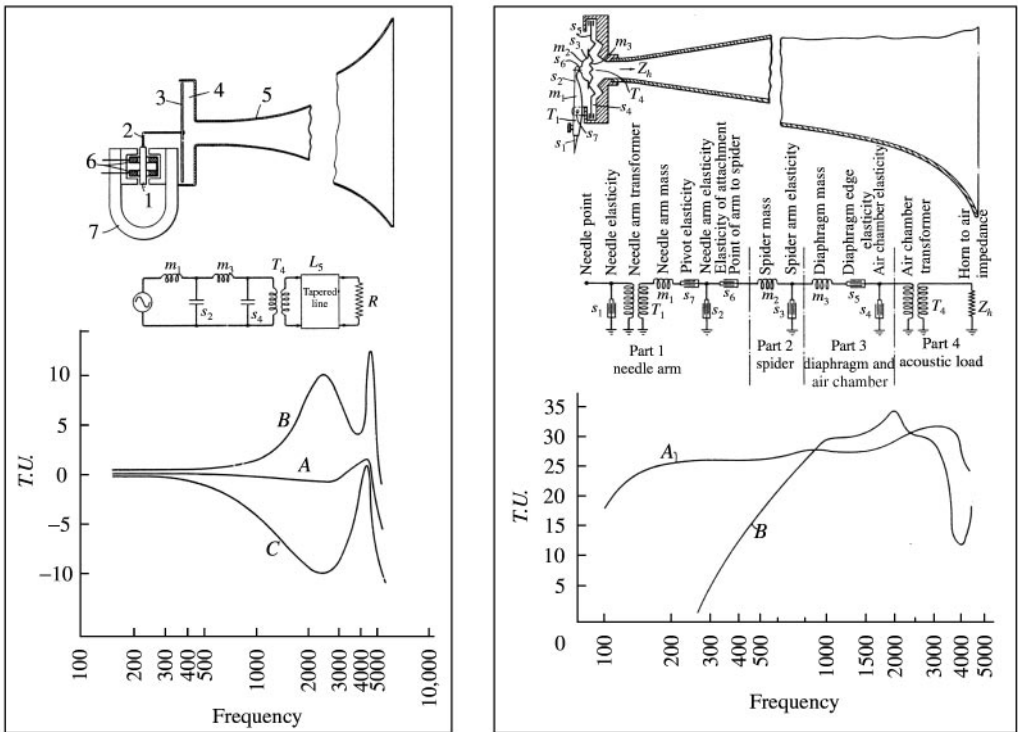


Figure 2. First two pages of the patent written by H.C. Harrison showing two inventions: an electric loudspeaker (left) and a phonograph (right). Below the pictures of the two inventions, the schematic diagram of the equivalent electric circuit to the electro-mechanical (left) and purely mechanical system (right) are shown (from reference [25]).

Because many of the early results were released as patents, work on electromechanical systems using network theory was not immediately noticed by the wider scientific community. The method eventually became known as the “direct analogy”, when papers and books dealing specifically with the analogy were first published [26, 27] and later on [28–32]. The approach was based on the following methodology: first, the electrical circuit, which is analogous to the mechanical problem to be solved was drawn; second, the analogous electrical problem was solved using the electric network theory and third, the electrical answer was reworked into mechanical terms. Current and e.m.f. of the equivalent electrical circuit represented velocity and force parameters of the mechanical system respectively [27]. Equivalent network impedances were derived from the corresponding mechanical impedances by assuming appropriate conversion factors and connecting rules [32].

Acoustical filters were first studied by Hershel in 1833 [33], but the need for better design of the telephone electromechanical transducers, gave renewed drive to the design of efficient electromechanical and acoustical/mechanical systems. The electromechanical analogy and the concept of a mechanical filter [25, 22, 27] were of considerable help in this endeavour. Stewart [34–36] showed that by combining resonators and tubes in an appropriate manner, it was possible to obtain the same transmission frequency characteristics as electrical filters. Studies on acoustical and mechanical filters using the electromechanical analogy were subsequently carried out by Mason [37, 38], Cady [39], Espenschied [40], Lindsay and White [41], Lindsay [42–44], Lindsay *et al.* [45] and Lakatos [46]. The origin of the

electromechanical analogy was therefore strongly linked to the development of electrical network theory and electric wave filter systems. As often happens in science, the concepts and ideas developed in one branch of science were transferred to another branch.

Once the electromechanical analogy had been established, scientists began to consider its limitations. Since the early studies on analogies, it had been apparent that the variables used in the differential equations of electrodynamic or electromagnetic systems were not consistent with those of purely electrical systems [18, 47]. It was concluded that electrodynamic and electromagnetic systems network diagrams could not be directly transposed to an equivalent purely electrical network. Indeed, the analogy based on Webster's definition of impedance for mechanical systems leads to some difficulties such as those listed below [48].

1. The force transmitted "through" a mechanical element is replaced by the e.m.f. "across" the equivalent electrical element while the velocity "across" a mechanical element is replaced by the current "through" the equivalent electrical element; therefore, the physical interpretation of the equivalent electric network variables with reference to the mechanical ones is inverted.

2. Mechanical elements in series are represented by electrical elements in parallel while mechanical elements in parallel are represented by electrical elements in series so that the composition of the analogue electrical circuit of a mechanical system differs from the procedure used to set the circuit chart of a purely electrical system.

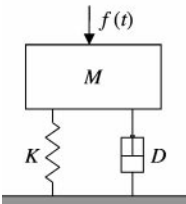
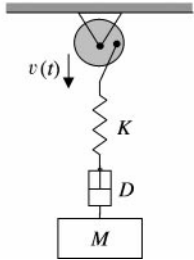
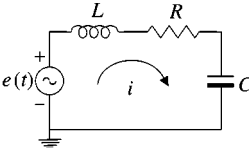
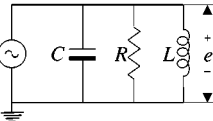
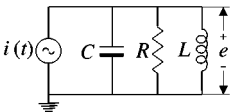
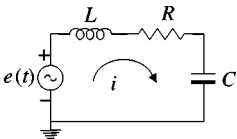
3. The resulting impedance of a series of mechanical elements is calculated as the reciprocal of the sum of the reciprocal impedance of the mechanical elements while the resulting impedance of a parallel of mechanical elements is given by the sum of each mechanical element impedance which is exactly the opposite formulation of that for the calculation of the total impedance of a series or parallel assembly of electrical elements.

4. The application of Kirchhoff's current law (sum of currents to a junction is zero) directly corresponds to the velocity law of mechanical systems (sum of velocity differences around a closed circuit is zero) while the application of Kirchhoff's voltage law (sum of e.m.f. around a mesh is zero) directly corresponds to the force law of mechanical systems (sum of force to junction is zero), so that, once more the physical explication of the analogue electric network is not simple and direct. On the contrary, it requires a certain ability to compare a junction-type law with a mesh-type law and *vice versa*.

Darriues [49] first mentioned the possibility of defining the analogy in a different way where force is equivalent to current rather than e.m.f. Two papers presenting a new analogy were subsequently published by Hähnle [50] and Firestone [48], in which a new definition "bar impedance" was proposed. This was the ratio of a kinematic variable over a dynamic variable and gave a new electromechanical analogy for solving vibroacoustic problems. The new analogy was free from limitations 1 to 4 above and became known as the "inverse analogy". The transformation of the mechanical bar impedances to electric impedances used new conversion factors and connecting rules as described by Bloch [32]. Because the physical principles behind this new approach for the solution of vibratory problems were not trivial, the new analogy was not adopted immediately by the scientific community. It was based on an alternative formulation of Newton's second law where the dynamical behaviour of a system was derived from kinematic equations rather than force equilibrium equations [51]. Le Corbeiller and Yeung [14] comment on this approach "The reader may wonder how it is possible to derive the dynamical behaviour of a system from kinematic equations. This question arises because we are accustomed in dynamics to a mistaken emphasis on forces, traceable to a metaphysical attitude current in Newton's times. From this emphasis on force considered as the cause of motion, there followed in electricity an

TABLE 1

Electromechanical analogies; the factors $\epsilon_f, \epsilon_v, \delta_f, \delta_v$ are the conversion factors from force or velocity variables to potential or current variables as from reference [32]

<i>Mechanical system</i>	M_f	DUAL OF	M_v
$M =$ mass (kg)		DUAL OF	
$K =$ stiffness (N/m)			
$D =$ dissipation factor (N/m s ⁻¹)			
$f(t) = F \exp(j\omega t)$ harmonic force (N)			
$v(t) = V \exp(j\omega t)$ harmonic velocity (m/s)	$M \frac{dv}{dt} + Dv + K \int v dt = f(t)$		$\frac{1}{K} \frac{df}{dt} + \frac{f}{D} + \frac{1}{M} \int f dt = v(t)$
Equivalent electric circuit from <i>Direct analogy</i>	E_v	DUAL OF	E_i
$f(t) \leftrightarrow e(t)$ $v(t) \leftrightarrow i(t)$		DUAL OF	
$L =$ inductance (H)			
$C =$ capacitance (F)			
$R =$ resistance (Ω)			
$e(t) = E \exp(j\omega t)$ Potential (V)	$L \frac{di}{dt} + Ri + C \int i dt = e(t)$		$C \frac{de}{dt} + \frac{1}{R} e + \frac{1}{L} \int e dt = i(t)$
$i(t) = I \exp(j\omega t)$ Current (A)	$L = M/\alpha^2, R = D/\alpha^2, C = \alpha^2/k$		where: $\alpha^2 = \epsilon_f/\epsilon_v; E = \epsilon_f F; I = \epsilon_v V$
Equivalent electric circuit from <i>the inverse analogy</i>	E_i	DUAL OF	E_v
$f(t) \leftrightarrow i(t)$ $v(t) \leftrightarrow e(t)$		DUAL OF	
$L =$ inductance (H)			
$C =$ capacitance (F)			
$R =$ resistance (Ω)			
$e(t) = E \exp(j\omega t)$ Potential (V)	$C \frac{de}{dt} + \frac{1}{R} e + \frac{1}{L} \int e dt = i(t)$		$L \frac{di}{dt} + Ri + C \int i dt = e(t)$
$i(t) = I \exp(j\omega t)$ Current (A)	$L = \beta^2/k, R = \beta^2/D, C = M/\beta^2$		where: $\beta^2 = \delta_f/\delta_v; I = \delta_f F; E = \delta_v V$

emphasis on electromotive force considered as the cause of the current flow. To one without such preconceptions, it should be clear that in Ohm's law $E = IR$, as well as in Newton's law $f = ma$, the general gas law $PV = nRT$, or any natural law whatsoever, there is no cause and no effect, there are only variable physical quantities permanently connected by a mathematical equation".

Starting from the known concept of duality between two electrical systems, Le Corbellier and Yeung [14] showed that the duality principle held for mechanical systems. Duality is a topological transformation exchanging node pairs and meshes, and this type of transformation together with the electromechanical analogies provide a set of mathematical tools for the analysis of electrodynamic systems.

Table 1 summarizes the main features of the direct and inverse electromechanical analogies. The first row shows two-single-degree-of-freedom mass-spring-damper systems. The one denoted \mathbf{M}_f is a parallel connected system and is excited by a force. The one denoted \mathbf{M}_v is a series connected system and is excited by a velocity source. Schematic diagrams of electrical networks shown below the mechanical systems are analogous or inversely analogous to the systems. The two mechanical systems depicted, and their direct or inverse analogue electric network pairs, are dual. The equation of motion of the mechanical system \mathbf{M}_f (top left) can be derived using Newton's second law directly; the applied force is equal to the sum of the inertial force (which is proportional to the absolute acceleration of the mass), the damping force (which is proportional to the relative velocity across the viscous damper) and the elastic force (which is proportional to the relative displacement across the spring). For this system, it is straightforward to derive the equivalent electrical circuit using the direct analogy where the forces from the three mechanical elements are represented by e.m.f. differences across an inductance, a resistor and a capacitor connected in series such that they balance the voltage generator e.m.f., \mathbf{E}_v (centre left).

Alternatively, the equation of motion of the mechanical system \mathbf{M}_v (top right) can be derived using the principle of compatibility. The sum of the absolute displacement of the mass and the relative displacements of the dash-pot and spring elements is equivalent to that generated by the scotch-yoke motion generator. In this case, it is easier to derive the equivalent electrical circuit by using the inverse analogy where the displacements of the three mechanical elements are represented by the e.m.f. differences across an inductance, a resistor and a capacitor connected in series in such a way to balance the voltage generator e.m.f., \mathbf{E}_v (bottom right). Thus, the choice of using the direct or inverse analogy depends on the topology of the mechanical system. Nevertheless, Table 1 clearly shows that, as advocated by Firestone, the electrical networks derived with the inverse analogy have the same topological features of the mechanical systems from which they are derived. Alternatively, the electric networks derived with the direct analogy have the node pair and meshes exchanged with reference to the mechanical system from which they are derived. Le Corbellier and Yeung [14] wrote about this matter the following comment: "The classical analogy makes the meshes of \mathbf{E}_v correspond to the node-pairs of \mathbf{M}_f , an awkward situation due to the topological innocence of our forebears".

Detailed introductions to electromechanical analogies can be found in books [30, 31, 52-55]. An interesting paper by Miles [56] compares the advantages and limitations of the two analogies.

4. IMPEDANCE AND MOBILITY METHODS FOR MECHANICAL SYSTEMS

Firestone hoped that the advantages introduced by the inverse analogy would slowly bring the direct analogy into disuse and subsequently, the new definition of impedance

would replace the one defined by Webster. Firestone [48] states “It has seemed advisable to introduce a new term, bar impedance” (*subsequently called mobility*) “which is equal to velocity across/force through. It is natural that the old analogists, having arrived on the ground first, should have chosen to define impedance as force/velocity since that fitted in with the other assumptions they had made. But in the author’s opinion, all of their assumptions were unwise and led to the left-handed result that while electrical impedances in series are additive, mechanical impedances in series must be added as the reciprocal of the sum of the reciprocals as was shown above. It would have been better if this new analogy had been thought of first, for in that case the quantity which we have been forced to call “bar impedance” would have been called impedance and would have been subject to the same laws of addition as are found in electrical circuits. It is now too late to change suddenly the unfortunate definition of impedance which has been so much used in the past, so it is recommended that the term “bar impedance” be used. Then if the new analogy should prove popular, the time may come when the old definition of impedance will have fallen into disuse, at which time the “bar impedance” may be shortened to “impedance” with the new definition”. Unfortunately, the new definition of impedance was not adopted, so Firestone [57] introduced a new parameter called mobility (ease of motion) which is the ratio between a kinematic and a dynamic parameter. A problem could now be formulated in terms of mechanical variables, and all the electric circuit laws, theorems and principles were defined for a purely mechanical network. Firestone summarized the main features of the mobility method as follows.

1. A set of conventionalized symbols with which the essential characteristics of a mechanical system can be set forth in the form of a schematic diagram.
2. The concept of the velocity *across* mechanical elements (velocity of one end of the element relative to the other end) as contrasted with the velocity of points in the system relative to ground; the advantage here is that the relationship between the velocity across an element and the force through it, depends only on the characteristics of the element itself and not on the characteristics of the rest of the system.
3. The use of complex numbers to represent simple harmonic velocities and forces, both the magnitude and phase of these quantities being represented by the absolute value and angle of the complex numbers.
4. The concept of “mobility” of an element (ease of motion), which is defined as the complex ratio of the velocity across an element to the force through the element; simple rules are developed for computing the mobility of series or parallel combinations of elements and the force can then be found simply as the velocity divided by the mobility, or the velocity can be found as the force multiplied by the mobility.

The formulation of the mobility method was completed by the description of force and velocity laws as the equivalent of Kirchhoff’s current and voltage laws. Equivalent Thévenin and Norton theorems for force or velocity sources were also described together with the mechanical forms of the principles of reciprocity, superposition and compensation. Thus, the mobility method proposed by Firestone had two main advantages: first, a problem could be formulated directly in terms of mechanical variables and second, a more intuitive formulation of mechanical network was possible.

The first referenced works where the dynamics of a system were analyzed using a mechanical formulation directly in terms of a mobility-type parameter, e.g. (kinematic variable)/(dynamic variable), or in terms of impedance-type parameter, e.g. (dynamic variable)/(kinematic variable), are those of Carter [58], Duncan [59, 60], Biot [61, 62], Manley [63, 64] and Miles [56]. Many other studies followed and an extensive bibliography is given by Gardonio and Brennan [65].

In a later paper, Firestone [66] proposed an extension of the mobility method. He reworked the direct and inverse analogies so that instead of drawing an analogous electrical network, the acoustical or mechanical diagrams were drawn directly using acoustical and mechanical impedance or mobility symbols. Two comprehensive tables were presented where symbols for lumped acoustical or lumped mechanical elements were given with reference to either the impedance or mobility parameters. The solution of a network problem was then carried out with reference to acoustical or mechanical units.

As with the electromechanical analogies, the choice of either the impedance or mobility methods is dependent upon the system of interest. Many mechanical systems can be modelled by rigidly connected lumped elements where the adjacent elements have the same velocity, or volume velocity in the acoustic case (\mathbf{M}_v system in Table 1). For this type of system, the mobility approach is convenient since the mechanical network can be drawn by inspection. When adjacent elements of a system have the same forces at their terminals, or pressures in the acoustic case, (\mathbf{M}_f system in Table 1) it is easier to derive the impedance network. In certain cases, it could be necessary to draw part of the system with the mobility method and part with the impedance method. These two different networks are then linked by means of specific couplers [67].

The concept of an entirely mechanical network was well received by the scientific community working on mechanical problems.

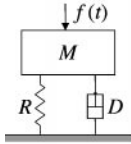
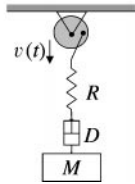
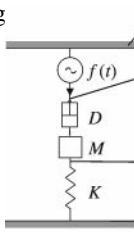
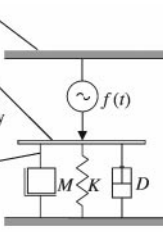
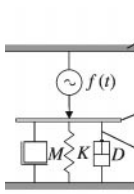
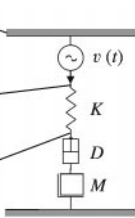
Table 2 shows the impedance (centre) and mobility (bottom) schematic diagrams of the two mechanical systems \mathbf{M}_f and \mathbf{M}_v considered in Table 1. The equivalent impedance and mobility schematic diagrams have been derived from the appropriate mechanical diagrams for either the impedance or mobility methods. The lines connecting the lumped elements have a different significance in the two diagrams. With the impedance method the connecting lines represent a "force junction" where there is equal force at the terminals of two adjacent elements. When the mobility method is employed, the lines connecting the elements represent a "velocity junction" that ensures equal velocity at the terminals of adjacent elements. The diagrams obtained with the mobility method retain the same topological features of the mechanical systems from which they are derived, so in general the mobility approach is more intuitive. This is true except in a few cases where the system is such that the transmission of force is imposed at the junction between the elements (\mathbf{M}_v in Table 2). In many cases, acoustical filters, with or without side branches, are better represented in terms of impedances.

In 1958, the American Society of Mechanical Engineers (ASME), organized a colloquium that presented the state of the art in the area of mechanical impedance and mobility methods. The papers presented during the colloquium were collected in a report by Plunket [68], and this is one of the key milestones in the development of impedance and mobility methods. In particular, a paper was presented by Crandall [69] which discussed the impedance and mobility approach for mechanical network analysis as suggested by Firestone but without any reference to an equivalent electric circuit. This was an important step in the consolidation of Firestone's ideas. It took some time, however, before the ideas introduced by Firestone were assimilated and exploited, which was probably because there was no standard nomenclature for mechanical network representation. Indeed, in most of the papers collected in Plunket's [68] or Belsheim's [70] proceedings or in other publications the nomenclature differs. In 1963, The American National Standards Institute was the first to introduce a standard nomenclature: "Nomenclature and Symbols for Specifying the Mechanical Impedance of Structures" (ANSI, S2.6-1963 R 1971).

There are few books that introduce the main principles of impedance or mobility methods. Perhaps the most complete presentation is in chapter 10 of the "Shock and

TABLE 2

Impedance and mobility diagrams

Mechanical system	M_f	M_v
<p>M = mass (kg)</p> <p>K = stiffness (N/m)</p> <p>D = dissipation factor (N/m s⁻¹)</p> <p>$f(t) = F \exp(j\omega t)$ harmonic force (N)</p> <p>$v(t) = V \exp(j\omega t)$ harmonic velocity (m/s)</p>		
	DUAL OF	
	$M \frac{dv}{dt} + Dv + K \int v dt = f(t)$	$\frac{1}{K} \frac{df}{dt} + \frac{f}{D} + \frac{1}{M} \int f dt = v(t)$
<p>Impedance schematic or tubing diagram from the</p> <p>Impedance method</p> <p>Z_{eq} = total impedance</p> <p>Z_m = impedance of the mass</p> <p>Z_d = impedance of the dissipator</p>	 <p>“Sky” an ideal dynamic reference which maintains constant the force or pressure.</p> <p>Hydraulic junction which ensures equal pressures (and forces if all tubes are of the same area) at all the terminals connected by the tree of tubes.</p> <p>Tube connector which is held stationary and it is filled with ideal fluid free from mass, viscosity or compressibility.</p>	 <p>“Sky” an ideal dynamic reference which maintains constant the force or pressure.</p> <p>Hydraulic junction which ensures equal pressures (and forces if all tubes are of the same area) at all the terminals connected by the tree of tubes.</p> <p>Tube connector which is held stationary and it is filled with ideal fluid free from mass, viscosity or compressibility.</p>
	DUAL OF	
<p>Z_k = impedance of the spring</p>	$Z_{eq} = Z_m + Z_d + Z_k$	$Z_{eq} = \frac{1}{1/Z_m + 1/Z_d + 1/Z_k}$
<p>Mobility schematic or wiring diagram from the</p> <p>mobility method</p> <p>Y_{eq} = total mobility</p> <p>Y_m = mobility of the mass</p> <p>Y_d = mobility of the dissipator</p> <p>Y_k = mobility of the spring</p>	 <p>“Earth” an ideal kinematic reference which uses an inertial system as the reference for velocity or volume velocity.</p> <p>Rigid junction which ensures equal velocities of all the terminals connected by the tree of rods.</p> <p>Rod connector which is a rigid element free from mass, friction or compliance.</p>	 <p>“Earth” an ideal kinematic reference which uses an inertial system as the reference for velocity or volume velocity.</p> <p>Rigid junction which ensures equal velocities of all the terminals connected by the tree of rods.</p> <p>Rod connector which is a rigid element free from mass, friction or compliance.</p>
	DUAL OF	
	$Y_{eq} = \frac{1}{1/Y_m + 1/Y_d + 1/Y_k}$	$Y_{eq} = Y_m + Y_d + Y_k$

Vibration Handbook” edited by Harris and Crede, entitled “Mechanical Impedance”, by E. L. Hixson [71]. A detailed description of mobility and impedance concepts of purely mechanical systems consisting of lumped elements is given. The schematic representation of the systems are derived using only the mobility approach even though both impedance and mobility formulae for the solutions of the systems are described.

5. IMPEDANCE, MOBILITY AND TRANSMISSION MATRIX METHODS FOR FLEXIBLE, DISTRIBUTED SYSTEMS

One of the first vibration problems studied using electromechanical analogies or impedance/mobility methods was vibration isolation. This type of problem consists of a vibration source connected to a receiving structure, via a mounting system. Simple impedance or mobility networks consisting of lumped bilateral elements (mass, spring and damping elements) allow the formulation of basic vibration isolation theory. However, three important features of isolator systems cannot be investigated: first, the distributed and flexible nature of the elements composing the whole systems [72–76]; second, the effects due to a multiple mounting isolator or multiple connecting system [77, 78]; and third the multiple degrees-of-freedom vibration transmission at connecting points [78, 79]. Thus, there was the need to extend the impedance/mobility approach to networks that included multiple terminal elements. Each terminal should represent a specific component of the kinematic (linear or angular velocity) or dynamic parameters (force or moment excitation). This was possible using a mechanical network theory where each element had $2 \times (6 \times n)$ terminals, where 6 was the number of allowable degrees of freedom at a junction, n was the number of element connection points and 2 takes into account both kinematic and dynamic parameters.

To solve problems of this type, mechanical engineers once more referred to work previously carried out by electrical engineers. They adapted the transmission matrix method developed for the study of electrical lines. This approach used block diagrams, where complicated electric circuits were simplified by means of single entities called “black boxes”. These entities were characterized by relatively simple algebraic equations relating the input and output e.m.f. and current. Initial studies considered only a single pair of input and output terminals in which case, the block diagram was called “a four-pole network”, which was linked to Campbell’s study on electrical filters. In his paper “Cissoidal Oscillations”, Campbell [11] showed that passive four-pole networks are equivalent to T or Π circuits. In this way he established the link between the classical network theory and the four-pole networks. From this first study the theory of electric networks including four pole networks evolved. Several papers and books were published in the period 1920s–1950s [12, 80–87].

The first study of a mechanical system by means of four-pole networks was probably carried out by Mason [37], for an acoustical filter, and by MacLean [88] for an electromechanical system. Many other studies on specific problems followed these first examples of mechanical four-pole network studies [89]. However, it was some time before a four-pole theory was systematically developed for purely mechanical systems. Molloy [90, 91] presented two similar papers where he gave definitions of four-pole mechanical elements and presented four-pole matrices of lumped elements (mass, spring, damper) and some distributed elements (cantilever beam, rubber in shear). He also presented the connecting rules for systems with series and parallel elements and considered both force and velocity generators. The study of four-pole networks for mechanical systems followed the same course of mechanical network studies. In the beginning, mechanical systems were converted into equivalent electrical networks using either the direct or inverse analogy. Later, however, mechanical systems were drawn in a way similar to the impedance and mobility methods [71, 90–92].

The mechanical four-pole network theory made analysis easier and in some cases made the study of systems composed of distributed elements possible. Exact solutions for the flexible elements was facilitated by the use of T or Π circuits where the input and output impedances and the velocity and forces ratios between the two terminals were derived

analytically [71, 75]. When engineers began to study vibration isolation systems with multiple mounts they found that to obtain an accurate analysis, multiple-degrees-of-freedom models were required. In many cases, all six kinematic and dynamic parameters at the connecting points were needed so that coupling between different types of vibration (flexural, longitudinal, shear, torsional vibrations) could be taken into account. Neither the T or Π impedance/mobility networks nor the four-pole networks were suitable for modelling systems with multiple input and output parameters, thus the impedance/mobility and four-pole network theories were developed so that multiple-degrees-of-freedom systems could be studied. In place of bilateral or four-pole elements, black box elements with multiple terminals were introduced [93–95]. These block elements were described in terms of matrix relations. O’Hara defined mobility as: “a tensor (or a tensor component) which operationally describes the effects upon the resultant velocity (or several velocities) of the application of a force or an array of forces”. He also defined the impedance as: “a tensor (or a tensor component) which operationally describes the effects upon the resultant force (or several forces) of the application of a velocity or an array of velocities”. The diagonal terms of these tensors were defined as the driving point mobilities and driving point impedances respectively. The off-diagonal terms were called transfer or cross mobilities and impedances respectively. Rubin [93, 94] extended the four-pole theory to a multiple terminal approach for mechanical systems, where the input and output variables were related by a transmission matrix. Rubin also presented the relationship between mobility, impedance and transmission matrices. It is important to note that both O’Hara and Rubin worked out their formulations directly in terms of mechanical variables rather than in terms of analogue electrical parameters since, at that time, the ideas of mobility and impedance methods were widely accepted by the scientific community.

The multiple terminal block diagram approach allowed the possibility of analyzing complicated networks by means of the input and output parameters of each element only. As a consequence, papers began to be published where mobility, impedance or transmission matrix elements were given using either measured data or analytical formulae. In particular, mobility, impedance or transmission functions were derived for simple structures as rods, shafts, beams, plates and cylindrical shells. A detailed list of references relative to this type of work is given in reference [65]. Recent work on applications has involved complex rotor dynamics systems [96] and structural–acoustic coupled systems [97, 98].

6. RECENT RESEARCH

Fundamental research into mobility and impedance methods is still underway, particularly with reference to the concentrated excitation of structures, for example references [99, 100], and line or surface excitations for the so-called “line” or “strip” mobilities, for example references [101–104]. An up-to-date bibliography of recent work is contained in reference [65].

Regrettably, there are few books dealing with the matrix representation of impedance/mobility and transmission theory. The most complete work is probably by Neubert [105]. Bishop and Johnson’s book [106] gives the driving point and transfer mobilities/impedances for one-dimensional flexible systems for longitudinal, torsional and flexural vibration. A general modal formulation for the calculation of plate and shell mobilities, longitudinal, shear and out-of-plane flexural vibration is presented in Soedel’s book [107]. Finally, Cremer *et al.* [108] give mobility or impedance formulae for one- and bi-dimensional distributed elements of infinite extent. A good introduction to transmission matrix theory is given in books by Pestel and Leckie [109] and Hatter [110].

7. CONCLUDING REMARKS

This paper gives an account of the conception and evolution of *mobility* and *impedance* methods in structural dynamics. The large number of papers and reports reviewed have indicated that the origin of the mechanical impedance concept was inspired by the work done by scientists at the end of the 19th century for electrical systems. Starting from the concept of electric impedance, Professor Arthur G. Webster first defined the mechanical impedance during a meeting of the Physical Society held in Philadelphia in 1914. Since that time the well-established electrical network theory has been used to convert mechanical systems into their electrical analogues. Two theories arose which were the direct and the inverse electromechanical analogies. In 1938, Firestone presented a paper describing an *ad hoc* network theory for mechanical systems. This was a major step forward in the formulation of a new network theory that is now a days known as mobility-impedance approach in structural dynamics. Firestone's theory has been completed by extending the mobility and impedance relations to multi-terminal structural elements using mobility, impedance and transmission matrices.

At present, there is still research in progress for the definition and characterization of mobility/impedance parameters of distributed flexible mechanical systems. In particular, much work is being carried out for the analysis of systems composed by one- or two-dimensional flexible systems (beams, plates and shells) connected along line or surface junctions.

REFERENCES

1. F. V. HUNT 1982 *Electroacoustics. The Analysis of Transduction, and its Historical Background*. New York: Published by the American Institute of Physics for the Acoustical Society of America; second edition.
2. W. P. MASON 1940 *Bell System Technical Journal* **20**, 405–414. Electrical and mechanical analogies.
3. A. E. KENNELLY 1923 *Electrical Vibration Instruments*. New York: Macmillan.
4. O. HEAVISIDE 1892 *Electrical Papers*, Vols. I, II. London: Macmillan.
5. G. R. KIRCHHOFF 1891 in *Vorlesungen über mathematische Physik*, Vol. III *Electricität und magnetismus* (M. Plank, editor). Leipzig: B.G. Teubner.
6. M. L. THÉVENIN 1883 *Rendus hebdomadaires de séances de l'Academie des sciences*, Vol. XCVII, 159–161. Paris: Gauthier-Villars.
7. D. F. TUTTLE, Jr 1958 *Network Synthesis*, Vol. I, 19–20. New York: John Wiley & Sons, Inc.
8. E. A. GUILLEMIN 1931 *Communication Networks*, Vol. I. New York: John Wiley & Sons, Inc.
9. E. A. GUILLEMIN 1935 *Communication Networks*, Vol. II. New York: John Wiley & Sons, Inc.
10. E. A. GUILLEMIN 1953 *Introductory Circuit Theory*. New York: McGraw-Hill.
11. G. A. CAMPBELL 1903 *Philosophical Magazine* **5** (Sixth series), 313–330. On loaded lines in telephonic transmission.
12. G. A. CAMPBELL 1922 *The Bell System Technical Journal* **1**, 1–32. Physical theory of the electric wave-filter.
13. A. G. WEBSTER 1919 *Proceedings of the National Academy of Sciences (Washington)* **5**, 275–282. Acoustical impedance and theory of horns and of the phonograph.
14. P. LE CORBEILLER and Y. W. YEUNG 1952 *Journal of the Acoustical Society of America* **24**, 643–648. Duality in mechanics.
15. W. HÄHNEMANN and H. HECHT 1919 *Physikalische Zeitschrift* **20**, 104–114. Schallgeber und Schallempfänger I.
16. W. HÄHNEMANN and H. HECHT 1919 *Physikalische Zeitschrift* **20**, 245–251. Schallgeber und Schallempfänger II.
17. W. HÄHNEMANN and H. HECHT 1920 *Physikalische Zeitschrift* **21**, 104–114. Schallgeber und Schallempfänger III.
18. R. L. Wegel 1921 *Journal of the American Institute of Electrical Engineers* **40**, 791–802. Theory of magneto-mechanical systems as applied to telephone receivers and similar structures.

19. J.-B. POMEY 1921 *Analogies Mécaniques de l'Électricité*. Paris: Gauthier-Villars Et Cie, Éditeurs.
20. H. LICHTÉ 1926 *E.N.T.* **5**, 183–189. Ein Beitrag zur theorie des elektromagnetischen telephons.
21. J. P. MAXFIELD and H. C. HARRISON 1925 *U.S.A. Patent No.* 1,535,538, filed 3 January 1923, issued 28 April 1925. Vibratory system.
22. E. L. NORTON 1928 *U.S.A. Patent No.* 1,681,554, filed 24 November 1924, issued 21 August 1928. Wave filter.
23. C. R. HANNA 1928 *Transactions of the American Institute of Electrical Engineers* **47**, 607–616. Loudspeakers of high efficiency and load capacity.
24. R. FRANKE 1930 *EineVergleichende Schalt und Getriebelehre*. München: R. Oldenbourg.
25. M. HARRISON 1929 *U.S.A. Patent No.* 1,730,425, filed 11 October 1927, issued 8 October 1929. Acoustic device.
26. C. A. NICKLE 1925 *Transactions AIEE* **44**, 844–856. Oscillographic solution of electromechanical systems.
27. J. P. MAXFIELD and H. C. HARRISON 1926 *Transaction of the American Institute Electrical Engineers* **45**, 334–348. Methods of high quality recording and reproducing of music and speech based on telephone research. (also in *Bell System Technical Journal* **5**, 493–523).
28. R. J. HERBERT 1932 *Review of Scientific Instruments* **3**, 287. A mechanical analogy for coupled electrical circuits.
29. M. PAWLEY 1937 *Franklin Institute Journal* **223**, 179–198. The design of a mechanical analogy for the general linear electrical network with lumped parameters.
30. W. P. MASON 1942 *Electromechanical Transducers and Wave Filters*. London: Van Nostrand Reinhold Company.
31. H. F. OLSON 1943 *Dynamical Analogies*. State College, PA: Jostens Printing and Publishing Company.
32. A. BLOCH 1945 *Journal of the Institution of Electrical Engineers* **92**, 157–169. Electromechanical analogies and their use for the analysis of mechanical and electromechanical systems.
33. W. HERSHEL 1912 in *The Scientific Papers of Sir William Herschel*. London: Royal Society and the Royal Astronomical Society.
34. G. W. STEWART 1922 *Physical Review* **20**, 528–551. Acoustic wave filters.
35. G. W. STEWART 1924 *Physical Review* **23**, 520–524. Acoustic wave filters: attenuation and phase factors.
36. G. W. STEWART 1925 *Physical Review* **25**, 90–98. Acoustic wave filters; an extension of the theory.
37. W. P. MASON 1927 *Bell System Technical Journal* **6**, 258–294. A study of the rectangular combination of acoustic elements, with applications to recurrent acoustics filters, tapered acoustic filters, and horns.
38. W. P. MASON 1934 *Bell System Technical Journal* **13**, 405–452. Electrical wave filters employing quartz crystals as elements.
39. W. G. CADY 1929 *U.S.A. Patent No.* 1,450,246, filed 28 January 1920, issued 3 April 1923. Reissued Re. 17,358, reissue filed 6 August 1925, reissued 2 July 1929.
40. L. ESPENSCHIED 1931 *U.S.A. Patent No.* 1,795,204, filed 3 January 1927, issued 3 March 1931. Electrical wave filter.
41. R. B. LINDSAY and F. E. WHITE 1932 *Journal of the Acoustical Society of America* **2**, 155–168. The theory of acoustic filtration in solid rods.
42. R. B. LINDSAY 1934 *Journal of the Acoustical Society of America* **5**, 196–201. Filtration of elastic waves in solid rods.
43. R. B. LINDSAY 1938 *Journal of Applied Physics* **9**, 612. The filtration of sound, I.
44. R. B. LINDSAY 1939 *Journal of Applied Physics* **10**, 620. The filtration of sound, II.
45. R. B. LINDSAY, C. R. LEWIS and R. D. ALBRIGHT 1934 *Journal of the Acoustical Society of America* **5**, 202–205. Acoustic filtration in non-homogeneous media.
46. E. LAKATOS 1939 *U.S.A. Patent No.* 2,166,359, filed 30 March 1937, issued 18 July 1939. Magnetostrictive device.
47. P. LE CORBEILLER 1929 *Annales des Postes Télégraphes* **18**, 1–22. Origine des terms gyroscopiques.
48. F. A. FIRESTONE 1933 *Journal of the Acoustical Society of America* **4**, 249–267. A new analogy between mechanical and electrical systems.
49. M. DARRIEUS 1929 *Bulletin de La Societe Francaise d'electriciens* **96**, 794–809. Les modèles mécaniques en électrotechnique leur application aux problèmes de stabilité.
50. W. HÄHNLE 1932 *Wiss Verloff and Siemens-Kohzerh* **11**, 1–23. Die darstellung elektromechanischer gebilde durch rein elektrische schaltbilder.

51. H. M. TRENT 1952 *Transactions of the American Society of Mechanical Engineers Journal of Applied Mechanics* **74**, 147–150. An alternative formulation of the laws of mechanics.
52. L. L. BERANEK 1954 *Acoustics*. New York: McGraw-Hill; Chapter 3. Electro-mechano-acoustical circuits.
53. H. E. KOENIG and W. A. BLACKWELL 1961 *Electromechanical System Theory*. New York: McGraw-Hill.
54. W. C. HURTY and M. F. RUBINSTEIN 1964 *Dynamics of Structures*. Englewood Cliffs, NJ: Prentice-Hall, Inc.
55. E. SKUDRZYK 1968 *Simple and Complex Vibratory Systems*. London: The Pennsylvania State University Press.
56. J. MILES 1943 *Journal of the Acoustical Society of America* **14**, 183–192. Applications and limitations of mechanical–electrical analogies, new and old.
57. F. A. FIRESTONE 1938 *Journal of Applied Physics* **9**, 373–387. The mobility method of computing the vibration of linear mechanical and acoustical systems: mechanical–electrical analogies.
58. B. C. CARTER 1936 *Reports and Memoranda of the Aeronautical Research Committee, No. 1758*. The vibration of airscrew blades with particular reference to the response to harmonic torque impulses in the drive.
59. W. J. DUNCAN 1941 *Philosophical Magazine, Series 7* **32**, 401–409. The admittance method for obtaining the natural frequencies of systems.
60. W. J. DUNCAN 1943 *Philosophical Magazine, Series 7* **34**, 49–63. Free and forced oscillations of continuous beams.
61. M. A. BIOT 1940 *Journal of the Aeronautical Sciences* **7**, 107–112. Vibration of crankshaft–propeller systems. New method of calculation.
62. M. A. BIOT 1940 *Journal of the Aeronautical Sciences* **7**, 376–382. Coupled oscillations of aircraft engine–propeller systems.
63. R. G. MANLEY 1941 *Journal of the Royal Aeronautical Society* **45**, 342–348. The mechanical impedance of damped vibrating systems.
64. R. G. MANLEY 1943 *Journal of the Royal Aeronautical Society* **47**, 22–26. The electro-mechanical analogy in oscillation theory.
65. P. GARDONIO and M. J. BRENNAN 2000 *ISVR Technical Report No. 289, University of Southampton*. Mobility and impedance methods in structural dynamics: an historical review.
66. F. A. FIRESTONE 1956 *Journal of the Acoustical Society of America* **28**, 1117–1153. Twixt earth and sky with road and tube; the mobility and classical impedance analogies.
67. B. B. BAUER 1953 *Journal of the Acoustical Society of America* **25**, 837–840. Transformer couplings for equivalent network synthesis.
68. R. PLUNKET 1958 (editor). *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations. Presented at the ASME Annual Meeting, New York, December 2*. The American Society of Mechanical Engineers.
69. S. H. CRANDALL 1958 in *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations* (R. Plunkett, editor), 5–18. *Presented at the ASME Annual Meeting, New York, December 2*. The American Society of Mechanical Engineers. Impedance and mobility analysis of lumped parameters systems.
70. R. O. BELSHEIM 1962 (editor) *Mechanical Impedance. Collection of Papers Presented at the 30th Symposium on Shock, Vibration and Associated Environments, Statler-Hilton Hotel, Detroit, MI, October 10–12, 1961. Shock and Vibration Bulletin* **30**.
71. E. L. HIXSON 1961 in *Shock and Vibration Handbook* (C. M. Harris and C. E. Crede, editors). New York: McGraw-Hill; Chapter 10. Mechanical impedance.
72. M. HARRISON, A. O. SYKES and M. MARTIN 1952 *Journal of the Acoustical Society of America* **24**, 62–71. Wave effects in isolation mounts.
73. E. J. SKUDRZYK 1958 *Journal of the Acoustical Society of America* **30**, 1140–1152. Vibrations of a system with a finite or an infinite number of resonances.
74. E. J. SKUDRZYK 1959 *Journal of the Acoustical Society of America* **31**, 68–74. Theory of noise and vibration insulation of a system with many resonances.
75. D. V. WRIGHT 1958 in *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations* (R. Plunkett, editor), 19–42. *Presented at the ASME Annual Meeting, New York, December 2*. The American Society of Mechanical Engineers. Impedance analysis of distributed mechanical systems.

76. J. E. RUZICKA and R. D. CAVANAUGH 1958 in *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations* (R. Plunkett, editor), 109–124. Presented at the ASME Annual Meeting, New York, December 2. The American Society of Mechanical Engineers. Vibration isolation of non-rigid bodies.
77. J. I. SOLIMAN and M. G. HALLAM 1968 *Journal of Sound and Vibration* **8**, 329–351. Vibration isolation between non-rigid machines and non-rigid foundations.
78. O. SYKES 1971 in *Synthesis of Vibrating Systems* (R. Plunkett, editor), 22–37. Presented at the Winter ASME Annual Meeting, Washington DC, November 30. The American Society of Mechanical Engineers. Application of admittance and impedance concepts in the synthesis of vibrating systems.
79. V. H. NEUBERT and W. H. EZELL 1958 in *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations* (R. Plunkett, editor), 77–86. Presented at the ASME Annual Meeting, New York, December 2. The American Society of Mechanical Engineers. Dynamic behaviour of a foundation-like structure.
80. P. LE CORBEILLER and CH LANGE 1923 *L'Onde Electrique* **ii**, 560–570. Etude sur les lignes en T. dissymetriques et application aux filters de bandes.
81. F. STRECKER and R. FELDTKELLER 1929 *Electrische Nachrichten-Technik* **6**, 93–103. GrundLagen der theorie des allgemeinen vierpoles.
82. L. BRILLOUIN 1936 *Revue Générale de l'electricite* **4**, 3–16. Les filtres electriques et le theorie des matrices.
83. S. DARLINGTON 1939 *Journal of Mathematical Physics* **18**, 257. Synthesis of reactance four-poles which produce prescribed insertion loss characteristics.
84. L. A. PIPES 1940 *Philosophical Magazine* **30**, 370–395. The matrix theory of four-terminal networks.
85. L. C. PETERSON 1948 *The Bell System Technical Journal* **27**, 593–622. Equivalent circuits of linear active four-terminal networks.
86. M. B. REED 1955 *Electrical Network Synthesis*. Englewood Cliffs, NJ: Prentice-Hall.
87. E. WEBER 1956 *Linear Transien Analysis, II*. New York: John Wiley and Sons.
88. N. W. MACLEAN 1940 *Journal of the Acoustical Society of America* **12**, 140–146. Absolute measurement of sound without a primary standard.
89. L. C. PETERSON and B. P. BOGERT 1950 *Journal of the Acoustical Society of America* **22**, 369–381. A dynamical theory of the cochlea.
90. C. T. MOLLOY 1957 *Journal of the Acoustical Society of America* **29**, 842–853. Use of four-pole parameters in vibration calculations.
91. C. T. MOLLOY 1958 in *Colloquium on Mechanical Impedance Methods for Mechanical Vibrations* (R. Plunkett, editor), 43–68. Presented at the ASME Annual Meeting, New York, December 2. The American Society of Mechanical Engineers. Four pole parameters in vibration analysis.
92. J. C. SNOWDON 1971 *Journal of Sound and Vibration*, **15**, 307–323. Mechanical four-pole parameters and their application.
93. S. RUBIN 1964 *Transactions of the American Society of Mechanical Engineers. Journal of Engineering for Industry* **86**, 9–21. Transmission matrices for vibration and their relation to admittance and impedance.
94. S. RUBIN 1967 *Journal of the Acoustical Society of America* **41**, 1171–1179. Mechanical immitance- and transmission-matrix concepts.
95. G. J. O'HARA 1966 *Journal of the Acoustical Society of America* **41**, 1180–1184. Mechanical impedance and mobility concepts.
96. P. BONELLO and M. J. BRENNAN 2001 *Journal of Sound and Vibration*, **239**, 445–466. Modelling the dynamic behaviour of a supercritical rotor on a flexible foundation using the mechanical impedance technique.
97. P. GARDONIO and S. J. ELLIOTT 1999 *Journal of Aircraft*, **36**, 1023–1032. Active control of structure-borne and air-borne sound transmission through double pannels.
98. S. M. KIM and M. J. BRENNAN 1999 *Journal of Sound and Vibration* **223**, 97–113. A compact matrix formulation using the impedance and mobility approach for the analysis of structural-acoustic systems.
99. B. A. T. PETERSSON and M. HECKL 1996 *Journal of Sound and Vibration* **196**, 295–321. Concentrated excitation of structures.
100. B. A. T. PETERSSON 1999 *Journal of Sound and Vibration* **224**, 243–266. Concentrated force excitation of deep beams.
101. B. A. T. PETERSSON 1983 *Journal of Sound and Vibration* **91**, 219–238. An approximation for the point mobility at the intersection of two perpendicular plates.

102. B. A. T. PETERSSON 1986 *Journal of Sound and Vibration* **108**, 471–485. A thin-plate model for the moment mobility at the intersection of two perpendicular plates.
103. B. A. T. PETERSSON 1997 *Journal of Sound and Vibration* **202**, 511–537. Geometrical and spatial effects on effective mobilities of annular interfaces.
104. K. H. HERON 1999 in *IUTAM Symposium on Statistical Energy Analysis* (F. J. Fahy and W. G. Price, editors), 107–118. London: Kluwer Academic Publisher. Predictive SEA using line wave impedances.
105. V. H. NEUBERT, 1987 *Mechanical Impedance: Modelling/analysis of Structures*. State College, PA: Jostens Printing and Publishing Company.
106. R. E. D. BISHOP and D. C. JOHNSON 1960 *The Mechanics of Vibration*. Cambridge: University Press Cambridge.
107. W. SOEDEL 1993 *Vibrations of Shells and Plates*. New York: Marcel Dekker Inc; second edition.
108. L. CREMER, M. HECKL and E. E. UNGAR 1988 *Structure-Borne Sound*. Berlin: Springer-Verlag second edition.
109. E. C. PESTEL and F. A. LECKIE 1963. *Matrix Methods in Elastomechanics*. New York: McGraw-Hill Book Company, Inc.
110. D. J. HATTER 1973 *Matrix Computer Methods of Vibration Analysis*. London: Butterworth & Co. Ltd.