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LETTERS TO THE EDITOR

COUPLING LOSS FACTORS FOR COUPLED ANISOTROPIC PLATES

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1. INTRODUCTION

Statistical energy analysis (SEA) represents a framework for analyzing the high-frequency response of complex structures to mechanical or acoustical excitation [1]. In SEA, a structure is divided into subsystems and the energy flow between these subsystems is described using coupling loss factors. In case of coupled plate subsystems, coupling loss factors are often expressed in terms of wave transmission coefficients [1, 2]. Expression for coupling loss factors are derived by combining the transmission coefficient and the intensity incident upon the junction on the source plate. The incident intensity depends on the distribution of the total energy over all directions in the source plate and is found to be proportional to the product of the energy density (energy per unit area) and the group velocity of the corresponding wave type [1].

For anisotropic plates, the derivation of the coupling loss factor should take into account the angle dependence of the wavenumber. This dependence affects the energy distribution over all directions in a reverberant field, as well as the direction of the energy flow associated with wave propagation. Auld [3] described the energy flow in anisotropic media in terms of a Poynting vector. This vector, which is parallel to the heading of the group velocity, is oriented normal to the curve obtained by plotting the wavenumber as a function of the wave heading. As illustrated in Figure 1, the wave heading θ_i is not parallel to the heading θ_e of the group velocity c_g, except for some discrete values of θ_i.

This behaviour is not restricted to anisotropic media, since it can also be observed for cylindrical shells consisting of isotropic materials [4]. In fact, a general expression for the coupling loss factor applicable to anisotropic components has first been derived by Langley [5] for junctions of curves panels. The expression was obtained by following the procedure briefly discussed above and the result was written in terms of the wave transmission coefficient, the group velocity and the phase velocity on the source plate. Later, Bosmans *et al.* [6, 7] presented a different expression which could be calculated directly from the transmission coefficient without requiring an evaluation of the group velocity. Although their results was derived for orthotropic plates, it can easily be extended to anisotropic

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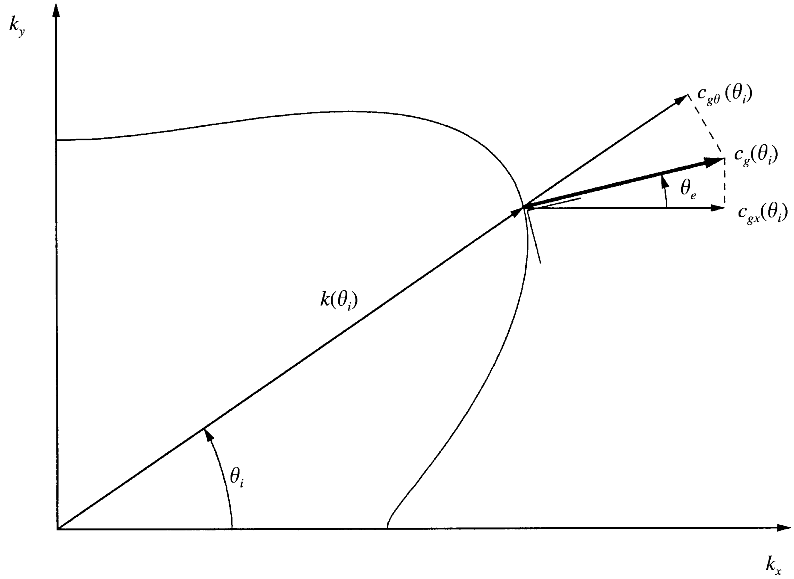

Figure 1. Wavenumber k as a function of the wave heading θ_i in the wavenumber plane. Also indicated are the group velocity $c_g(\theta_i)$ and its projections on the x-axis and the direction of wave propagation, $c_{gx}(\theta_i)$ and $c_{g\theta}(\theta_i)$ respectively.

plates by increasing the angular range over which the transmitted power is integrated from $\pi/2$ to π. The difference in appearance and derivation of the expressions given by Langley and Bosmans *et al.* may have raised some questions concerning their applicability. In this paper, it will be shown that both expressions are identical, and that the coupling loss factor presented by Bosmans *et al.* [6, 7] can be derived directly from Langley's formulation [5] without loss of generality.

2. ANALYSIS

Consider the junction of two anisotropic plates i and j shown in Figure 2. The response of both finitesized plates is assumed to be reverberant, and the SEA ensemble average of the resulting vibration field is described as a superposition of plane waves travelling in all directions [1]. This mode–wave duality allows for the exchange of vibrational energy at the junction of finite plates to be quantified by the transmission coefficient obtained by modelling the interaction of plane waves at a corresponding junction of semi-infinite plates. In this semi-infinite plate model, a unit amplitude plane wave w_i is assumed to be travelling towards the junction with angle of incidence θ_i. This incident wave causes a transmitted wave w_j on plate j. The intensity, i.e., energy flow per unit width, carried by the incident and transmitted waves in the x direction is equal to $I_{xi}(\theta_i)$ and $I_{xj}(\theta)$ respectively. The transmission coefficient of the junction is then defined as

$$\tau_{ij}(\theta_i) = \frac{I_{xj}(\theta_i)}{I_{xi}(\theta_i)}. \tag{1}$$

According to Langley [5], the energy flow P_{ij} through the junction of finite plates in Figure 2 is expressed in terms of the transmission coefficient as follows:

$$P_{ij} = E_i \omega \eta_{ij} = L_{ij} \int_{\theta_i} e_i(\theta_i) c_{gxi}(\theta_i)\, \tau_{ij}(\theta_i)\, \mathrm{d}\theta_i, \tag{2}$$

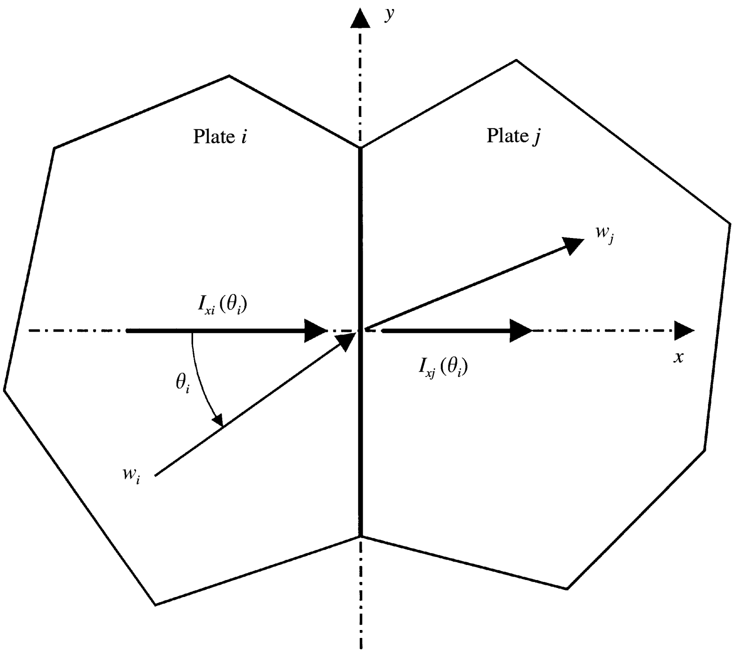

Figure 2. Junction of two anisotropic plates i and j. Incident wave w_i with heading θ_i on plate i, transmitted wave w_j on plate j, and corresponding intensities in the x direction $I_{xi}(\theta_i)$ and $I_{xj}(\theta_i)$.

where E_i is the total energy of plate i, ω the circular frequency, η_{ij} the desired coupling loss factor, L_{ij} represents the junction length and $c_{gxi}(\theta_i)$ the x-component of the group velocity $c_{gi}(\theta_i)$ on plate i (see Figure 1). The energy density $e_i(\theta_i)$ associated with waves travelling in the direction θ_i is defined as [5]

$$e_i(\theta_i) = \frac{E_i\omega}{4\pi^2 n_i(\omega)} \frac{1}{c_i(\theta_i)\, c_{g\theta i}(\theta_i)}. \tag{3}$$

In equation (3), $c_i(\theta_i)$ denotes the phase velocity and $c_{g\theta i}(\theta_i)$ the group velocity in the direction of the wave heading θ_i (see Figure 1). The modal density $n_i(\omega)$ for anisotropic structural components is given by Langley [8] as

$$n_i(\omega) = \frac{S_i}{2\pi^2} \int_0^{\pi} k_i(\theta_i) \frac{\partial k_i(\theta_i)}{\partial \omega} \, \mathrm{d}\theta_i, \tag{4}$$

where S_i represents the surface area of plate i and k_i the wavenumber corresponding to the incident wave. The product $e_i(\theta_i)\, \mathrm{e}_{gxi}(\theta_i)$ in equation (2) is equal to the incident intensity in the x direction corresponding to the wave heading θ_i. While the x-component of the group velocity $c_{gxi}(\theta_i)$ accounts for the fact that the energy propagates in a direction different from that of the associated wave, the energy density $e_i(\theta_i)$ incorporates the probability distribution of the propagation direction over all angles in the reverberant field. The final expression for the coupling loss factor for anisotropic components is found by combining equations (2) and (3) [5]:

$$\eta_{ij} = \frac{L_{ij}}{4\pi^2 n_i(\omega)} \int_{\theta_i} \left[\frac{c_{gxi}(\theta_i)\, \tau_{ij}(\theta_i)}{c_i(\theta_i)\, c_{g\theta i}(\theta_i)} \right] \mathrm{d}\theta_i\,. \tag{5}$$

In case of an isotropic plate, the integration in equation (5) should be carried out from $-\pi/2$ to $+\pi/2$. For anisotropic components, incident waves with heading in the interval $[-\pi/2, +\pi/2]$ may actually carry energy away from the junction. Consequently, the integration in equation (5) should include all angles between $-\pi$ and $+\pi$ for which the group velocity $c_{gxi}(\theta_i)$ is positive [5].

In the approach of Bosmans *et al.* [6, 7], the distribution of the vibrational energy over all directions of the reverberant field is described using a weighting function $D_{wi}(\theta_i)$. This function quantifies the probability distribution of the propagation direction in a reverberant wave field and its use was first suggested by Lyon *et al.* [1]. Bosmans [7] derived an expression for $D_{wi}(\theta_i)$ in the case of an orthotropic plate, which may be rewritten for the case of anisotropic plates by adjusting the integration limits.

$$D_{wi}(\theta_i) = \frac{\pi\, k_i(\theta_i)\,(\partial k_i(\theta_i)/\partial\omega)}{\int_0^\pi k_i(\theta_i)\,(\partial k_i(\theta_i)/\partial\omega)\,\mathrm{d}\theta_i}. \tag{6}$$

After rewriting equation (6) using the equations

$$c_i(\theta_i) = \frac{\omega}{k_i(\theta_i)}, \quad c_{g\theta i}(\theta_i) = \frac{\partial\omega}{\partial k_i(\theta_i)}, \tag{7}$$

the resulting expression can be combined with equations (3) and (4) to obtain an expression for the corresponding energy density associated with wave heading θ_i:

$$e_i(\theta_i) = \frac{E}{2\pi S_i}\, D_{wi}(\theta_i). \tag{8}$$

Equation (8) shows that the total energy density E_i/S_i is equally distributed over all angles in the plate when $D_{wi}(\theta_i) = 1$. The latter conditions is satisfied for an isotropic plate and corresponds to an ideal diffuse wave field.

Substitution of equation (8) into equation (2) yields

$$\eta_{ij} = \frac{L_{ij}}{2\pi\omega S_i}\int_{\theta_i} c_{gxi}(\theta_i)\,\tau_{ij}(\theta_i)\,D_{wi}(\theta_i)\,\mathrm{d}\theta_i. \tag{9}$$

It is interesting to note that the latter equation can be considered as the generalization of the coupling loss factor for isotropic plates, which can easily be derived from equation (9) by setting $D_{wi}(\theta_i) = 1$ and $c_{gxi}(\theta_i) = c_g \cos\theta_i$. Equation (9) can be further simplified by considering that the incident intensity $I_{xi}(\theta_i)$ in equation (1) can also be written as the product of energy density and group velocity. Since $I_{xi}(\theta_i)$ corresponds to a plane wave with unit amplitude, the associated energy density e'_i is [2]

$$\mathrm{e}'_i = \frac{m''_i\,\omega^2}{2}, \tag{10}$$

where m''_i denotes the mass per unit area of plate i. As a result, the x-component of the incident intensity can be expresses as

$$I_{xi}(\theta_i) = \mathrm{e}'_i\, c_{gxi}(\theta_i) = \frac{m''_i\,\omega^2}{2}\, c_{gxi}(\theta_i). \tag{11}$$

Finally, substitution of equations (1) and (11) into equation (9) leads to the expression for the coupling loss factor of anisotropic components as presented by Bosmans *et al.* in references [6, 7]:

$$\eta_{ij} = \frac{L_{ij}}{\pi\omega^3 M_i} \int_{\theta_i} I_{xi}(\theta_i) D_{wi}(\theta_i)\, \mathrm{d}\theta_i, \tag{12}$$

where M_i equals the total mass of plate i. Also, in this case, the integration in equation (12) should include all angles for which the incident intensity $I_{xi}(\theta_i)$ is positive. It should be noted that an expression for the group velocity is not needed to evaluate $I_{xi}(\theta_i)$, since the incident intensity in the x direction can be expressed in terms of the forces F_n and corresponding velocities v_n at a cross-section perpendicular to the x-axis [2]:

$$I_{xi}(\theta_i) = \frac{1}{2} \sum_n \mathrm{Re}\{F_n v_n^*\}, \tag{13}$$

where "*" denotes complex conjugate and the summation is taken over the degrees-of-freedom involved.

Since equations (5) and (12) have both been derived from equation (2), the coupling loss factors proposed by Langley [5] and Bosmans *et al.* [6, 7] are essentially the same. The implementation of equation (12) may appears to be more practical, since it does not explicitly require an expression for the group velocity on the source plate. However, equation (5) is not much more difficult to implement because the group velocity in the direction of wave propagation $c_{g\theta i}(\theta_i)$ can be easily derived from expression (7), and the x-component of the group velocity $c_{gxi}(\theta_i)$ can be obtained by combining equation (11) and (13).

3. CONCLUSIONS

Two previously published formulations for the coupling loss factor applicable to coupled anisotropic components were discussed. The first one was derived in the context of coupled cylindrical panels and was based on an extensive theoretical analysis. The second one was established in the context of orthotropic plates and was the result of a simpler formulation where the coupling loss factor could be deducted directly from the transmission coefficient. Although both expressions appeared to be very different, it was shown that they are essentially identical and that there is no apparent reason for preferring one formulation over the other.

REFERENCES

1. R. H. Lyon and R. G. DeJong 1995 *Theory and Application of Statistical Energy Analysis*. Boston: Butterworth-Heinemann; second edition.
2. L. Cremer, M. Heckl and E. E. Ungar 1998 *Structure-Borne Sound*. Berlin: Springer-Verlag; second edition.
3. B. A. Auld 1973 *Acoustics Fields and Waves in Solids*. Vol. 1. New York: Wiley.
4. R. S. Langley 1994 *Journal of Sound and Vibration* **169**, 29–42. Wave motion and energy flow in cylindrical shells.
5. R. S. Langley 1994 *Journal of Sound and Vibration* **169**, 297–318. Elastic wave transmission coefficients and coupling loss factors for structural junctions between curved panels.
6. I. Bosmans, P. Mees and G. Vermeir 1996 *Journal of Sound and Vibration* **191**, 75–90. Structure-borne sound transmission between thin orthotropic plates: analytical solutions.
7. I. Bosmans 1998 *Ph.D. Dissertation, Faculty of Applied Sciences, K.U. Leuven.* Analytical modelling of structure-borne sound transmission and modal interaction at complex plate functions.
8. R. S. Langley 1996 *Journal of the Acoustical Society of America* **99**, 3481–3487. The modal density of anisotropic structural components.