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ADAPTIVE-PASSIVE ABSORBERS USING SHAPE-MEMORY ALLOYS

K. Williams, G. Chiu and R. Bernhard

Ray W. Herrick Laboratories, Purdue University, West Lafayette, IN 47907-1077, *U.S.A.*
E-mail: *keith@ecn.purdue.edu*

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The adaptive-passive vibration absorber shows promise for combining the stability and low complexity of passive tuned absorbers with the robust performance of active vibration control schemes. Previous adaptive tuned vibration absorbers (ATVA) had been complex and bulky. Shape memory alloys (SMA), with their variable material properties, offer an alternative adaptive mechanism. Heating an SMA causes a change in the elastic modulus of the material. An ATVA using spring elements composed of three pairs of SMA wires and one pair of steel wires was constructed and tested. On–off actuation of the SMA elements created an ATVA with four discrete tuned frequencies. Characterization testing of the absorber showed variation of the natural frequency of the ATVA of approximately 15%. The ATVA was applied to a primary system and the frequency response of the system at various states of ATVA actuation was determined. Manual tuning of the ATVA actuation during a stepped-sine base excitation of the primary system showed a wider notch of attenuation than was possible with a non-adaptive absorber. Results of the tests indicate that an adaptive absorber incorporating SMA as a tuning element has potential as a simple, high-performance adaptive-passive technique for vibration control.

1. INTRODUCTION

Vibration control has been and remains an important field of study in engineering. Most commonly, the goal is the attenuation of the vibration of a primary system. Motivation includes the reduction of dynamic stresses of machinery, the isolation of precision devices from shock and vibration, and the reduction of vibration-induced sound. This paper presents a novel design for an adaptive tuned vibration absorber (ATVA) that uses shape-memory alloy (SMA) elements to achieve on-line adaptation. In the first part of the paper, the use of tuned vibration absorbers (TVA) for vibration control is introduced along with a brief discussion of the properties of SMA and its prior use as an adaptive structural material in vibration control applications. In the second part of the paper, the design and construction of an ATVA incorporating SMA elements are described. Experimental results of the testing of an ATVA incorporating SMA are presented in the third part of the paper. Results are given for both the characterization of the ATVA as a dynamic system and for the implementation of the device to control the vibration of a primary system. Comparisons with predicted results are given. In the fourth part of the paper, conclusions are provided along with a description of areas for future study.

1.1. TUNED VIBRATION ABSORBERS AND VIBRATION CONTROL

The TVA is a well-known device for achieving a reduction in the vibration of a primary system subject to harmonic disturbance inputs. A comprehensive treatment of the theory of

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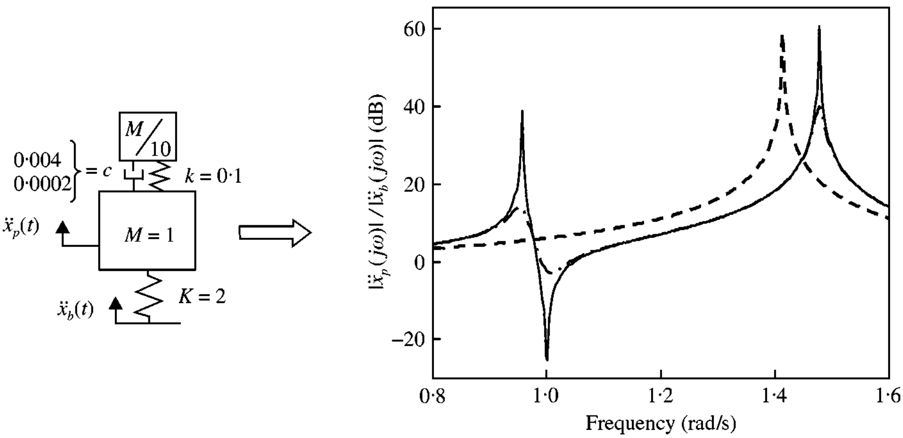

Figure 1. Application of a TVA to a primary system: —, 0·1% damped TVA; — - —, 2% damped TVA; – –, no TVA.

the TVA is given in the 1928 paper by Ormondroyd and Den Hartog [1]. A more recent study, along with examples of TVAs in industry, is given in the paper by Sun *et al.* [2]. In its simplest form, the TVA consists of a secondary mass and spring assembly attached to a primary system that is being acted upon by an external tonal excitation. Tuning the natural frequency of the TVA to the excitation frequency will result in attenuation of the vibration of the primary mass. The frequency response of a primary system with and without the implementation of TVAs is shown in Figure 1. The natural frequency of the primary system is $\omega_p = 1{\cdot}414$ rad/s and the undamped natural frequency of the TVA is $\omega_{abs} = 1{\cdot}0$ rad/s. The TVA mass was set at 10% of the primary system mass. The expectation is that the primary system has been designed without a resonance at the excitation frequency and that the absorber will be used to attenuate the response of the primary system at the excitation frequency.

The TVA is classified as a passive vibration control device as it does not require external control or actuation during operation. Under certain conditions, the TVA is a very effective vibration control device. In many situations, a lightweight TVA is highly desirable. However, to achieve substantial vibration reduction, the lightweight TVA must be very lightly damped and precisely tuned. A consequence of the lightweight and light damping is the introduction of a new resonant peak in the frequency response of the primary system near the vibration reduction notch, as shown in Figure 1. To avoid amplification of the vibration of the primary system, it is important that the TVA be precisely tuned to the external excitation frequency. This limits the use of the passive TVA to applications where the excitation frequency is known and constant and the TVA can be precisely tuned.

Active vibration control may be used in applications where the excitation frequency is unknown or varying. Active vibration control often involves driving the primary system in opposition to an external disturbance such that in the ideal case, the two inputs cancel to produce no net vibration of the primary mass. Inman distinguishes between active and passive control methods based on the need for external actuators and sensors for the implementation of active control [3]. Active vibration control has the advantage of being able to control vibration across wider bands of operating frequencies, which implies robustness to changes in operating frequencies. However, active control techniques often require complex combinations of sensors, actuators, and controllers. Additionally, active control has the potential for introducing instabilities into a system.

An alternative to both passive and active techniques is adaptive-passive vibration control. Adaptive-passive control methods attempt to integrate the positive aspects of both the passive and active schemes into a single vibration control system. Typically, adaptive techniques are used to modify the passive characteristics of the primary system. The adaptive TVA is a good example of an adaptive-passive approach to vibration control. In most manifestations of an adaptive TVA, the TVA stiffness is adapted to change the TVA natural frequency to match the excitation frequency. In this paper, the acronym ATVA will be used to refer to an adaptive TVA. An overview of ATVAs is provided in the paper by Sun *et al.* [2].

A good example of an ATVA is the device described by Franchek *et al.* [4]. In this device, the stiffness of an ATVA spring is adjusted by means of screwing a helical spring through a hole in a fixed plate. The spring stiffness is inversely dependent on the number of coils. A softer spring is achieved by making the spring longer. The resulting variable stiffness enables the operation of the ATVA over a range of frequencies.

While the ATVA is an attractive alternative to active and passive vibration control techniques, it is difficult to find an adaptive design that is both simple to control and easy to manufacture. According to Sun *et al.*, many adaptive-passive absorber designs suffer reliability limitations due to the complexity of their design and operation. To address these limitations, Sun *et al.* [2] proposed the use of smart materials to realize simpler device designs. Sun *et al.*'s definition of smart materials is "materials that respond to stimulus instantly with a stiffness change". Smart materials have the benefit of incorporating both actuators and structural elements in a single unit. Shape-memory alloys are a class of materials that fits this definition and will be described next.

1.2. THE SHAPE-MEMORY ALLOY AND VIBRATION CONTROL

Shape-memory alloys (SMAs) are a class of alloys exhibiting the shape-memory effect (SME). Detailed descriptions of the mechanics of the SME and the constitutive relations of SMA are given by Liang and Rogers [5–7]. For the purpose of this paper, it is sufficient to understand that SMA is a class of alloys that undergoes reversible changes in crystalline structure. In this paper, we shall consider only the SMA Nitinol, an alloy of nickel and titanium, discovered at the Naval Ordinance Laboratories [8]. The crystalline structure of SMA is dependent on its temperature. The two crystal structures are martensite and austenite, which generally correspond to the "cold" and "hot" states of the material. In the martensitic state, the metal exhibits a relatively low elastic modulus and yield strength, beyond which the material may be plastically deformed. Subsequent heating of the material induces the change to austenite, with a corresponding higher elastic modulus and yield strength. In it "hot" state, SMA exhibits an elastic modulus that is as much as three times that of the "cold" state. Additionally, during the transition from martensite to austenite, the heated material will "remember" its original undeformed condition and will attempt to revert to that shape. Restraining the metal during heating will induce stresses in the material, with the magnitude of the stresses dependent upon the initial plastic strain. Hodgson [8] provides values of the elastic moduli of Nitinol in his monograph on SMA. These values are shown in Table 1, along with the elastic modulus for structural steel, as given by Beer and Johnston [9].

SMA materials have been used in vibration control applications. Baz *et al.* [10] described the use of SMA actuators to achieve active control of the first mode of a flexible beam. The SMA wires were attached to a flexible beam. The wires were placed at offset distances from the top and bottom of the beam. Deflection of the beam stretched one of the two SMA

TABLE 1

Elastic moduli of Nitinol and steel

Material	Elastic modulus (GPa)
Nitinol:	
Martensite	28–41
Austenite	83
Structural steel	200

wires. The stretched SMA wire was then heated such that it contracted and produced a restoring moment in the beam.

Liang and Rogers explored two different techniques for the SMA in vibration control. The first was active properties tuning (APT), where the change in the elastic modulus of SMA with heating was used to modify the dynamic characteristics of a composite plate in which the SMA wires had been embedded [11, 12]. The resulting stiffening of the plate was used to attenuate the transmission of sound through the plate. The other technique used by Liang and Rogers was the active strain energy tuning (ASET). In that application, SMA elements were given initial plastic strains before insertion into a composite material. Heating the SMA then resulted in in-plane forces within the plate, with subsequent changes in the structure's natural frequency and mode shapes.

The APT and ASET techniques are considered adaptive-passive approaches to vibration control. Passive characteristics of the composite plates and adapted to change its dynamic characteristics. Both techniques involve adaptation of a primary system. In some applications it is not practical to redesign an entire structural system to accommodate this control technique. As an alternative approach, the adaptive properties of an SMA may be used to realize an ATVA. An SMA spring element whose stiffness is directly dependent on the elastic modulus of the spring material can be constructed as the adaptive spring in an ATVA. A variable stiffness SMA spring was demonstrated by Liang and Rogers [7]. Variation of the spring constant by a factor greater than 2·5 was achieved across a narrow temperature range (5°C). Liang and Rogers considered SMA springs with two distinct spring constants that corresponded to "cold" and "hot" springs. The temperature threshold associated with the phase change of the SMA varies with applied stress. As such, SMA springs were only examined for completely "on" or "off" temperatures.

Implementation of SMA spring elements in an ATVA results in an ATVA that would be applicable to situations where implementation of a TVA is desirable, but is not possible due to the uncertainties or variations in the excitation frequency. The acronym SMA ATVA will be used to identify such a device. Following the work of Liang and Rogers, an SMA ATVA was designed and constructed that implemented SMA springs as "on–off" elements with two discrete stiffness per spring. The development and testing of that SMA ATVA is the focus of this work and is described in the next section.

2. SMA ATVA DESIGN AND CONSTRUCTION

2.1. SMA ATVA DESIGN

To use the variable elastic modulus of the SMA to realize an adaptive spring, the spring stiffness must be dependent on the elastic modulus of the spring material. One simple

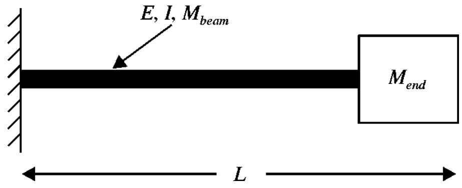

Figure 2. Mass-ended cantilevered beam TVA.

TABLE 2

Natural frequency with various levels of SMA actuation

Stiffness	Equivalent stiffness	Natural frequency
$3K_{cold}$	$3K_{cold} = K_1$	ω_1
$2K_{cold} + 1K_{hot}$	$5K_{cold} = \frac{5}{3}K_1$	$1{\cdot}29\omega_1$
$1K_{cold} + 2K_{hot}$	$7K_{cold} = \frac{7}{3}K_1$	$1{\cdot}53\omega_1$
$3K_{hot}$	$9K_{cold} = \frac{9}{3}K_1$	$1{\cdot}73\omega_1$

realization of such a spring is in the mass-ended cantilevered beam. The first natural frequency of a mass-ended cantilevered beam, ω_n, is given by Blevins [13] to be

$$\omega_n^2 = \frac{3EI}{L^3(M_{end} + 0{\cdot}24m_{beam})}, \tag{1}$$

where E is the elastic modulus of the beam material, l the cross-sectional inertia of the beam, m_{beam} the mass of the beam, L the length of the beam, and M_{end} the concentrated mass at the end of the beam, as shown in Figure 2. The variable elastic modulus of SMA may be used to provide a direct modification of the beam's spring stiffness, through the heating and cooling of SMA beam elements. In the extreme case of a beam composed entirely of SMA elements, a change in beam stiffness by a factor of three would result in an increase in the first natural frequency of approximately 73%, assuming that the SMA elastic modulus varies by a factor of three when heated.

To realize an SMA ATVA to attenuate the vibration of a primary system across a range of frequencies, multiple SMA elements may be used and actuated independently to create a set of discrete tuning conditions. An SMA ATVA incorporating three independent SMA elements would have four discrete, achievable natural frequencies, corresponding to the cases of no actuation or actuation of one, two, or three SMA wires. The natural frequencies of a three SMA-element system are shown in Table 2 for the case of three SMA beam elements of identical cross-sectional inertia, I_{SMA}, where $E_{hot} = 3E_{cold}$.

The SMA Nitinol has a relatively low yield strength (70–140 MPa [8], as compared with 250 MPa for structural steel [9]). With a low yield strength, care must be taken not to plastically deform the SMA elements. Plastic yielding may add hysteretic losses to the system, resulting in a higher equivalent damping in the SMA ATVA. On method of dealing with this limitation is to increase the overall stiffness of the beam elements through the use of additional, non-SMA material in parallel with the SMA. In particular, steel elements in parallel with SMA elements will substantially increase the SMA ATVA spring stiffness. This has the effect of limiting the range of the natural frequency of the ATVA and increases the

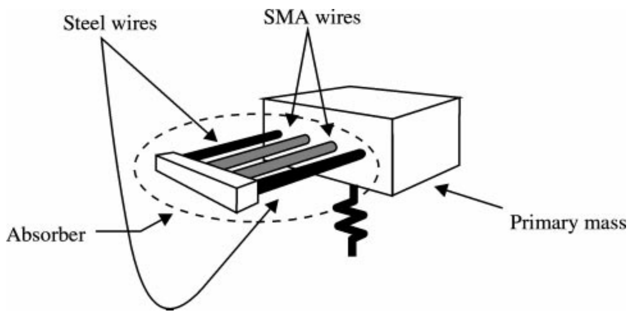

Figure 3. Conceptual SMA ATVA design.

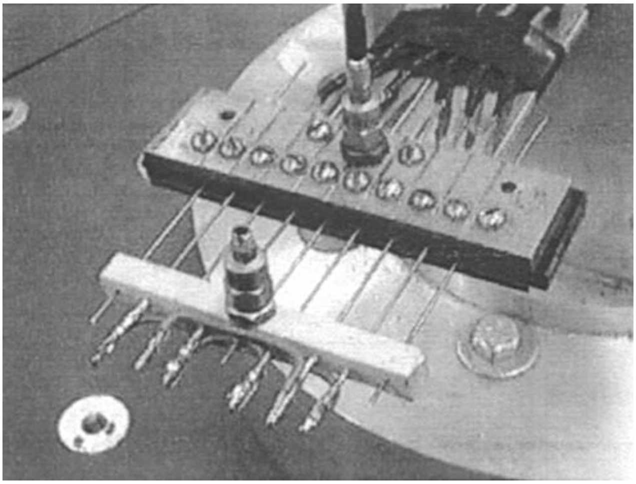

Figure 4. Photograph of SMA ATVA and fixture.

tuning resolution about the base natural frequency. The base natural frequency is determined by the "cold state" of the SMA ATVA, where none of the SMA elements are actuated. A conceptual design of an SMA ATVA incoroporating SMA and steel elements in parallel is shown in Figure 3.

2.2. SMA ATVA CONSTRUCTION

An SMA ATVA was constructed according to the guidelines in the previous section. Three pairs of 0·71 mm SMA wires were used in parallel with three 0·77 mm steel wires. The wires were embedded in a 4·65 g end-mass. An accelerometer was mounted on the end-mass, along with the circuitry for actuating the SMA wires. The total mass of the end-mass was approximately 7 g. A picture of the SMA ATVA mounted on a brass fixture is shown in Figure 4.

Actuation of the SMA wires was accomplished through the use of a current-controlled power supply. Analysis of the heat transfer between the SMA wires and the ambient air predicted a required current of approximately 1 A to maintain SMA temperature above the threshold transition temperature of 50°C. Testing showed this to be an adequate current,

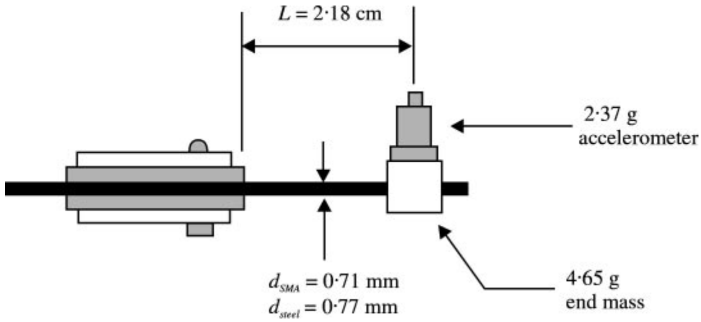

Figure 5. SMA ATVA and fixture combination.

although during tests of the ATVA, higher currents were used to reduce the transient heating response time of the SMA wires. The SMA wires were actuated in pairs of two reasons. First, with the planar geometry shown in Figure 4, actuation of an off-center SMA would result in a non-symmetric distribution of stiffness about the vertical axis of the device. It was believed that this would have the effect of creating undesired torsional modes of vibration.

The SMA ATVA was mounted in a fixture consisting of two brass plates separated by two insulating phenolic ("Bakelite") plates. The length of the SMA and steel wires from the base of the fixture to the center of the end-mass was 2·18 cm. The ATVA and fixture combination is shown schematically in Figure 5.

3. SMA ATVA TEST RESULTS

3.1. CHARACTERIZATION OF THE SMA ATVA

Characterization testing of the SMA ATVA was performed on an electromagnetic shaker. The SMA ATVA fixture was clamped to the top of the shaker, as shown in Figure 6. Actuation of the SMA ATVA was accomplished through resistive heating of the wires via a current-regulating DC power supply. The supply voltage was put through a switch box that ran an identical current through one, two, or three pairs of the SMA wires, depending on the switch position. The electrical circuit used for heating the SMA wires is shown in Figure 7. For cold tests, the DC power supply was not turned on.

Excitation of the SMA ATVA was accomplished using a 0·5 g stepped-sine input from 59 to 131 Hz, in 0·125 Hz increments. The frequency response functions for the SMA ATVA at the various levels of actuation were calculated using a digital signal analyzer, with the shaker acceleration as the input signal and the SMA ATVA end-mass acceleration as the output signal. The frequency response functions are shown in Figure 8.

The natural frequency of the SMA ATVA varied from approximately 83·55 Hz in the "All Cold" state, to 98·0 Hz when all SMA wire pairs were actuated, resulting in an increase in the SMA ATVA natural frequency of approximately 17·4%. With no actuation, the frequency response shows the characteristic shape of the response of a "softening spring". With greater actuation, the response peaks grow in magnitude and the general shape appears more typical of a linear system. One possible reason for the non-linear behavior of the SMA ATVA in the cold state may be plastic yielding of the SMA material that imparts the softening spring effect. This effect may be avoided by reducing the input excitation amplitudes or increasing the size of the ATVA.

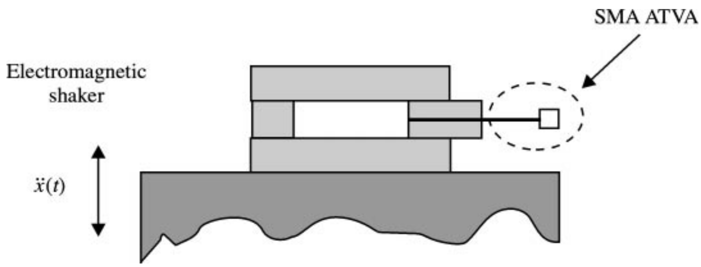

Figure 6. Characterization testing schematic.

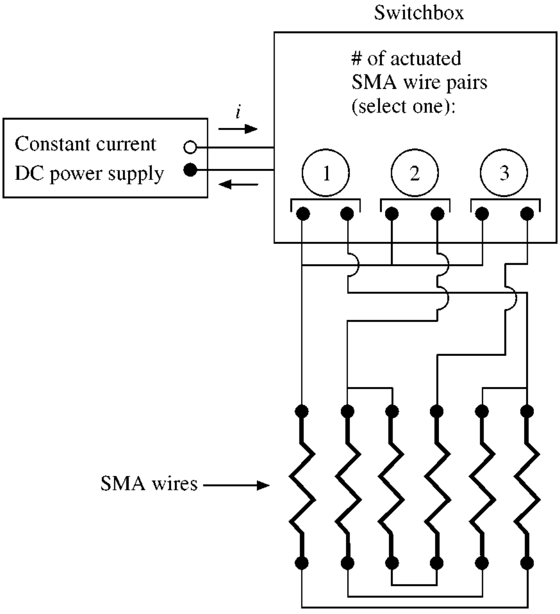

Figure 7. The SMA wire heating circuit.

To predict the natural frequencies of the SMA ATVA, the equivalent lumped mass and stiffness of the device were estimated for each level of actuation. The equivalent stiffnesses for each of the four levels of actuation, K_0, K_1, K_2, and K_3 (where K_0 corresponds to no pairs actuated and K_3 corresponds to all three pairs being actuated) are

$$K_0 = \frac{3(2E_{steel}I_{steel} + 6E_{cold}I_{sma})}{L^3}, \qquad K_1 = \frac{3(2E_{steel}I_{steel} + 4E_{cold}I_{sma} + 2E_{hot}I_{sma})}{L^3}, \quad (2a, b)$$

$$K_2 = \frac{3(2E_{steel}I_{steel} + 4E_{cold}I_{sma} + 4E_{hot}I_{sma})}{L^3}, \qquad K_3 = \frac{3(2E_{steel}I_{steel} + 6E_{hot}I_{sma})}{L^3}, \quad (2c, d)$$

where E_{steel} is the elastic modulus and I_{steel} the cross-sectional inertia of each of the steel wires respectively. E_{cold}, E_{hot}, and I_{SMA} are the "hot" and "cold" elastic moduli and the cross-sectional inertia of the SMA wires respectively. The beam length L is slightly modified

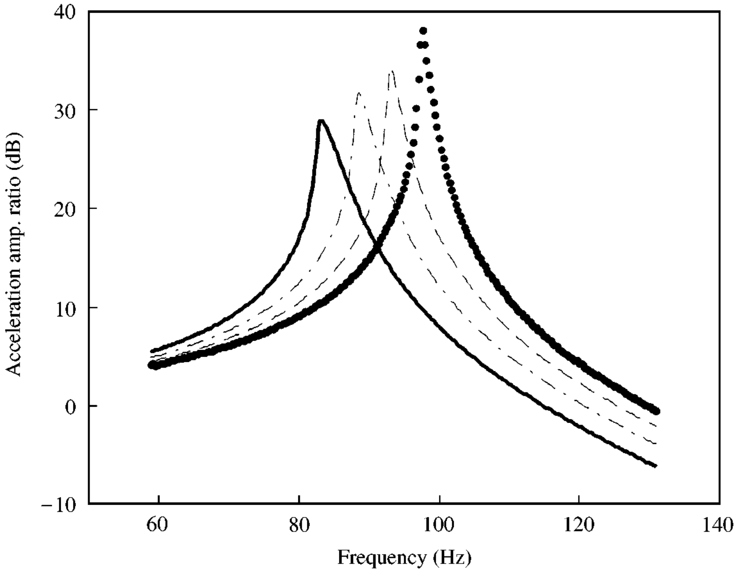

Figure 8. The SMA ATVA frequency responses: ——, no pairs hot; —-—, two pairs hot; - - - -, three pairs hot; ·····, all pairs hot.

TABLE 3

Predicted and measured absorber natural frequencies

Actuation level	Predicted ω_n(Hz)	Measured ω_n (Hz)
None	88·8	83·5
One pair	95·0	88·9
Two pairs	100·8	93·5
Three pairs	106·3	98·0

from Blevins' original equation [13]. In his text, Blevins shows the length of the beam measured to the outer edge of the lumped mass. For this case, it was found that a more appropriate measure of the beam length would be from the base of the SMA ATVA to the center of the end-mass. The measured and predicted natural frequencies are shown in Table 3.

To illustrate the on-line adaptation of the SMA elements, the SMA ATVA was driven at 83·5 Hz in the "All Cold" state. Once the steady state was achieved, all three pairs of SMA wires were simultaneously actuated. This corresponded to a switch from the "All Cold" to the "All Hot" state of the SMA ATVA. This test was performed at three current levels: 1·5, 3·0, and 4·5 A. Actuation occurred at 1 s after the start of data recording. The time-domain responses are shown in Figure 9.

Following the actuation of the SMA wires, the magnitude of the acceleration of the SMA ATVA decreased to approximately 10% of its magnitude before actuation. For the 4·5 A response, the new state was achieved in less than 0·5 s . For the 3·0 A response, a full second was required to achieve the actuated state. For the 1·5 A level, the adaptation was much slower and did not achieve the fully actuated state within 5 s of data shown in the record. Based on the different responses, a tuning strategy may be developed that uses an initially

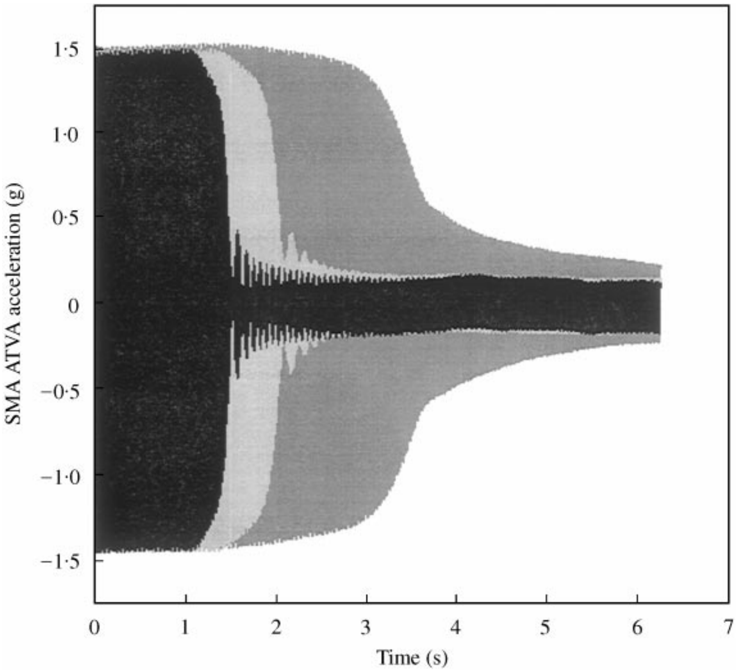

Figure 9. Actuation off of a resonant condition: ■, 4·5 A; ■, 3·0 A; ■, 1·5 A.

high current level to achieve the actuated state. Once that state is achieved, the current level may be scaled back to a minimum level required to maintain the temperature of the SMA wires above the threshold transition temperature of the material.

3.2. SMA ATVA IMPLEMENTATION OF A PRIMARY SYSTEM

The SMA ATVA was implemented on a primary system to illustrate the effect of the ATVA on the dynamic response of the primary system. A second mass-ended cantilevered beam system was used as the primary system. The brass and phenolic fixture of the SMA ATVA represented the lumped mass of the primary system (the primary mass). The primary system's spring element was an aluminum beam to which the end-mass was mounted. The natural frequency for the primary system alone (no SMA ATVA attached) was approximately 142·5 Hz. The schematic of this new set-up is shown in Figure 10.

Accelerometers were mounted on the primary mass and the shaker, which was driven with a stepped-sine input, from 60 to 190 Hz. The resolution of the measurements was 0·25 Hz from 60 to 75 Hz, 0·125 Hz from 75 to 110 Hz, and 0·25 Hz from 110 to 190 Hz. The accelerometer outputs were inputs to the signal analyzer, which was used to calculate the resulting frequency response function between the acceleration of the shaker and the acceleration of the primary mass. The frequency response functions of the primary mass with the SMA ATVA at the different levels of actuation are shown in Figure 11. In Figure 12, the primary system response is shown over the operating range of the SMA ATVA.

The different levels of SMA ATVA actuation resulted in reduction of the acceleration of the primary mass from 7 dB to almost 20 dB, across a 13 Hz band of frequencies. With the

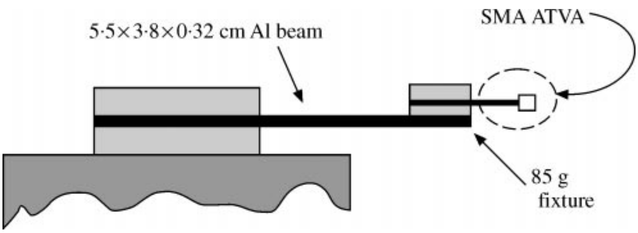

Figure 10. Implementation of the SMA ATVA on a primary system.

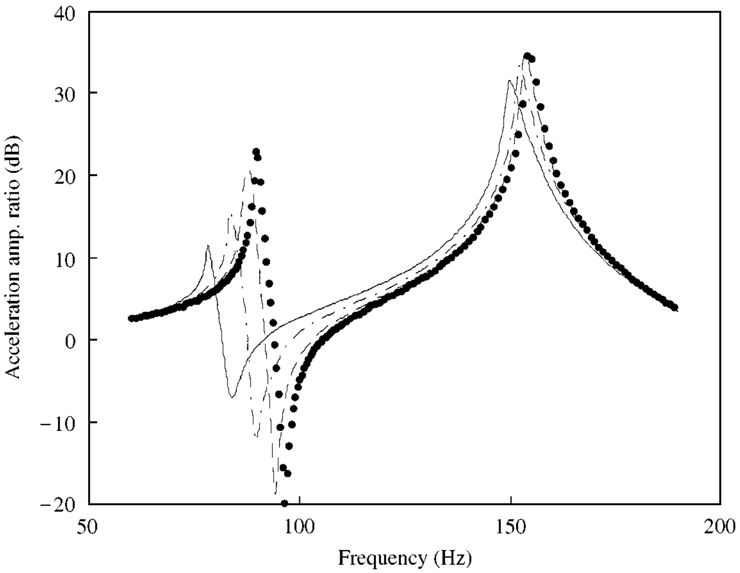

Figure 11. Primary system transfer functions with SMA ATVA actuation: ——, no pairs hot; —-—, two pairs hot; ---- three pairs hot; ·····, all pairs hot.

increasing actuation of the SMA wires, the responses show increasingly sharp "peaks" and "notches". This is attributed to the reduced effective damping of the SMA ATVA with the stiffening of the SMA wires.

Based on the results shown in Figure 12, a simple approach to tuning the SMA ATVA in response to a harmonic excitation of increasing frequency is to actuate additional SMA wire pairs when a higher stiffness will result in a lower acceleration response of the primary mass. For example, the "All Cold" state is the most appropriate state for frequencies up to approximately 88 Hz. For higher frequencies, the "One Pair Actuated" state of the SMA ATVA will result in greater attenuation than the "All Cold" state. Similar advantageous transition frequencies between the "One Pair Actuated" and "Two Pairs Actuated" and the "Two Pairs Actuated" and "All Hot" states can be found. Actuation of increasing pairs of SMA wires at known cross-over frequencies provides a simple method of tuning the SMA ATVA to achieve the minimal response of the primary system.

To test the tuning strategy, the stepped-sine input was applied to the system a second time. No SMA wires were actuated until the excitation frequency reached 88 Hz, at which time the first pair of SMA wires was heated. The second and third pairs of wires were

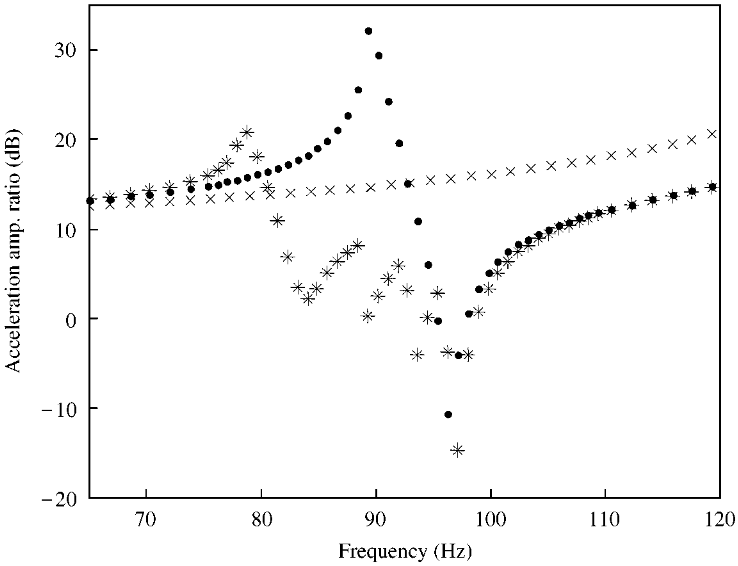

Figure 12. Close-up of Figure 11: ——, no pairs hot; —-—–, two pairs hot; ----, three pairs hot; ·····, all pairs hot.

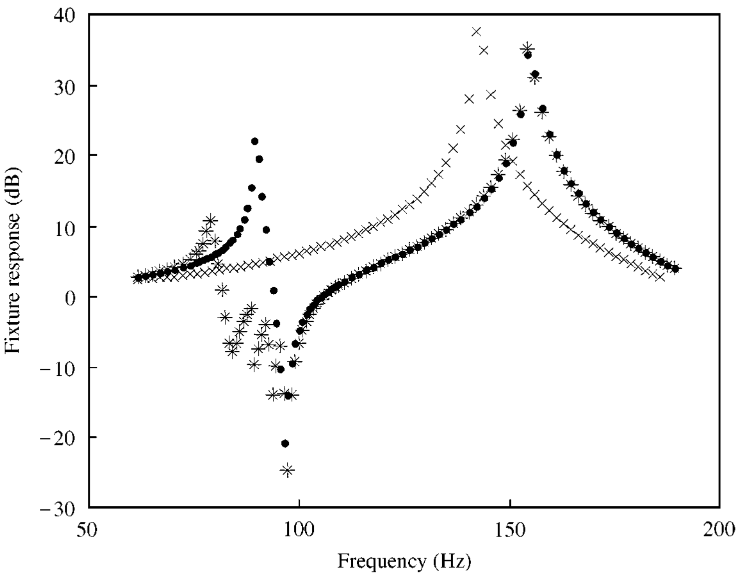

Figure 13. Performance plot of a manually tuned SMA ATVA: *, manual tuning; •, fixed TVA; ×, baseline.

actuated at 92·625 and 95·625 Hz respectively. The resulting performance plot of the system is shown in Figure 13 and is labelled "Manual Tuning". Two additional performance plots are also shown for comparison. The fixed tuning case is the response when the SMA ATVA is actuated at the "All Hot" state for the entire test. The test was intended to show the maximum attenuation possible with a comparable fixed-tuning TVA. For the "Baseline" case, the SMA ATVA was removed from its fixture for the duration of the test to measure the frequency response function of the untreated mass-ended cantilevered beam alone.

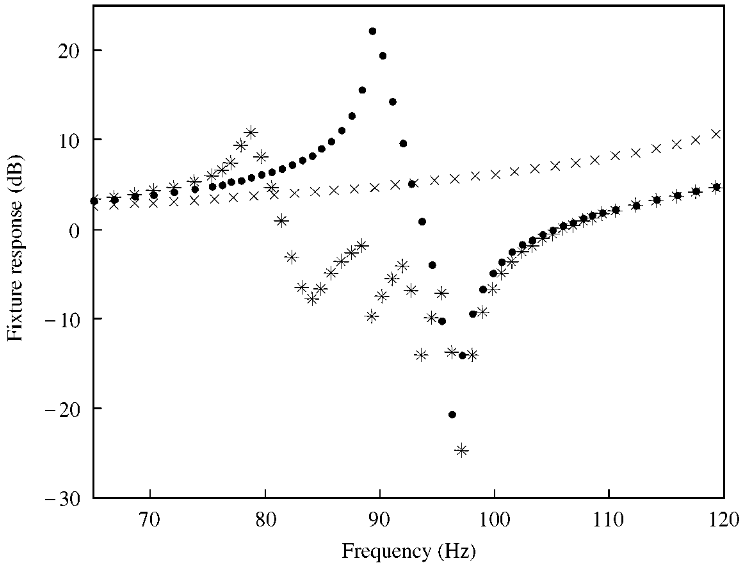

Figure 14. Close-up of Figure 13: *, manual tuning; •, fixed TVA; ×, baseline.

An important distinction must be made between the "performance plots" shown in Figures 13 and 14 and the frequency response plots shown in Figures 11 and 12. The frequency response plots are indications of the response of the primary system to excitation at the different frequencies. This excitation may occur across a band of frequencies, or it may be random excitation. In contrast, each point on the performance plots shows the response of the primary system to a tonal input at a single fixed frequency. The performance plots do not represent the response of the primary system to broadband excitation.

4. CONCLUSIONS

The design, characterization, and testing of a novel ATVA for use in adaptive-passive vibration control applications has been presented. The SMA ATVA requires no moving parts or complex mechanisms. The manually tuned SMA ATVA was successful in achieving attenuation of tonal vibration of the primary system across a range of frequencies. When applied to a primary system, the absorber can be used to attenuate the primary system vibration across a wider band of frequencies with greater attenuation than is possible with a similar non-adaptive TVA. Control of the system was achieved using a human operator in the loop. However, if the transition frequencies are known in advance, it may be possible to automate the system such that it will tune itself as the different levels of actuation become more or less appropriate. Performance of the device did degrade at frequencies intermediate to the deep "notches" in the performance plot. An important research direction is the development of a continuously tuned SMA ATVA such that the discrete notches in the performance plot may be expanded to a continuous band of attenuation.

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