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TRANSVERSE VIBRATIONS OF CLAMPED AND SIMPLY SUPPORTED CIRCULAR PLATES OF DEFORMED BOUNDARIES

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1. INTRODUCTION

The present study deals with the determination of the fundamental frequency of transverse vibration of thin, elastic plates whose boundary in the z-plane is mapped onto a unit circle in the ζ-plane by means of the analytic function

$$z = f(\zeta) = \frac{a}{1+m}(\zeta + m\zeta^{n+1}), \quad m \leqslant \frac{1}{n+1}. \tag{1}$$

For $m = 0$, one has the case of the circular plate of radius a. On the other hand, for $m = 1/(n+1)$, a cusp is originated and the results must be considered as first order approximations in view of the fact that a cusp or a re-entrant corner generates a severe stress concentration field when the plate is subjected to bending. The phenomenon has been treated by Leissa and his students in a pioneering effort [1, 2].

The solution of the problem is tackled by means of a combination conformal mapping–variational approach [3] and in the case where $m = 1/(n+1)$, the fundamental eigenvalue is determined using the finite-element method [4].

2. APPROXIMATE SOLUTION

In the case of normal modes of vibration, the problem is governed by the functional

$$J(W) = D\iint_P [(W_{x^2} + W_{y^2})^2 - 2(1-\upsilon)(W_{x^2}W_{y^2} - W_{xy}^2)]\,\mathrm{d}x\,\mathrm{d}y - \rho h\omega^2 \iint_P W^2\,\mathrm{d}x\,\mathrm{d}y \tag{2}$$

and appropriate boundary conditions.

Expressing equation (1) in the form

$$z = f(\zeta), \qquad z = x + yi, \qquad \zeta = \xi + \eta i \tag{3}$$

and by substituting into equation (2), one obtains

$$J(W) = D\iint_c \left\{\frac{1+\upsilon}{2}\frac{(W_{\xi^2} + W_{\eta^2})}{|f'(\zeta)|^4} + \frac{1-\upsilon}{2}\frac{|(W_{\xi^2} - W_{\eta^2} - 2W_{\xi\eta}i)f'(\zeta) - 2(W_\xi - W_\eta i)f''(\zeta)|^2}{|f'(\zeta)|^6}\right\}|f'(\zeta)|^2\,\mathrm{d}\xi\,\mathrm{d}\eta - \rho h\omega^2 \iint_c W^2 |f'(\zeta)|^2\,\mathrm{d}\xi\,\mathrm{d}\eta. \tag{4}$$

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In the case of simply supported plates, one adopts the approximating expression [4]

$$W_a = \sum_{j=1}^{N} C_j(1 - r^{p+j-1}) + C_{N+1}(1 - r^p) r^2 \cos n\theta, \tag{5}$$

while, when the boundary is clamped, it was found convenient to use

$$W_a = \sum_{j=1}^{N} C_j(1 - r^{p+j-1})^2 C_{N+1} + (1 - r^p)^2 r^2 \cos n\theta. \tag{6}$$

By applying the classical Rayleigh–Ritz method, one obtains a determinant equation whose lowest root constitutes the fundamental frequency of the plate,

$$\Omega_1 = \sqrt{\frac{\rho h}{D}}\,\omega_1 a^2.$$

By minimizing Ω_1 with respect to "p", one obtains an optimized value of the fundamental eigenvalue.

3. NUMERICAL RESULTS

All calculations were determined for the Poisson ratio (ν) equal to 0·3. Five co-ordinate functions were used when applying the analytical methodology.

Table 1 depicts the values of fundamental frequency coefficients of simply supported and clamped plates. For $n = 1$, 4 and 12 and $m = \frac{1}{2}, \frac{1}{5}$ and $\frac{1}{13}$, respectively, the values of Ω were determined using the finite-element method, see Figure 1. Table 2 depicts the values of the

TABLE 1

Fundamental frequency coefficients of simply supported and clamped circular plates of deformed boundaries

		Simply supported		Clamped	
n	m	Finite element	Analytical	Finite element	Analytical
1	1/2	10·53	—	16·98	—
	1/3	—	9·04	—	17·41
3	1/6	—	8·62	—	14·54
	1/10	—	6·31	—	12·53
	1/30	—	5·30	—	10·92
4	1/5	15·86	—	16·93	—
	1/6	—	12·34	—	15·23
	1/10	—	7·06	—	12·71
	1/30	—	5·34	—	10·94
12	1/13	12·68	—	12·76	—
	1/15	—	11·71	—	12·25
	1/30	—	7·35	—	11·04

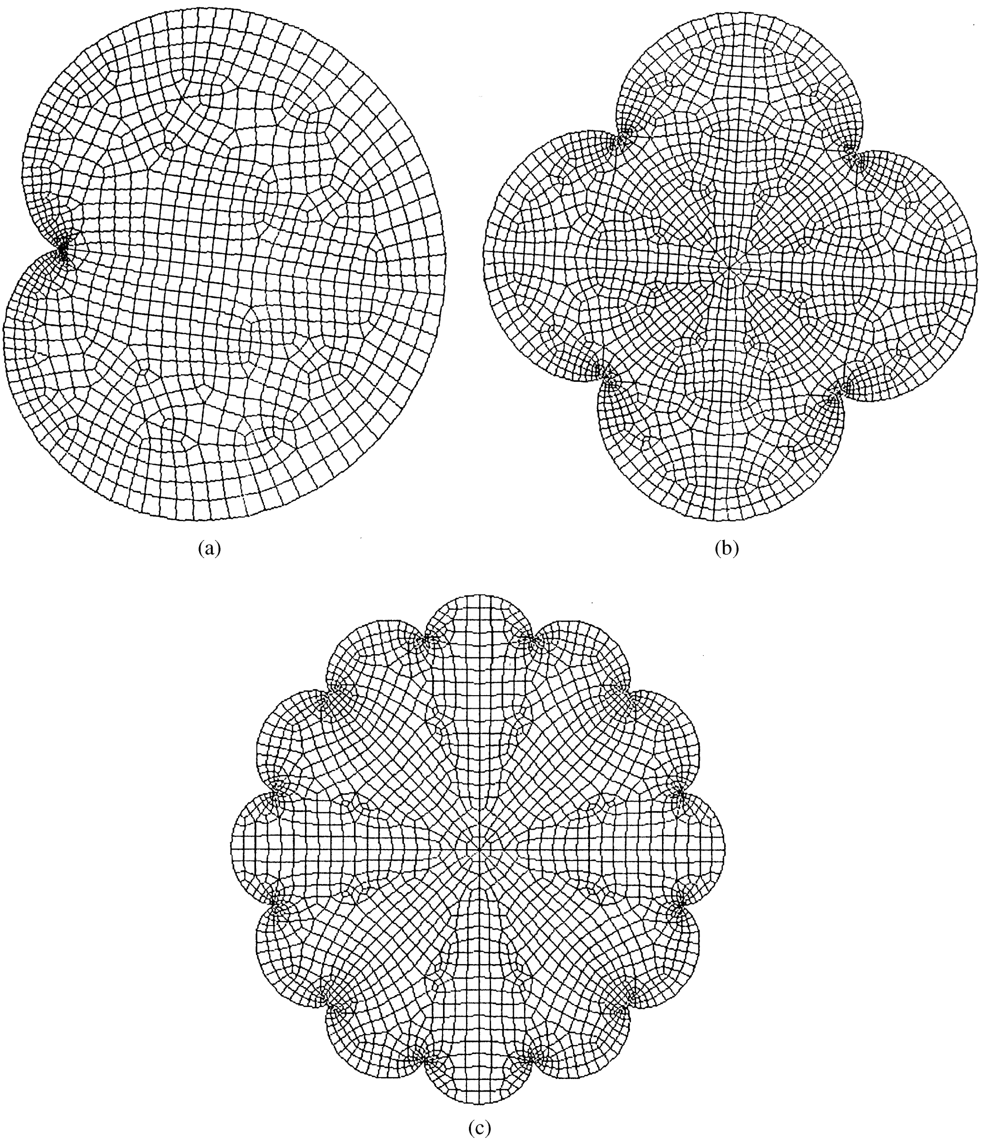

Figure 1. Configurations with cusps and finite element meshes: (a) $n = 1$, $m = 1/2$, 1044 elements; (b) $n = 4$, $m = 1/5$, 1880 elements; (c) $n = 12$, $m = 1/13$, 1220 elements.

first three frequency coefficients in the case of deformed boundaries with cusps. As expected, the difference of values of the frequency coefficients between the simply supported and the clamped situation becomes very small as the number of corrugations increase. On the other hand, as m becomes close to the value which yields a cusp the values obtained analytically approach the values obtained by means of the finite-element method.

An interesting conclusion is the fact that for a high degree of waviness (say $n = 12$, $m = \frac{1}{30}$), the effect upon the fundamental frequency is much larger in the case of a simply supported edge than for a clamped edge with respect to a perfect circular plate (40% and 10% respectively).

TABLE 2

Values of Ω_i in the case of circular plates with cusps obtained by means of the finite-element method

		Simply supported			Clamped		
n	m	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
1	1/2	10·53	19·43	27·64	16·98	30·06	37·78
4	1/5	15·86	25·35	27·23	16·93	31·61	39·83
12	1/13	12·68	26·13	42·19	12·76	26·40	42·97

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