Journal of Sound and Vibration (2002) **250**(2), 368–377
doi:10.1006/jsvi.2001.3764, available online at http://www.idealibrary.com on IDEAL®

NATURAL FREQUENCIES OF TENSIONED PIPES CONVEYING FLUID AND CARRYING A CONCENTRATED MASS

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(*Received* 8 *February* 2001, *and in final form* 14 *May* 2001)

1. INTRODUCTION

The understanding of the vibrations of axially moving continuous medium is important in the design of high-speed magnetic tapes, band-saws, power transmission chains and belts, textile and composite fibers, aerial cable tramways, flexible robotic manipulators with prismatic joints, flexible appendages on spacecraft, paper sheets during processing, pipes and beams conveying fluid, etc. Especially, pipe lines are used in conveying gas, oil, water, dangerous liquids in chemical plants, cooling water in nuclear power plants, and in many other places. Ulsoy *et al.* [1] and Wickert and Mote [2] reviewed the relevant work up to 1978 and 1988 respectively. Wickert and Mote [3, 4] showed that the energy flux at a fixed support is the product of the string tension and the convective component of a velocity and investigated the transverse vibrations by complex modal analysis. Pakdemirli *et al.* [5] re-derived the equations of motion for an axially accelerating strip using Hamilton's Principle and investigated the stability using Floquet theory. Pakdemirli and Batan [6] analyzed the constant acceleration-type motion. Pakdemirli and Ulsoy [7] obtained approximate analytical solutions by using the method of multiple scales and showed that direct perturbation yields better results for higher order expansions with respect to discretization–perturbation method. Öz *et al.* [8] studied the transition behavior from string to beam for an accelerating material and determined stability borders for variable velocity and studied principal parametric resonance case. The transverse vibrations of an axially accelerating beam on different supports for different flexural stiffness coefficients were considered [9–12].

The subject of the vibrations of pipes conveying fluid has been under much consideration. Benjamin [13] made an analysis by neglecting fluid friction effects. Nemat-Nasser *et al.* [14] and Gregory and Paidoussis [15] found the destabilizing effect of dissipation in a cantilevered, fourth order beam conveying fluids. Paidoussis and Li [16] reviewed the dynamics of pipes conveying fluid. Lee and Mote [17, 18] by neglecting gravity and pressure effects, studied energetics of translating one-dimensional uniform strings, highly tensioned pipes with vanishing bending stiffness and tensioned beams. Öz and Boyaci [19] considered the problem in reference [17] and investigated principal parametric resonances of tensioned pipes conveying fluid with harmonic velocity using the method of multiple scales (a perturbation technique). The authors made a stability analysis for different fluid mass to the total mass ratios and for different end conditions.

There are some studies about axially moving continua with masses. Hill and Swanson [20] investigated the stability of a fluid-conveying cantilever pipe having concentrated

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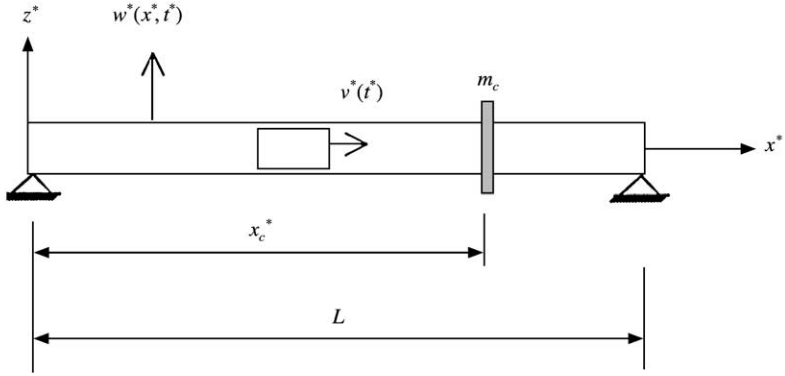

Figure 1. Schematics of the tensioned pipe conveying fluid with a concentrated mass.

masses. Wu and Raju [21] showed that mass at the mid-span of a simply supported pipe could change frequencies and mode shapes. Chen and Jendrzejczyk [22] experimentally studied the natural frequencies and mode shapes of a cantilever pipe with a tip mass. Wickert and Mote [23] modelled a monocable ropeway system as an axially moving string that transports an attached mass particle and investigated frequency and amplitude variation by using the method of strained parameters. Stylianou and Tabarrok [24, 25] used a finite element formulation to show the accuracy of variable-domain beam element, considered translational and rotary inertia effects of the tip mass and made a stability analysis. Chen [26] studied a moving string problem in contact with a stationary load system and made a stability analysis. Borglund [27] considered the stability and optimal design of a beam subject to forces induced by fluid flow through attached pipes with a tip mass by using a finite element formulation. Lee and Mote [28] analyzed energy transfer and mode localization in a translating string coupled to a stationary system using travelling waves. Kang [29] investigated the effect of rotary inertia on the natural frequencies of a fourth order pipe by using the Galerkin method.

In this study, the transverse vibrations of highly tensioned pipes conveying fluid is investigated. The pipe carries a concentrated mass and is fixed at both ends. The fluid velocity is assumed constant. The pipe is assumed to have negligible flexural stiffness. So the equations of motion become second order. The linear equations of motion are solved analytically by means of direct application of the strained parameters method (a perturbation technique). The natural frequencies are found analytically depending on fluid velocity and ratio of fluid mass to total mass per unit length and are drawn for the first two modes. Assuming the concentrated mass is small, the correction term is calculated at the second order of perturbation analysis and perturbed natural frequencies are obtained. The results are compared with other studies. The effect of mass on different locations is investigated.

2. EQUATIONS OF MOTION

For the tensioned pipe conveying fluid in Figure 1, x^* and z^* are the spatial co-ordinates, w^* is transverse displacement. v^* is constant fluid velocity. The tension force in the pipe is P_0 and the length is L. In the analysis, the following assumptions are made: (1) the pipe has negligible flexural stiffness; (2) the transverse displacement is assumed small compared with length L; (3) the tension force is assumed to be sufficiently large compared to the effects arising from additional elongation due to transverse motion; (4) the extensional stiffness is sufficiently large so that the longitudinal deformation resulting from the pretension is negligible; (5) variation of cross-sectional dimensions during vibration is not considered; (6) gravity, pressure and viscous effects of the fluid are neglected and the sub-critical region is considered. Then, a pipe conveying fluid is considered to be a string conveying fluid. Let us denote the time by t^*, the derivatives with respect to the spatial variable by $(\,)'$ and the derivatives with respect to time by $(\cdot)$. The linear equation of transverse motion of the tensioned pipe is [17]

$$(m_f + m_p)\ddot{w}^* + 2m_f v^* \dot{w}^{*\prime} + m_f v^{*2} w^{*\prime\prime} - P_0 w^{*\prime\prime} = 0, \tag{1}$$

where m_f and m_p are the fluid and pipe masses per unit length respectively. The effect of concentrated mass placed at $x^* = x_c^*$ can be modelled as [30, 31]

$$m_c \delta(x^* - x_c^*)\ddot{w}^*, \tag{2}$$

where m_c is the concentrated mass and δ is the Dirac delta function. This equation represents the inertia force due to the lateral acceleration of the concentrated mass. Then, the governing equation for the pipe conveying incompressible fluid and having a concentrated mass can be expressed as

$$\{m_f + m_p + m_c\delta(x^* - x_c^*)\}\ddot{w}^* + 2m_f v^* \dot{w}^{*\prime} + m_f v^{*2} w^{*\prime\prime} - P_0 w^{*\prime\prime} = 0 \tag{3}$$

and the fixed–fixed boundary condition is

$$w^*(0, t^*) = w^*(L, t^*) = 0. \tag{4}$$

Introducing dimensionless parameters, the equation of motion and boundary conditions become

$$\{1 + \alpha\delta(x - x_c)\}\ddot{w} + 2\sqrt{\beta} v \dot{w}' + (v^2 - 1)w'' = 0, \qquad w(0, t) = w(1, t) = 0, \tag{5, 6}$$

where the dimensionless parameters are

$$\alpha = \frac{m_c}{L(m_f + m_p)}, \quad \beta = \frac{m_f}{m_f + m_p}, \quad w = \frac{w^*}{L}, \quad x = \frac{x^*}{L}, \quad x_c = \frac{x_c^*}{L},$$

$$t = \frac{t^*}{L}\sqrt{\frac{P_0}{m_f + m_p}}, \quad v = v^* \sqrt{\frac{m_f}{P_0}}. \tag{7}$$

In equation (5), $\ddot{w}$, $2\dot{w}'v$ and $v^2 w''$ denote local, Coriolis and centrifugal acceleration components respectively.

3. APPROXIMATE SOLUTION

Solutions of the approximate eigenvalue problem are restricted to systems in which mass ratio α, is small. The transition of solutions from those of the tensioned pipe conveying fluid to those of the tensioned pipe conveying fluid and carrying a concentrated mass system is studied by the method of strained parameters to determine a first order perturbation solution for small α. Let us assume the solution to equation (5) to be

$$w(x, t) = Y(x)e^{\lambda t} \tag{8}$$

and make the expansions for both the complex eigenfunction Y and eigenvalue λ,

$$Y(x, \alpha) = Y^{(0)}(x) + \alpha Y^{(1)}(x) + O(\alpha^2), \qquad \lambda = \lambda^{(0)} + \alpha\lambda^{(1)} + O(\alpha^2). \tag{9,10}$$

Substituting equations (9) and (10) into equations (5) and (6), one obtains coupled equations in two orders of α by neglecting higher order of perturbations.

Order α^0:

$$(v^2 - 1)Y^{(0)''} + 2\sqrt{\beta}v\lambda^{(0)}Y^{(0)'} + \lambda^{(0)^2}Y^{(0)} = 0, \qquad Y^{(0)}(0) = Y^{(0)}(1) = 0. \tag{11,12}$$

Order α^1:

$$(v^2 - 1)Y^{(1)''} + 2\sqrt{\beta}v\lambda^{(0)}Y^{(1)'} + \lambda^{(0)^2}Y^{(1)} = -2\lambda^{(1)}(\lambda^{(0)}Y^{(0)} + \sqrt{\beta}vY^{(0)'}) - \delta(x - x_c)\lambda^{(0)^2}Y^{(0)}, \tag{13}$$

$$Y^{(1)}(0) = Y^{(1)}(1) = 0. \tag{14}$$

The solution of equation (11) can be assumed as

$$Y_n^{(0)}(x) = c e^{k_{sn}x}, \tag{15}$$

where k_{sn} is the wave number, $s = d, u$ (downstream and upstream, respectively), and n is the mode number. Since equation (11) is a second order differential equation, the solution gives two wave numbers. Substituting the shape function into equation of order α^0, the dispersion relation is obtained,

$$(v^2 - 1)k_{sn}^2 + 2\sqrt{\beta}v\lambda^{(0)}k_{sn} + \lambda^{(0)^2} = 0. \tag{16}$$

By following the solutions similar to Öz and Boyaci [19] the solution of second order equation (16) gives real downstream, upstream wave numbers and natural frequencies, respectively, as follows:

$$\bar{k}_d = \frac{\bar{\lambda}_n^{(0)}}{\sqrt{\beta}v + \sqrt{1 + (\beta - 1)v^2}}, \qquad \bar{k}_u = \frac{\bar{\lambda}_n^{(0)}}{-\sqrt{\beta}v + \sqrt{1 + (\beta - 1)v^2}},$$

$$\bar{\lambda}_n^{(0)} = \frac{n\pi(1 - v^2)}{\sqrt{1 + (\beta - 1)v^2}}, \tag{17–19}$$

where

$$\bar{k}_d = ik_d, \quad \bar{k}_u = ik_u, \quad \bar{\lambda}_n^{(0)} = i\lambda^{(0)}, \quad i = \sqrt{-1}. \tag{20}$$

The solution of order α^1 gives the first order correction term to the natural frequency of a tensioned pipe conveying fluid, due to the concentrated mass. In terms of new parameters the shape function is

$$Y_n^{(0)}(x) = c(e^{-i\bar{k}_d x} - e^{i\bar{k}_u x}). \tag{21}$$

Since the homogeneous problems described by equations (11) and (12) have a non-trivial solution, the inhomogeneous equation (13) has a non-secular solution if and only if the

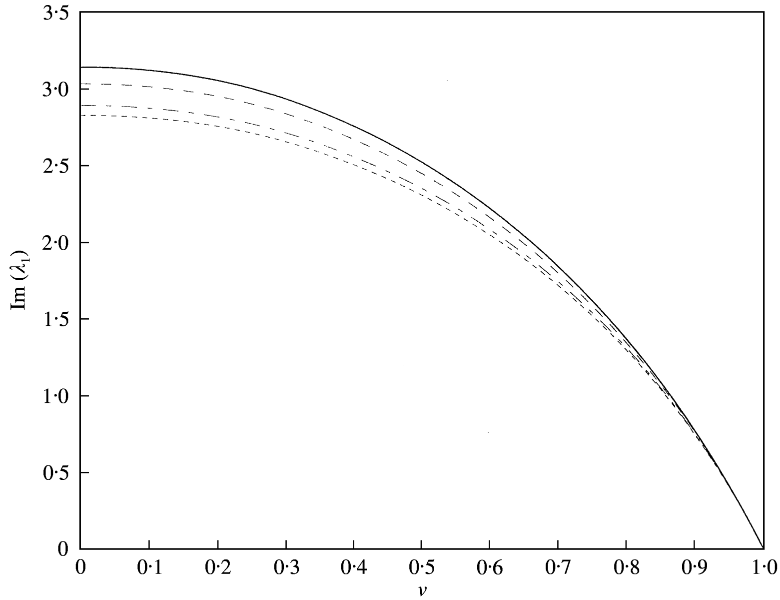

Figure 2. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus flow velocity ($n = 1$, $\beta = 0{\cdot}5$, no mass (——), $\alpha = 0{\cdot}1$, ($x_c = 0{\cdot}20$ (– – –), 0·35, (–.–.–.), 0·50 (·······))).

following solvability condition is satisfied (see reference [32]):

$$\int_0^1 \{2\lambda_n^{(1)}[\lambda_n^{(0)} Y_n^{(0)} + \sqrt{\beta} v Y_n^{(0)'}] + \delta(x - x_c)\lambda_n^{(0)^2} Y_n^{(0)}\} \, \bar{Y}_n^{(0)} \, \mathrm{d}x = 0, \tag{22}$$

where $\bar{Y}_n^{(0)}$ is the complex conjugate of shape function (15). The correction term $\lambda_n^{(1)}$ can be obtained from the solvability condition. The perturbed eigenvalue or natural frequency is

$$\lambda_n = \mathrm{i}\bar{\lambda}_n^{(0)} + \alpha \frac{\bar{\lambda}_n^{(0)^2} | Y_n^{(0)} \bar{Y}_n^{(0)} |_{x=x_c}}{2\{\mathrm{i}\bar{\lambda}_n^{(0)} \int_0^1 Y_n^{(0)} \bar{Y}_n^{(0)} \, \mathrm{d}x + \sqrt{\beta} v \int_0^1 Y_n^{(0)'} \bar{Y}_n^{(0)} \, \mathrm{d}x\}}. \tag{23}$$

The imaginary part of the perturbed eigenvalue corresponds to the frequency of oscillation and the real part is related to amplitude variation.

4. NUMERICAL ANALYSIS

In this section, numerical solutions of the governing equations for a tensioned pipe conveying fluid and carrying a concentrated mass will be given.

If one considers equation (5), several cases can be discussed for restricted parameter values. Without a concentrated mass, $\alpha = 0$, for the limiting case of $\beta = 1$ the equation of motion for a travelling string with constant velocity is obtained as a special case of the second order fluid–pipe system. For a stationary fluid, $\beta = 1$ and $v = 0$, the equation for a linear stationary string is obtained. When α is small and $\beta = 1$, the system becomes a uniform travelling string with a stationary mass constraint.

In Figures 2 and 3 ($\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$), 4 and 5 ($\alpha = 0{\cdot}2$, $\beta = 0{\cdot}5$), the perturbed natural frequency, in other words, the natural frequency of a tensioned pipe conveying fluid system

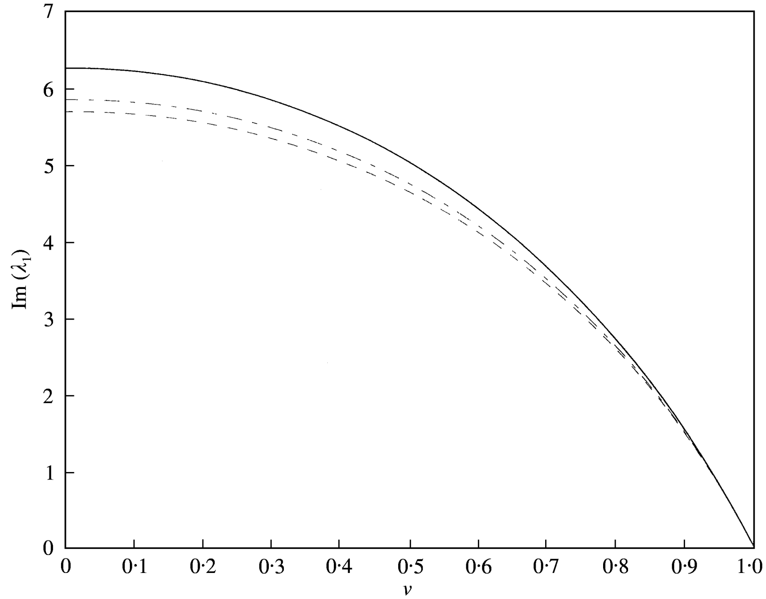

Figure 3. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus flow velocity ($n = 2$, $\beta = 0{\cdot}5$, no mass (——), $\alpha = 0{\cdot}1$, ($x_c = 0{\cdot}20$ (– – –), 0·35, (– . – . – .))).

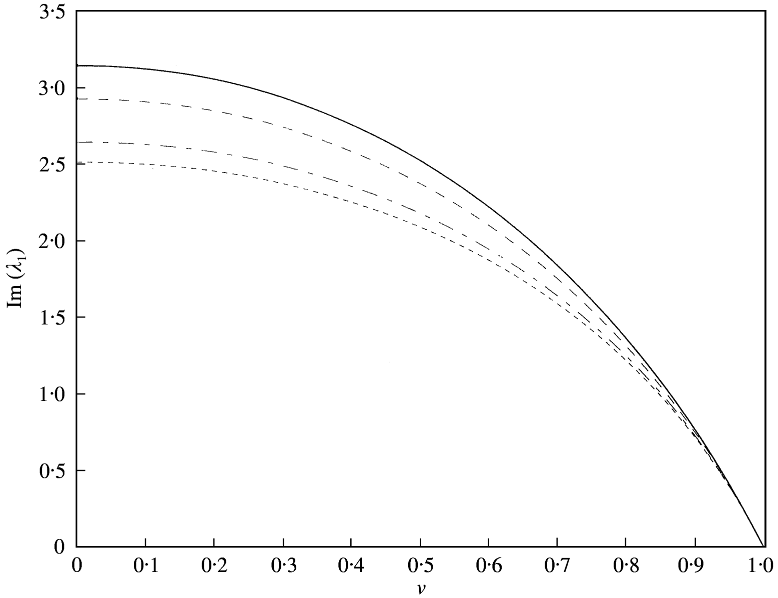

Figure 4. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus flow velocity ($n = 1$, $\beta = 0{\cdot}5$, no mass (——), $\alpha = 0{\cdot}2$, ($x_c = 0{\cdot}20$ (– – –), 0·35, (– . – . – .), 0·50 (·······))).

with a concentrated mass is plotted by taking the imaginary part of equation (23) for the first two modes. The concentrated mass decreases the natural frequencies of all modes if the mass is not at the nodal point of the particular mode. The comparisons of Figure 2 with Figure 4 and Figure 3 with Figure 5 show that an increase in mass ratio α decreases the

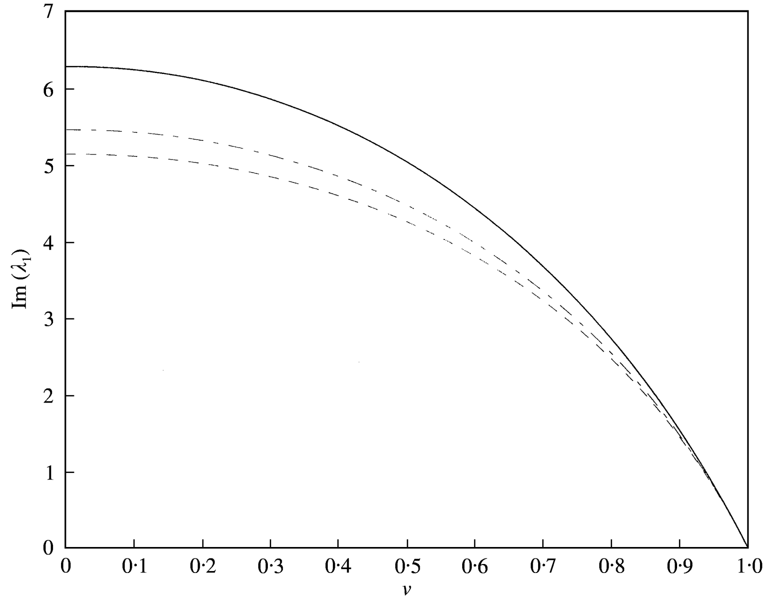

Figure 5. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus flow velocity ($n = 2$, $\beta = 0{\cdot}5$, no mass (——), $\alpha = 0{\cdot}2$, ($x_c = 0{\cdot}20$ (– – –), 0·35, (–.–.–.))).

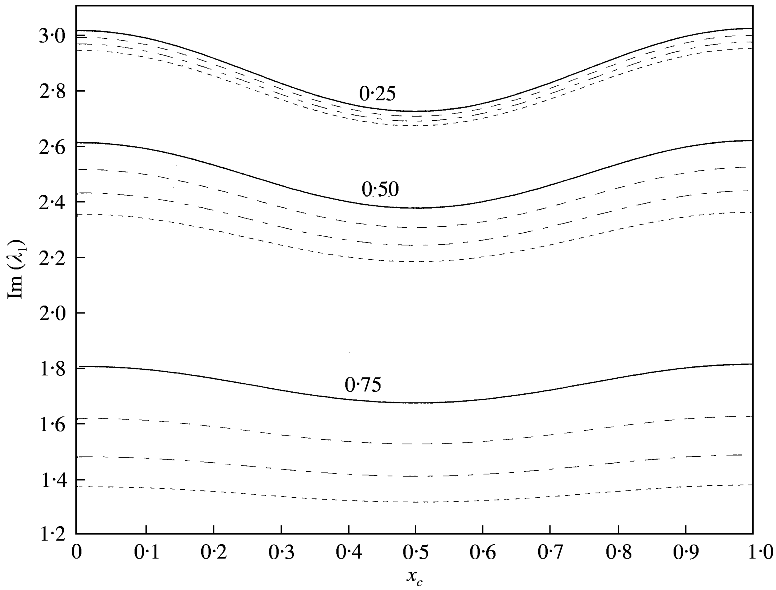

Figure 6. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus position of concentrated mass ($n = 1$, $\alpha = 0{\cdot}1$, ($v = 0{\cdot}25$, 0·50, 0·75), ($\beta = 0{\cdot}25$, (——), 0·50 (– – –), 0·75, (–.–.–.), 1·00 (········))).

frequency for the same modes [23, 26, 28, 29]. Locating the concentrated mass towards the middle of the pipe reduces the frequency of the first mode for the same flow velocity. For the second mode of vibration, locating the mass in the middle does not affect the frequency. In Figures 6 and 7, the perturbed natural frequency variation with the position of the

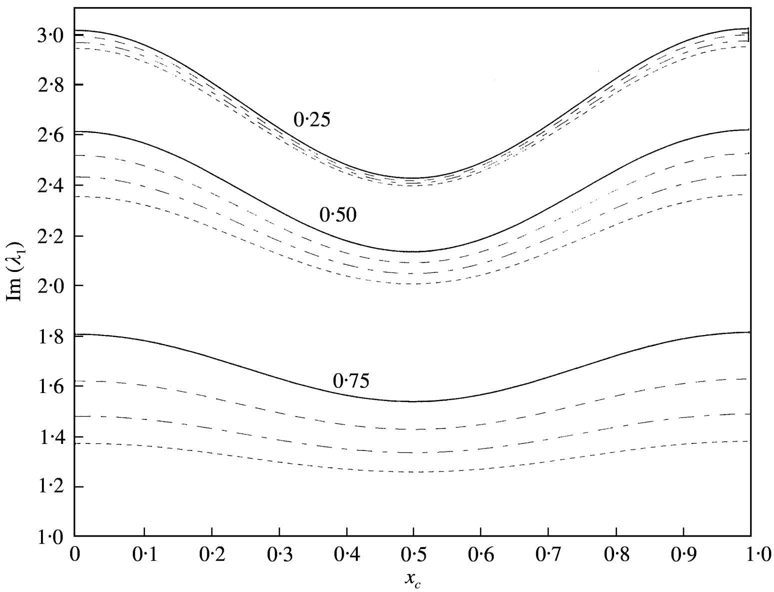

Figure 7. Natural frequency of the tensioned pipe conveying fluid with concentrated mass versus position of concentrated mass ($n = 1$, $\alpha = 0{\cdot}2$, ($v = 0{\cdot}25$, 0·50, 0·75), ($\beta = 0{\cdot}25$, (——), 0·50 (– – –), 0·75, (–.–.–.), 1·00 (········))).

concentrated mass is plotted for different flow velocities (0·25, 0·50, 0·75 shown in the figures), α (0·1, 0·2) and β (0·25, 0·50, 0·75, 1·00) values for the first mode. As shown in Figure 6, the mass effect lowers the frequencies; increasing the flow velocity also lowers the frequency. The effect of β is more important in high fluid velocities. Due to the symmetric boundary conditions, the perturbed frequency variation shows symmetry about the middle point. An increase in α, has much effect towards the middle of the pipe since this section has a larger displacement for the first mode. The real parts of eigenvalues Re(λ_n), are not affected by the concentrated mass and always remain zero. In other words, the amplitude variation is zero; the concentrated mass does not affect the stability of the system as indicated by Chen [26] for an axially travelling string with a stationary load system problem.

5. CONCLUSIONS

The linear transverse vibration of highly tensioned pipes conveying fluid with constant velocity is considered. The pipe has a negligible flexural stiffness and is fixed at both ends. Also, it carries a concentrated mass. The equations of motion are solved analytically applying the strained parameters method. The natural frequencies are determined for different fluid velocities and ratios of fluid mass to total mass per unit length without a concentrated mass. The effect of the value and position of the concentrated mass is investigated for different parameters. Assuming that the concentrated mass is small, the correction term is calculated at the second order of perturbation analysis and perturbed natural frequencies are obtained. The concentrated mass decreases the frequencies through all flow velocities. Its effect increases for higher flow velocities and the ratio of fluid mass to the fluid and pipe masses per unit length. No amplitude variation exists due to the concentrated mass.

ACKNOWLEDGMENTS

This work is supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under NATO B1 scholarship.

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