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MODELLING AND CONTROL WITH PIEZOACTUATORS FOR A SIMPLY SUPPORTED BEAM UNDER A MOVING MASS

Y.-G. Sung

Department of Mechanical Engineering, Chosun University, 375 Seosuk-dong, Dong-gu, Kwanju, 501-759, Korea. E-mail: sungyg@chosun.ac.kr

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In the paper, the dynamic modelling and control are presented for a simply supported beam under a moving mass. The equations of motion are obtained based on the Euler–Bernoulli beam theory by including the dynamic effect of a moving mass travelling along a vibrating path. The equations of motion are discretized by using the assumed modes method with the static deflection of the beam. In order to reduce the deflection of the beam under a moving mass, a controller with full state feedback is designed based on linearized equations of motion. Two piezoelectric actuators are bonded along the bottom of the beam at different locations determined by the minimization of an optimal cost functional. Numerical simulations are performed with respect to different constant velocities and different moving masses. The controller with two piezoelectric actuators shows excellent performance under unknown disturbances to the system.

1. INTRODUCTION

Many researchers have been concerned with the dynamic stability and control of flexible structure under a moving mass such as a high-speed train bridge, ceiling crane, etc. [1, 2]. Due to the low mass–size ratio and high velocity of a moving mass, the stability of the system is an important issue for system safety and passenger comfort. For the accurate dynamic modelling of a simply supported beam under a moving mass, Lin [3] claimed that the effect of a moving mass should be accounted for carefully in the dynamic formulation since the mass is moving along a vibrating path. Abdel-Rohman and Leipholz [4] presented the active control of a simply supported beam under a moving mass by using bending moment in terms of tension and compression forces with a single actuator. Unlike the active control, the passive control approaches have been proposed in civil engineering. Kwon *et al.* [5] presented an approach to reduce the deflection of a beam under a moving load by means of adjusting the parameters of a conceptually second order damped model attached to a flexible structure. Recently, the piezoactuator has been intensively used to reduce the deflection of flexible structures such as space structures, helicopter blades, robot manipulators, etc. Devasia *et al.* [6] presented the approaches to determine the length and placement of piezoactuators in terms of the optimization of damping effect under collocated damping control, linear quadratic cost functional in the initial condition with the assumption of detectability and stabilizability, and minimum eigenvalue of controllability grammian.

In this paper, the equations of motion are, firstly, presented based on the Euler–Bernoulli beam theory by including the dynamic effect of a moving mass travelling along a deflecting path. Secondly, a multi-input–multi-output deflection controller is presented to actively reduce the structural deflection under a moving mass by using piezoactuators. Lastly, a comparison between uncontrolled and controlled cases can be found in the numerical simulation.

2 DYNAMIC MODELLING WITH PIEZOACTUATORS

In Figure 1, a conceptual model is presented for a simply supported beam under a moving mass and $y_s(x,0)$ and $y_1(x,t)$ are the initial beam deflection and total beam deflection respectively. v is the velocity of the moving mass. EI, m and M are the elastic modulus, beam mass per unit length, and moving mass respectively.

With the assumption of the small thickness ratio of piezoactuator to beam, the stress σ_x and displacement distribution are shown along the y-axis in Figure 2. The t and h are piezoactuator thickness and beam thickness respectively [7]. The relationship between the beam and piezoceramic actuator can be obtained using the moment equation

$$\int_0^h \sigma_b y \, \mathrm{d}y = \int_h^{h+t} \sigma_p y \mathrm{d}y, \tag{1}$$

where σ_b and σ_p are stresses for beam and piezoactuator respectively. The moment M_x acting on the beam is expressed in equation (2) which is used in the equations of motion.

$$M_x = C_0 V(t), \tag{2}$$

where

$$C_0 = \frac{2h^2 E_b d_{31}}{3\ R\ t}, \qquad R = 1 - \frac{E_b}{K_0 E_p}, \qquad K_0 = \frac{3t(2h+t)}{2h^2},$$

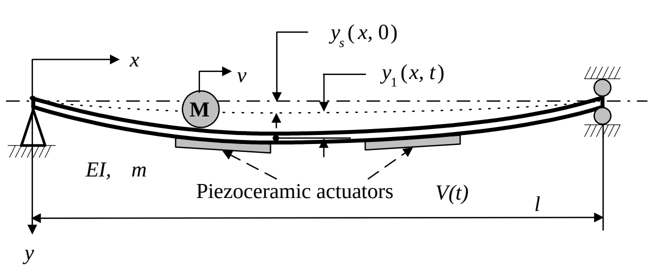

Figure 1. System configuration for modelling and control of beam.

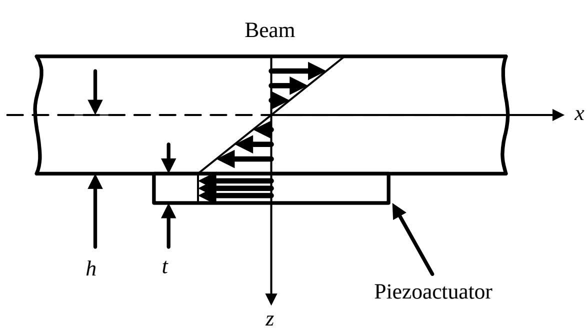

Figure 2. Stress and strain relationship between piezoactuator and beam.

$V(t)$, E_b, E_p and d_{31} are supply voltage, Young's modulus coefficient of the beam, piezoceramic modulus coefficient and piezoceramic constant respectively. The equations of motion and boundary conditions can be written as

$$\begin{aligned} EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} &= mg + Mg\delta(x - vt) \\ &\quad - M\left(\frac{\partial^2 y}{\partial x^2}v^2 + 2\frac{\partial^2 y}{\partial x \partial t}v + \frac{\partial y}{\partial x}\dot{v} + \frac{\partial^2 y}{\partial t^2}\right)\delta(x - vt) \\ &\quad + C_0 V_1(t)[\delta(x - x_1) - \delta(x - x_2)] \\ &\quad + C_0 V_2(t)[\delta(x - x_3) - \delta(x - x_4)], \end{aligned} \tag{3}$$

$$y(0,t) = \frac{\partial^2 y}{\partial x^2}(0,t) = y(l,t) = \frac{\partial^2 y}{\partial x^2}(l,t) = 0, \tag{4}$$

where v is the velocity of the moving mass, $V_1(t)$ and $V_2(t)$ are control input voltages. The third term [3] on the right-hand side of equation (3) is the centripetal acceleration of the moving mass, Coriolis acceleration, the acceleration component in the vertical direction when the moving mass speed is not a constant, the support beam acceleration in successive order. $(x_2 - x_1)$ and $(x_4 - x_3)$ imply the length of the piezoceramic actuator. g is the gravity.

A general solution $y(x,t)$ is expanded in the finite series

$$y(x,t) = \sum_{n=1}^{\infty} [A_n + q_n(t)] \sin\frac{n\pi x}{l}, \tag{5}$$

where $q_n(t)$ are generalized co-ordinates and the static deflection coefficient due to the beam weight is

$$A_n = \sum_{n=1}^{\infty} \frac{2mgl^4[1 - (-1)^n]}{n^5 EI\pi^5}, \tag{6}$$

where the assumed mode shape is $\phi_n = \sin n\pi x/l$. Substituting equation (5) into equation (3) and non-dimensionalizing it, the equations of motion are expressed as

$$\begin{aligned} &\ddot{\varphi}_n + n^4\varphi_n + 2m_0 \sum_{i=1}^{\infty} [(\ddot{\varphi}_i - i^2 v_0^2 \varphi_i)\sin iv_0\tau + i(2v_0\dot{\varphi}_i + a_0\varphi_i)\cos iv_0\tau] \sin nv_0\tau \\ &\quad = m_0\left[\frac{\pi}{2} + v_0^2 \sum_{i=1}^{\infty} \frac{[1 - (-1)^i]}{i^3} \sin iv_0\tau \right. \\ &\qquad \left. - a_0 \sum_{i=1}^{\infty} \frac{[1 - (-1)^i]}{i^4} \cos iv_0\tau \right] \sin nv_0\tau \\ &\qquad + \frac{2C_0}{\delta\omega_1^2 ml}\left(\cos\frac{n\pi x_1}{l} - \cos\frac{n\pi x_2}{l}\right)V_1(t) \\ &\qquad + \frac{2C_0}{\delta\omega_1^2 ml}\left(\cos\frac{n\pi x_3}{l} - \cos\frac{n\pi x_4}{l}\right)V_2(t). \end{aligned} \tag{7}$$

In equation (7), the non-dimensional parameters are defined as

$$m_0 = \frac{M}{ml}, \qquad \delta = 4\frac{mgl^4}{EI\pi^5}, \qquad \tau = \omega_1 t, \qquad \varphi_n = \frac{q_n}{\delta},$$

$$\omega_1 = \sqrt{\frac{EI\pi^2}{ml^2}}, \qquad v_0 = \frac{v}{v_c r} = \frac{\pi v}{\omega_1 l}, \qquad a_0 = \frac{\pi \dot{v}}{\omega_1^2 l}.$$

The non-dimensional external disturbance term can be written simply as

$$d = M_t \ddot{\varphi}_n + C_t \dot{\varphi}_n + K_t \varphi_n - f, \tag{8}$$

where M_t, C_t, K_t, and f are time-varying non-linear terms associated with the moving mass. As a result, the non-dimensional equations of motion are expressed in vector and matrix form as

$$\ddot{\varphi}_n + K_0 \varphi_n = B_0 F + d, \tag{9}$$

where K_0 and B_0 are a constant diagonal stiffness matrix and a constant matrix resulting from two piezoactuator loactions respectively. $F = [V_1, V_2]^{\mathrm{T}}$ is the control input vector. By defining the state vector $x = [\varphi_n, \dot{\varphi}_n]^{\mathrm{T}}$, the state and output equations are given as

$$\dot{x} = Ax + Bu + D, \qquad y = Cx, \tag{10, 11}$$

where

$$A = \begin{bmatrix} 0 & I \\ -K_0 & 0 \end{bmatrix}, \qquad B = [0 \quad B_0 F]^{\mathrm{T}}, \qquad C = \begin{bmatrix} \delta \cdot \phi_n & 0 \\ 0 & \delta \cdot \phi_n \end{bmatrix}, \qquad D = [0, -d]^{\mathrm{T}}.$$

Note that the external disturbance did not affect the beam dynamics when the moving mass passed the end of the beam because $m_0 = 0$.

3. DEFLECTION REDUCTION CONTROLLER

In order to design the state feedback optimal controller, an objective function is a quadratic functional of the plant states and control inputs. With a linear system, the quadratic objective function is expressed as

$$J_c = \int_0^{T_f} (x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u)\,\mathrm{d}t, \tag{12}$$

where Q is a symmetric semi-definite matrix with regard to the state vector, R is a symmetric positive-definite matrix with respect to the control input and T_f is the final time. The minimization of J_c with respect to the control input u is known as the linear quadratic regulator. The control law to minimize is given as

$$u(t) = -K(t)\, x, \tag{13}$$

where $K(t) = R^{-1} B^{\mathrm{T}} P(t)$ is a linear time-varying gain matrix of state feedback form. For an asymptotically stable closed-loop system [8], a steady state gain matrix can be expressed as

$$K = R^{\mathrm{T}} B^{\mathrm{T}} P, \tag{14}$$

where P is the positive-definite solution of the algebraic Riccati equation

$$PA + A^{\mathrm{T}} P + Q - PBR^{-1} B^{-1} P = 0. \tag{15}$$

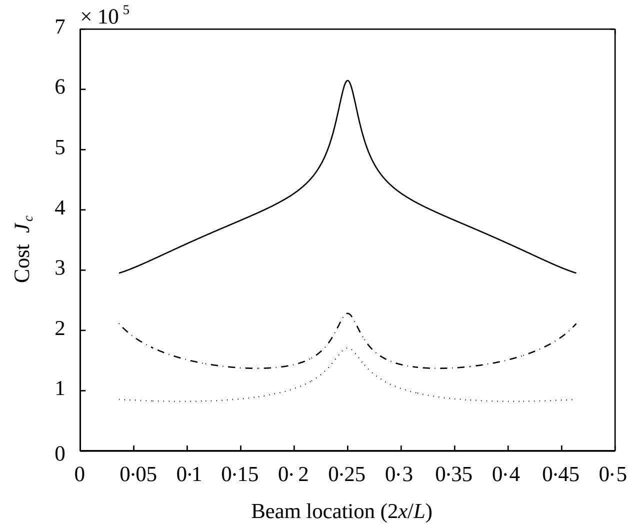

Figure 3. Cost of each mode. ——, First mode; ·—·—, second mode; · · · ·, third mode.

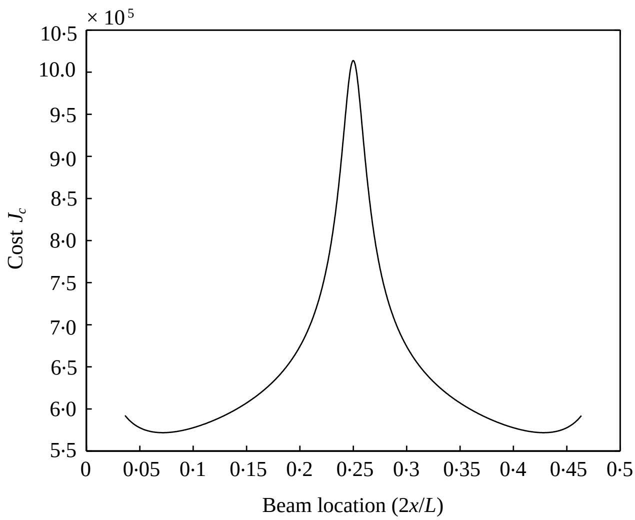

Figure 4. Added cost value.

4. PIEZOACTUATOR PLACEMENT

In this section, the linear quadratic regulator (LQR) formulation based on [6] is used for deciding upon good placement and length of the piezoactuators. Provided system equations (10) and (11) satisfy the conditions of stabilizability and detectability, the minimum cost is given as

$$\min J_c = x_0^{\mathrm{T}} P x_0. \tag{16}$$

The minimization of the cost functional J_c is employed for the worst-case initial condition so that the following optimization is posed to determine the best placement x_p and size l_p of

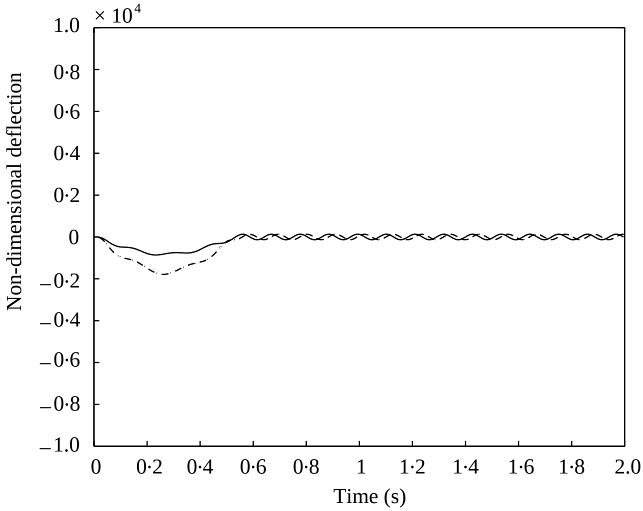

Figure 5. Uncontrolled time response at the center of the beam with $v_0 = 0{\cdot}1$. ——, $m_0 = 0{\cdot}5$; —·—, $m_0 = 1{\cdot}0$.

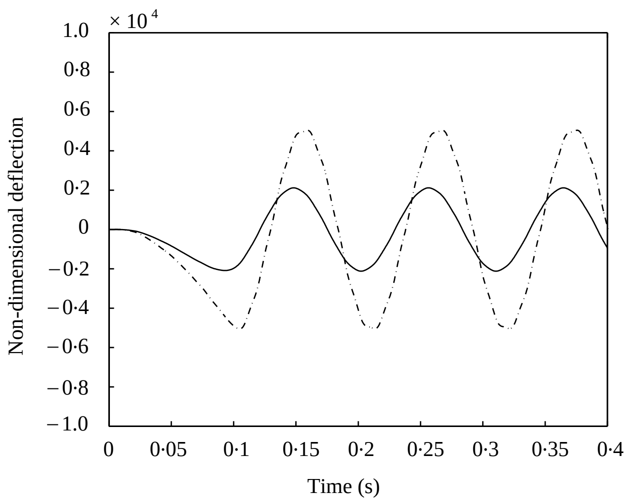

Figure 6. Uncontrolled time response at the center of the beam with $v_0 = 0{\cdot}5$. ——, $m_0 = 0{\cdot}5$; ·—·, $m_0 = 1{\cdot}0$.

the piezoactuator as

$$\min_{\substack{l_p < [0, l] \\ x_p < [l_p/2, l - l_p/2]}} \max_{\|x_0\| = 1} x_0^{\mathrm{T}} P x_0, \; x_0 = x_0(T_f), \tag{17}$$

where P is the solution for the algebraic Ricatti equation of the given system. The method optimizes performance uniformly in initial conditions. Note that the nodal point of flexible modes should be eliminated efficiently to obtain deflection reduction.

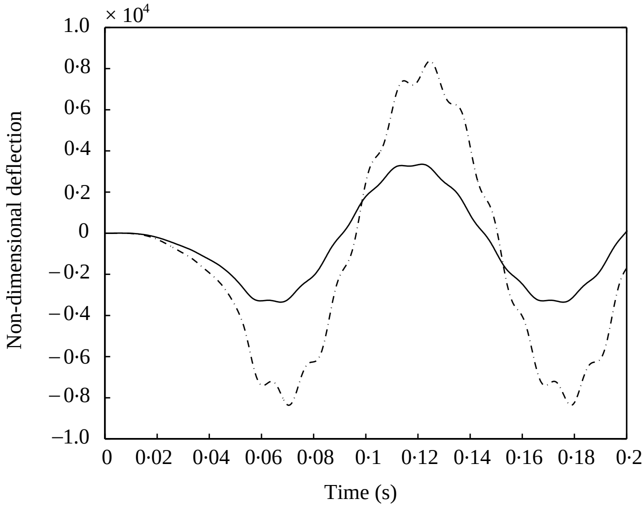

Figure 7. Uncontrolled time response at the center of the beam with $v_0 = 1{\cdot}0$. ——, $m_0 = 0{\cdot}5$; —·–·, $m_0 = 1{\cdot}0$.

5. NUMERICAL SIMULATION

In the numerical simulation, the placement of piezoactuator is determined and then the controller with two piezoactuators bonded on the bottom of the beam is evaluated for deflection reduction under constant high-speed cases. The mechanical properties for the slender beam are given as thickness $t = 4$ mm, density $\rho = 2700$ kg/m^3, Young's modulus of beam $E_b = 6{\cdot}5 \times 10^{10}$ Pa. The dimension of the aluminum beam is chosen as length $l = 1$ m and width $b = 32$ mm. In the numerical simulation, the first three modes are included due to the most dominant vibrational modes. The natural frequencies of the beam are given as $\omega_1 = 9{\cdot}21$ Hz, $\omega_2 = 36{\cdot}85$ Hz and $\omega_3 = 82{\cdot}91$ Hz. Meanwhile, the physical dimension of the piezoactuator is chosen as length 72·4 mm and width 32 mm with Young's modulus $E_p = 6{\cdot}6 \times 10^{10}$ Pa and the deformation coefficient $d_{31} = -190 \times 10^{-12}$ m/V. Notably, the thickness of the piezoactuator is related to the amplification coefficient C_0 in equation (2) so that the thickness 2 mm is selected for the best deflection reduction at the given system configuration.

The weighting matrices of Q and R are selected as follows with the emphasis on the least displacement and equal input weight

$$Q = \begin{bmatrix} 10^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-5} \end{bmatrix},$$

$$R = 50^{-2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

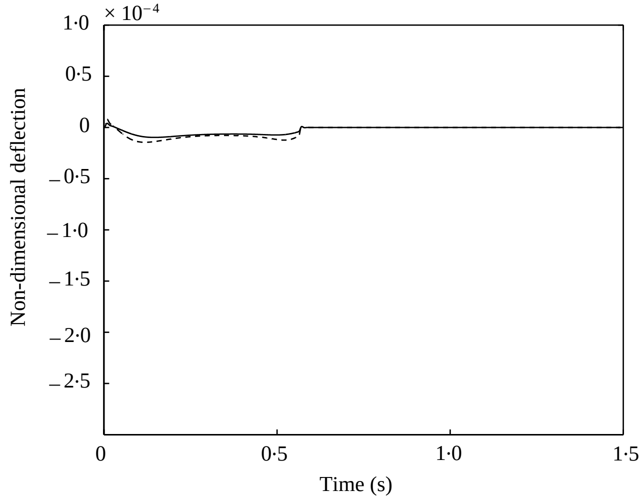

Figure 8. Controlled time response at the center of the beam with $v_0 = 0{\cdot}1$. ——, $m_0 = 0{\cdot}5$; —·—·, $m_0 = 1{\cdot}0$.

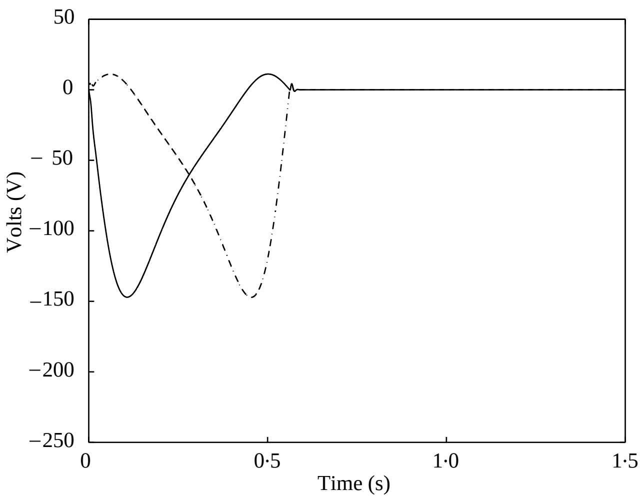

Figure 9. Control inputs with $m_0 = 0{\cdot}5$ and $v_0 = 0{\cdot}1$. ——, V_1; —·—·, V_2.

In this work, the piezoactuators are symmetrically located with respect to the center of the beam so that the cost of the functional J_c is shown only for the half of the beam in Figures 3 and 4. In Figure 3, the cost of functional J_c is shown corresponding to the piezoactuator location for each mode. The added cost to obtain the best actuator placement is shown in Figure 4. In Figure 3, the first mode requires the most control energy and then the second and third modes respectively. The best location for the first and third modes is at the center $l/2$ of the beam which is physically meaningful. In the meantime, the optimal location is at around $2l/10$ for the third mode. In Figure 4, the best center locations of the two piezoactuators are symmetrically located at $3l/8$ and $5l/8$ with the exclusion of the nodal point for each mode.

In Figures 5–7, the controller is not operated and the dynamic responses are shown at the center of the beam. In Figure 5, the maximum deflection is 11 mm and the residual oscillation appears after the moving mass passes the end of the beam. As the speed of the

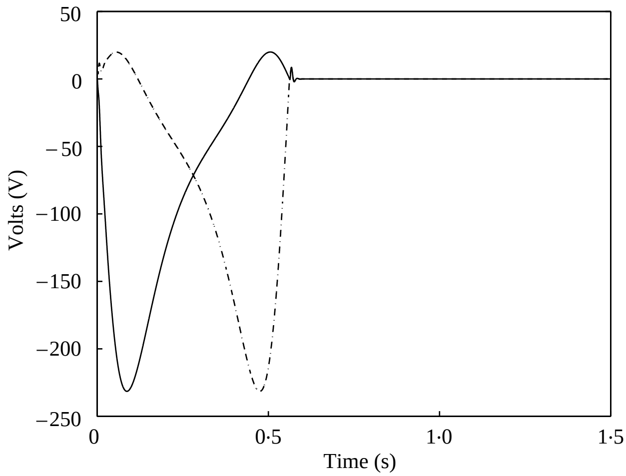

Figure 10. Control inputs with $m_0 = 1{\cdot}0$ and $v_0 = 0{\cdot}1$. ——, V_1; —·—, V_2.

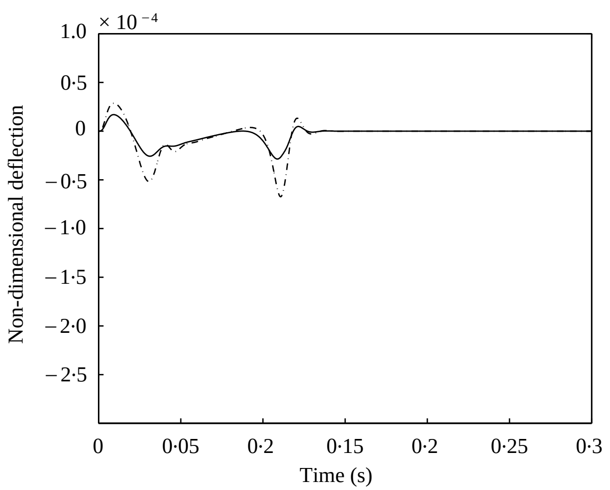

Figure 11. Controlled time response at the center of the beam with $v_0 = 0{\cdot}5$. ——, $m_0 = 0{\cdot}5$; —·—, $m_0 = 1{\cdot}0$.

moving mass goes up in the case of Figures 6 and 7, large amplitudes are shown as a result of the inertial effect of the moving mass.

From Figures 8–12, the designed controller performance is evaluated. In Figures 8, 11 and 12, the maximum deflection is from 0·08 mm to 0·2 mm so that the controller quickly suppresses the beam deflection and firmly counteracts the inertia effect of the moving mass. In Figures 9 and 10, the control input voltages show that the magnitude is dependent upon the moving mass and location. For the most extreme cases shown in Figures 11 and 12, the maximum voltage of the control input was around 800 V to obtain the current performance.

6. CONCLUSIONS

In this paper, the dynamic modelling and control were presented for a simply supported beam under a moving mass. The equations of motion were presented for a simply supported

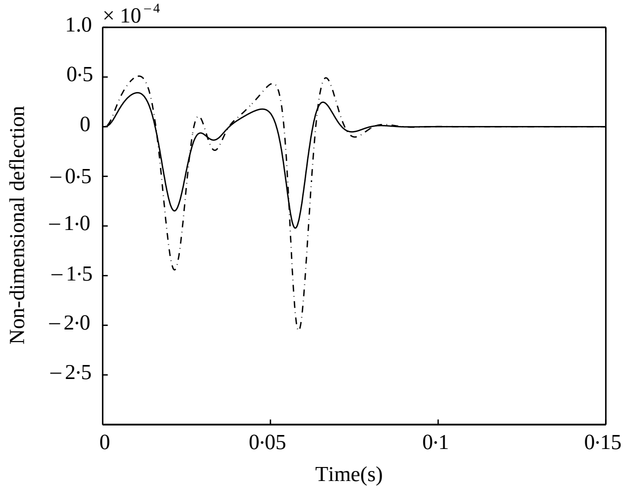

Figure 12. Controlled time response at the center of the beam with $v_0 = 1{\cdot}0$. ——, $m_0 = 0{\cdot}5$; —·—, $m_0 = 1{\cdot}0$.

beam under a moving mass travelling along a deflecting path. Using piezoactuators bonded to the beam, the controller with full state feedback was designed to reduce the structural deflection. In order to determine the optimal placements of piezoactuators, the LQR-based formulation was employed. The best actuator placement was determined from the aspect of the entire dynamics. In the time response comparisons of uncontrolled and controlled cases, excellent controller performance was shown even with higher moving speeds, in that the acceleration effect becomes large.

In future research, a non-linear controller will be designed to account for non-linear effects and it is necessary to compare it with the control performance of the deflection reduction controller.

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