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VIBRATION PARAMETERS FOR DAMAGE DETECTION IN STRUCTURES

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1. INTRODUCTION

The fact that defects cause changes in the dynamic characteristic of a structure has been widely used for safety inspection and control of production. The basic idea of non-destructive damage detection is to measure this dynamic characteristic during the lifetime of the structure and use it as a basis for the identification of structural damage.

The standard procedure involves conducting several vibration surveys on the structural system. The first one is conducted preferably before any important structural damage has occurred. This first test is then utilized as a baseline, and all subsequent tests are compared to it. Deviation of the tests results from the baseline provides an indication of structural damage. From this mismatch, procedures have been developed to estimate both the location and the extent of structural damage.

The aim of this paper is to examine how damage indicators are sensitive to changing number of frequencies and mode shapes and also to number and location of measurement points. The influence of measurement errors was also analyzed for all cases. For the analysis, those damage indicators are chosen which use changes in such modal parameters as natural frequencies and mode shapes and also differences between the curvatures of the damaged and undamaged structures in a given frequency range. All numerical calculations are based on a mathematical model of a cracked beam presented by Ostachowicz and Krawczuk [1].

2. NATURAL VIBRATIONS

Natural vibrations, as a solved description of a structure's physical characteristics (in terms of its mass, stiffness and damping), describe the behaviour of the structure as a set of natural frequencies with corresponding mode shapes and modal damping factors. This solution always describes the various ways in which the structure is particularly sensitive to vibrations and possible resonances.

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2.1. NATURAL FREQUENCIES

Changes in natural frequencies if any may be called the classical damage indicators. They are without any doubt the most used damage indicators both formerly and nowadays. The main reason for their great popularity is that natural frequencies are rather easy to determine with a relatively high level of accuracy. In fact, one sensor placed on a structure and connected to a frequency analyzer gives estimates for several natural frequencies. Further, natural frequencies are sensitive to all kinds of damage—local and global.

2.1.1. *Cawley–Adams criterion*

One of the first methods that claimed to be able to detect damage in an elastic structure by using natural frequencies was introduced by Cawley and Adams [2]. They proposed a method of predicting the site of damage based on changes in the natural frequencies.

The main idea of this method is that the change in stiffness is independent of frequency and the ratio of frequency in two modes is therefore only a function of the damage location. Positions where this theoretically determined ratio equals the experimentally measured value are therefore possible damage sites. In summary, a matching error, $e_s(p, q)$, associated with any pair of modes, p and q, at a possible damage site, s, can be found from the following formulae:

$$e_s(p, q) = \frac{\delta\Omega_p/\delta\Omega_q}{\delta\omega_p(s)/\delta\omega_q(s)} - 1 \quad \text{if} \quad \frac{\delta\omega_p(s)}{\delta\omega_q(s)} \leqslant \frac{\delta\Omega_p}{\delta\Omega_q}, \tag{1}$$

$$e_s(p, q) = \frac{\delta\omega_p(s)/\delta\omega_q(s)}{\delta\Omega_p/\delta\Omega_q} - 1 \quad \text{if} \quad \frac{\delta\omega_p(s)}{\delta\omega_q(s)} \geqslant \frac{\delta\Omega_p}{\delta\Omega_q},$$

where $\delta\omega_p(s)$ is the theoretical prediction of the change in the natural frequency for mode p with damage at location s and $\delta\Omega_p$ is the actual change in the natural frequency for mode p with unknown damage. The total matching error for all mode pairs is

$$e_s = \sum_{p=1}^{m-1} \sum_{p=q=1}^{m} e_s(p, q). \tag{2}$$

The predicted damage location is indicated by the minimum value using

$$E_s = \frac{(e_s)_{min}}{e_s}. \tag{3}$$

It will be seen that the predicted damage location is indicated by $E_s = 1$.

It follows from equation (1) that if the actual damage on the structure is identical to one of the cases in the database and the measurements are free of errors, then the algorithm will always predict the exact location correctly.

2.1.2. *Damage location assurance criterion* (*DLAC*)

The next criterion based on changes in natural frequencies is damage location assurance criterion (DLAC). This criterion is proposed by Messina *et al.* [3] in the following form:

$$DLAC(s) = \frac{|\{\delta\Omega\}^{\mathrm{T}}\{\delta\omega_s\}|^2}{(\{\delta\Omega\}^{\mathrm{T}}\{\delta\Omega\})(\{\delta\omega_s\}^{\mathrm{T}}\{\delta\omega_s\})}, \tag{4}$$

where $\{\delta\Omega\}$ is the trial "experimental" frequency change vector and $\{\delta\omega_s\}$ is the theoretical frequency change for damage at location s. DLAC values lie in the range 0–1, with 0 indicating no correlation and 1 indicating an exact match between the patterns of frequency changes. The value of s giving the highest DLAC values determines the predicted damage site.

The use of percentage frequency change data (rather than absolute changes) provides the best results.

Problems remain when the level of damage is low. The presence of measurement error will result in a degradation of the ability to predict the damage site accurately. Nevertheless, experience shows that the method is capable of giving a prediction with sufficient confidence to give a useful warning of a problem. If the predicted site is confirmed by subsequent measurements, then the method will still be seen as a valid early warning exercise.

2.2. MODE SHAPES

The mode shape is a unique characteristic of a mechanical structure and it is known as the spatial description of the amplitude of each resonance. It is common knowledge that local damage will cause a change in the derivatives of the mode shapes at the position of the damage. The idea of using the mode shape as a damage indicator is to detect the change of mode shapes obtained from successive tests. This fact has resulted in an increase of changes in mode shapes as damage indicators.

Unfortunately, to get estimates of the mode shape one has to perform a measurement at each of the points where estimates are wanted. Thus, the duration of a measurement session will increase considerably if a detailed mode shape were estimated. This is probably the main disadvantage in using mode shapes as damage indicators.

2.2.1. *Modal assurance criterion*

In several publications, it was suggested that it is possible to apply other criteria based on measurements of mode shapes. The first is the modal assurance criterion (MAC), a simple quantitative method for comparing mode shapes, which is defined as [4]

$$MAC(\phi_i, \phi_j) = \frac{|\phi_i^{\mathrm{T}}\phi_j|^2}{\phi_i^{\mathrm{T}}\phi_i\phi_j^{\mathrm{T}}\phi_j}, \tag{5}$$

where ϕ_i and ϕ_j are two eigenvectors of a discrete system or two eigenfunctions for a continuous system. This calculation, a scalar product between two complex unit vectors, results in a single number. The MAC is a correlation which varies between 0 and 1. A value of 1 indicates identical shapes—the constructional element is uncracked, and 0 indicates that they are orthogonal to and very unlike one another.

Although it indicates that there is a disparity between the two sets of data, it does show explicitly where the source of the discrepancy in the structure lies.

The MAC principle can be extended in several ways; thus, increasing its field of applications. Obviously, it is of greater importance to know the position, or at least the region, of the error rather than the modes which contribute to the lack of overall correlation.

2.2.2. *Co-ordinate modal assurance criterion*

The Co-ordinate modal assurance criterion (COMAC) identifies the co-ordinates at which two sets of mode shapes do not agree. The COMAC factor at a point i between two sets of the mode shape in states $A(\phi^A)$ and $B(\phi^B)$ is defined by [5]

$$COMAC(i) = \frac{[\sum_{j=1}^{N} |\phi_{i,j}^A \phi_{i,j}^B|]^2}{\sum_{j=1}^{N} (\phi_{i,j}^A)^2 \sum_{j=1}^{N} (\phi_{i,j}^B)^2}, \tag{6}$$

where N is the number mode shapes, $\phi_{i,j}^A$ and $\phi_{i,j}^B$ denote the values of jth mode shape at a point i for the states A and B respectively.

The COMAC has been developed from the original MAC concept in such a way that the correlation is related to degrees of freedom of the structure rather than to the mode numbers. The resultant COMAC values have been shown to be of considerable help in showing where the errors on the structure occur.

From the literature it emerges that the MAC and COMAC are highly dependent on the geometry of a structure and the location of damage. It is also proved that these criteria are not sensitive enough to detect damage at the earlier stage.

3. FORCED VIBRATIONS

Forced vibration allows one to analyze how the structure will vibrate under a given excitation, especially with what amplitudes. Clearly, this will depend not only upon the structure's inherent properties but also on the nature and magnitude of the imposed excitation. To obtain this information, frequency response functions (FRFs) are used, which express the relationship between the response (displacement (x_j), velocity or acceleration) at point j and a given load F_k at point k. For a multi-degree-of-freedom system the general element in the FRF matrix is defined as follows:

$$\alpha_{ij}(\omega) = \frac{x_j(\omega)}{F_k(\omega)}, \tag{7}$$

where $\alpha_{ij}(\omega)$ is the FRF of the system.

3.1. THE FREQUENCY RESPONSE CURVATURE METHOD

This method is based on the differences between the curvatures of the damaged and undamaged structures in a given frequency range to assess the damage [6]. This method has the advantage of simplicity and has no need for modal identification.

In this method, the curvature for each frequency can be computed by a central difference approximation, given by

$$\alpha''(\omega)_{i,j} = \frac{\alpha(\omega)_{i+1,j} - 2\alpha(\omega)_{i,j} + \alpha(\omega)_{i-1,j}}{h^2}, \tag{8}$$

where h is the distance between locations and $\alpha_{i,j}$ is the receptance measured at location i for a force input at location j.

The absolute difference between the FRF curvatures of the damaged and undamaged structures at location i, along the chosen frequency range, is calculated for an applied force at point j:

$$\Delta\alpha''_{i,j} = \sum_{\omega} |\alpha''_d(\omega)_{i,j} - \alpha''(\omega)_{i,j}|. \tag{9}$$

Finally, it can be summed up that for several force location cases:

$$\Sigma \Delta\alpha'' = \sum_{j} \Delta\alpha''_{i,j}. \tag{10}$$

4. MATHEMATICAL MODEL OF THE CRACKED BEAM

In this section, the mathematical model of the cracked beam used in numerical tests will be presented. The physical model of such a beam is shown in Figure 1. The beam is divided into two segments connected by an elastic element, the stiffness of which is calculated according to the law of fracture mechanics (see reference [1]).

The equation of natural vibration for a Bernoulli–Euler beam is as follows:

$$EI \frac{\partial^4 y(x, t)}{\partial x^4} + \rho F \frac{\partial^2 y(x, t)}{\partial t^2} = 0, \tag{11}$$

where ρ is the material density, F denotes the cross-sectional area of the beam, $y(x, t)$ is the the deflection of the beam, I the geometrical moment of inertia of the beam cross-section and E is Young's modulus.

The solution of equation (11) is sought in the form

$$y(L, t) = y(L) \sin \omega t, \tag{12}$$

where $L = x/l$.

Substituting this solution into equation (11), and after a simple algebraic transformation, one has

$$y^{\mathrm{IV}}(L) - k^4 y(L) = 0, \tag{13}$$

where

$$k^4 = \omega^2 \rho F / l^4 EI. \tag{14}$$

Taking the function $y(L)$ in the form of a sum of two functions,

$$y_1(L) = A_1 \cosh(k \cdot L) + B_1 \sinh(k \cdot L) + C_1 \cos(k \cdot L) + D_1 \sin(k \cdot L), \quad L \in [0, lp), \tag{15}$$

$$y_2(L) = A_2 \cosh(k \cdot L) + B_2 \sinh(k \cdot L) + C_2 \cos(k \cdot L) + D_2 \sin(k \cdot L), \quad L \in (lp, l]. \tag{16}$$

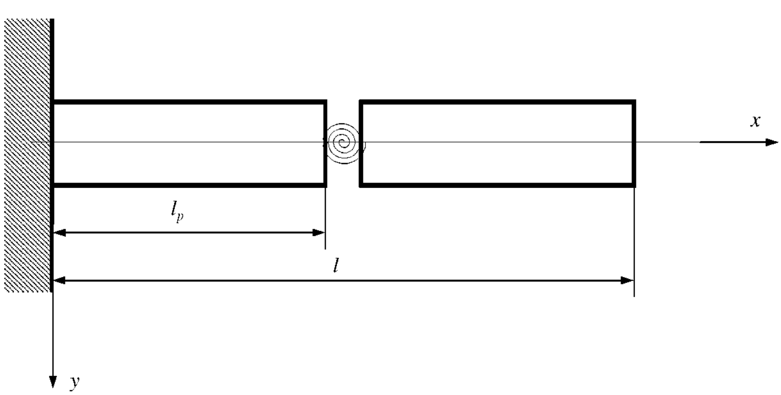

Figure 1. The model of a cracked beam, with a crack at l_p location.

The boundary conditions in terms of the non-dimensional beam length $Lp = x/l$, can be expressed as follows:
$y_1(0) = 0$—zero displacement of the beam at the restraint point,
$y'_1(0) = 0$—zero angle of rotation of the beam at the restraint point,
$y'_1(L_p) = y'_2(L_p)$—compatibility of the displacement of the beam at the location of the crack,
$y'_2(L_p) - y'_1(L_p) = \theta y''_2(L_p)$—total change of the rotation angle of the beam at the location of the crack,
$y''_1(L_p) = y''_2(L_p)$—compatibility of the bending moments at the location of the crack,
$y'''_1(L_p) = y'''_2(L_p)$—compatibility of the shearing forces at the location of the crack,
$y''_2(1) = 0$—zero bending moment at the end of the beam,
$y'''_2(1) = 0$—zero shearing force at the end of the beam.

Taking into account the boundary conditions, one obtains the characteristic equation which can be solved to determine the characteristic roots:

$$\det \begin{vmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{D} & \mathbf{E} \\ \mathbf{0} & \mathbf{F} \end{vmatrix} = 0. \tag{17}$$

The submatrices **C**, **D**, **E** and **F** of the characteristic matrix have the following forms:

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \tag{18}$$

$$\mathbf{D} = \begin{bmatrix} \cosh(A) & \sinh(A) & \cos(A) & \sin(A) \\ \sinh(A) & \cosh(A) & \sin(A) & \cos(A) \\ \cosh(A) & \sinh(A) & -\cos(A) & \sin(A) \\ \sinh(A) & \cosh(A) & \sin(A) & -\cos(A) \end{bmatrix}, \tag{19}$$

$$\mathbf{E} = \begin{bmatrix} -\cosh(A) & -\sinh(A) & -\cos(A) & -\sin(A) \\ -\sinh(A) + B\cosh(A) & -\cosh(A) + B\sinh(A) & \sinh(A) - B\cosh(A) & -\cosh(A) - B\sinh(A) \\ -\cosh(A) & -\sinh(A) & \cos(A) & \sin(A) \\ -\sinh(A) & -\cosh(A) & -\sin(A) & \cos(A) \end{bmatrix}, \tag{20}$$

$$\mathbf{F} = \begin{bmatrix} \cosh(k) & \sinh(k) & -\cos(k) & -\sin(k) \\ \sinh(k) & \cosh(k) & \sin(k) & -\cos(k) \end{bmatrix}, \tag{21}$$

where $A = k \cdot L_p$, $B = \theta \cdot k$ and θ denotes the beam flexibility at the crack location.

The roots k_i of the characteristic equation are used for the calculation of the natural vibration frequencies

$$\omega_i = \left(\frac{k_i}{L}\right)^2 \sqrt{\frac{EI}{\rho F}}, \quad i = 1, 2, \ldots, n, \tag{22}$$

where ω_i is the ith natural frequency of the beam and k_i is the ith characteristic root.

5. EXAMPLE RESULTS

All numerical tests were carried out for a cantilever beam with dimensions as follows: the length 1 (m), the height 0·01 (m), the width 0·01 (m), Young's modulus 210 (GPa), and mass density 7860 (kg/m^3). For all numerical tests, it was assumed that modal parameters for the cracked beam were known. The aim of the searching process was to find the location and depth of the crack from the whole possible range of damage location (0–1 (m)) and depth (0–0·006 (m)) using different damage indicators. For all examples, it was assumed that the location and depth of the crack correlate with the maximum value of the function describing the examined criterion.

Those criteria which use natural vibration parameters were examined as functions of the number of natural frequencies and the number of measurement points (for estimating mode shapes). Great interest was also put on examining the influence of the measurement error on the accuracy of predicting the damage location.

Damage indicators based on changes in forced vibrations were examined as functions of the parameters of the excitation force and number of measurement points. As for natural vibrations, great interest was put on examining the influence of the measurement error on the accuracy of predicting the damage location. The results of numerical calculations are shown in Figures 2–7.

Figure 2 presents the results for the Cawley–Adams criterion. The crack with depth equal to 30% of the beam height was located 0·8 (m) from the fixed end of the beam. The first column shows the results for the first two and first four natural frequencies without measurement errors. The second column presents the results for two and four natural frequencies with measurement error. The first frequency was measured with $-$ 1% error; the second with + 1·5% error, the third with $-$ 2·5% error and the fourth with + 2% error. From Figure 2 it can be seen that when there are no measurement errors the crack parameters are precisely found both for two and four natural frequencies. Measurement errors resulted in crack parameters (marked as location and depth in Figure 2(b)) being properly found only for four natural frequencies.

Figure 3 presents the result for the DLAC criterion. In this example, the crack with depth equal to 20% of the beam height was located 0·8 (m) from the fixed end of the beam. The first column shows results for the first two and first four natural frequencies without measurement errors. The second column presents results for two and four natural frequencies with measurement errors. The first frequency was measured with $-$ 0·1% error, the second with + 0·15% error, the third with $-$ 0·25% error and the fourth with + 0·2% error. From Figure 3 it can been seen that even very small measurement errors cause improper prediction of damage parameters (marked as depth and location in Figures 3(b)).

Figure 4 illustrates the results for the MAC criterion obtained for the crack with depth equal to 15% of the beam height and located 0·1 (m) from the fixed end of the beam. In this case, the influence of measurement error was not analyzed. The first column shows the results for four mode shapes, the second for two mode shapes. The number of mode shapes

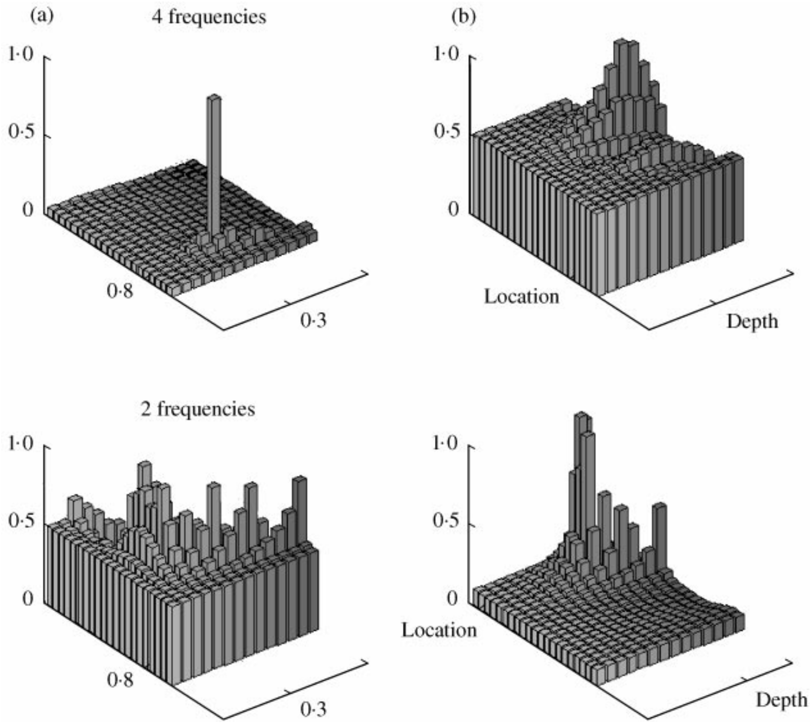

Figure 2. CA criterion: (a) without measurement errors, (b) with $\Delta f_1 = -1\%$; $\Delta f_2 = 1{\cdot}5\%$; $\Delta f_3 = -2{\cdot}5\%$ and $\Delta f_4 = 2\%$ measurement errors.

used for the process of damage assessing does not influence the accuracy of the results obtained. The location and depth of the crack (marked as location and depth in Figure 4(b)) were properly found for four as well as for two mode shapes.

Example calculations for the COMAC criterion are shown in Figure 5. The crack analyzed in this example had a depth equal to 15% of the beam height and was located 0·9 (m) from the fixed end of the beam. In this case, the influence of measurement error was not analyzed. The first column shows the results calculated for four mode shapes, the second—for two mode shapes estimated by ten points. The lower number of mode shapes used for the COMAC criterion does not seem to influence the damage parameter prediction process; however, for two mode shapes the results are not as clear as for four mode shapes.

Figures 6 and 7 present example results for the FRCM criterion. In the first case, the crack with depth equal to 20% of the beam height, located 0·8 (m) from the fixed end of the beam was analyzed. The left column shows the results of calculation without taking into account the measurement errors. In the results in the right column a $+2\%$ measurement error of the excitation force amplitude was analyzed. The curvatures were obtained for seven and three measurement points, uniformly located along the beam. It is easily seen that if more measurement points are taken into consideration, the prediction of the crack location and depth is clearer. The excitation force error causes disturbances in assessing the location of the crack (see Figure 6(b) where the location and depth denote crack parameters obtained as a result of the searching process).

In the second numerical example for the FRCM criterion, the analyzed crack parameters are the same as in the previous example. The left column of Figure 7 presents results without the measurement error whilst the right column presents results obtained for $+5\%$ error in

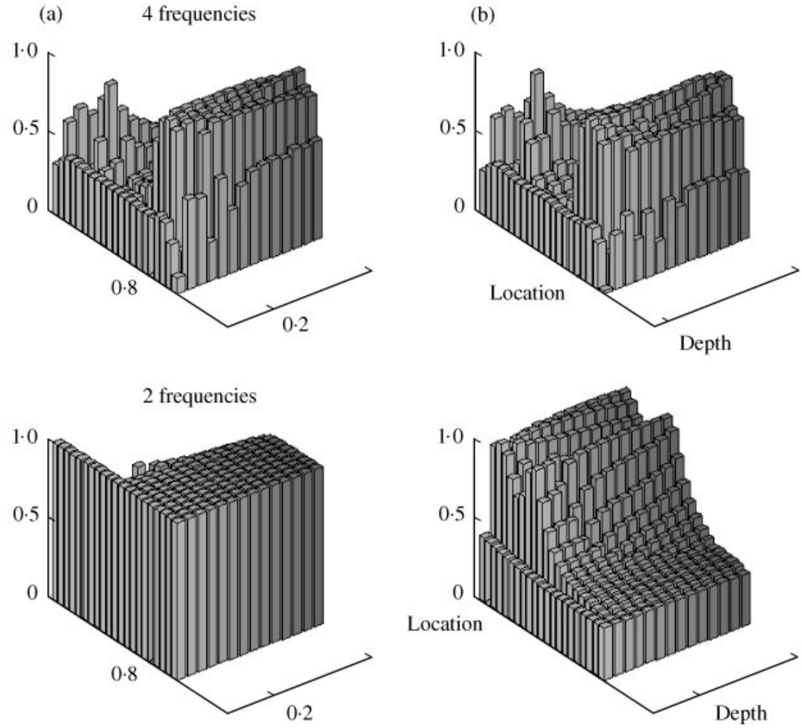

Figure 3. DLAC criterion: (a) without measurement errors, (b) with $\Delta f_1 = -0{\cdot}1\%$; $\Delta f_2 = 0{\cdot}15\%$; $\Delta f_3 = -0{\cdot}25\%$; $\Delta f_4 = 0{\cdot}2\%$ measurement errors.

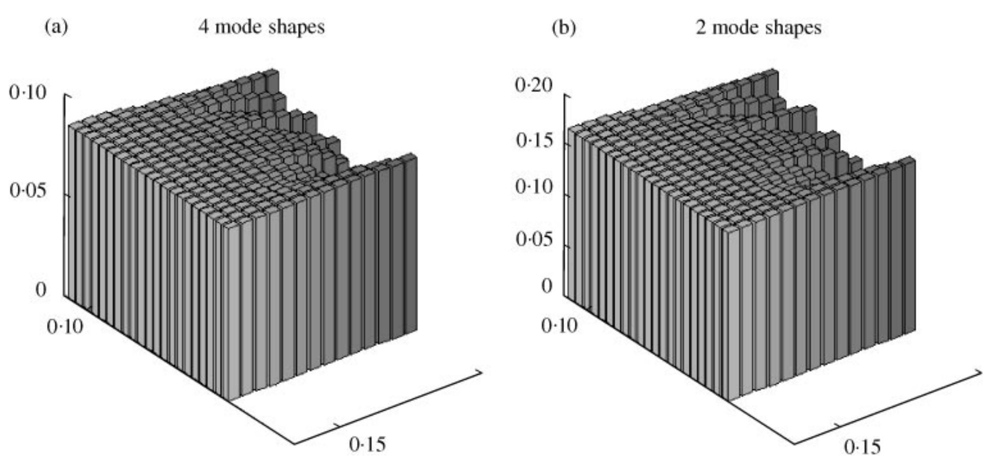

Figure 4. MAC criterion: (a) four mode shapes and (b) two mode shapes.

curvatures. Measurements were done at seven and three points uniformly located along the beam. It is easily seen that if more measurement points are taken into the numerical calculations, the prediction of the crack location and depth is better. Measurement errors cause disturbances in proper assessment of the crack location for a lesser number of measurement points (see Figure 7(b)).

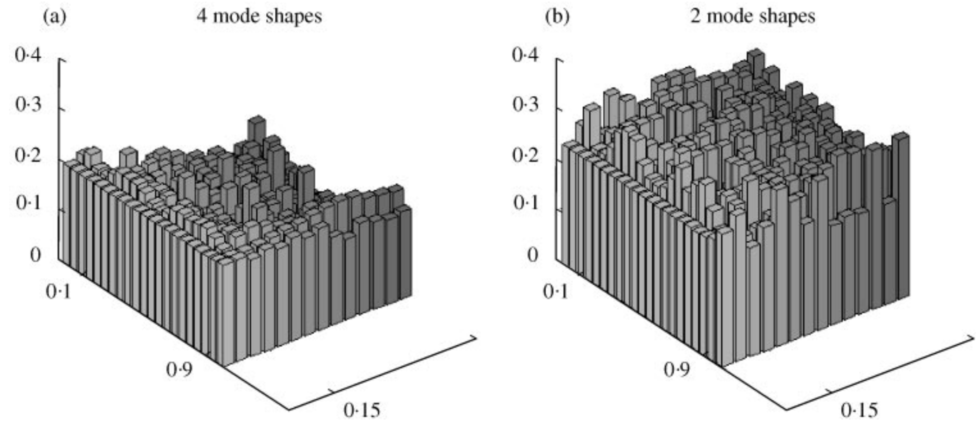

Figure 5. COMAC criterion: (a) four mode shapes and (b) two mode shapes.

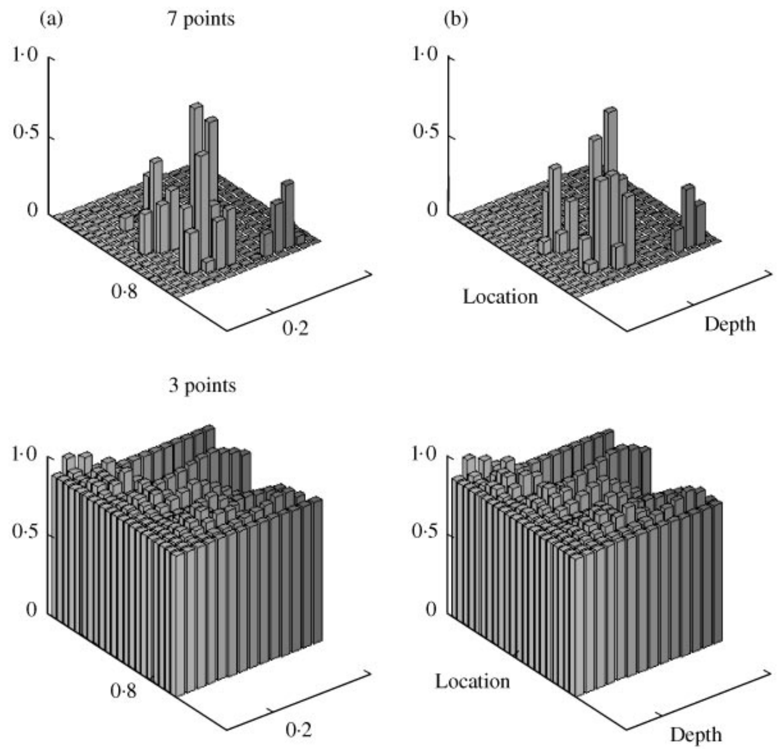

Figure 6. FRCM: (a) without measurement error and (b) with excitation force amplitude measurement error + 2%.

6. CONCLUSIONS

Taking into account the results of the numerical investigations the following conclusions can be drawn:

FOR CA CRITERION

- An increasing number of frequencies gives a better damage location prediction, locating damage with two natural frequencies only for measurements without errors,

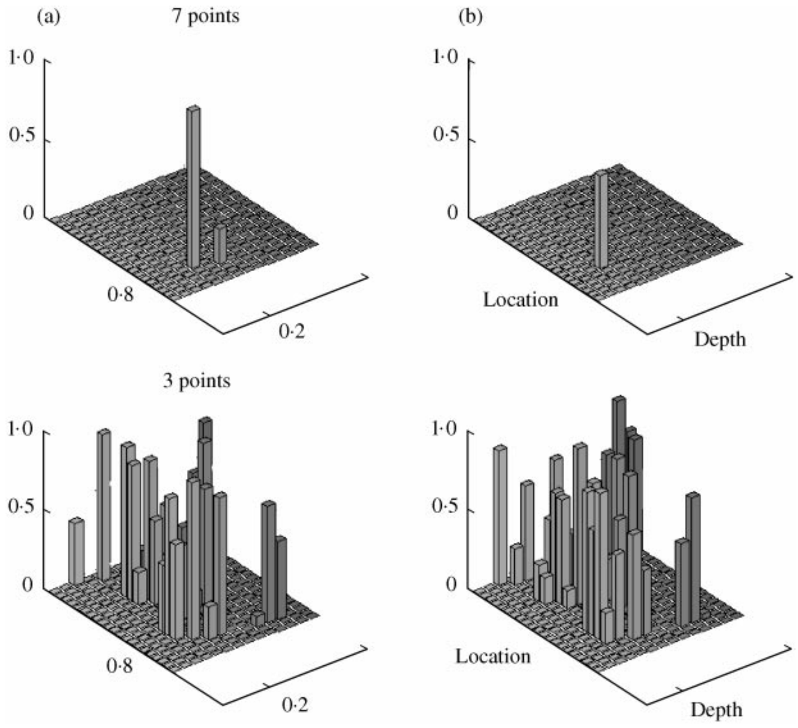

Figure 7. FRCM: (a) without measurement error and (b) with curvature measurement error $+5\%$.

- the method is very sensitive for even small errors in measured frequencies ($\pm$ 0·1%) when cracks are lesser than 20% of the height of the beam; for bigger cracks, the influence of the measurement error does not disturb proper damage location estimation, especially when a relatively high number of frequencies is known (>4).

FOR DLAC CRITERION

- An increasing number of frequencies in the numerical calculations gives a definitely better damage location prediction,
- there is a bigger chance of overestimating the damage location in comparison with CA criterion,
- this criterion is very sensitive even for very small frequency measurement errors (0·1%).

FOR MAC AND COMAC CRITERIA

- The damage location prediction is clearer when more than two mode shapes are taken into consideration,
- when the MAC and COMAC criteria are built from two mode shapes the results do not depend on the number of the modes which are used (it means that if the first and second mode shapes are taken into account, the result is the same as, for example, for the second and fourth mode shapes),

- for the MAC criterion a smaller number of points of the examined mode shapes gives a slightly worse damage location prediction, whereas the COMAC criterion seems to be more resistant to such limitation.

FOR FRCM CRITERION

- An increasing number of measurement points always gives better damage location prediction,
- measurement errors of the exciting force amplitude do not allow one to predict the crack parameters properly for a small number of measurement points,
- curvature measurement errors (even bigger than 1%) do not influence the process of predicting the crack parameters much.

On the basis of the above-mentioned numerical investigation, one can conclude that each one of the examined methods gives a proper estimation for relatively large disturbances in the structure (more than 20% of the height of the beam). The accuracy of the obtained numerical results depends on the level of precision during modal parameter measurement. Recent scientific research shows that the wavelet analysis methods [7, 8], as well as analysis of mechanical wave propagation in structures [9, 10] are more promising in the detection of very small cracks (even 0·5% of the height of the beam). The above-mentioned methods with genetic algorithms shorten the time needed for obtaining a satisfactory solution [11, 12].

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