*Journal of Sound and Vibration* (2002) **250**(3), 503–517
doi:10.1006/jsvi.2001.3922, available online at http://www.idealibrary.com on IDEAL®

# EXACT BUCKLING AND VIBRATION SOLUTIONS FOR STEPPED RECTANGULAR PLATES

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(*Received* 26 *March* 2001, *and in final form* 10 *July* 2001)

This paper is concerned with the determination of exact buckling loads and vibration frequencies of multi-stepped rectangular plates based on the classical thin (Kirchhoff) plate theory. The plate is assumed to have two opposite edges simply supported while the other two edges can take any combination of free, simply supported and clamped conditions. The proposed analytical method for solution involves the Levy method and the state-space technique. By using this analytical method, exact buckling and vibration solutions are obtained for rectangular plates having one- and two-step thickness variations. These exact solutions are extremely useful as benchmark values for researchers developing numerical techniques and software for analyzing non-uniform thickness plates.



## 1. INTRODUCTION

Varying thickness plates are frequently used in order to economize on the plate materials or to lighten the plates, especially when used in wings for high-speed, high-performance aircrafts. By carefully designing the thickness distribution, a substantial increase in stiffness, buckling and vibration capacities of the plate may be obtained over its uniform thickness counterpart.

Focusing our attention on the buckling and vibration of non-uniform thickness plates of a rectangular planform, we find that researchers have investigated various forms of thickness variations that include

(a) a linear function along one direction (e.g., references [1, 2]),
(b) a non-linear function along one direction (e.g., references [3–5]) or in both directions (e.g., references [6, 7]),
(c) piecewise constant step functions in one direction (e.g., references [8–12]), or in both directions (e.g. references [13, 14]),
(d) piecewise linear functions (e.g., reference [15]).

It should be remarked that there are also many papers, in the open literature, dealing with buckling and vibration of circular and annular plates of non-uniform thickness (e.g., references [16–20]).

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When dealing with non-uniform thickness plates, it is generally difficult to obtain exact solutions. Thus, it is not surprising that many of the above-mentioned references reported the use of numerical methods for determining the buckling and vibration solutions. When establishing the convergence, validity and accuracy of numerical methods developed for analyzing non-uniform thickness plates, it is crucial to have exact solutions as benchmarks. So far, exact vibration and buckling solutions of stepped rectangular plates based on the classical thin (Kirchhoff) plate theory have been derived for the sole case of all edges simply supported [9, 12]. Therefore, this study aims to provide much needed exact buckling and vibration solutions of stepped rectangular plates for other boundary conditions as well as an analytical method for exact solutions. Here the stepped plates consist of $n$-step variation in one direction parallel to the plate edges while the thickness is constant in the other direction. By considering two opposite edges to be simply supported in the direction of the stepped variation, the Levy method may be combined with the state-space technique to produce an analytical approach that will enable our objectives to be fulfilled.

## 2. ANALYTICAL MODELLING FOR STEPPED PLATES

Consider an isotropic, elastic, stepped rectangular plate of length $aL$, width $L$, modulus of elasticity $E$, Poisson's ratio $v$ and shear modulus $G = E/[2(1 + v)]$. As shown in Figure 1, the plate has a constant thickness in the $y$ direction and $n$ steps in the $x$ direction, with thickness $h_i$ ($i = 1, 2, \ldots, n$) for the $i$th step. The origin of the co-ordinate system is set at the centre of the bottom edge $BC$ of the plate as shown in Figure 1. The plate is simply supported along two opposite edges that are parallel to the $x$-axis, i.e., edges $AD$ and $BC$. The other two edges $AB$ and $CD$ may be both free or simply supported or clamped. The plate may be subjected to either a uni- or a bi-axial in-plane compressive load. The problem at hand is to determine the critical buckling loads and the vibration frequencies for such an $n$-stepped rectangular plate.

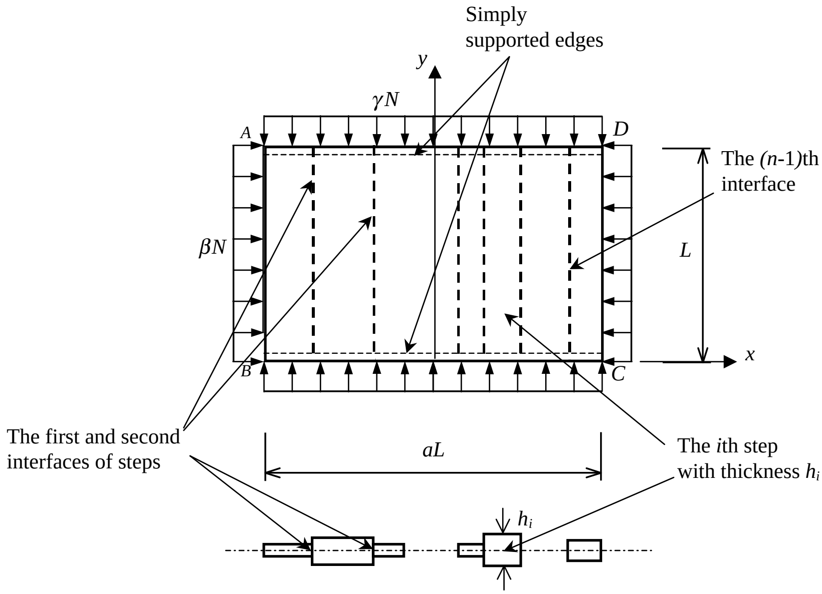


Figure 1. Geometry and co-ordinate system for a multi-stepped rectangular plate.

Based on the classical thin plate theory, the governing differential equation for the $i$th step in harmonic vibration is given by [21]

$$D_i\left(\frac{\partial^4 w_i}{\partial x^4}+2\frac{\partial^4 w_i}{\partial^2 x\partial^2 y}+\frac{\partial^4 w_i}{\partial y^4}\right)+\beta N\frac{\partial^2 w_i}{\partial x^2}+\gamma N\frac{\partial^2 w_i}{\partial y^2}-\rho h_i\omega^2 w_i=0,\quad i=1,2,\ldots,n, \tag{1}$$

in which the subscript $i(=1,2,\ldots,n)$ refers to the $i$th step in the plate, $w_i(x,y)$ is the transverse displacement, $x$ and $y$ are the Cartesian co-ordinates, $D_i=Eh_i^3/[12(1-v^2)]$ is the flexural rigidity of the step, $N$ is the in-plane compressive load, $\rho$ is the mass density of the plate, $\omega$ is the angular frequency of vibration and $\beta$ and $\gamma$ are tracers that take values of either 0 or 1 for different in-plane load combinations.

The essential and natural boundary conditions for the two simply supported edges at $y=0$ and $L$ associated with the $i$th span are [21]

$$w_i=0,\qquad (M_y)_i=0, \tag{2, 3}$$

where $(M_y)_i$ is the bending moment as defined by

$$(M_y)_i=D_i\left(\frac{\partial^2 w_i}{\partial y^2}+v\frac{\partial^2 w_i}{\partial x^2}\right). \tag{4}$$

The essential and natural boundary conditions for the other two edges at $x=-aL/2$ and $aL/2$ (see Figure 1) are given by

$$w_i=0,\qquad (M_x)_i=D_i\left(\frac{\partial^2 w_i}{\partial x^2}+v\frac{\partial^2 w_i}{\partial y^2}\right)=0\quad\text{if the edge is simply supported,} \tag{5a, b}$$

$$w_i=0,\quad \frac{\partial w_i}{\partial x}=0\ \text{ if the edge is clamped,} \tag{6a, b}$$

$$(M_x)_i=D_i\left(\frac{\partial^2 w_i}{\partial x^2}+v\frac{\partial^2 w_i}{\partial y^2}\right)=0,$$

$$(V_x)_i=D_i\left(\frac{\partial^3 w_i}{\partial x^3}+(2-v)\frac{\partial^3 w_i}{\partial x\partial y^2}\right)+\beta N\frac{\partial w_i}{\partial x}=0\quad\text{if the edge is free,} \tag{7a, b}$$

in which the subscript $i$ takes the value of either 1 or $n$, $(M_x)_i$ is the bending moment and $(V_x)_i$ the effective shear force. Note that the free edge condition for the effective shear force $(V_x)_i$ involves the in-plane load $\beta N$. The effect of this in-plane force term on the buckling capacity of plates was discussed in an earlier paper by Liew *et al.* [22].

Adopting the Levy method, the displacement function for the $i$th step of the plate can be expressed as

$$w_i(x,y)=\sin\left(\frac{m\pi}{L}y\right)X_i(x),\quad i=1,2,\ldots,n, \tag{8}$$

where $m$ is the number of half-waves of the buckling or vibration mode in the $y$ direction and $X_i(x)$ is an unknown function to be determined. Equation (8) satisfies the boundary conditions [equations (2) and (3)] for the two simply supported edges at $y=0$ and $L$.

Using the state-space technique, a homogenous differential equation system for the $i$th step can be derived in view of equations (8) and (1):

$$\mathbf{\Psi}_i'-\mathbf{H}_i\mathbf{\Psi}_i=\mathbf{0},\quad i=1,2,\ldots,n, \tag{9}$$

in which

$$\boldsymbol{\Psi}_i = \begin{Bmatrix} X_i \\ X_i' \\ X_i'' \\ X_i''' \end{Bmatrix} \tag{10}$$

and the prime denotes differentiation with respect to $x$, $\boldsymbol{\Psi}_i'$ is the first derivative of $\boldsymbol{\Psi}_i$, and $\mathbf{H}_i$ is a $4 \times 4$ matrix. The non-zero elements of $\mathbf{H}_i$ can be derived as

$$(H_{12})_i = (H_{23})_i = (H_{34})_i = 1, \tag{11}$$

$$(H_{41})_i = -\left(\frac{m\pi}{L}\right)^4 + \frac{\gamma N}{D_i}\left(\frac{m\pi}{L}\right)^2 + \frac{\rho h_i \omega^2}{D_i}, \tag{12}$$

$$(H_{43})_i = 2\left(\frac{m\pi}{L}\right)^2 - \frac{\beta N}{D_i}. \tag{13}$$

The procedure for solving equation (9) has been elaborated in the papers by Xiang *et al.* [23] and Liew *et al.* [22]. The solution for equation (9) may be expressed as

$$\boldsymbol{\Psi}_i = \mathbf{e}^{\mathbf{H}_i x}\mathbf{c}_i, \tag{14}$$

in which $\mathbf{e}^{\mathbf{H}_i x}$ is a general matrix solution for equation (9), $\mathbf{c}_i$ a $4 \times 1$ constant column matrix that is to be determined using the plate boundary conditions [equations (5)–(7)] for the two side steps and/or interface conditions between steps.

Along the interface between the $i$th step and the $(i+1)$th step, the following continuity conditions must be satisfied:

$$w_i = w_{i+1}, \qquad \frac{\partial w_i}{\partial x} = \frac{\partial w_{i+1}}{\partial x}, \qquad (M_x)_i = (M_x)_{i+1}, \qquad (V_x)_i = (V_x)_{i+1}, \tag{15–18}$$

where $(M_x)_i$ and $(M_x)_{i+1}$, and $(V_x)_i$ and $(V_x)_{i+1}$ are the bending moments and effective shear forces for the $i$th and $(i+1)$th steps respectively. The continuity conditions for bending moment and shear force at the step as given by Chopra [8] and Lam and Amrutharaj [10] are, however, not correct.

In view of equation (14), a homogeneous system of equations can be derived by implementing the boundary conditions of the plate along the two edges parallel to the $y$-axis [equations (5)–(7)] and the interface conditions between two steps [equations (15)–(18)] when assembling the steps to form the whole plate

$$\mathbf{K}\begin{Bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_i \\ \mathbf{c}_{i+1} \\ \vdots \\ \mathbf{c}_n \end{Bmatrix} = \{\mathbf{0}\} \tag{19}$$

where $\mathbf{K}$ is a $4n \times 4n$ matrix. The buckling load $N$ ($\omega$ is set to be zero) or the angular frequency $\omega$ ($N$ is set to be zero) is evaluated by setting the determinant of $\mathbf{K}$ in equation (19) to be zero.

## 3. RESULTS AND DISCUSSIONS

The proposed method is used to determine exact buckling and vibration solutions for Levy rectangular plates of multiple steps in the $x$ direction. The number of steps and the lengths of steps may have any feasible combination along the $x$ direction. The buckling load $N$ and the angular frequency $\omega$ are expressed in non-dimensional forms, namely, non-dimensional buckling factor $\lambda = NL^2/(\pi^2 D_1)$ and non-dimensional frequency parameter $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$, respectively, where $h_1$ and $D_1$ are the thickness and the flexural rigidity of the first step, respectively.

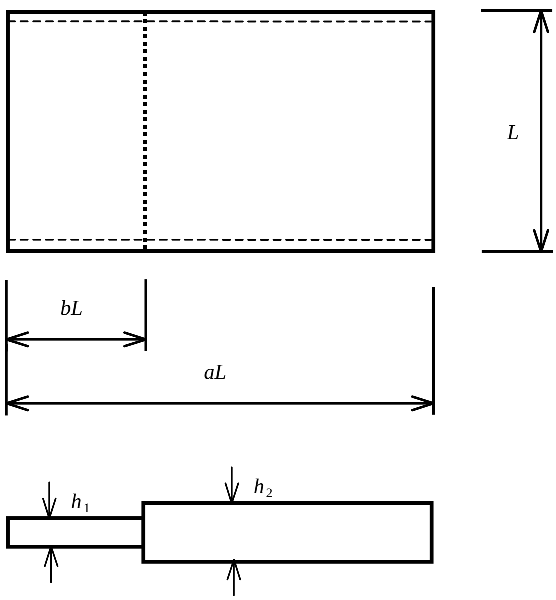


Figure 2. A one-step rectangular Levy plate.

TABLE 1

*Comparison of buckling factors $\lambda = NL^2/\pi^2 D_1$ for a one-step, SS rectangular plate subjected to uniaxial inplane load* $[(\beta, \gamma) = (1, 0),\ a = 2{\cdot}0,\ b = 0{\cdot}5,\ \nu = 0{\cdot}25]$

| | Sources | |
|---|---|---|
| $h_2/h_1$ | Reference [12] | Present study |
| 0·4 | 0·8619 | 0·3083 |
| 0·6 | 1·0245 | 1·0246 |
| 0·8 | 2·3442 | 2·3442 |
| 1·0 | 4·0000 | 4·0000 |
| 1·2 | 4·5324 | 4·5325 |
| 1·4 | 4·6663 | 4·6663 |
| 1·6 | 4·7292 | 4·7292 |
| 1·8 | 4·7652 | 4·7652 |
| 2·0 | 4·7877 | 4·7878 |
| 2·2 | 4·8026 | 4·8027 |

TABLE 2

*Buckling factors $\lambda = NL^2/\pi^2 D_1$ for one-step SS and FF rectangular plates subjected to either uniaxial or biaxial inplane compressive loads*

| | | | *SS* plate *b* | | | *FF* plate *b* | | |
|---|---|---|---|---|---|---|---|---|
| ($\beta$, $\gamma$) | $a$ | $h_2/h_1$ | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| (1, 0) | 1 | 1·2 | 5·7436 | 4·9654 | 4·5131 | 2·5640 | 2·3468 | 2·2768 |
| | | 1·5 | 7·6886 | 6·0456 | 5·1516 | 3·0733 | 2·4783 | 2·3473 |
| | | 2·0 | 10·430 | 7·7696 | 5·8978 | 3·5939 | 2·5589 | 2·3815 |
| | 2 | 1·2 | 5·0009 | 4·5310 | 4·3008 | 2·3757 | 2·3262 | 2·3127 |
| | | 1·5 | 5·8262 | 4·7003 | 4·3989 | 2·4172 | 2·3448 | 2·3186 |
| | | 2·0 | 6·8621 | 4·7862 | 4·4553 | 2·4417 | 2·3647 | 2·3244 |
| (0, 1) | 1 | 1·2 | 5·9819 | 5·1996 | 4·6049 | 1·3839 | 1·2421 | 1·1287 |
| | | 1·5 | 9·5017 | 7·0659 | 5·5938 | 2·2127 | 1·7034 | 1·3819 |
| | | 2·0 | 16·437 | 10·878 | 7·5927 | 4·2200 | 2·5593 | 1·7741 |
| | 2 | 1·2 | 2·3690 | 2·0088 | 1·7467 | 1·3037 | 1·1627 | 1·0865 |
| | | 1·5 | 3·7755 | 2·6211 | 1·9847 | 1·6744 | 1·2998 | 1·1550 |
| | | 2·0 | 6·5860 | 3·5793 | 2·3002 | 2·2368 | 1·4433 | 1·2123 |
| (1, 1) | 1 | 1·2 | 2·9547 | 2·5602 | 2·2867 | 1·2597 | 1·1276 | 1·0523 |
| | | 1·5 | 4·4051 | 3·3264 | 2·7171 | 1·7300 | 1·3225 | 1·1489 |
| | | 2·0 | 6·6870 | 4·7291 | 3·4449 | 2·4948 | 1·5552 | 1·2505 |
| | 2 | 1·2 | 1·8462 | 1·5589 | 1·3789 | 1·1130 | 1·0348 | 1·0044 |
| | | 1·5 | 2·6356 | 1·8821 | 1·5106 | 1·2416 | 1·0731 | 1·0209 |
| | | 2·0 | 3·8359 | 2·2513 | 1·6401 | 1·3822 | 1·1073 | 1·0338 |

TABLE 3

*Buckling factors $\lambda = NL^2/\pi^2 D_1$ for one-step CC and SF rectangular plates subjected to either uniaxial or biaxial inplane compressive loads*

| | | | *CC* plate *b* | | | *SF* plate *b* | | |
|---|---|---|---|---|---|---|---|---|
| ($\beta$, $\gamma$) | $a$ | $h_2/h_1$ | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| (1, 0) | 1 | 1·2 | 10·212 | 8·4000 | 7·6096 | 4·0009 | 3·8018 | 3·4939 |
| | | 1·5 | 14·888 | 10·328 | 8·7916 | 6·9785 | 5·7575 | 4·9043 |
| | | 2·0 | 19·699 | 13·812 | 9·8568 | 10·391 | 7·7614 | 5·8884 |
| | 2 | 1·2 | 6·7887 | 5·7546 | 5·2077 | 3·9867 | 3·9497 | 3·7560 |
| | | 1·5 | 8·5901 | 6·3081 | 5·3578 | 5·8232 | 4·6979 | 4·3986 |
| | | 2·0 | 11·199 | 6·5874 | 5·4188 | 6·8609 | 4·7850 | 4·4513 |
| (0, 1) | 1 | 1·2 | 11·801 | 9·9392 | 8·5789 | 2·2647 | 2·0928 | 1·8682 |
| | | 1·5 | 19·269 | 13·206 | 9·8104 | 4·0820 | 3·4098 | 2·6906 |
| | | 2·0 | 34·010 | 18·822 | 11·526 | 8·5470 | 6·3343 | 4·3993 |
| | 2 | 1·2 | 2·9501 | 2·4848 | 2·1447 | 1·8028 | 1·6439 | 1·4398 |
| | | 1·5 | 4·8172 | 3·3015 | 2·4526 | 3·2322 | 2·4324 | 1·8283 |
| | | 2·0 | 8·6174 | 4·7056 | 2·8816 | 6·2400 | 3·5087 | 2·2387 |
| (1, 1) | 1 | 1·2 | 5·7457 | 4·8730 | 4·3978 | 1·7386 | 1·6289 | 1·4685 |
| | | 1·5 | 8·8406 | 6·2352 | 5·2751 | 3·1433 | 2·6077 | 2·0875 |
| | | 2·0 | 13·063 | 8·7744 | 6·4524 | 5·8700 | 4·1991 | 3·0086 |
| | 2 | 1·2 | 2·2687 | 1·8772 | 1·6286 | 1·6558 | 1·4850 | 1·3090 |
| | | 1·5 | 3·4997 | 2·3657 | 1·8086 | 2·6051 | 1·8689 | 1·4902 |
| | | 2·0 | 5·4894 | 3·0360 | 2·0038 | 3·8304 | 2·2483 | 1·6338 |

TABLE 4

*Buckling factors $\lambda = NL^2/\pi^2 D_1$ for one-step CF and CS rectangular plates subjected to either uniaxial or biaxial inplane compressive loads*

| | | | *CF* plate $b$ | | | *CS* plate $b$ | | |
|---|---|---|---|---|---|---|---|---|
| $(\beta, \gamma)$ | $a$ | $h_2/h_1$ | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| (1, 0) | 1 | 1·2 | 4·1093 | 3·9825 | 3·6674 | 7·7409 | 6·8258 | 5·8235 |
| | | 1·5 | 7·8834 | 7·0730 | 5·9511 | 13·078 | 9·0189 | 7·0847 |
| | | 2·0 | 16·534 | 11·967 | 8·8707 | 18·790 | 12·358 | 9·0650 |
| | 2 | 1·2 | 3·9867 | 3·9498 | 3·7618 | 6·5313 | 5·6097 | 5·0247 |
| | | 1·5 | 7·7730 | 6·2768 | 5·2902 | 8·5518 | 6·2882 | 5·2974 |
| | | 2·0 | 11·187 | 6·5803 | 5·3833 | 11·194 | 6·5839 | 5·3997 |
| (0, 1) | 1 | 1·2 | 2·6573 | 2·4956 | 2·2381 | 8·6604 | 7·6638 | 6·6916 |
| | | 1·5 | 4·8115 | 4·1528 | 3·2932 | 14·186 | 10·558 | 8·1744 |
| | | 2·0 | 10·308 | 7·8700 | 5·5126 | 25·216 | 15·979 | 11·061 |
| | 2 | 1·2 | 1·8719 | 1·7372 | 1·5203 | 2·6850 | 2·2937 | 1·9598 |
| | | 1·5 | 3·4806 | 2·7652 | 2·0336 | 4·5104 | 3·1418 | 2·2920 |
| | | 2·0 | 7·3365 | 4·4057 | 2·6515 | 8·3090 | 4·5832 | 2·7653 |
| (1, 1) | 1 | 1·2 | 1·8992 | 1·8151 | 1·6500 | 4·1201 | 3·6310 | 3·1403 |
| | | 1·5 | 3·5538 | 3·1647 | 2·5416 | 6·8334 | 4·9233 | 3·8281 |
| | | 2·0 | 7·8354 | 5·7974 | 4·0789 | 11·251 | 7·1159 | 5·1219 |
| | 2 | 1·2 | 1·7059 | 1·5990 | 1·3997 | 2·1193 | 1·7873 | 1·5312 |
| | | 1·5 | 3·1576 | 2·2769 | 1·7013 | 3·4043 | 2·3168 | 1·7430 |
| | | 2·0 | 5·4163 | 3·0064 | 1·9590 | 5·4486 | 3·0151 | 1·9717 |

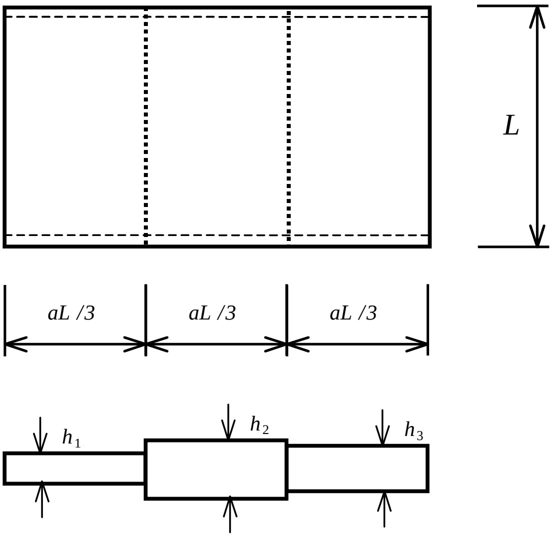


Figure 3. A two-even-step rectangular Levy plate.

For brevity, the letters $F$, $S$ and $C$ are used to denote a free edge, a simply supported edge and a clamped edge respectively. A two-letter symbol is used to describe the plate boundary conditions on the two edges parallel to the $y$-axis. For instance, an $SF$ plate has the edge $AB$ simply supported and the edge $DC$ free (see Figure 1). The Poisson ratio $\nu = 0{\cdot}3$ is adopted for all cases in the paper.

TABLE 5

*Buckling factors $\lambda = NL^2/\pi^2 D_1$ for two-even-step rectangular plates subjected to either uniaxial or biaxial inplane compressive loads*

| $(\beta, \gamma)$ | $a$ | $h_2/h_1$ | $h_3/h_1$ | Cases | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *SS* | *FF* | *CC* | *SF* | *CF* | *CS* |
| (1, 0) | 1 | 1·2 | 1·0 | 4·9317 | 2·2463 | 8·8337 | 2·6073 | 2·6221 | 5·8590 |
| | | 1·5 | 1·0 | 6·0992 | 2·5795 | 11·534 | 2·9102 | 2·9198 | 7·0771 |
| | | 1·2 | 1·5 | 6·1721 | 2·5988 | 11·231 | 5·7874 | 6·9218 | 9·4137 |
| | | 1·5 | 2·0 | 7·9559 | 2·9533 | 14·990 | 7·9410 | 13·018 | 13·978 |
| | 2 | 1·2 | 1·0 | 4·5759 | 2·3251 | 5·9622 | 2·3567 | 2·3571 | 4·8364 |
| | | 1·5 | 1·0 | 5·2811 | 2·3684 | 7·3397 | 2·3863 | 2·3866 | 5·5090 |
| | | 1·2 | 1·5 | 4·9389 | 2·3621 | 6·6836 | 4·9371 | 6·6410 | 6·6412 |
| | | 1·5 | 2·0 | 5·5753 | 2·3906 | 8·1340 | 5·5733 | 8·1181 | 8·1265 |
| | 3 | 1·2 | 1·0 | 4·4698 | 2·3252 | 5·4852 | 2·3268 | 2·3268 | 4·5317 |
| | | 1·5 | 1·0 | 4·6804 | 2·3447 | 6·2289 | 2·3448 | 2·3448 | 4·6975 |
| | | 1·2 | 1·5 | 4·5435 | 2·3270 | 5·6703 | 4·5433 | 5·6677 | 5·6702 |
| | | 1·5 | 2·0 | 4·7006 | 2·3450 | 6·2898 | 4·7006 | 6·2897 | 6·2898 |
| (0, 1) | 1 | 1·2 | 1·0 | 5·0074 | 1·1544 | 10·223 | 1·6840 | 1·9527 | 7·1017 |
| | | 1·5 | 1·0 | 6·5167 | 1·5784 | 13·662 | 2·2488 | 2·5622 | 9·1060 |
| | | 1·2 | 1·5 | 7·0442 | 1·6489 | 13·107 | 3·2452 | 3·9199 | 10·463 |
| | | 1·5 | 2·0 | 11·319 | 2·6274 | 20·514 | 6·2503 | 7·7794 | 17·368 |
| | 2 | 1·2 | 1·0 | 2·0474 | 1·1510 | 2·5556 | 1·2759 | 1·2957 | 2·2521 |
| | | 1·5 | 1·0 | 2·8899 | 1·4280 | 3·6474 | 1·5513 | 1·5632 | 3·1795 |
| | | 1·2 | 1·5 | 2·6064 | 1·3304 | 3·2768 | 2·3667 | 2·6370 | 3·0779 |
| | | 1·5 | 2·0 | 3·9384 | 1·6180 | 5·1286 | 3·8100 | 4·6391 | 4·9420 |
| | 3 | 1·2 | 1·0 | 1·6442 | 1·1263 | 1·8109 | 1·1706 | 1·1730 | 1·7116 |
| | | 1·5 | 1·0 | 2·2752 | 1·2735 | 2·6104 | 1·3003 | 1·3008 | 2·3854 |
| | | 1·2 | 1·5 | 1·9005 | 1·1852 | 2·1219 | 1·8866 | 2·0785 | 2·1069 |
| | | 1·5 | 2·0 | 2·5837 | 1·3071 | 3·0664 | 2·5812 | 3·0542 | 3·0605 |
| (1, 1) | 1 | 1·2 | 1·0 | 2·4882 | 1·1078 | 4·8755 | 1·2282 | 1·3105 | 3·2792 |
| | | 1·5 | 1·0 | 3·1677 | 1·4536 | 6·2950 | 1·5520 | 1·6233 | 4·1112 |
| | | 1·2 | 1·5 | 3·3652 | 1·3293 | 6·6129 | 2·5248 | 2·9941 | 4·9832 |
| | | 1·5 | 2·0 | 4·8753 | 1·7501 | 9·6985 | 4·2757 | 5·9978 | 7·9762 |
| | 2 | 1·2 | 1·0 | 1·6194 | 1·0634 | 1·9871 | 1·0882 | 1·0912 | 1·7456 |
| | | 1·5 | 1·0 | 2·1884 | 1·1810 | 2·7934 | 1·1907 | 1·1914 | 2·3492 |
| | | 1·2 | 1·5 | 1·9250 | 1·1011 | 2·4191 | 1·9034 | 2·2781 | 2·3456 |
| | | 1·5 | 2·0 | 2·5713 | 1·1992 | 3·4631 | 2·5649 | 3·4038 | 3·4239 |
| | 3 | 1·2 | 1·0 | 1·4426 | 1·0290 | 1·5999 | 1·0350 | 1·0351 | 1·4891 |
| | | 1·5 | 1·0 | 1·8211 | 1·0715 | 2·1815 | 1·0731 | 1·0731 | 1·8619 |
| | | 1·2 | 1·5 | 1·5512 | 1·0365 | 1·7680 | 1·5506 | 1·7634 | 1·7651 |
| | | 1·5 | 2·0 | 1·8837 | 1·0736 | 2·3192 | 1·8836 | 2·3184 | 2·3186 |

### 3.1. BUCKLING OF STEPPED PLATES

Consider a one-step *SS* plate as shown in Figure 2. The plate is subjected to a uniaxial inplane compressive load in the $x$ direction (i.e. $\beta = 1$, $\gamma = 0$). Table 1 compares our exact results with the very accurate ones computed by Eisenberger and Alexandov [12], who used exact beam stability functions in the stiffness method and performed the analysis in two directions in cycles. The two sets of results are in excellent agreement, with the exception of the case $h_2/h_1 = 0{\cdot}4$. The difference is attributed to the fact that Eisenberger and Alexandov

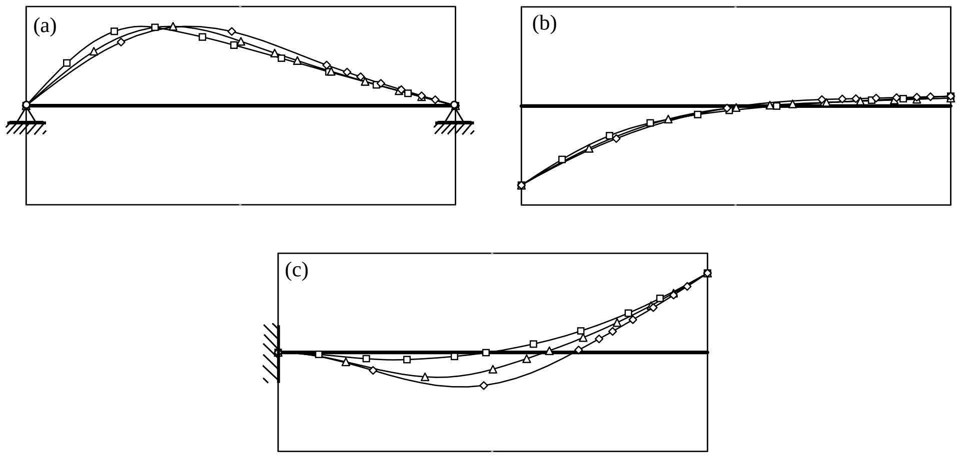


Figure 4. Normalized buckling modal shapes in the $x$ direction for one-step *SS*, *FF* and *CF* square plates with varying step length parameter $b$. The step thickness ratio is $h_2/h_1 = 1{\cdot}5$. The number of half-waves in the $y$ direction is $m = 1$ for all cases. The plates are subjected to uniaxial load in the $x$ direction ($\beta = 1, \gamma = 0$). (a) *SS* plates, (b) *FF* plates, (c) *CF* plates: □, $b = 0{\cdot}3$; △, $b = 0{\cdot}5$; ◇, $b = 0{\cdot}7$.

TABLE 6

*Comparison of frequency parameters $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$ for a one-step SS rectangular plate*

| | | | | Mode number | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $a$ | $b$ | $h_2/h_1$ | Sources | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0·25 | 0·5 | Reference [9] | 1·29333 | 2·87183 | 2·89981 | 4·92249 | 4·41555 | 5·67965 |
| | | | Present | 1·29333 | 2·87182 | 2·89981 | 4·92248 | 5·41555 | 5·67965 |
| | | 0·8 | Reference [9] | 1·70392 | 4·18715 | 4·19685 | 6·76611 | 8·25094 | 8·48212 |
| | | | Present | 1·70392 | 4·18715 | 4·19685 | 6·76611 | 8·25094 | 8·48212 |
| | 0·75 | 0·5 | Reference [9] | 1·62903 | 4·04892 | 4·34142 | 6·86923 | 8·57562 | 8·71333 |
| | | | Present | 1·62904 | 4·04892 | 4·3414 | 6·86923 | 8·57562 | 8·71333 |
| | | 0·8 | Reference [9] | 1·88936 | 4·68981 | 4·78334 | 7·56023 | 9·40069 | 9·62732 |
| | | | Present | 1·88936 | 4·68981 | 4·78334 | 7·56024 | 9·40069 | 9·62731 |
| 2 | 0·5 | 0·5 | Reference [9] | 0·89787 | 1·40673 | 2·34063 | 2·50701 | 3·40224 | 3·66570 |
| | | | Present | 0·89787 | 1·40673 | 2·34063 | 2·50701 | 3·40224 | 3·66570 |
| | | 0·8 | Reference [9] | 1·11745 | 1·79546 | 2·89624 | 3·68986 | 4·48273 | 4·54319 |
| | | | Present | 1·11745 | 1·79546 | 2·89625 | 3·68986 | 4·48273 | 4·54318 |

[12] obtained the buckling load factor that corresponds to the third buckling mode while the authors obtained the correct value for the first buckling mode. Tables 2–4 present sample buckling factors for *SS* and *FF* plates, *CC* and *SF* plates and *CF* and *CS* plates respectively. The plates are subjected to either a uniaxial inplane load in the $x$ direction (i.e., $\beta = 1$, $\gamma = 0$) or a uniaxial inplane load in the $y$ direction (i.e., $\beta = 0$, $\gamma = 1$) or biaxial in-plane loads (i.e., $\beta = 1, \gamma = 1$). The results show the significant differences in the buckling loads with respect to changing step-lengths and thicknesses. These influencing factors may

TABLE 7

*Frequency parameters $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$ for a one-step SS and FF rectangular plates*

| | | | *SS* plate $b$ | | | *FF* plate $b$ | | |
|---|---|---|---|---|---|---|---|---|
| $a$ | $h_2/h_1$ | Mode | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| 1 | 1·5 | 1 | 2·6289 | 2·4471 | 2·3108 | 1·3289 | 1·2266 | 1·1361 |
| | | 2 | 6·5380 | 6·1338 | 5·5291 | 2·2613 | 2·0624 | 1·8960 |
| | | 3 | 6·7603 | 6·2229 | 5·5639 | 4·9784 | 4·4695 | 4·2152 |
| | | 4 | 10·724 | 9·8576 | 9·0309 | 5·0123 | 4·5048 | 4·2724 |
| | | 5 | 13·432 | 11·801 | 10·716 | 6·4950 | 6·2066 | 5·8001 |
| | | 6 | 13·501 | 11·948 | 11·364 | 9·4696 | 8·8575 | 8·2737 |
| | 2·0 | 1 | 3·1452 | 2·9015 | 2·6709 | 1·7251 | 1·4928 | 1·2923 |
| | | 2 | 8·0892 | 7·1156 | 5·8447 | 2·9303 | 2·4441 | 2·1574 |
| | | 3 | 8·4235 | 7·1830 | 6·0116 | 5·8006 | 4·7767 | 4·3934 |
| | | 4 | 13·515 | 11·254 | 10·082 | 5·9243 | 5·1756 | 4·7678 |
| | | 5 | 16·511 | 12·864 | 11·088 | 8·3368 | 7·7109 | 6·8686 |
| | | 6 | 16·536 | 13·785 | 12·032 | 11·133 | 9·7827 | 8·9592 |
| 2 | 1·5 | 1 | 1·6901 | 1·5335 | 1·3910 | 1·2446 | 1·1262 | 1·0681 |
| | | 2 | 2·6809 | 2·4644 | 2·2577 | 1·6238 | 1·5516 | 1·4500 |
| | | 3 | 4·3543 | 3·9634 | 3·6606 | 2·3674 | 2·2144 | 2·0684 |
| | | 4 | 5·5759 | 4·8031 | 4·4517 | 3·6974 | 3·4490 | 3·1954 |
| | | 5 | 6·5669 | 6·1128 | 5·6307 | 4·3885 | 4·1476 | 4·0691 |
| | | 6 | 6·8001 | 6·3178 | 5·6822 | 5·7174 | 5·2297 | 4·8184 |
| | 2·0 | 1 | 2·1059 | 1·7957 | 1·5029 | 1·4501 | 1·1942 | 1·0984 |
| | | 2 | 3·3787 | 2·8135 | 2·5205 | 2·0842 | 1·9277 | 1·7172 |
| | | 3 | 5·2651 | 4·6813 | 3·8991 | 2·8602 | 2·6516 | 2·3110 |
| | | 4 | 6·4307 | 5·0140 | 4·5252 | 4·5498 | 3·8887 | 3·5969 |
| | | 5 | 7·8135 | 6·8045 | 5·9839 | 4·5826 | 4·1855 | 4·0831 |
| | | 6 | 8·6194 | 7·3314 | 6·1850 | 6·9874 | 5·9087 | 5·0433 |

be optimally designed to economize on the plate material. As expected, plates associated with a greater supporting restraint on the boundaries have higher buckling loads. The plate boundary conditions associated with buckling factors in ascending order of magnitude are *FF*, *SF*, *CF*, *SS*, *CS* and *CC*.

Next, we consider two-step rectangular plates with equal step length as shown in Figure 3. Table 5 presents sample buckling factors $\lambda = NL^2/\pi^2 D_1$ for such two-step rectangular plates subjected to either uniaxial or biaxial in-plane compressive loads.

The buckling modal shapes are examined for one-step *SS*, *FF* and *CF* square plates (with $b = 0{\cdot}3, 0{\cdot}5, 0{\cdot}7$) subjected to uniaxial inplane load in the $x$ direction. Figure 4 shows the normalized buckling modal shapes for the plates along the line $y = L/2$, parallel to the $x$-axis. It can be seen that the buckling modal shapes, for the stepped *SS* plates, are not symmetrical about the $y$-axis of the plates. These modal shapes are skewed towards the weaker (left) portion of the plate and more so as $b$ takes on smaller values. For the *FF* plates, the deflection mainly occurs at the left portion of the plates where the plates are of the smaller thickness value $h_1$ and the modal shapes are almost the same for the $b$ values considered. The buckling modal shapes for the *CF* plates reveal that the plate with a larger step length parameter ($b = 0{\cdot}7$) has a greater deflection at the mid-span. By knowing the modal shapes, the engineer can make an informed decision on where to place the internal

TABLE 8

*Frequency parameters $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$ for a one-step CC and SF rectangular plates*

| | | | *CC* plate | | | *SF* plate | | |
|---|---|---|---|---|---|---|---|---|
| | | | $b$ | | | $b$ | | |
| $a$ | $h_2/h_1$ | Mode | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| 1 | 1·5 | 1 | 3·7938 | 3·5610 | 3·4866 | 1·6672 | 1·5752 | 1·4702 |
| | | 2 | 7·4960 | 6·8275 | 6·2024 | 3·7003 | 3·4988 | 3·2496 |
| | | 3 | 9·0923 | 8·7199 | 7·8316 | 6·0260 | 5·6474 | 5·1578 |
| | | 4 | 12·776 | 11·769 | 10·752 | 8·0345 | 7·5529 | 6·9172 |
| | | 5 | 14·195 | 12·409 | 11·118 | 8·3047 | 7·7256 | 7·0154 |
| | | 6 | 17·473 | 15·621 | 14·703 | 12·925 | 11·705 | 10·614 |
| | 2·0 | 1 | 4·4503 | 4·1711 | 4·0439 | 2·1252 | 1·9604 | 1·7828 |
| | | 2 | 9·2716 | 8·0867 | 6·7397 | 4·4600 | 4·2453 | 3·6734 |
| | | 3 | 11·032 | 9·9047 | 8·5478 | 7·7778 | 6·8894 | 5·8298 |
| | | 4 | 15·992 | 13·276 | 11·568 | 10·115 | 8·5588 | 7·6000 |
| | | 5 | 17·659 | 13·835 | 11·951 | 10·436 | 9·1223 | 8·1146 |
| | | 6 | 21·712 | 18·045 | 15·440 | 16·211 | 12·850 | 11·066 |
| 2 | 1·5 | 1 | 1·8740 | 1·7069 | 1·5506 | 1·5065 | 1·4119 | 1·2895 |
| | | 2 | 3·1940 | 2·9422 | 2·6880 | 2·0086 | 1·8882 | 1·7539 |
| | | 3 | 5·2638 | 4·7599 | 4·3771 | 3·2312 | 2·9364 | 2·7613 |
| | | 4 | 5·8247 | 4·9549 | 4·5262 | 5·0314 | 4·6562 | 4·2307 |
| | | 5 | 7·1092 | 6·5752 | 5·8590 | 5·5626 | 4·8006 | 4·4458 |
| | | 6 | 7·8558 | 7·2549 | 6·7277 | 6·1728 | 6·0170 | 5·5096 |
| | 2·0 | 1 | 2·3179 | 2·0217 | 1·6849 | 1·9445 | 1·7224 | 1·4575 |
| | | 2 | 3·9980 | 3·3189 | 2·9877 | 2·5287 | 2·2806 | 2·0286 |
| | | 3 | 6·4516 | 5·2381 | 4·6154 | 4·0528 | 3·3822 | 3·1180 |
| | | 4 | 6·9588 | 5·5929 | 4·6410 | 6·0004 | 5·0139 | 4·5243 |
| | | 5 | 8·9943 | 7·6968 | 6·2563 | 6·4299 | 5·4615 | 4·6273 |
| | | 6 | 9·2623 | 8·0498 | 7·2097 | 8·0986 | 7·2862 | 5·9677 |

restraint or support that will enhance the buckling load. The best place for internal supports is usually in the vicinity of the nodal lines of the modal shapes (see reference [24]).

### 3.2. VIBRATION OF STEPPED PLATES

We first consider one-step *SS* plates. Inspection of the first six natural frequencies given in Table 6 shows total agreement with the exact results obtained by Yuan and Dickinson [9]. Tables 7–9 present sample vibration frequencies for *SS* and *FF* plates, *CC* and *SF* plates and *CF* and *CS* plates respectively. As in the buckling problem, the vibration frequencies of the plate varies significantly with respect to the step-lengths, the step-thicknesses and the boundary conditions. Table 10 shows the frequency parameters for square and rectangular plates with two even steps.

Figure 5 depicts the first six vibration modal shapes in the $x$ direction for the two-even-step *SS*, *FF* and *CF* rectangular plates. The plate aspect ratio is set to be $a = 3$ and the plate step thickness ratios are $h_2/h_1 = 1·5$ and $h_3/h_1 = 2·0$. The influence of the steps on the modal shapes of the plates can be seen from Figure 5. These higher modal shapes should provide useful information to engineers as to where the internal restraints are best positioned.

TABLE 9

*Frequency parameter* $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$ *for a one-step CF and CS rectangular plates*

| | | | *CF* plate $b$ | | | *CS* plate $b$ | | |
|---|---|---|---|---|---|---|---|---|
| $a$ | $h_2/h_1$ | Mode | 0·3 | 0·5 | 0·7 | 0·3 | 0·5 | 0·7 |
| 1 | 1·5 | 1 | 1·7973 | 1·7072 | 1·5940 | 3·1406 | 2·9175 | 2·7634 |
| | | 2 | 4·4020 | 4·1227 | 3·8722 | 7·1939 | 6·5719 | 5·9446 |
| | | 3 | 6·1574 | 5·8338 | 5·3408 | 7·6985 | 7·4291 | 6·6141 |
| | | 4 | 8·6412 | 8·1247 | 7·5055 | 11·685 | 10·891 | 9·8553 |
| | | 5 | 9·5773 | 9·1726 | 8·0704 | 14·073 | 12·350 | 11·045 |
| | | 6 | 13·408 | 12·165 | 10·894 | 15·233 | 13·721 | 13·099 |
| | 2·0 | 1 | 2·2938 | 2·1098 | 1·9208 | 3·7451 | 3·3789 | 3·1878 |
| | | 2 | 5·2187 | 4·9121 | 4·4271 | 8·9838 | 7·8593 | 6·5543 |
| | | 3 | 8·0045 | 7·2940 | 6·2261 | 9·2329 | 8·7240 | 6·9324 |
| | | 4 | 10·780 | 9·8738 | 8·5783 | 14·638 | 12·509 | 10·922 |
| | | 5 | 11·744 | 10·362 | 8·7032 | 17·580 | 13·814 | 11·538 |
| | | 6 | 17·219 | 13·780 | 11·504 | 19·067 | 15·439 | 14·007 |
| 2 | 1·5 | 1 | 1·5394 | 1·4585 | 1·3352 | 1·7985 | 1·6430 | 1·4861 |
| | | 2 | 2·1603 | 2·0312 | 1·8764 | 2·9212 | 2·7228 | 2·4638 |
| | | 3 | 3·5213 | 3·2367 | 3·0092 | 4·8001 | 4·3354 | 4·0433 |
| | | 4 | 5·5489 | 4·9483 | 4·5134 | 5·8172 | 4·9529 | 4·5214 |
| | | 5 | 5·7725 | 5·0646 | 4·6335 | 6·9742 | 6·5084 | 5·8130 |
| | | 6 | 6·2579 | 6·0868 | 5·6378 | 7·2587 | 6·7147 | 6·1900 |
| | 2·0 | 1 | 2·0011 | 1·8235 | 1·5565 | 2·2460 | 1·9648 | 1·6386 |
| | | 2 | 2·6950 | 2·4684 | 2·1446 | 3·6594 | 3·1272 | 2·7305 |
| | | 3 | 4·4390 | 3·6868 | 3·4138 | 5·9573 | 5·0734 | 4·3126 |
| | | 4 | 6·7772 | 5·2375 | 4·6129 | 6·9573 | 5·2379 | 4·6141 |
| | | 5 | 6·9530 | 5·9959 | 5·0093 | 8·5683 | 7·5321 | 6·2421 |
| | | 6 | 8·1682 | 7·5689 | 6·2160 | 8·8824 | 7·6735 | 6·7943 |

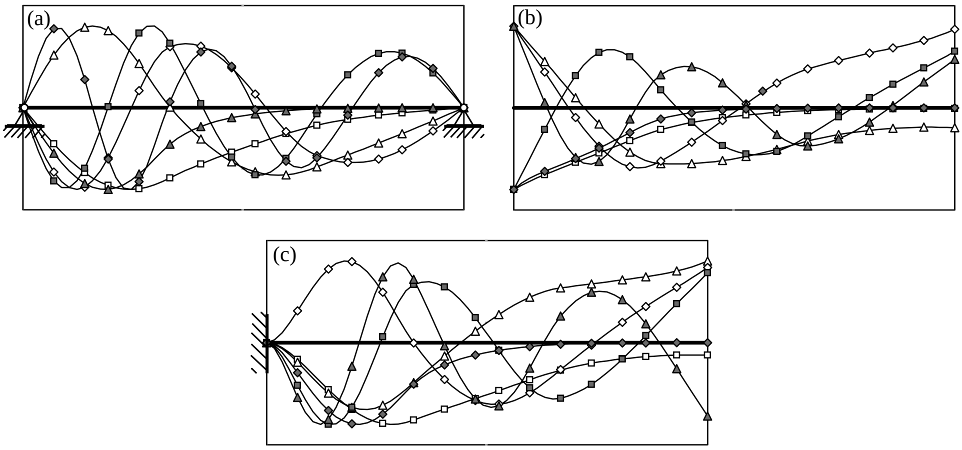


Figure 5. Normalized vibration modal shapes in the $x$ direction for two-even-step *SS*, *FF* and *CF* rectangular plates ($a = 3$). The step thickness ratios are $h_2/h_1 = 1·5$ and $h_3/h_1 = 2·0$. The number of half-waves in the $y$ direction is $m = 1$ for all case except for the cases marked with $m = 2$. (a) *SS* plates: □, Mode 1; △, Mode 2; ◇, Mode 3; ■, Mode 4; ▲, Mode 5 ($m = 2$); ◆, Mode 6. (b) *FF* plates: □, Mode 1; △, Mode 2; ◇, Mode 3; ■, Mode 4; ▲, Mode 5; ◆, Mode 6 ($m = 2$). (c) *CF* plates: □, Mode 1; △, Mode 2; ◇, Mode 3; ■, Mode 4; ▲, Mode 5; ◆, Mode 6 ($m = 2$).

TABLE 10

*Frequency parameters $\Lambda = (\omega L^2/\pi^2)\sqrt{\rho h_1/D_1}$ for two-even-step rectangular plates*

| $a$ | $h_2/h_1$ | $h_3/h_1$ | Mode | Cases | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *SS* | *FF* | *CC* | *SF* | *CF* | *CS* |
| 1 | 1·5 | 1·0 | 1 | 2·2633 | 1·1780 | 3·2169 | 1·4154 | 1·5260 | 2·6654 |
| | | | 2 | 5·5848 | 1·9883 | 6·7268 | 3·2647 | 3·8373 | 6·3668 |
| | | | 3 | 6·1005 | 4·3628 | 7·7080 | 4·8313 | 4·8628 | 6·5978 |
| | | | 4 | 9·3393 | 4·6021 | 11·152 | 6·9918 | 7·4595 | 10·179 |
| | | | 5 | 11·833 | 5·2594 | 13·097 | 7·0142 | 8·0549 | 12·733 |
| | | | 6 | 12·484 | 8·2013 | 15·392 | 10·145 | 10·150 | 13·436 |
| | 1·5 | 2·0 | 1 | 2·8840 | 1·4694 | 4·2262 | 1·9370 | 2·0971 | 3·4481 |
| | | | 2 | 7·1034 | 2·5193 | 7·9292 | 4·0918 | 4·8415 | 7·6895 |
| | | | 3 | 7·1047 | 4·9734 | 9·8627 | 6·7651 | 7·1132 | 8·3622 |
| | | | 4 | 11·712 | 5·3283 | 13·867 | 8·9799 | 9·7224 | 12·798 |
| | | | 5 | 13·534 | 7·6126 | 14·368 | 9·1191 | 10·348 | 14·324 |
| | | | 6 | 14·571 | 10·196 | 19·025 | 13·493 | 14·239 | 16·804 |
| 2 | 1·5 | 1·0 | 1 | 1·5251 | 1·1505 | 1·6817 | 1·2078 | 1·2157 | 1·5917 |
| | | | 2 | 2·3348 | 1·3149 | 2·7880 | 1·7479 | 1·8649 | 2·5448 |
| | | | 3 | 3·7993 | 2·0503 | 4·6198 | 2·7461 | 2·9900 | 4·1727 |
| | | | 4 | 5·2516 | 3·1728 | 5·4526 | 4·3216 | 4·3218 | 5·3202 |
| | | | 5 | 5·6235 | 4·3018 | 6·0418 | 4·3412 | 4·7995 | 5·8644 |
| | | | 6 | 5·6469 | 4·3437 | 6·7528 | 5·3891 | 5·6010 | 6·1841 |
| | 1·5 | 2·0 | 1 | 1·7758 | 1·2433 | 1·9823 | 1·6913 | 1·7783 | 1·9224 |
| | | | 2 | 2·9279 | 1·9031 | 3·4667 | 2·2798 | 2·4306 | 3·1995 |
| | | | 3 | 4·6643 | 2·6029 | 5·6303 | 3·5504 | 3·8911 | 5·2076 |
| | | | 4 | 5·4441 | 4·0619 | 5·7140 | 5·3758 | 5·7113 | 5·7129 |
| | | | 5 | 7·0846 | 4·3250 | 7·6432 | 5·4434 | 5·9659 | 7·6040 |
| | | | 6 | 7·3036 | 6·1626 | 8·3925 | 7·2191 | 7·4516 | 7·7276 |
| 3 | 1·5 | 1·0 | 1 | 1·3871 | 1·1101 | 1·4552 | 1·1272 | 1·1278 | 1·4148 |
| | | | 2 | 1·6687 | 1·1497 | 1·8528 | 1·4651 | 1·5266 | 1·7575 |
| | | | 3 | 2·3239 | 1·6284 | 2·6407 | 1·8955 | 1·9928 | 2·4701 |
| | | | 4 | 3·1424 | 2·1209 | 3·5780 | 2·6025 | 2·7828 | 3·3596 |
| | | | 5 | 4·3396 | 2·9249 | 4·8761 | 3·5100 | 3·7201 | 4·5997 |
| | | | 6 | 4·7814 | 3·8512 | 4·9199 | 4·1476 | 4·1476 | 4·7980 |
| | 1·5 | 2·0 | 1 | 1·5038 | 1·1329 | 1·5965 | 1·4993 | 1·5821 | 1·5916 |
| | | | 2 | 2·1317 | 1·7144 | 2·3139 | 1·9672 | 2·0238 | 2·2557 |
| | | | 3 | 2·8785 | 2·0936 | 3·2199 | 2·4343 | 2·5843 | 3·0847 |
| | | | 4 | 3·9974 | 2·7171 | 4·4919 | 3·2660 | 3·4800 | 4·2481 |
| | | | 5 | 4·8026 | 3·6239 | 4·9520 | 4·4823 | 4·7551 | 4·9520 |
| | | | 6 | 5·4247 | 4·1477 | 6·1210 | 4·8026 | 4·9520 | 5·7670 |

## 4. CONCLUSIONS

This paper presents an analytical approach that combines the Levy method and the state-space technique for determining exact buckling and vibration solutions of unidirectional multi-stepped rectangular plates having two parallel edges simply supported while the remaining two edges can take any combination of free, simply supported and clamped conditions. Sample results for buckling and vibration of one- and two-step rectangular plates subjected to uni- and bi-axial in-plane loads are obtained. The number of steps, step-thicknesses, step-lengths and the boundary conditions influence significantly the buckling and vibration behaviour of the stepped plates. The exact results presented herein provide valuable benchmark solutions for researchers who are developing numerical techniques and software for buckling and vibration analysis of non-uniform thickness plates.

## ACKNOWLEDGMENT

This work was supported by the University of Western Sydney through a Research Grant Scheme 01790-7266.

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