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WAVE PROPAGATION THROUGH A CYLINDRICAL BORE CONTAINED IN A MICROSTRETCH ELASTIC MEDIUM

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Propagation of waves in a cylindrical bore filled with viscous liquid embedded in a microstretch elastic medium is investigated. Frequency equation for the surface wave propagation near the surface of the cylindrical bore is obtained, characterizing the dispersive nature of the wave. Significant effects of viscosity, microstretch and micropolarity are observed. Some special cases have been deduced.

1. INTRODUCTION

The problem of surface wave propagation near the bore hole in elastic host medium is of great practical importance. Since valuable organic and inorganic deposits beneath the earth's surface are difficult to detect by drilling randomly, the wave propagation technique is the simplest and most economical and does not require any drilling through the earth. Almost all the oil companies rely on seismic interpretation for selecting the sites for exploratory oil wells. Seismic wave methods also have higher accuracy, higher resolution and are more economical, as compared to drilling which is a costly and time-consuming affair.

Modern engineering structures are often made up of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. Classical elasticity is inadequate to represent the behavior of such materials. The analysis of such materials requires incorporation of the theory of oriented media. "Micropolar elasticity" termed by Eringen [1] is used to describe the deformation of elastic media with oriented particles. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Typical examples of such materials are granular media and multimolecular bodies, whose microstructures act as an evident part in their macroscopic responses. The physical nature of these materials needs an asymmetric description of deformation, while theories for classical continua fail to accurately predict their physical and mechanical behavior.

Eringen [2] introduced the theory of microstretch elastic solids as a generalization of the micropolar theory. The material points of microstretch solids can stretch and contract independent of their translations and rotations. A microstretch continuum can model composite materials reinforced with chopped elastic fibers and various porous solids. Different authors [3–12] discussed different problems in microstretch/micropolar elastic/elastic medium. The present study is concerned with the problem of surface wave propagation in a cylindrical bore filled with viscous liquid and contained in a microstretch elastic medium.

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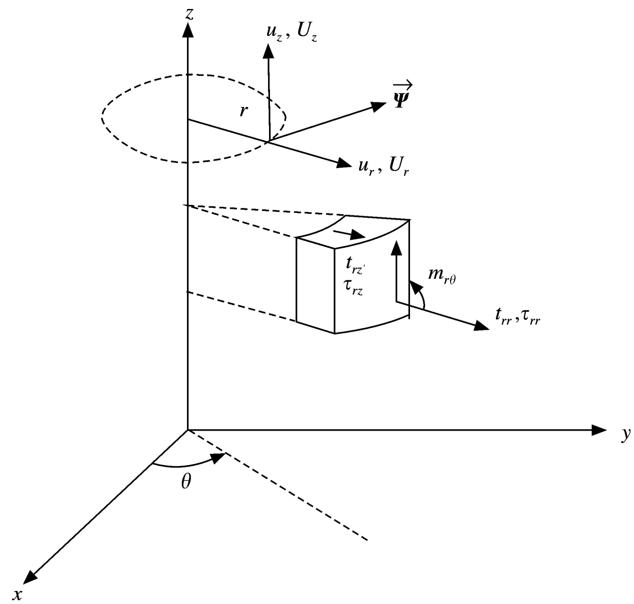


Figure 1. Cylindrical co-ordinate system, z along the axis of the bore.

2. PROBLEM FORMULATION AND SOLUTION

We consider a cylindrical bore of radius "a" having a circular cross-section in a microstretch elastic medium of infinite extent. We are studying the propagation of axial symmetric waves which are pure sinusoidal along the axial direction. Cylindrical polar co-ordinates (r, θ, z) are considered with z-axis pointing upwards (Figure 1).

Following Eringen [13], the field equations and constitutive relations in a microstretch elastic solid are given by

$$(\mu + K)\ \nabla^2\mathbf{u} + (\lambda + \mu)\ \nabla(\nabla\cdot \mathbf{u}) + K\,\nabla\times \boldsymbol{\phi} + \lambda_0\ \nabla\phi^* = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

$$(\alpha + \beta + \gamma)\ \nabla(\nabla\cdot \boldsymbol{\phi}) - \gamma\,\nabla x\ (\nabla x\boldsymbol{\phi}) + K\ \nabla x\mathbf{u} - 2K\boldsymbol{\phi} = \rho j\,\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}, \tag{2}$$

$$c_6^2\ \nabla^2\,\boldsymbol{\phi}^* - c_7^2\,\phi^* - c_8^2\ \nabla\cdot \mathbf{u} = \frac{\partial^2\phi^*}{\partial t^2} \tag{3}$$

and

$$t_{ij} = \lambda\,\delta_{ij}\,u_{k,k} + \mu(u_{i,j} + u_{j,i}) + K\,[u_{j,i} - \varepsilon_{ijk}\,\phi_K] + \lambda_0\,\delta_{ij}\phi^*, \tag{4}$$

$$m_{ij} = \alpha\,\phi_{k,k}\,\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i}, \tag{5}$$

$$\lambda_k = \alpha_0\,\phi^*_{,k}, \tag{6}$$

where

$$c_6^2 = \frac{2\alpha_0}{3\rho j}, \qquad c_7^2 = \frac{2\lambda_1}{9\rho j}, \qquad c_8^2 = \frac{2\lambda_0}{9\rho j} \tag{7}$$

and $\lambda, \mu, K, \alpha, \beta, \gamma, \alpha_0, \lambda_0, \lambda_1$ are material moduli, ρ the density, and j the microinertia. $\mathbf{u}$ and $\boldsymbol{\phi}$ denote the displacement and microrotation vectors, while ϕ^* is the scalar microstretch, t is the time, t_{ij} and m_{ij} are components of force stress and couple stress respectively. λ_k is the microstress that induces an extension to the microelements.

We are discussing a two-dimensional problem with symmetry about the z-axis, so all the partial derivatives with respect to the variable θ would be zero. Therefore, we have

$$\mathbf{u} = (u_r, 0, u_z), \ \boldsymbol{\phi} = (0, \phi_\theta, 0) \quad \text{and} \quad \partial/\partial\theta = 0, \tag{8}$$

where u_r, u_z and ϕ_θ are related in terms of potential functions ϕ', ψ and Γ as

$$u_r = \frac{\partial \phi'}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z}, \quad u_z = \frac{\partial \phi'}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\psi, \quad \phi_\theta = -\frac{\partial \Gamma}{\partial r}. \tag{9}$$

On substituting equations (9) into equations (1)–(3) with the help of equation (8), then eliminating Γ and ϕ^* from the resulting expressions, we obtain

$$[\Box_2 \, \Box_3 + K^2 \, \nabla^2] \, [\psi, \Gamma] = 0 \tag{10}$$

and

$$\left[\Box_1 \, \Box_4 + \lambda_0 \frac{c_8^2}{c_6^2}\right] [\phi', \phi^*] = 0, \tag{11}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2},$$

$$\Box_1 = (\lambda + 2\mu + K)\, \nabla^2 - \rho \frac{\partial^2}{\partial t^2}, \qquad \Box_2 = (\mu + K)\, \nabla^2 - \rho \frac{\partial^2}{\partial t^2},$$

$$\Box_3 = \gamma \nabla^2 - 2K - \rho j \frac{\partial^2}{\partial t^2}, \qquad \Box_4 = \nabla^2 - \frac{c_7^2}{c_6^2} - \frac{1}{c_6^2}\frac{\partial^2}{\partial t^2}. \tag{12}$$

The time-harmonic wave solutions of equations (10) and (11) for the wave propagating in the positive direction of z-axis are given by

$$\psi = [A_1 K_0 (q_1 r) + A_1' K_0 (q_2 r)] \, \mathrm{e}^{\imath(kz - \omega t)}, \tag{13}$$

$$\Gamma = [A_2 K_0 (q_1 r) + A_2' K_0 (q_2 r)] \, \mathrm{e}^{\imath(kz - \omega t)}, \tag{14}$$

$$\phi' = [A_3 K_0 (q_3 r) + A_3' K_0 (q_4 r)] \, \mathrm{e}^{\imath(kz - \omega t)}, \tag{15}$$

$$\phi^* = [A_4 K_0 (q_3 r) + A_4' K_0 (q_4 r)] \, \mathrm{e}^{\imath(kz - \omega t)}. \tag{16}$$

where $K_0\,()$ is a modified Bessel function of order zero and of second kind, $\omega\,(=kc)$ is the frequency of the wave, k is the wave number and c is the phase velocity. $q_{1,2}^2$ and $q_{3,4}^2$ are the roots of equations (10) and (11), respectively, and are given by

$$q_{1,2}^2 = k^2 - \tfrac{1}{2}\,[(\sigma_4^2 + \sigma_2^2 + \eta_4^2 - v^2) \mp \{(\sigma_4^2 - \sigma_2^2 - v^2 + \eta_4^2)^2 + 4\eta_4^2\,\sigma_2^2\}^{1/2}],$$

$$q_{3,4}^2 = k^2 - \tfrac{1}{2}\,[(\sigma_5^2 - \eta_1^2 + \eta_2^2 + \eta_3^2) \mp \{(\sigma_5^2 + \eta_1^2 - \eta_2^2 - \eta_3^2)^2 + 4\eta_3^2\,\sigma_5^2\}^{1/2}], \tag{17}$$

where

$$\sigma_4^2 = \frac{\omega^2}{c_4^2}, \quad \sigma_2^2 = \frac{\omega^2}{b^2}, \quad v^2 = \frac{2K}{\gamma}, \quad \eta_4^2 = \frac{K^2}{(\mu + K)\,\gamma},$$

$$\sigma_5^2 = \frac{\omega^2}{c_1^2}, \quad \eta_1^2 = \frac{c_7^2}{c_6^2}, \quad \eta_2^2 = \frac{\omega^2}{c_6^2}, \quad \eta_3^2 = \frac{\lambda_0\, c_8^2}{\rho c_1^2\, c_6^2},$$

$$c_4^2 = \frac{\gamma}{\rho j}, \quad b^2 = \frac{\mu + K}{\rho}, \quad c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \tag{18}$$

A_i and A_i' $(i = 1, \ldots, 4)$ are related as

$$A_2 = b_1\, A_1, \qquad A_2' = b_2\, A_1',$$

$$A_4 = b_3\, A_3, \qquad A_4' = b_4\, A_3', \tag{19}$$

where

$$b_i = (k^2 - \sigma_2^2 - q_i^2)/p^*,$$

$$b_j = (k^2 - \sigma_5^2 - q_j^2)\Big/\left(\frac{\lambda_0}{\rho c_1^2}\right), \quad p^* = \frac{K}{(\mu + K)}, \quad i = 1, 2, \quad j = 3, 4. \tag{20}$$

Following Fehler [14], the equations for wave propagation in a viscous medium are

$$\left(K' + \frac{4}{3}\eta\,\frac{\partial}{\partial t}\right) \nabla^2 \phi^0 = \rho'\,\frac{\partial^2 \phi^0}{\partial t^2}, \tag{21}$$

$$\eta\,\frac{\partial}{\partial t}\,(\nabla^2 \psi^0) = \rho'\,\frac{\partial^2 \psi^0}{\partial t^2}, \tag{22}$$

where, K' is the bulk modulus, η is the fluid viscosity and ρ' is the fluid density.

The stresses in viscous liquid medium are given by

$$\tau_{ij} = \left(K' - \frac{2}{3}\eta\,\frac{\partial}{\partial t}\right) U_{k,k}\,\delta_{ij} + \eta\,\frac{\partial}{\partial t}\,(U_{i,j} + U_{j,i}), \tag{23}$$

where, $\mathbf{U}$ is the displacement vector.

Since the problem is axi-symmetric, we take

$$\mathbf{U} = (U_r, 0, U_z), \tag{24}$$

where, the displacement components in viscous liquid medium are given by

$$U_r = \frac{\partial \phi^0}{\partial r} + \frac{\partial^2 \psi^0}{\partial r \partial z}, \quad U_z = \frac{\partial \phi^0}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right) \psi^0. \tag{25}$$

The time-harmonic wave solutions of equations (21) and (22) may be written as

$$\phi^0 = A_5\, \mathrm{I}_0\,(r\zeta)\, \exp\,[\imath(kz - \omega t)], \tag{26}$$

$$\psi^0 = A_6\, \mathrm{I}_0\,(r\zeta')\, \exp\,[\imath(kz - \omega t)], \tag{27}$$

where

$$\zeta = \sqrt{k^2 - \frac{\omega^2}{c_\ell^2}}, \quad \zeta' = \sqrt{k^2 - \frac{\imath\omega}{c_{\ell_0}^2}},$$
$$c_\ell^2 = (K' - \tfrac{4}{3}\,\imath\omega\eta)/\rho', \quad c_{\ell_0}^2 = \eta/\rho' \tag{28}$$

and $\mathrm{I}_0()$ is the modified Bessel function of order zero and first kind.

3. BOUNDARY CONDITIONS

The boundary conditions are the continuity of stresses and displacements at the interface $r = a$, between the microstretch elastic medium and viscous liquid. Since couple stresses and stress moment do not support the viscous liquid, therefore these must vanish there.

Mathematically, these boundary conditions can be expressed as

$$\begin{matrix} t_{rr} = \tau_{rr}, & t_{rz} = \tau_{rz}, & m_{r\theta} = 0 \\ u_r = U_r, & u_z = U_z, & \lambda_r = 0 \end{matrix} \quad \text{at } r = a, \tag{29}$$

where t_{rr}/τ_{rr} and t_{rz}/τ_{rz} are the radial and tangential stress components, respectively, u_r/U_r and u_z/U_z are the radial and tangential displacement components, respectively, $m_{r\theta}$ is the torsional couple stress and λ_r is the radial component of stress moment.

Using equations (4)–(9), (13)–(16), (19), (23) and (25)–(27) in the above boundary conditions (29), we shall obtain a system of six homogeneous equations in unknowns A_1, A_1', A_3, A_3', A_5 and A_6. The elimination of these unknowns gives the frequency equation, which is dispersive in nature as follows:

$$\begin{aligned} & g_1 g_3\,[d_2\,\{f_4 A_1 - f_5 A_6 - f_6 A_7\} - d_4\,\{f_2 A_1 - f_5 A_2 - f_6 A_3\} \\ & + d_5\,\{f_2 A_6 - f_4 A_2 + f_6 B_4\} - d_6\,\{-f_2 A_7 + f_4 A_3 + f_5 B_4\}] \\ & - g_1 g_4\,[d_2\,\{f_3 A_1 - f_5 A_8 - f_6 A_9\} - d_3\,\{f_2 A_1 - f_5 A_2 - f_6 A_3\} \\ & + d_5\,\{f_2 A_8 - f_3 A_2 + f_6 B_3\} - d_6\,\{-f_2 A_9 + f_3 A_3 + f_5 B_3\}] \\ & - g_2 g_3\,[d_1\,\{f_4 A_1 - f_5 A_6 - f_6 A_7\} - d_4\,\{f_1 A_1 - f_5 A_5 - f_6 A_4\} \\ & + d_5\,\{f_1 A_6 - f_4 A_5 + f_6 B_2\} - d_6\,\{-f_1 A_7 + f_4 A_4 + f_5 B_2\}] \\ & + g_2 g_4\,[d_1\,\{f_3 A_1 - f_5 A_8 - f_6 A_9\} - d_3\,\{f_1 A_1 - f_5 A_5 - f_6 A_4\} \\ & + d_5\,\{f_1 A_8 - f_3 A_5 + f_6 B_1\} - d_6\,\{-f_1 A_9 + f_3 A_4 + f_5 B_1\}] = 0, \end{aligned} \tag{30}$$

where

$$d_i = \frac{q_i^2\,a^2}{k^2\,a^2}\left[e_i + \frac{1}{q_i a}\right],$$

$$d_j = \left[\frac{q_j^2\,a^2}{k^2\,a^2} + \frac{\lambda}{(2\mu + K)}\left(\frac{q_j^2\,a^2}{k^2\,a^2} - 1\right)\right] e_j + \frac{q_j^2\,a^2}{k^2\,a^2} + \frac{\lambda_0\,b_j\,e_j}{(2\mu + K)\,k^2},$$

$$\begin{aligned} d_5 = {} & \left(\frac{2\eta\,\omega}{(2\mu + K)}\right)\left(\frac{Y_2 - \imath x_2}{ka}\right) - \left[\frac{K'}{(2\mu + K)}(x_2^2 - y_2^2 - 1 + 2\imath x_2 y_2)\right. \\ & \left. + \frac{8}{3}\left(\frac{\eta\,\omega}{(2\mu + K)}\right)\left\{x_2 y_2 + \frac{1}{2}\left(x_2^2 - y_2^2 + \frac{1}{2}\right)\right\}\right] e_5, \end{aligned}$$

$$d_6 = \left(\frac{2\eta\,\omega}{(2\mu+K)}\right)\left[\left(\frac{x_1 - \imath y_1}{ka}\right) - e_6\,(x_1^2 - y_1^2 + 2\imath x_1 y_1)\right],$$

$$f_i = \left[1 + \left(\frac{K}{(2\mu+K)}\right)\frac{b_i}{k^2} + \left(\frac{\mu+K}{(2\mu+K)}\right)\left(\frac{\lambda_i^2\,a^2}{k^2\,a^2} - 1\right)\right]\frac{\lambda_i\,a}{ka},$$

$$f_j = \frac{\lambda_j\,a}{ka}, \quad f_5 = \left(\frac{2\eta\,\omega}{(2\mu+K)}\right)(y_2 - \imath x_2),$$

$$f_6 = \left(\frac{2\mu\,\omega}{(2\mu+K)}\right)[x_1\,(x_1^2 - 3y_1^2 + 1) + \imath y_1\,(3x_1^2 - y_1^2 + 1)],$$

$$g_i = \frac{q_i^2\,a^2}{k^2\,a^2}\left[\frac{1}{q_i\,a} + \frac{\gamma}{\beta+\gamma}\,e_i\right]\frac{b_i}{k^2}, \quad g_j = \frac{b_j}{k^2}\frac{q_j a}{ka},$$

$$h_i = \frac{q_i\,a}{k\,a}, \quad h_j = \frac{q_j\,a}{k\,a}, \quad h_5 = x_2 + \imath y_2, \quad h_6 = -[y_1 - \imath x_1],$$

$$g_5 = [2x_1 y_1 + \imath\,(y_1^2 - x_1^2)]\,e_6,$$

$$e_i = \frac{K_0\,(q_i\,a)}{K_1\,(q_i\,a)}, \quad e_j = \frac{K_0\,(q_j\,a)}{K_1\,(q_j\,a)}, \quad e_5 = \frac{\mathrm{I}_0\,(\zeta a)}{\mathrm{I}_1\,(\zeta a)}, \quad e_6 = \frac{\mathrm{I}_0\,(\zeta'\,a)}{\mathrm{I}_1\,(\zeta'\,a)}, \quad i = 1,2, \quad j = 3,4.$$

$$A_1 = h_5 g_5 + e_5 h_6, \quad A_2 = h_2 g_5 - e_2 h_6, \quad A_3 = h_2 e_5 + e_2 h_5,$$

$$A_4 = h_1\,e_5 + e_1 h_5, \quad A_5 = h_1 g_5 - e_1 h_6, \quad A_6 = h_4\,(g_5 - h_4 h_6 e_4),$$

$$A_7 = h_4\,(e_5 + h_4\,h_5\,e_4), \quad A_8 = h_3\,(g_5 - h_3 h_6 e_3), \quad A_9 = h_3\,(e_5 + h_3 h_5 e_3),$$

$$B_1 = h_3\,(h_1\,h_3 e_3 - e_1), \quad B_2 = h_4\,(h_1\,h_4\,e_4 - e_1), \quad B_3 = h_3\,(h_2 h_3 e_3 - e_2),$$

$$B_4 = h_4\,(h_2 h_4 e_4 - e_2),$$

where

$$x_1 = \frac{1}{\sqrt{2}}\left[1 + \left(1 + \frac{c^4}{c_{\ell_0}^4\,\omega^2}\right)\right]^{1/2}, \quad x_2 = \frac{1}{\sqrt{2}}\,[1 - e_7 + e_8]^{1/2},$$

$$y_1 = \frac{1}{\sqrt{2}}\left[\left(1 + \frac{c^4\rho' K'}{c_{\ell_0}^4\,\omega^2}\right)^{-1}\right]^{1/2}, \quad y_2 = \frac{1}{\sqrt{2}}\,[-1 + e_7 + e_8]^{1/2},$$

$$e_7 = \frac{c^2\rho' K'}{(K'^2 + \frac{16}{9}\,\omega^2\eta^2)}, \quad e_8 = \left(1 + \frac{c^2\rho'\,(c^2\rho' - 2K')}{(K'^2 + \frac{16}{9}\,\omega^2\eta^2)}\right)^{1/2}. \tag{31}$$

4. SPECIAL CASES

(1) If $\eta \to 0$, we obtain the frequency equation in a microstretch elastic medium containing a cylindrical bore filled with a homogeneous, inviscid liquid, as

$$(a_1 D_2 - a_2 D_1)\,[D_3^* b_4 q_4 - D_4^* b_3 q_3] - (2\mu+K)^2\,k^2\,[q_3\,b_4 q_4 - q_4 b_3 q_3]\,D_5 = 0. \tag{32}$$

where

$$a_i = (2\mu + K)\, k^2 + K b_i + (\mu + K)\,(q_i^2 - k^2),$$

$$D_i = b_i q_i \left[\frac{\beta + \gamma}{q_i a} + \gamma\, e_i\right], \quad D_j^* = D_j - M q_j,$$

$$D_j = (2\mu + K)\, q_j^2 \left[e_j + \frac{1}{q_j\, a}\right] + [\lambda\,(q_j^2 - k^2) + \lambda_0 b_j]\, e_j,$$

$$D_5 = \left[D_2 q_1 \left(e_1 + \frac{1}{q_1 a}\right) - \frac{M D_2}{(2\mu + K)}\right] - \left[D_1 q_2 \left(e_2 + \frac{1}{q_2\, a}\right) - \frac{M D_1}{(2\mu + K)}\right],$$

$$M = \frac{\rho_0 \omega^2}{\zeta_0} \frac{\mathrm{I}_0\,(\zeta_0\, a)}{\mathrm{I}_1\,(\zeta_0\, a)}, \quad \zeta_0^2 = k^2 \left(1 - \frac{c^2}{y^2}\right) \quad (i = 1, 2,\ j = 3, 4), \tag{33}$$

$y\,[\,= (K_0/\rho_0)^{1/2}]$ is the velocity of dilatational wave in liquid. ρ_0 and K_0 are the density and bulk modulus of liquid.

(2) If the bore is empty, the boundary conditions (29) take the form

$$t_{rr} = t_{rz} = m_{r\theta} = \lambda_r = 0 \quad \text{at } r = a \tag{34}$$

and we obtain the frequency equation for wave propagation in an empty bore in a microstretch elastic medium as

$$(a_1 D_2 - a_2 D_1)\,(D_3 b_4 q_4 - D_4 b_3 q_3) - (2\mu + K)^2\, k^2\, [q_3\, b_4\, q_4 - q_4 b_3 q_3]\, D_5' = 0, \tag{35}$$

where D_5' is same as D_5 with $M = 0$.

(3) If we neglect the stretch effect in equation (30), then the problem reduces to that of wave propagation in a cylindrical bore filled with viscous liquid and situated in a micropolar elastic medium. The dispersion equation obtained in this case coincides with the dispersion equation obtained by Deswal *et al.* [15].

(4) If we put $\alpha_0 = \lambda_0 = \lambda_1 = 0$ in equation (32), we obtain

$$\begin{aligned} &a_3\,[a_1 D_2 - a_2 D_1] - (2\mu + K)^2\, k^2\, q_3' D_5' - M q_3'\,[D_2\,\{a_1 - (2\mu + K)\, k^2\} \\ &\quad - D_1\,\{a_2 - (2\mu + K)\, k^2\}] = 0, \end{aligned} \tag{36}$$

where

$$a_3 = (2\mu + K)\, q_3'^2 \left[\frac{1}{q_3'\, a} + \frac{K_0\,(q_3' a)}{K_1\,(q_3'\, a)}\right] + \lambda\,(q_3'^2 - k^2)\, \frac{K_0\,(q_3'\, a)}{K_1\,(q_3'\, a)}, \quad q_3'^2 = k^2 - \frac{\omega^2}{c_1^2}. \tag{37}$$

Equation (36) is the frequency equation in a micropolar medium of infinite extent containing a cylindrical bore filled with homogeneous, inviscid liquid and is the same dispersion equation as obtained by Banerji and Sengupta [16].

By taking $K = 0$, in equation (36), we obtain the frequency equation as

$$\begin{aligned} &\left[2\mu\sigma^2 \left(\frac{1}{\sigma a} + \frac{K_0\,(\sigma a)}{K_1\,(\sigma a)}\right) + \lambda\,(\sigma^2 - k^2)\, \frac{K_0\,(\sigma a)}{K_1\,(\sigma a)}\right] (k^2 + q_1'^2) \\ &\quad - 4\mu k^2 \sigma\, q_1' \left[\frac{1}{q_1'\, a} + \frac{K_0\,(q_1'\, a)}{K_1\,(q_1'\, a)}\right] = \frac{\omega^2 \rho_0}{\zeta_0} \frac{\mathrm{I}_0\,(\zeta_0\, a)}{\mathrm{I}_1\,(\zeta_0\, a)}\, \sigma\,(q_1'^2 - k^2) \end{aligned} \tag{38}$$

and

$$\frac{1}{q_2'\, a} + \frac{\gamma}{\beta + \gamma} \frac{K_0\,(q_2'\, a)}{K_1\,(q_2'\, a)} = 0. \tag{39}$$

where

$$\sigma^2 = k^2 - \frac{\omega^2}{c_3^2}, \quad c_3^2 = \frac{(\lambda + 2\mu)}{\rho}, \quad q_1'^2 = k^2 - \frac{\omega^2}{c_2^2},$$

$$c_2^2 = \frac{\mu}{\rho}, \quad q_2'^2 = k^2 - \frac{\omega^2}{c_4^2}. \tag{40}$$

Equation (38) is the same dispersion equation as obtained as Biot [17]. Equation (39) refers to a hypothetical medium in which only rotation and couple stresses may exist and is obtained by using the boundary condition $m_{r\theta} = 0$.

(5) By neglecting stretch effect in equation (35), we obtain the frequency equation in a micropolar elastic medium containing an empty cylindrical bore as

$$a_3 (a_1 D_2 - a_2 D_1) - (2\mu + K)^2 k^2 q_3' D_5' = 0. \tag{41}$$

Equation (41) is the same frequency equation as obtained by Banerji and Sengupta [16].

(6) If we neglect the microstretch effect of the medium by putting $K = \beta = \gamma = \alpha_0 = \lambda_0 = \lambda_1 = 0$, in the frequency equation (30), we obtain the frequency equation for wave propagating in the bore hole filled with viscous liquid and situated in an elastic medium as obtained by Deswal *et al.* [15].

(7) In order to reduce the problem of wave propagation in a cylindrical bore filled with homogeneous, inviscid liquid and passing through an elastic medium of infinite extent we make the micropolar constants equal to zero in equation (36). The resulting dispersion equation corresponds to the dispersion equation obtained by Biot [17] for the relevant problem.

(8) By neglecting micropolar effect in equation (41), we obtain the frequency equation in an elastic medium containing empty cylindrical bore as obtained by Biot [17].

5. NUMERICAL RESULTS AND DISCUSSION

Frequency equation (30) is solved for phase velocity $C_0\ (= c/c_1)$ by giving different values of the wave number in non-dimensional form to study the viscous and stretch effects on dispersion equation numerically.

Following reference [18], the physical constants used are

$$\lambda = 7{\cdot}59 \times 10^{10}\ \text{dyn/cm}^2,$$

$$K = 0{\cdot}0149 \times 10^{10}\ \text{dyn/cm}^2,$$

$$\gamma = 2{\cdot}63 \times 10^{8}\ \text{dyn},$$

$$\rho = 2{\cdot}192\ \text{g/cm}^3,$$

$$\lambda_0 = \lambda_1 = 11{\cdot}37 \times 10^{10}\ \text{dyn/cm}^2,$$

$$\mu = 1{\cdot}89 \times 10^{10}\ \text{dyn/cm}^2$$

$$\beta = 2{\cdot}26 \times 10^{8}\ \text{dyn},$$

$$j = 0{\cdot}196 \times 10^{-2}\ \text{cm}^2$$

$$\omega^2/\omega_0^2 = 8{\cdot}0,$$

$$\alpha_0 = 1{\cdot}61 \times 10^{10}\ \text{dyn}.$$

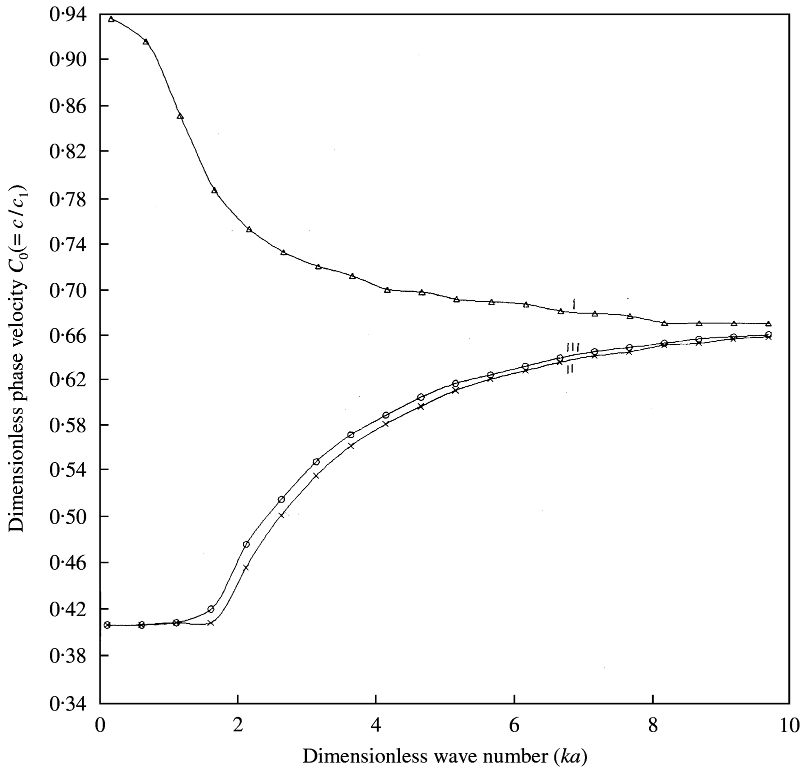


Figure 2. Variations of dimensionless phase velocity versus dimensionless wave number: (△△△△△), I, microstretech, viscous liquid; (× × × × ×), II, microstretch, inviscid liquid; (○○○○○), III, microstretch, empty.

For viscous liquid, the values of elastic parameters for kerosense oil [19, pp. 363 and 366] are taken as

$$c_\ell = 1320 \times 10^2\,\text{cm/s}, \quad \rho' = 0{\cdot}8201\ \text{g cm}^3, \quad \eta = 0{\cdot}0164\ \text{P}.$$

The following values are taken from Ewing *et al.* [20] for inviscid liquid as

$$K_0 = 2{\cdot}14 \times 10^{10}\ \text{dyn/cm}^2, \quad \rho_0 = 1{\cdot}0\ \text{g/cm}^3.$$

The values of dimensionless phase velocity C_0 ($= c/c_1$) are obtained as a function of dimensionless wave number ka to study the effects of viscosity, microstretch and micropolarity on dispersion curves in Figures 2–4. Figure 2 shows the effect of viscosity on dispersion curves. The values of phase velocity, when the bore is filled with viscous liquid, decrease monotonically with the increase in wave number and finally attain a constant value beyond $ka = 8{\cdot}0$. The values of phase velocity, in this case, lie in a higher range in comparison to the case of bore filled with inviscid liquid. These variations are shown by curves I and II in Figure 2 and depict the viscous effect on phase velocity. It is observed that curves I and II have opposite behavior due to viscous effect. Curve III represents the dispersion curve when the cylindrical bore is empty and situated in a microstretch elastic medium.

Microstretch effect on dispersion curves is shown in Figure 3. When the bore is filled with viscous liquid, the values of phase velocity vary in lower range in microstretch elastic medium in comparison to elastic medium, whereas in the case of empty bore and bore filled with inviscid liquid, the reverse happens after a small range ($0 \leqslant ka \leqslant 1{\cdot}8$), i.e., phase

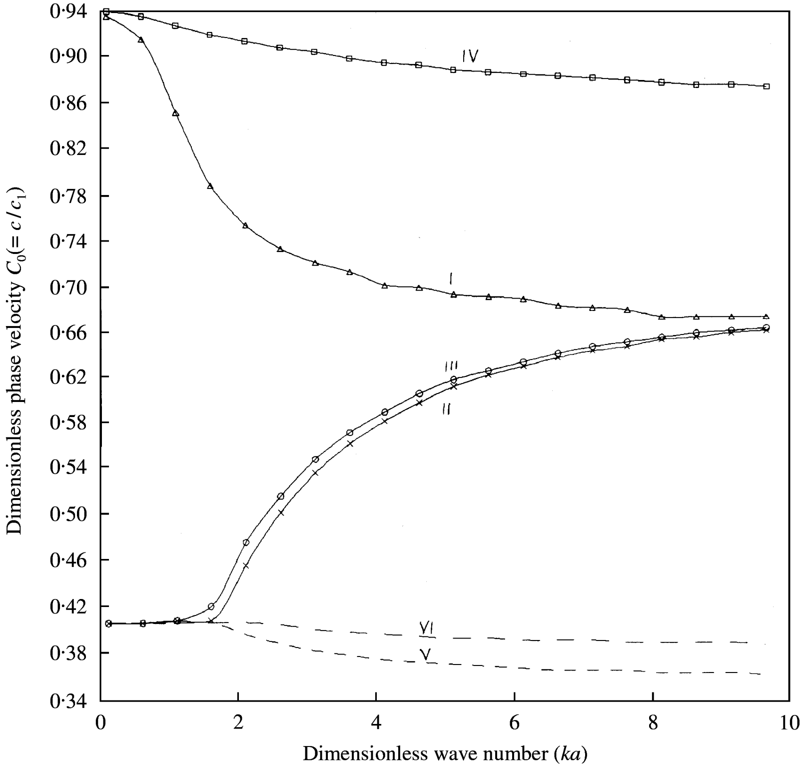
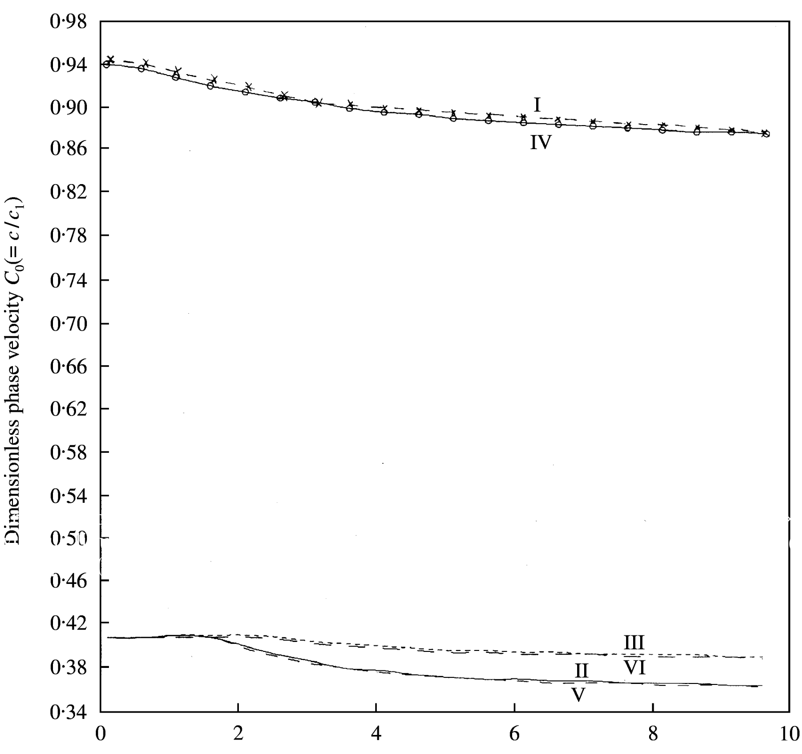


Figure 3. Variations of dimensionless phase velocity versus dimensionless wave number: (△△△△△), I, microstretech, viscous liquid; (× × × × ×), II, microstretch, inviscid liquid; (○○○○○), III, microstretch, empty; (□□□□□), IV, elastic, viscous liquid; (– – – –), V, elastic, inviscid liquid; (— —), VI, elastic empty.



velocities for microstretch elastic medium are more than the elastic medium. In Figure 4, the effect of micropolarity is observed by comparing curves, I, II, III with IV, V and VI respectively. It is observed that in all three cases, i.e., when the bore is filled with viscous liquid, inviscid liquid and empty bore, the values of phase velocity in the elastic medium are slightly less than the micropolar elastic medium.

6. CONCLUSIONS

A mathematical study is presented here to determine the effect of viscosity, microstretch and micropolarity on surface wave dispersion in bore holes. The frequency equations for two particular cases (micropolar elastic medium and elastic medium) are obtained from the frequency equation of the present problem. Numerical computations are performed to solve the frequency equation and it is seen that the phase velocity of wave propagation depends on wave number, showing that the frequency equations are dispersive. Also, the effect of viscosity and microstretch are more significant than the effect of micropolarity on dispersion curves.

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Figure 4. Variations of dimensionless phase velocity versus dimensionless wave number: (× × × ×), I, micropolar viscous liquid; (——), II, micropolar, inviscid liquid; (-------), III, micropolar, empty; (○ ○ ○ ○), IV, elastic, viscous liquid; (– – – –), V, elastic, inviscid liquid; (— —), VI, elastic, empty.

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