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DYNAMIC ANALYSIS OF AN ACTIVE FLEXIBLE SUSPENSION SYSTEM

Y. G. Sun, K. J. Song and Y. H. Mao

College of Mechanical Engineering, Shandong University, Jinan 250061, *People's Republic of China. E-mail*: *yuguo_sun@263.net*

AND

L. Lin

System Engineering Research Institute, Beijing 100036, *People's Republic of China*

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1. INTRODUCTION

The active isolation of vibration from a motor to a flexible supporting structure has received much attention in recent years [1–8]. According to Pan and Gardonio's reports [1–4], feedforward control strategy provides superior performance when the source of vibration is deterministic. In this paper, the effect of moving isolator locations on the feedforward control strategy is examined theoretically and numerically. The numerical results show that the low-frequency disturbances and certain high-frequency disturbances can be isolated more effectively by moving isolator locations, in the case where the active controller is unalterable.

2. DESCRIPTION OF THE SYSTEM

Figure 1 shows the active flexible suspension system to be studied. A motor is supported on a flexible base through n active mounts. The motor is assumed to be excited by harmonic vertical force (F_0) and moment (T_0), $\mathbf{F}_e = [F_0 \; T_0]^{\mathrm{T}}$. The actuators are parallel with the passive isolators, and generate only axial control forces, the control force vector is denoted as $\mathbf{U} = [u_1 \; u_2 \; \cdots \; u_n]^{\mathrm{T}}$. The force vector and velocity vector on the interface between the motor and the mounts can be written as $\mathbf{F}_t$, $\mathbf{V}_t$. Similarly, the force vector and velocity vector on the junction between the base and the mounts are defined as $\mathbf{F}_b$, $\mathbf{V}_b$. The dynamics of the motor can be described in its velocity mobility matrix form as

$$\begin{bmatrix} \mathbf{V}_e \\ \mathbf{V}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}_e \\ \mathbf{F}_t \end{bmatrix}. \tag{1}$$

The dynamic behavior of the active mounts are defined by

$$\mathbf{F}_t = \mathbf{B}(\mathbf{V}_t - \mathbf{V}_b) + \mathbf{U}, \qquad \mathbf{F}_b = \mathbf{F}_t, \tag{2, 3}$$

$$\mathbf{B} = \mathrm{diag}\left(-\frac{\bar{k}_1}{\mathrm{i}\omega} - \frac{\bar{k}_2}{i\omega} \cdots -\frac{\bar{k}_n}{\mathrm{i}\omega} \right). \tag{4}$$

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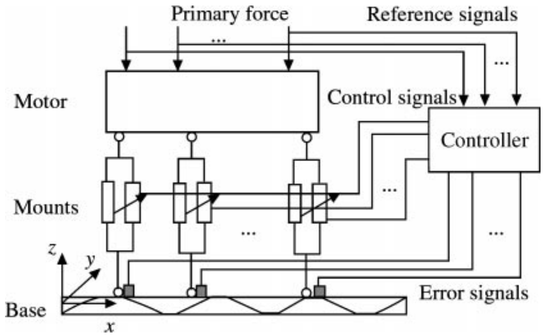

Figure 1. The active flexible suspension system.

Here, k_i is the complex stiffness of the passive isolator. The dynamics of the flexible foundation can be expressed as

$$\mathbf{V}_b = \mathbf{CF}_b. \tag{5}$$

Here, **C** is the velocity mobility matrix of the flexible foundation.

The time-average power flow transmitted into the flexible base is

$$Qtr = 0{\cdot}5\ real\,(\mathbf{F}_b^{\mathrm{H}}\,\mathbf{V}_b). \tag{6}$$

Here, the superscript H denotes the Hermitian of the vector. We now choose the error signal to be the sum of the squared force transmitted to the base defined by

$$J = \mathbf{F}_b^{\mathrm{H}}\,Q\mathbf{F}_b + \mathbf{U}^{\mathrm{H}}\,R\mathbf{U}. \tag{7}$$

Here, Q and R are positive-definite symmetric weighting matrices. Substituting equations (1)–(3) and (5) into equation (6), J can also be written in standard Hermitian quadratic format as

$$J = \mathbf{U}^{\mathrm{H}}\mathbf{AU} + \mathbf{U}^{\mathrm{H}}b + b^{\mathrm{H}}\mathbf{U} + c. \tag{8}$$

Here,

$$A = \Omega^{\mathrm{H}}Q\Omega + R, \qquad b = \Omega^{\mathrm{H}}Q\Omega BA_{21}F_e, \tag{9, 10}$$

$$\Omega = [\mathbf{I}_{n\times n} + \mathbf{B}(\mathbf{C} - \mathbf{A}_{22})]^{-1}. \tag{11}$$

The optimal control force vector is [5]

$$U = -A^{-1}b. \tag{12}$$

3. DYNAMIC ANALYSIS

In this paper, we consider isolation of the vibration of a motor supported on a simply supported rectangle plate by a pair of active mounts, and only the vibration in the X–Z plane is examined. According to the general isolation design principle, the mounts are arranged symmetrically to the motor's gravity center. The performance of the active suspension system is evaluated in terms of power flow.

The weighting matrix: $Q = \mathrm{diag}(10, 10)$, $R = \mathrm{diag}(1\ 1)$. The mass and moment of inertia of the motor are 8·388 kg and 0·07 kg m^2 respectively. The stiffness and loss factor of the

Table 1

The natural frequency of the coupled system (units: Hz) (mount frequencies: 15·54 Hz, 17·03 Hz)

Mod. no.	1	2	3	4	5	6	7
Flexible base	18·50	37·00	55·50	67·84	74·00	104·84	111·02
Coupled system	12·05	16·70	23·02	37·93	55·94	68·34	74·15

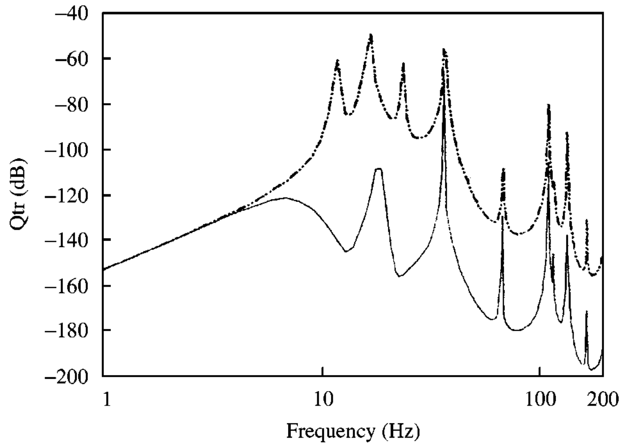

Figure 2. Power flow transmitted to the flexible base: (–··–··–··–), passive control; (---), active control.

passive isolator are 4×10^4 N/m and 0·05 respectively. The density, Young's modulus, Poisson's ratio and loss factor of the plate ($1{\cdot}4\,\mathrm{m} \times 1\,\mathrm{m} \times 0{\cdot}005\,\mathrm{m}$) are $7{\cdot}7 \times 10^3$ kg/m^3, $1{\cdot}95 \times 10^{11}$ N/m, 0·28 and 0·01 respectively.

As is shown in Table 1, When the motor's mount frequencies are close or over the first mode frequency of the supporting structure, the motor–sprung subsystem and the flexible base will constitute a strong dynamic coupled system. The bounce motion of the motor is coupled with the first mode of the base obviously (coupled frequencies: 12·05 and 23·02 Hz).

As is shown in Figure 2, in the passive isolation system, at low frequencies, the motor's bounce and pitch motions transmit most of the vibration energy. At high frequencies, power flow transmission is mostly attributed to the flexible base modes. Using feedfoward control strategy, the bounce motion of the motor and the first mode of the base are controlled effectively, but the pitch motion of the motor and the plate even modes (37·9, 68·3, 104·9 Hz) excited by the moment (T_0) still transmit considerable power after active control. The reason is that the actuators generate only vertical secondary force, and these coupled modes are uncontrolled. For example, the fourth order coupled mode (37·97 Hz, corresponding to the foundation's second mode) is antisymmetric to the $X = a/2$ in the plate's local co-ordinate system (X, Y, Z), the secondary forces acting points ($\sigma_1 = (0{\cdot}6\,\mathrm{m}, 0{\cdot}5\,\mathrm{m})$ and $\sigma_2 = (0{\cdot}8\,\mathrm{m}, 0{\cdot}5\,\mathrm{m})$) are symmetrical to the $X = a/2$, and this order mode's mode control force is zero.

If the dominant disturbance frequency is around this order mode frequency (37·97 Hz), we may change the isolator locations on the base to improve the effectiveness of active control as the dashed line in Figure 3 shows. The isolator locations ($\sigma_1 = (0{\cdot}7\,\mathrm{m}, 0{\cdot}4\,\mathrm{m})$ and

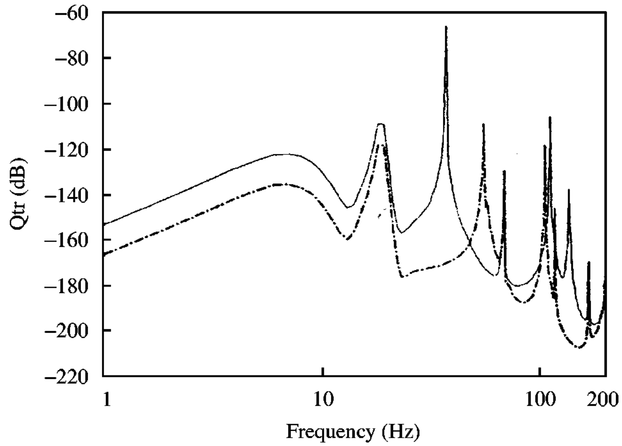

Figure 3. The effect of moving the isolator locations.

$\sigma_2 = (0{\cdot}7\,\mathrm{m}, 0{\cdot}6\,\mathrm{m})$) are now on the nodal line ($X = a/2 = 0{\cdot}7\,\mathrm{m}$) of the second mode shape of the flexible base; then the peak of power flow transmitted to the foundation around this order mode frequency disappears. From Figure 3, it can also be seen that the active control effectiveness at low-frequency bands is improved evidently after modifying the isolator locations: at 1 Hz, the power flow transmitted to the base is attenuated about 10 dB. This is because the bending stiffness of the foundation in the $X = a/2$ direction is larger than that in the $Y = b/2$ direction.

Our studies also indicate that the modulus of the secondary force is not sensitive to the isolator locations in this model, if the weighting matrix Q and R in equation (6) are not changed.

4. CONCLUSIONS

Based on the computer simulation above, the following conclusions can be drawn: (1) The isolators should be located in the direction where the base's stiffness is larger, which is important to enhance the isolation performance of the low-frequency disturbances. (2) The pitch mode of the motor and some antisymmetric modes of the flexible base excited by the moment still transmit considerable power after active control of vertical forces. (3) If $R = 0$ in equation (6), no power should then be transmitted to the flexible base in this single-axis model, but, the secondary force will be a very large value at low frequencies under this situation.

ACKNOWLEDGMENTS

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