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A SYNTHETIC ANALYSIS ON DESIGN OF OPTIMUM CONTROL FOR AN OPTIMIZED INTELLIGENT STRUCTURE

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A hybrid optimization algorithm is developed to minimize the control energy and the structural weight under the constraints of the structural dynamic property requirements. A 72-bar space truss with two piezoelectric actuators is used to illustrate the complete process of this algorithm. It is shown that the control energy and the structural weight are clearly reduced using the proposed method.

1. INTRODUCTION

Among the modern structures of huge space vehicles and aircraft, the truss is one of the most commonly used structures. In the design of such structures, two essential requirements should be met. The first is that they must possess excellent dynamic behaviors to ensure the safety and stability of the structures, and be able to keep the instruments and equipment they carry in good condition. The second is that the structural weight must be as light as possible in order to reduce the cost of launching and to increase the payload. However, these two requirements are often contradictory.

With the development of modern aerospace techniques, structural dynamic characteristics play an increasingly important role. For example, in order to catch a 1–10 m^2 target on the ground by electronic and optical microscope or laser–radar set on the observation satellite, the orientation and stability errors of the supporting shelf should not exceed 10^{-4} rad [1]. On the other hand, with the increment of structural weight, the cost of launching a space vehicle increases rapidly. Therefore, improvement of the dynamic behavior and decrement of the structural weight are very important for space vehicles.

Recently, in order to achieve better dynamic properties of the structure, great attention has been paid to the adaptive control of structural vibration using intelligent structures [2]. However, in order to realize the adaptive control of structural vibration, the weight of the structure will increase due to the additional hardware, such as piezoelectric actuators and electronic components. Generally speaking, the structural weight will increase obviously along with the increase of the required control force. For example, a piezoelectric actuator made in Japan, which can be used to control the vehicle structural vibration, weighs up to 2 kg and its actuating force is about 1000 N [3]. Because piezoelectric elements consume less power and can be operated easily by electric field, they nowadays act as actuators in the vibration control of intelligent space vehicle structures. However, since mass density of the piezoelectric materials is large, the additional weight to the space structure is not negligible.

Many researchers have been studying the optimal vibration control and structural optimization for a long time. Jin and Schmit [4] presented a method of integrating the design space for structural/control system optimization problems in the case of linear state feedback control, using a variety of dynamic behavior constraints, such as closed-loop eigenvalues, peak transient displacements, and peak actuator forces. Taday and Minami [5] studied the weight minimization problem of a 3-D truss structure with constraints on the stresses and natural frequencies of specified modes, and where the truss members were optimized by sequential linear programming. Ou and Kikuchi [6] proposed an integrated design procedure composed of structural design, control design, and actuator locations design. They used an independent modal space control algorithm (IMSC) to reduce the dynamic response and to minimize the control force, while still keeping the same modal response for the controlled modes. Kim [7] developed a method of structure/control simultaneous optimization for the active vibration control of bridge towers, platforms, ocean vehicles, etc. The simultaneous design method is used to achieve optimal system performance using linear matrix inequality (LMI).

In the present paper, a powerful hybrid optimization approach is developed. The minimization of the control energy and the structural weight is dealt with simultaneously under the constraints of the structural dynamic properties. Only a few papers related to this subject have been reported so far.

2. THEORETICAL ANALYSIS OF HYBRID OPTIMIZATION

Assume that the design variables for structural weight minimization are the cross-sectional areas s_j of the intelligent truss structure, whose vibration can be controlled using piezoelectric actuators, and the range of variation of the areas $s_j (j = 1, 2, \ldots, N)$ is given by

$$s_{dj} < s_j < s_{uj}, \quad j = 1, 2, \ldots, N, \tag{1}$$

where s_{dj} and s_{uj} are the lower and upper bounds of the cross-sectional area of each bar, respectively, and N is the total number of bars composing the truss structure.

The equation of motion for a vibrating truss with control force can be modelled using finite element method as

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}_0\mathbf{u}, \\ &\mathbf{y} = \mathbf{C}_0\dot{\mathbf{q}}, \\ &\dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0, \mathbf{q}(0) = \mathbf{q}_0, \end{aligned} \tag{2}$$

where **M**, **K** and **D** are the mass matrix, the stiffness matrix and the damping matrix of the system, respectively; **u** is the vector of control force produced by the actuator made of piezoelectric bars; $\mathbf{B}_0$ is the configuration matrix of the input control force; $\mathbf{C}_0$ is the measuring matrix of node velocity; **q** is the vector of node displacement and the overdot represents the derivation with respect to time.

Here, only the excitation with non-zero initial conditions is considered, because the excitation on the space vehicle structure is mainly the transient pulse, and the behaviors of the responses caused by the transient pulse is similar to those caused by the excitation with non-zero initial conditions.

Suppose that the system possesses proportional damping, i.e.,

$$\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K},$$

where α and β are proportional damping coefficients. The matrices $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{D}$ are functions of the design variables s_j, i.e.,

$$\mathbf{M} = \mathbf{M}(s_j), \qquad \mathbf{D} = \mathbf{D}(s_j), \quad \mathbf{K} = \mathbf{K}(s_j).$$

The configuration matrix $\mathbf{B}_0$ and measuring matrix $\mathbf{C}_0$ are only related to the locations of the piezoelectric actuators and sensors, respectively, and are irrelevant to s_j.

In equation (2), let

$$\mathbf{x} = \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix}.$$

The state-space equations of the structure to be controlled can then be written as

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}\mathbf{u},$$

$$\mathbf{y} = [0 \ \ \mathbf{C}_0]\mathbf{x}, \qquad \mathbf{x}_0 = \begin{Bmatrix} \mathbf{q}_0 \\ \dot{\mathbf{q}}_0 \end{Bmatrix}. \tag{3}$$

Introducing the state feedback gives

$$\mathbf{u} = \mathbf{G}\mathbf{x}, \tag{3$'$}$$

and equation (3) can be rewritten as

$$\dot{\mathbf{x}} = \left\{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}\mathbf{G}\right\}\mathbf{x},$$

$$\mathbf{y} = [0 \ \ \mathbf{C}_0]\mathbf{x}, \qquad \mathbf{x}_0 = \begin{Bmatrix} \mathbf{q}_0 \\ \dot{\mathbf{q}}_0 \end{Bmatrix}. \tag{4}$$

When the structure is imposed by the ith initial condition x_{0i} ($i = 1, 2, \dots, n$), its state response vector is denoted by $\mathbf{x}(i, t)$. Supposing that at time t the required input control force vector is $\mathbf{u}(i, t)$, and the measured output vector is $\mathbf{y}(i, t)$, the constraint on structural dynamics through the whole dynamic response process can be written as

$$\sqrt{\sum_{i=1}^{n} \int_0^{\infty} [y_j(i, t)]^2 \, \mathrm{d}t} \leqslant Y_j, \quad j = 1, 2, \dots, n_y, \tag{5}$$

where $y_j(i, t)$ is the jth component of the vector $\mathbf{y}(i, t)$, and Y_j ($j = 1, 2, \dots, n_y$) is a restricted value of vibration response at measurement point j.

In order to meet the condition expressed by equation (5), the truss structure should possess enough stiffness. As an initial step for structure design, this can be realized by choosing the allowable upper bound of the cross-sectional area for each bar, i.e.,

$$s_j = s_{uj}, \quad j = 1, 2, \dots, N.$$

Since the selected s_j can make equation (5) valid, $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ in equation (4) can be determined. $\mathbf{B}_0$ and $\mathbf{C}_0$ are given by the locations of the actuators and sensors respectively.

If the structural parameters of a controllable system are invariable, the linear quadratic optimal control scheme can be used to obtain the optimal control input and the minimal structural vibration response simultaneously. However, generally speaking, the obtained structural system is not optimal, and its structural parameters can be sequentially optimized to minimize its weight. Thus, an iterative method must be adopted.

In the initial design of a controller, the energy of the control input is regarded as the object function

$$J = \sum_{i=1}^{n} \int_{0}^{\infty} \mathbf{u}^{\mathrm{T}}(i, t)\mathbf{R}\mathbf{u}(i, t)\,\mathrm{d}t, \tag{6}$$

subjected to the constraints of equations (4) and (5). In equation (6), $\mathbf{R}$ is the weighting matrix, and it can be determined according to the ability of the actuator. In order to minimize the objective function J, according to equation (3′), $\mathbf{u}$ in equation (6) is replaced by $\mathbf{Gx}$, the initial optimum matrix $\mathbf{G}$ of control gain can be obtained by solving equations (4) and (6) based on the optimum control theory. On the other hand, if equation (5) is regarded as the objective function, and equations (4) and (6) are regarded as the constraint conditions, it is also feasible.

The structural optimization aims at decreasing the cross-sectional area of the bars in truss within the allowable range of design variables denoted by equation (1), and still keeping equation (5) valid. This means that when a smaller cross-sectional area for the bars is chosen, if the dynamic response $\mathbf{x}(t)$ of the structure system still remains the same as before, equation (5) will be satisfied automatically, and as a result, the weight of the structure will definitely be decreased. Therefore, the required control energy may also be reduced. Consequently, this provides the possibility for further optimization.

Assume that the variation of the cross-sectional area of each bar is $\tilde{s}_j\,(j = 1, 2, \ldots, N)$; then the new cross-sectional area of each bar is

$$s_j = s_{uj} + \tilde{s}_j, \quad j = 1, 2, \ldots, N.$$

If the variation of the structural parameters matrices caused by $\tilde{s}_j = 1$ can be written as $\mathbf{M}_j$, $\mathbf{D}_j$ and $\mathbf{K}_j$, the structural parameter matrices of the new system will become

$$\begin{aligned} \tilde{\mathbf{M}}(s_j) &= \mathbf{M}(s_{uj}) + \sum_{j=1}^{N} \tilde{s}_j \mathbf{M}_j, \\ \tilde{\mathbf{D}}(s_j) &= \mathbf{D}(s_{uj}) + \sum_{j=1}^{N} \tilde{s}_j \mathbf{D}_j, \\ \tilde{\mathbf{K}}(s_j) &= \mathbf{K}(s_{uj}) + \sum_{j=1}^{N} \tilde{s}_j \mathbf{K}_j, \end{aligned} \tag{7}$$

where $\mathbf{M}(s_{uj})$, $\mathbf{D}(s_{uj})$ and $\mathbf{K}(s_{uj})$ are the structural parameter matrices of the initially designed system. Let $\mathbf{u}_a$ and $\tilde{\mathbf{G}}$ denote the required control input vector and controller gain matrix of the new system, respectively; the new structure system, which possesses the same dynamic response as the initial system, can be written as follows:

$$\dot{\mathbf{x}} = \left\{ \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}} & -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{M}}^{-1}\mathbf{B}_0 \end{bmatrix} \tilde{\mathbf{G}} \right\} \mathbf{x},$$

$$\mathbf{y} = [\mathbf{0} \;\; \mathbf{C}_0]\mathbf{x}, \qquad \mathbf{x}_0 = \begin{Bmatrix} \mathbf{q}_0 \\ \dot{\mathbf{q}}_0 \end{Bmatrix}.$$

Since the new system and the initially designed one have the same response, the matrices of the two closed-loop systems should be equivalent, i.e.,

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}} & -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{M}}^{-1}\mathbf{B}_0 \end{bmatrix}\tilde{\mathbf{G}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}\mathbf{G}, \quad (8)$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{D}}$, $\tilde{\mathbf{K}}$ and $\tilde{\mathbf{G}}$ are the function matrices of variables $\tilde{s}_j\,(j = 1, 2, \ldots, N)$.

According to the modern control theory [8], if the initially and newly designed systems are both controllable and observable, the constraint equation can be obtained as follows:

$$(\mathbf{I} - \mathbf{B}_0\mathbf{B}_0^+)\left[\sum_{i=1}^{N} \mathbf{M}_i\tilde{\mathbf{s}}_i \;\; \sum_{i=1}^{N} \mathbf{K}_i\tilde{\mathbf{s}}_i \;\; \sum_{i=1}^{N} \mathbf{D}_i\tilde{\mathbf{s}}_i\right]\boldsymbol{\Delta} = \mathbf{0},$$

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{M}^{-1}\mathbf{B}_0\mathbf{G}_p - \mathbf{M}^{-1}\mathbf{K}, & \mathbf{M}^{-1}\mathbf{B}_0\mathbf{G}_r - \mathbf{M}^{-1}\mathbf{D} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad (9)$$

where $[\mathbf{G}_p, \mathbf{G}_r] = \mathbf{G}$, and $\mathbf{B}_0^+$ is the pseudo-inverse matrix of $\mathbf{B}_0$.

It has been proved that equation (9) is the sufficient and necessary condition for the existence of a solution for matrix $\tilde{\mathbf{G}}$ [9], and the new control gain matrix $\tilde{\mathbf{G}}$ can be expressed as

$$\tilde{\mathbf{G}} = \mathbf{G} + \mathbf{B}_0^+\left[\sum_{i=1}^{N} \mathbf{M}_i\tilde{s}_i \;\; \sum_{i=1}^{N} \mathbf{K}_i\tilde{s}_i \;\; \sum_{i=1}^{N} \mathbf{D}_i\tilde{s}_i\right]\boldsymbol{\Delta}. \quad (9')$$

Now, our task is to seek $\tilde{s}_j$ in the new system in order to minimize the control input energy $\tilde{J}$, i.e.,

$$\tilde{J}_{\min} = \sum_{i=1}^{n} \int_0^\infty \mathbf{u}_a^{\mathrm{T}}(i, t)\mathbf{R}\mathbf{u}_a(i, t)\,\mathrm{d}t. \quad (10)$$

For the controller of the new system, $\mathbf{u}_a = \tilde{\mathbf{G}}\mathbf{x}$, the objective function $\tilde{J}$ can be written as

$$\begin{aligned} \tilde{J} &= \sum_{i=1}^{n} \int_0^\infty \mathbf{x}^{\mathrm{T}}(i, t)\tilde{\mathbf{G}}^{\mathrm{T}}\mathbf{R}\tilde{\mathbf{G}}\mathbf{x}(i, t)\,\mathrm{d}t \\ &= \mathrm{tr}\left(\tilde{\mathbf{G}}^{\mathrm{T}}\mathbf{R}\tilde{\mathbf{G}} \sum_{i=1}^{n} \int_0^\infty \mathbf{x}(i, t)\mathbf{x}^{\mathrm{T}}(i, t)\,\mathrm{d}t\right) \\ &= \mathrm{tr}(\tilde{\mathbf{G}}^{\mathrm{T}}\mathbf{R}\tilde{\mathbf{G}}\mathbf{X}), \end{aligned} \quad (11)$$

where tr denotes the trace of a matrix and $\mathbf{X}$ is the state covariance matrix [9], i.e.,

$$\mathbf{X} = \sum_{i=1}^{n} \int_0^\infty \mathbf{x}(i, t)\mathbf{x}^{\mathrm{T}}(i, t)\,\mathrm{d}t.$$

The state covariance matrix $\mathbf{X}$ can be calculated using the Riccati equation as follows:

$$\mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{A}^{\mathrm{T}} + \mathbf{X}_0 = \mathbf{0}, \quad (11')$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -M^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}\mathbf{G}, \;\; \mathbf{X}_0 = \mathrm{diag}\,[x_{01}^2, x_{02}^2, \ldots, x_{0n}^2].$$

The variation of the cross-sectional areas $\tilde{s}_j$ is expressed as a vector by

$$\tilde{\mathbf{s}} = \{\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N\}^{\mathrm{T}}$$

and the constraint condition equation (9) can be deduced as follows [8]:

$$\mathbf{F}\tilde{\mathbf{s}} = \mathbf{0}. \tag{12}$$

Similarly, the objective function $\tilde{J}$ of equation (11) can be transformed into a function of the unknown vector $\tilde{\mathbf{s}}$, i.e.,

$$\tilde{J} = \tilde{\mathbf{s}}^{\mathrm{T}}\mathbf{H}_2\tilde{\mathbf{s}} + \mathbf{H}_1^{\mathrm{T}}\tilde{\mathbf{s}} + \mathbf{H}_0. \tag{13}$$

The matrices $\mathbf{F}$, $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ in equations (12) and (13) are known matrices, because they can be obtained from matrices $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$, $\mathbf{G}$ and $\mathbf{X}$, as well as $\mathbf{M}_j$, $\mathbf{D}_j$ and $\mathbf{K}_j$ via some matrix operation.

In order to obtain the concrete expressions of matrices $\mathbf{F}$, $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$, two definitions f matrix operation can be adopted as follows.

(**1**) The definition of the Kronecker product of a matrix:

If $\mathbf{A}$ is a matrix of $n \times m$ dimension, and $\mathbf{B}$ is a matrix of arbitrary dimension, the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$ is defined as

$$\mathbf{A}\otimes\mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \cdots & a_{1m}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \cdots & a_{2m}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}\mathbf{B} & a_{n2}\mathbf{B} & \cdots & a_{nm}\mathbf{B} \end{bmatrix},$$

where a_{ij} $(i = 1, 2, \dots, n;\ j = 1, 2, \dots, m)$ is the element of matrix $\mathbf{A}$.

(**2**) The definition of the vector function of a matrix:

If $\mathbf{A}$ is a matrix of $n \times m$ dimension and $\mathbf{a}_i$ $(i = 1, 2, \dots, m)$ is the ith column of $\mathbf{A}$, the vector function $Vec\,(\mathbf{A})$ of $\mathbf{A}$ is defined as

$$Vec(\mathbf{A}) = \{\mathbf{a}_1^{\mathrm{T}}, \mathbf{a}_2^{\mathrm{T}}, \dots, \mathbf{a}_m^{\mathrm{T}}\}^{\mathrm{T}}.$$

Using definitions (1) and (2), the matrix $\mathbf{F}$ in constraint equation (12) can be written as

$$\mathbf{F} = [\mathbf{P}^{\mathrm{T}}\otimes(\mathbf{B}_0\mathbf{B}_0^{+})]\mathbf{U},$$

where

$$\mathbf{U} = [Vec(\mathbf{W}_1), Vec(\mathbf{W}_2), \dots, Vec(\mathbf{W}_N)], \qquad \mathbf{W}_j = [\mathbf{K}_j\ \ \mathbf{D}_j\ \ \mathbf{M}_j],$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{B}_0\mathbf{G}_P - \mathbf{M}^{-1}\mathbf{K}, & \mathbf{M}^{-1}\mathbf{B}_0\mathbf{G}_r - \mathbf{M}^{-1}\mathbf{D} \end{bmatrix}.$$

Assume $\mathbf{Z} = [\mathbf{P}^{\mathrm{T}}\otimes\mathbf{B}_0^{+}]\mathbf{U}$, then

$$\mathbf{H}_2 = \mathbf{Z}^{\mathrm{T}}(\mathbf{X}\otimes\mathbf{R})\mathbf{Z}, \qquad \mathbf{H}_1 = 2\mathbf{Z}^{\mathrm{T}}(\mathbf{X}\otimes\mathbf{R})\,Vec(\mathbf{G}), \qquad \mathbf{H}_0 = [Vec(\mathbf{G})]^{\mathrm{T}}(\mathbf{X}\otimes\mathbf{R})\,[Vec(\mathbf{G})].$$

Thus, the optimum design of a new structure is simplified to solve the minimum value problem of quadratic form as follows:

$$\tilde{J}_{\underset{\mathbf{S}}{\min}} = \tilde{\mathbf{s}}^{\mathrm{T}}\mathbf{H}_2\tilde{\mathbf{s}} + \mathbf{H}_1^{\mathrm{T}}\tilde{\mathbf{s}} + \mathbf{H}_0,$$

$$\text{s.t. } \mathbf{F}\tilde{\mathbf{s}} = \mathbf{0}. \tag{14}$$

Because equation (14) is the extremum problem of generic quadratic form with constraint condition, it can be solved using common numeric algorithms and existing programs.

When the variation of $\tilde{s}_j\,(j = 1, 2, \ldots, N)$ is calculated from equation (14), the cross-sectional areas of the bars of the optimized structure can be expressed by

$$s_j = s_{uj} + \tilde{s}_j, \quad j = 1, 2, \ldots, N.$$

Then, the structural parameter matrix $\tilde{\mathbf{M}}$, $\tilde{\mathbf{D}}$ and $\tilde{\mathbf{K}}$ for the optimized structure can be worked out using equation (7). The optimum feedback gain matrix $\tilde{\mathbf{G}}$ for the optimized system can be obtained using equation (9′).

3. EXAMPLE OF THE INTEGRATED OPTIMIZATION OF AN INTELLIGENT TRUSS

A 72-bar intelligent truss with two piezoelectric actuators (Figure 1) is used for analysis. The structural vibration can be actively controlled by the piezoelectric actuators located at bars 55 and 61 through the application of a controllable electric field. The feedback input voltage into the actuators are automatically determined using a computer according to the measured vibration response of the truss.

The output force of the piezoelectric actuators I and II is 8.50 N/V, their Young's modulus $E_P = 139$ GPa, and their mass density $\rho_p = 7500\ \mathrm{kg/m^3}$. The truss bars are made of aluminum alloy. Their Young's modulus $E = 70$ GPa and their mass density $\rho = 2700\ \mathrm{kg/m^3}$. In the initial structure design, the constrained root-mean square values of the dynamic response amplitude of the measurement points n_y are $Y_j = 0{\cdot}001$ m, $j = 1, 2, \ldots, n_y$. In addition, the upper bound of the cross-sectional area of each bar is $s_{uj} = 2{\cdot}0\ \mathrm{cm^2}$, and the lower bound is $s_{dj} = 1{\cdot}0\ \mathrm{cm^2}$. The maximum allowable voltage input to the piezoelectric actuators is $V_{max} = 100$ V.

Firstly, we made the design of an optimum controller for the initial structure and ascertained that the required maximal input voltages are $u_1 = 65$ V and $u_2 = 24{\cdot}5$ V for actuators I and II, respectively. Secondly, the dynamic structural optimization of the initial

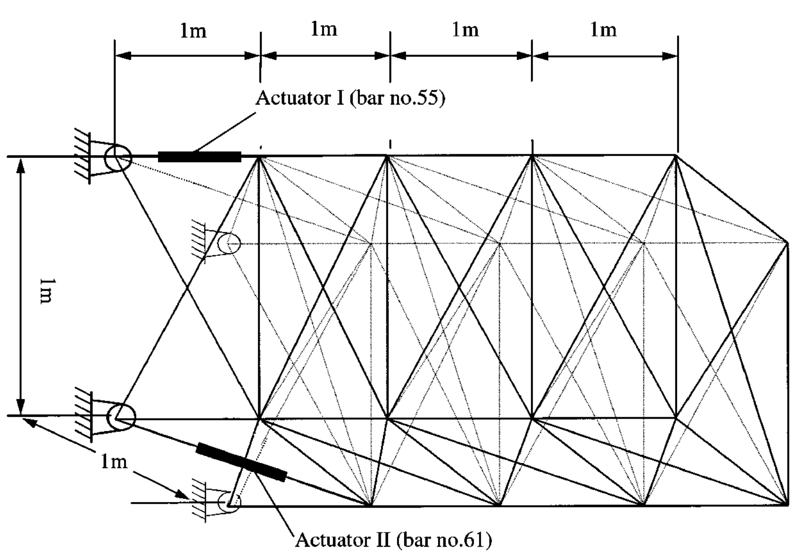

Figure 1. An intelligent truss with actuators for vibration control.

TABLE 1

Cross-sectional areas of bars of the initial and optimized truss structures

Number of bars	1–4	5–12	13–16	17–18	19–22	23–30	31–34	35–36	37–40	41–48	49–52	53–54	55–58	59–66	67–70	71–72
Init(ial s_{uj} (cm^2)	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0	2·0
Optimized s_j (cm^2)	1·64	1·35	1·55	1·10	1·58	1·25	1·41	1·48	1·27	1·47	1·13	1·00	1·21	1·38	1·31	1·0

TABLE 2

Comparison of optimal controllers and structural weight between the initial and optimized structures

		Initial truss	Opt. truss
Struct. total mass		44·6 kg	29·3 kg
Piezoelectric actuator I	Max. input force (N)	552	430
	Max. exciting voltage (V)	65	50
	Mass (kg)	2·2	1·6
Piezoelectric actuator II	Max. input force (N)	208	166
	Max. exciting voltage (V)	24·5	19·6
	Mass (kg)	1·5	1·2

structure was carried out using the proposed methods and a new structure was designed and constructed.

The cross-sectional areas of the bars in the initial and optimized structures are shown in Table 1. For the optimized structure system, its optimum controller was designed again and the required maximal control input voltages are given in Table 2, where the total weights of the initial and new structures are also listed for convenience of comparison. Because the actuating force produced by the piezoelectric actuator is directly proportional to the cross-sectional area of the actuator, decreasing the required actuating force is equivalent to reducing the cross-sectional area of the actuator. Table 2 shows that the total structural weight decreases 34·5% from the initial to the optimum structure, and the required maximum actuating force produced by the piezoelectric actuators decreases by about 20%. This indicates the efficiency of the integrated optimization of the controller and the structural weight.

Table 3 presents four sets of data: Set I is the natural frequencies of the initial truss system without structural optimization; Set II is the natural frequencies of the optimized truss system; Set III is the eigenvalues of the closed-loop system ($\mathbf{A} + \mathbf{BG}$) for the initial truss without structural optimization but with optimal control input gain $\mathbf{G}$; and Set IV is the eigenvalues of the closed-loop system ($\tilde{\mathbf{A}} + \mathbf{B}\tilde{\mathbf{G}}$) for the optimized truss system with optimal control input gain $\tilde{\mathbf{G}}$. The natural frequencies in Table 3 show that the initial truss has more repetitive natural frequencies than the optimized truss, the natural frequencies of the

TABLE 3

Some of the natural frequencies and eigenvalues of the closed-loop system for the initial and optimized trusses

	Set I	Set II	Set III		Set IV	
Order	(Hz)	(Hz)	Real	Image	Real	Image
1	62·038	63·62	− 0·3897	− 389·79	− 0·3998	− 399·80
2	62·038	66·54	− 0·3897	389·79	− 0·3998	399·80
3	161·53	164·1	− 0·3897	− 389·79	− 0·4175	− 417·55
4	250·64	257·4	− 0·3897	389·79	− 0·4175	417·55
5	254·23	261·5	− 1·0149	− 1014·9	− 1·0310	− 1031·0
6	254·23	268·8	− 1·0149	1014·9	− 1·0310	1031·0
7	460·64	446·2	− 1·5748	− 1574·8	− 1·6174	− 1617·4
8	513·05	502·5	− 1·5748	1574·8	− 1·6174	1617·4
9	513·05	506·8	− 1·5973	1597·3	− 1·6432	− 1643·2
10	657·98	636·1	− 1·5973	− 1597·3	− 1·6432	1643·2
11	657·98	647·5	− 1·5973	− 1597·3	− 1·6887	− 1688·7
12	666·86	674·8	− 1·5973	1597·3	− 1·6887	1688·7
13	692·50	682·3	− 2·8943	2894·3	− 2·8036	− 2803·6
14	704·51	693·1	− 2·8943	− 2894·3	− 2·8036	2803·6
15	704·51	706·7	− 3·2236	3223·6	− 3·1573	− 3157·3
16	707·92	713·5	− 3·2236	− 3223·6	− 3·1573	3157·3
17	707·95	718·1	− 3·2236	3223·6	− 3·1841	− 3184·1
18	802·82	808·0	− 3·2236	− 3223·6	− 3·1841	3184·1
19	814·72	810·2	− 4·1342	4134·2	− 3·9966	− 3996·5
20	814·72	816·2	− 4·1342	− 4134·2	− 3·9966	3996·5
21	829·12	820·3	− 4·1342	− 4134·2	− 4·0685	− 4068·5
22	868·19	859·6	− 4·1342	4134·2	− 4·0685	4068·5
23	927·79	943·2	− 4·1900	4190·0	− 4·2399	− 4239·9
24	927·79	949·6	− 4·1900	− 4190·0	− 4·2399	4239·9
25	950·99	957·3	− 4·3511	4351·1	− 4·2871	− 4287·0
26	1000·1	1002	− 4·3511	− 4351·1	− 4·2871	4287·0
27	1014·8	1038	− 4·4266	4426·6	− 4·3552	− 4355·2
28	1058·6	1056	− 4·4266	− 4426·6	− 4·3552	4355·2
29	1058·6	1061	− 4·4266	− 4426·6	− 4·4401	− 4440·1
30	1059·7	1070	− 4·4266	4426·6	− 4·4401	4440·1
31	1076·4	1091	− 4·4480	− 4448·0	− 4·4830	− 4483·0
32	1108·1	1111	− 4·4480	4448·0	− 4·4830	4483·0
33	1137·4	1134	− 4·4482	− 4448·2	− 4·5123	− 4512·3
34	1177·4	1186	− 4·4482	4448·2	− 4·5123	4512·3
35	1177·4	1190	− 5·0443	5044·3	− 5·0769	− 5076·9
36	1179·1	1200	− 5·0443	− 5044·3	− 5·0769	5076·9
37	1200·4	1203	− 5·1190	− 5119·0	− 5·0907	− 5090·7
38	1200·4	1205	− 5·1190	5119·0	− 5·0907	5090·7

optimized system are not always higher than those of the same order of the initial system, and the variation of closed-loop eigenvalues of the initial and optimized systems possesses the same tendency. Although the cross-sectional areas of the bars in the optimized truss decrease, the stiffness distribution of the optimized truss is more reasonable, so that the optimized truss can still keep the same vibration response as that of the initial truss when it undergoes the same excitation. Moreover, the optimized system has less weight and consumes less energy of control input than the initial truss.

4. CONCLUSIONS

The results of numerical simulation for the integrated optimization of an intelligent truss show that the hybrid optimization algorithm proposed by the authors results in a considerable reduction of the weight of the optimized structure. As a result, the weight of the control actuators and the relevant instruments, as well as the required input control forces, will be reduced significantly. Obviously, this approach possesses important application merit in developing an intelligent structure technique for active vibration control. Therefore, it is worthy of further investigation.

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