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EFFECT OF PHASE ON HUMAN RESPONSES TO VERTICAL WHOLE-BODY VIBRATION AND SHOCK—ANALYTICAL INVESTIGATION

Y. MATSUMOTO† AND M. J. GRIFFIN

Human Factors Research Unit, Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, England. E-mail: mjg@isvr.soton.ac.uk

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The effect of the "phase" on human responses to vertical whole-body vibration and shock has been investigated analytically using alternative methods of predicting subjective responses (using r.m.s., VDV and various frequency weightings). Two types of phase have been investigated: the effect of the relative phase between two frequency components in the input stimulus, and the phase response of the human body. Continuous vibrations and shocks, based on half-sine and one-and-a-half-sine accelerations, each of which had two frequency components, were used as input stimuli. For the continuous vibrations, an effect of relative phase was found for the vibration dose value (VDV) when the ratio between two frequency components was three: about 12% variation in the VDV of the unweighted acceleration was possible by changing the relative phase. The effect of the phase response of the body represented by frequency weightings was most significant when the frequencies of two sinusoidal components were about 3 and 9 Hz. With shocks, the effect of relative phase was observed for all stimuli used. The variation in the r.m.s. acceleration and in the VDV caused by variations in the relative phase varied between 3 and 100%, depending on the nature of stimulus and the frequency weighting. The phase of the frequency weightings had a different effect on the r.m.s. and the VDV.

1. INTRODUCTION

Whole-body vibration may cause discomfort, interfere with activities and impair health. Subjective reactions are of principal interest when assessing some types of whole-body vibration and mechanical shocks, such as those experienced in vehicles and in buildings caused by traffic, machinery, or persons walking. Previous psychophysical studies have investigated the relation between subjective sensations and the physical characteristics of various types of oscillatory stimuli. From these studies it is possible to predict judgments of comfort from measures of vibration.

The prediction of discomfort from whole-body vibration is normally based on frequency weightings derived from equivalent comfort contours showing the acceleration required to produce a similar degree of discomfort at each frequency of oscillation (e.g., references [1–6], reviewed in reference [7]). The results were used to define frequency weightings for the evaluation of whole-body vibration with respect to comfort, perception and health in British Standard 6841 [8]. Similar frequency weightings reappear in International Standard

†Now at Department of Civil and Environmental Engineering, Saitama University, 255 Shimo-Ohkubo, Urawa, Saitama, 338-8570, Japan.

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2631-1 [9], although modified without explanation [10]. The frequency weightings are assumed to modify the acceleration time history in a manner representative of the dependency of human response on vibration frequency. The standards define the phase response of the weightings but only the modulus of the weightings is based on knowledge of human response: the phase is mainly based on convenience when defining filters to achieve the desired modulus.

International Standard 2631-1 [9] and British Standard 6841 [8] offer the frequency-weighted root-mean-square (r.m.s.) acceleration as a standard evaluation method. There is, however, a general consensus from previous studies and experience that the r.m.s. method can underestimate response to transient vibration and intermittent and repeated shocks. For example, Griffin and Whitham [11] and Howarth and Griffin [12] found that subjective responses to impulsive stimuli with various waveforms were better predicted from the fourth power of the acceleration than from the square of the acceleration, as with the r.m.s. value. The fourth power vibration dose value (VDV) with an appropriate frequency weighting is offered in ISO 2631-1 [9] and BS 6841 [8] as an additional evaluation method. International Standard 2631-1 [9] also provides a running r.m.s. method (by determining the "maximum transient vibration value", MTVV) for situations in which the overall r.m.s. method may not be appropriate.

There are differing opinions as to the suitability of r.m.s. and the alternative evaluation methods and also the suitability of the frequency weightings. Payne *et al.* [13] and Payne [14] investigated the correlation between subjective reactions to vertical vibration in off-road vehicles and evaluations of the motions using alternative frequency weightings, including filters based on lumped parameter models of the biodynamic responses of the body developed by the authors. It was claimed that correlation coefficients were lower for evaluation methods using the frequency weighting defined in ISO 2631-1 [9] than for those using a filter developed by the authors. It was suggested that the ISO 2631 frequency weighting (i.e., W_k) should be replaced with their filter. It was also suggested that the time history of the W_k frequency-weighted acceleration was unrealistic because of the non-causal nature of the frequency weighting at low frequencies: the phase of the W_k frequency weighting is defined by a series of four filters and is positive (i.e., a phase lead) at frequencies below about 5.5 Hz. Payne and Payne *et al.* also stated that the correlation coefficients between the VDV and the subjective responses were lower than those calculated with other evaluation methods.

The "phase" response of the body, whether it be considered a mechanical, neurological, physiological, or psychological system, is therefore of interest. In the present study, subjective responses to simple stimuli were predicted so as to estimate the maximum likely effect of phase on subjective responses. Continuous vibrations and shocks that consisted of two frequency components were used as the input stimuli. This analytical investigation was conducted prior to an experimental study [15] so as to systematically investigate the possible effects of phase and determine suitable conditions for an experimental study.

2. METHOD

The root-mean-square (r.m.s.) value, the running r.m.s. value (MTVV) and the vibration dose value (VDV) were calculated for acceleration signals that consisted of two sinusoidal components. The phase difference between two sinusoidal components was altered systematically so as to investigate the influence of the phase relation between the two components on the r.m.s., the MTVV and the VDV. The definitions of the r.m.s., the MTVV and the VDV provided in the standards are:

$$r.m.s. = \left[\frac{1}{T} \int_0^T \{a_w(t)\}^2 \, \mathrm{d}t \right]^{1/2}, \tag{1}$$

$$MTVV = \max \left[\left[\frac{1}{\tau} \int_{t_0 - \tau}^{t_0} \{a_\mathrm{w}(t)\}^2 \, \mathrm{d}t \right]^{1/2} \right], \tag{2}$$

$$VDV = \left[\int_0^T \{a_w(t)\}^4 \, \mathrm{d}t \right]^{1/4}, \tag{3}$$

where $a_w(t)$ is the time history of frequency-weighted acceleration (m/s^2); T the duration of measurement (s) ($T > 0$); t_0 the time of observation (instantaneous time); and τ the integration time for running averaging ($\tau = 1$ (s) is recommended).

The frequency weightings defined for the evaluation of vertical seat surface vibrations, W_k in ISO 2631-1 [9] and W_b in BS6841 [8], were applied. The mathematical definitions of the W_k and W_b frequency weightings are provided in Appendix A.

The dependence of human response on frequency might be represented by frequency weightings other than those defined in the standards, such as from some measure of the biodynamic response of the body. A filter was therefore developed from the apparent mass of seated human subjects exposed to vertical whole-body vibration as measured by Matsumoto and Griffin [16]. The filter was based on a two-degree-of-freedom lumped parameter model. The parameters of the filter were determined by comparing the apparent mass of the model with the median measured apparent mass, so that it was not greatly affected by extreme values in the experimental data. The transfer function of the "apparent mass filter" is presented with its parameters in Appendix B. The apparent mass filter was used as a frequency weighting, along with those defined in the standards. Figure 1 shows the W_k and W_b frequency weightings defined in the standards and the apparent mass filter developed here over the frequency range 0·5–20 Hz.

It was hypothesized that the r.m.s., the MTVV, or the VDV mentioned above would predict the subjective reactions to the various input stimuli. The results obtained analytically here may, therefore, indicate the influence of the phase relation between two frequency components in the input stimuli and of the phase response of the human body system on the subjective reactions. Three types of input stimuli, continuous accelerations and two shock-type accelerations, were investigated.

3. RESULTS

3.1. CONTINUOUS ACCELERATION

For an input signal consisting of two continuous "stationary" sinusoidal components, the r.m.s. and the VDV values were calculated as follows:

$$r.m.s. = \left[\frac{1}{T} \int_0^T \{A \sin(\omega t) + RA \sin(r\omega t + \phi)\}^2 \, \mathrm{d}t \right]^{1/2}, \tag{4}$$

$$VDV = \left[\int_0^T \{A \sin(\omega t) + RA \sin(r\omega t + \phi)\}^4 \, \mathrm{d}t \right]^{1/4}, \tag{5}$$

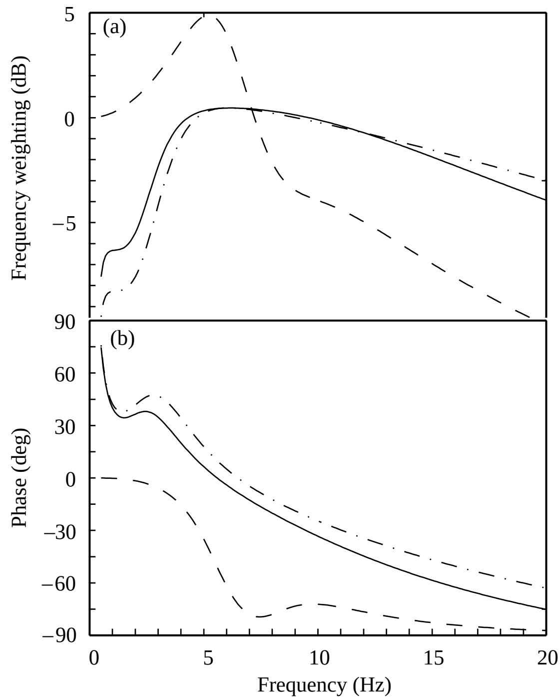

Figure 1. The W_k frequency weighting defined in ISO 2631-1 [9], the W_b frequency weighting defined in BS 6841 [8], and a filter based on measured apparent mass by Matsumoto and Griffin [16]: ——, W_k; $-\cdot-\cdot-\cdot-$ W_b; $----$, apparent mass filter.

where ω is the angular frequency of base component (rad/s) ($\omega > 0$); ϕ the phase of second component with respect to base component (rad); r the frequency ratio of second component to base component ($r \geqslant 1$); A the amplitude of base component (m/s^2) ($A > 0$); R the amplitude ratio of second component to base component ($R > 0$); T the duration (s) ($T > 0$).

Here, the effect of a frequency weighting, such as defined in ISO 2631-1 [9], are ignored so as to focus on the effect of phase difference between the two frequency components on the r.m.s and the VDV. The integrals in equations (4) and (5) were calculated and the equations can be expressed as

$$r.m.s. = A\left[\frac{N_{r.m.s.}(\omega, \phi, r, R, T)}{4\,r\,(r-1)(r+1)\,T\omega}\right]^{1/2} \tag{6}$$

$$VDV = A\left[\frac{N_{vdv}(\omega, \phi, r, R, T)}{32\,r\,(r-1)(r+1)(r-3)(r+3)(3r-1)(3r+1)\omega}\right]^{1/4}, \tag{7}$$

where the numerators, $N_{r.m.s.}(\omega, \phi, r, R, T)$ and $N_{vdv}(\omega, \phi, r, R, T)$, are functions containing trigonometric functions of ω, ϕ, r, and T. The forms of $N_{r.m.s.}(\omega, \phi, r, R, T)$ and $N_{vdv}(\omega, \phi, r, R, T)$ are presented in Appendix C. Equations (6), (7), (C1) and (C2) show that when $r = 1$ the denominators and the numerators of these equations were equal to zero ($r \geqslant 1$ in the definition above). When $r = 3$ the denominator and the numerator of equation (7) became zero.

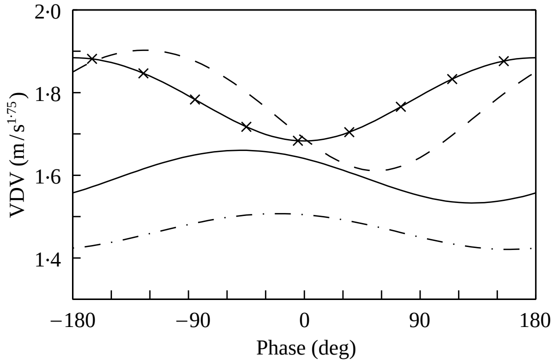

Figure 2. Effect of the phase of the second component with respect to the base component on the VDV calculated for continuous stimuli with different frequency weightings. Frequency of the base component: 3 Hz; amplitude of the two components: 1 m/s^2; duration: 20/3 s (i.e., 20 cycles for the base signal): —×—, no weighting; —— W_k; –·–·–·–, W_b; – – –, apparent mass filter.

When the duration of the input acceleration was an integer multiple of the period of the base acceleration (i.e., $T = 2n\pi/\omega$; n: positive integer) and the frequency ratio, r, was an integer, the r.m.s. and the VDV were obtained as

$$r.m.s. = \begin{cases} A\,\{(1+R^2)/2\}^{1/2} & (r \neq 1) \\ A\,[\{1+R^2+2R\cos(\phi\,)\}/2]^{1/2} & (r = 1) \end{cases} \tag{8}$$

$$VDV = \begin{cases} A[3n\pi(1+4R^2+R^4)/(4\omega)]^{1/4} & (r \neq 1, 3) \\ A[3n\pi(1+R^2+2R\cos(\phi)\}/(4\omega)]^{1/4} & (r = 1) \\ A[n\pi\{3(1+4R^2+R^4)-4R\cos(\phi)\}/(4\omega)]^{1/4} & (r = 3) \end{cases} \tag{9}$$

In equations (8) and (9), it is obvious that, when the frequency ratio between two frequency components is equal to unity (i.e., $r = 1$), the r.m.s. and the VDV are dependent on the phase difference between the two components; when the amplitudes of two frequency components are the same (i.e., $R = 1$), the base acceleration will be doubled when the phase, ϕ, is equal to 0, whereas the base acceleration will be cancelled out when the phase is equal to $(2n+1)\pi$ where n is an integer. It can be seen in equations (8) and (9) that, when the frequency ratio is equal to three (i.e., $r = 3$), the r.m.s. will be independent of the phase, ϕ, whereas the VDV will be dependent on the phase. The MTVV will also be independent of the phase when the r.m.s. is independent of the phase for stationary accelerations. When the amplitude ratio of two components is unity (i.e., $R = 1$), the maximum VDV will be 12% greater than the minimum VDV due to the phase difference between two frequency components. When the frequency ratio, r, is not an integer, terms dependent on the phase, ϕ, were included in the results. However, the effects of those phase-dependent terms were trivial compared to the other terms in the r.m.s. or the VDV and could be neglected.

Figure 2 shows the effect of the phase difference between two frequency components on the VDV, when the frequency of the second component is three times greater than the frequency of the base component (i.e., $r = 3$). The VDVs were calculated for input stimuli having the following parameters: $\omega/2\pi = 3$ Hz, $A = 1$ m/s^2, $R = 1$, $T = 40\pi/\omega$ s (i.e., 20 cycles for the base signal), $-\pi \leqslant \phi \leqslant \pi$ with an interval of $\pi/36$. In Figure 2, the VDVs obtained for frequency-weighted accelerations by three frequency weightings mentioned in

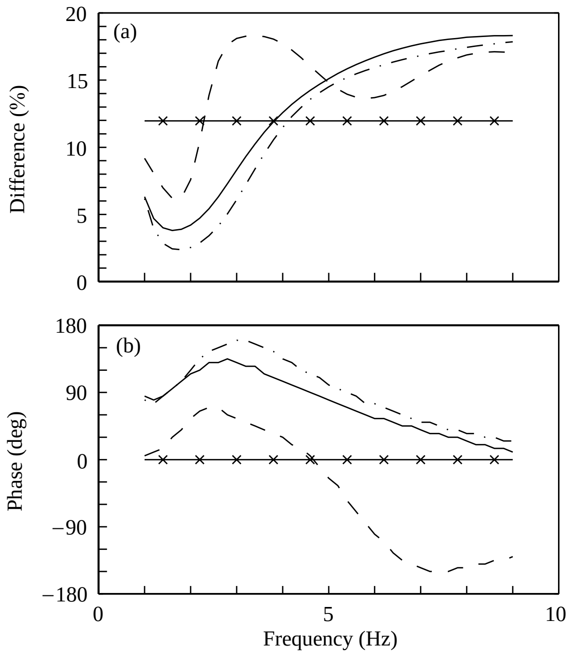

Figure 3. Effect of the base component frequency on the dependency of the VDV on the phase difference between two frequency components in continuous stimuli: (a) the percentage at which the maximum VDV is greater than the minimum VDV, and (b) the phase of the second component with respect to the base component at which the VDV is lowest. For the base component frequency between 1 and 9 Hz with the frequency ratio between two components fixed at 3: —×—, no weighting; ——, W_k; –·–·–·–, W_b; –––– apparent mass filter.

the preceding section are compared with the VDVs obtained without any frequency weighting, as in equation (5). It can be observed in Figure 2 that the VDV is dependent on the phase difference between two frequency components when a frequency weighting is not used as in equation (9). The dependency of the VDV on the phase of the input stimulus is also observed in the VDV obtained with frequency weightings, irrespective of the type of frequency weighting. However, the differences between the frequency weightings in the modulus and phase at 3 and 9 Hz, as seen in Figure 1, resulted in different trends in the dependency of the VDV on the phase of the input stimulus; the phase difference between two frequency components that gives the maximum or minimum VDV and the ratio between the maximum and minimum VDVs caused by the phase difference between two components are different for the three frequency weightings.

The trend of the dependency of the VDV obtained with frequency-weighted accelerations on the phase of the input stimulus differs for different frequencies of the base acceleration. This is caused by the frequency-dependent modulus and phase of the frequency weightings. Figure 3 shows the difference between the maximum and minimum VDVs caused by the phase difference between two input components and the phase of the second component at which the VDV is lowest. The frequency of base component was altered between 1 and 9 Hz. The phase of the second component at which the VDV was greatest was 180° greater or smaller than that shown in Figure 3(b). The frequency ratio of the second component to the base component was three, as in Figure 2 (i.e., $r = 3$). When no frequency weighting was used, the difference between the maximum and minimum VDVs and the phase for the minimum VDV were constant for all base component frequencies; about 12% and 0°, respectively, as seen in Figure 3 and also calculated by equation (9). The maximum difference in the VDV caused by the phase of the input stimulus was greatest (about 18%) at the base component frequency of around 8–9 Hz for the W_k and W_b weighted accelerations, and it was maximal (about 17–18%) at 3–3·5 Hz and at 8–9 Hz for the weighted acceleration when using the apparent mass filter (Figure 3(a)). It can be seen in Figure 3(b) that the differences in the phase between the unweighted acceleration and the weighted accelerations for the minimum VDV were greatest with a base component frequency of about 3 Hz for the W_k and W_b frequency weightings and maximal at about 2·5 and 7·5 Hz for the apparent mass filter.

If the duration of an input acceleration is not an integer multiple of the period of the base acceleration (i.e., $T \neq 2n\pi/\omega$; n: positive integer), the calculated r.m.s. and VDV have terms dependent on the phase difference between the two frequency components, ϕ, irrespective of the frequency ratio, r. However, the effect of phase-dependent terms caused by a non-integer multiple period reduces as the duration of the signal becomes longer. This is because these phase-dependent terms are caused only by a part of the stimulus corresponding to an incomplete cycle of the base component. When, for example, $r = 2$ and $R = 1$, the difference between the maximum and minimum r.m.s. values (without a frequency weighting) caused by the phase will be 57.6% of the minimum r.m.s. when the duration is half of the period of the base acceleration (i.e., $T = \pi/\omega$), although this stimulus cannot be called "stationary" or "continuous". When the duration increases to, for example, 20·5 periods of the base acceleration (i.e., $T = 41\pi/\omega$), the maximum difference in the r.m.s. due to the phase will be only 2·09%. For the VDV, the maximum value will be 5·80 times greater than the minimum value when $T = \pi/\omega$ (i.e., a half-cycle), while the difference between the maximum and minimum VDV will be only 3·70% when $T = 41\pi/\omega$ (i.e., 20·5 cycles). Therefore, if the duration is sufficiently long, the effects of phase on the r.m.s. and the VDV when the duration is not an integer multiple of the period of the base acceleration can be neglected. However, when $r = 3$, the effect of the phase-dependent term discussed for an integer multiple of the period, as seen in equation (9), applies for a non-integer multiple period even if the duration of the signal becomes long.

3.2. HALF-SINE ACCELERATION

It is stated in ISO 2631-1 [9] that the r.m.s. method may underestimate the effects of occasional shocks on humans. A half-sine acceleration shown in Figure 4 is often regarded as an example shock. The r.m.s. and the VDV were explicitly calculated for two half-sine accelerations superimposed with a phase of ϕ:

$$r.m.s. =$$

$$\left[\frac{1}{t_3}\left\{\int_0^{t_1}\{A\sin(\omega t)\}^2\,\mathrm{d}t + \int_{t_1}^{t_2}\{A\sin(\omega t) + RA\sin(r\omega t+\phi)\}^2\,\mathrm{d}t + \int_{t_2}^{t_3}\{A\sin(\omega t)\}^2\,\mathrm{d}t\right\}\right]^{1/2}, \tag{10}$$

$$VDV =$$

$$\left[\int_0^{t_1}\{A\sin(\omega t)\}^4\,\mathrm{d}t + \int_{t_1}^{t_2}\{A\sin(\omega t) + RA\sin(r\omega t+\phi)\}^4\,\mathrm{d}t + \int_{t_2}^{t_3}\{A\sin(\omega t)\}^4\,\mathrm{d}t\right]^{1/4}, \tag{11}$$

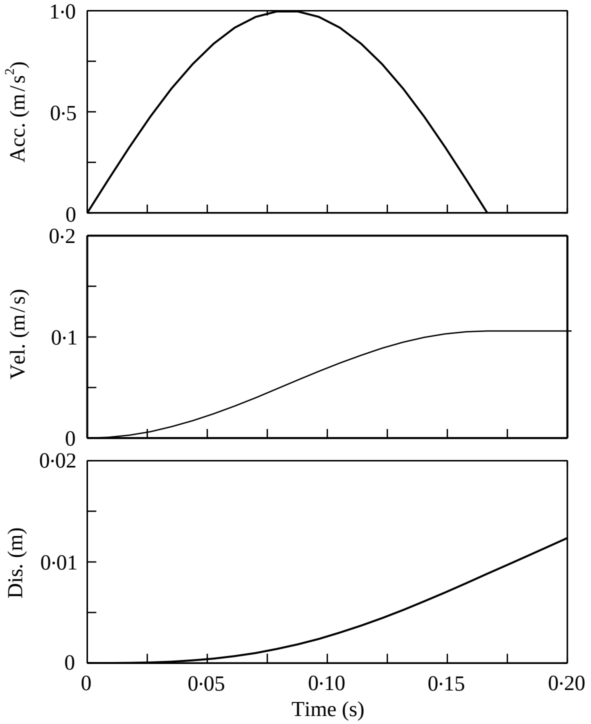

Figure 4. Example of half-sine acceleration and corresponding velocity and displacement obtained by the integration. Frequency is 3 Hz.

where $t_1 = -\phi/(r\omega)$ is the time when second shock begins (time delay); $t_2 = (-\phi + \pi)/(r\omega)$ is the time when second shock ends (delay + duration of second shock); $t_3 = \pi/\omega$ is the time when base shock ends (duration of base shock).

Here, the definitions of ω, ϕ, r, A, and R are the same as those in equations (4) and (5), except that the range of phase was limited as $\pi(1-r) \leqslant \phi \leqslant 0$ (i.e., the second shock always occurs within the duration of the base shock). The integrals in equations (10) and (11) were calculated; the equations can be expressed as

$$r.m.s. = A\left[\frac{\pi\,(r^2-1)(R^2+r) + 4r^2 R\cos(\pi/(2\,r))\sin\{(2\phi+\pi)/(2r)\}}{2\pi r(r-1)(r+1)}\right]^{1/2}, \tag{12}$$

$$VDV = A\left[\frac{N_{vdv}(\phi, r, R)}{8r(r-1)(r+1)(r-3)(r+3)(3r-1)(3r+1)\omega}\right]^{1/4}. \tag{13}$$

The numerator in equation (13), $N_{vdv}(\phi, r, R)$, is a function containing trigonometric functions of the phase ϕ and the frequency ratio r. The explicit form of $N_{vdv}(\phi, r, R)$ is presented in Appendix C. Unlike the r.m.s. and the VDV of continuous accelerations described above, the r.m.s. and the VDV for combined half-sine accelerations were dependent on the phase difference between two frequency components, irrespective of the frequency ratio r.

Figure 5 shows the r.m.s. and VDV obtained numerically for combined half-sine accelerations with $\omega/2\pi = 3$ Hz, $r = 3$, $A = 1$ m/s^2, and $R = 1$. The phase of the second frequency component with respect to the base component is represented as a "normalized

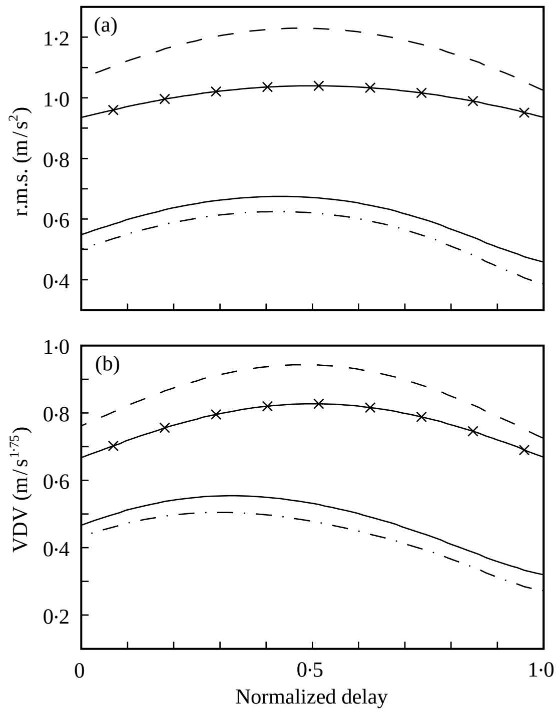

Figure 5. Effect of the phase of the second component with respect to the base component on the r.m.s. and the VDV calculated for half-sine stimuli with different frequency weightings. Frequency of the base component: 3 Hz; frequency ratio of the second component to the base component: 3; amplitude of the two components: 1 m/s^2. —×—, no weighting; ——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

delay"; the time delay of the second component was normalized by dividing it by the maximum time delay which is obtained when $\phi = \pi\,(1 - r)$, as defined above. A normalized delay of zero, therefore, means that the two components begin simultaneously; a normalized delay of unity means that the two components end at the same time. The r.m.s. values and the VDVs in Figure 5 were obtained without a frequency weighting and with the W_k and W_b frequency weightings and the apparent mass filter. The MTVV was not obtained because the duration of the signal is less than the integration time constant recommended in ISO 2631-1 [9]. It is clear in Figure 5 that the phase difference between the two components had effects on both the r.m.s. and the VDV, as seen in equations (12) and (13), whereas for continuous acceleration it only affected the VDV when $r = 3$, as described in the previous section.

The manner in which the r.m.s. and the VDV are dependent on the phase of the input stimulus (i.e., normalized delay) depends on the frequencies of the two components (i.e., the base component frequency, $\omega/2\pi$, and the frequency ratio between the two components, r). Figure 6 shows the maximum difference in the r.m.s. and the VDV caused by the phase between two components and the normalized time delay at which the r.m.s. and the VDV are greatest, for frequencies of the base component between 1 and 9 Hz. The frequency ratio between the two components was fixed at three (i.e., $r = 3$). The maximum difference was defined by the difference between the maximum value and the minimum value divided by the minimum value. The maximum differences of about 60% in the r.m.s. and about 90% in the VDV were caused by variations in the phase of the input stimulus with the W_b frequency-weighted acceleration at the base component frequency of about 2·4 Hz (Figures 6(a) and 6(c)). For the unweighted acceleration, the maximum differences in the

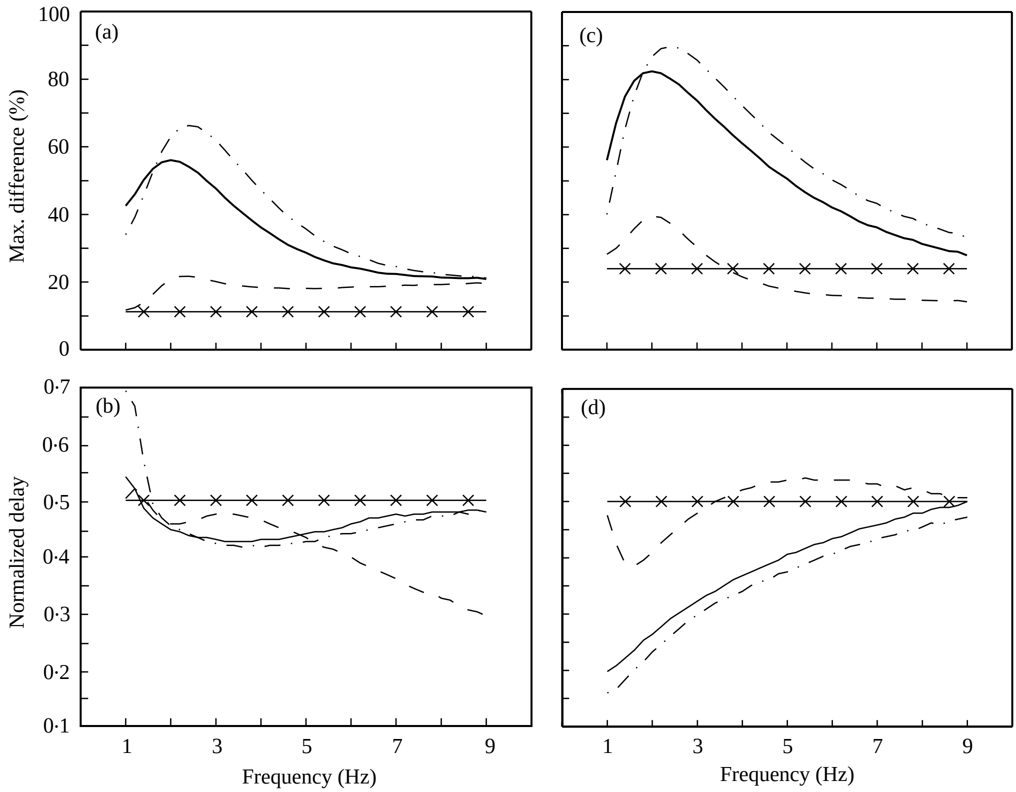

Figure 6. Effect of the base component frequency on the dependency of the r.m.s. and the VDV on the phase difference between two frequency components in half-sine stimuli: the percentage at which the maximum value is greater than the minimum value for (a) the r.m.s. and (c) the VDV, and the phase of the second component with respect to the base component at which the value is highest for (b) the r.m.s. and (d) the VDV. For the base acceleration frequency between 1 and 9 Hz with the frequency ratio between two components fixed at 3: —×—, no weighting; ——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

r.m.s. and the VDV were 11·3 and 24·2%, respectively, for all base component frequencies. The effects of the phase of the input stimulus on the maximum differences in the r.m.s. and the VDV were similar for the W_b and W_k frequency-weighted accelerations, with a difference of 10% at most. For the W_b and W_k frequency-weighted accelerations, the effect of the phase of the input stimulus on the VDV tended to be greater than that on the r.m.s. For the input stimuli weighted by the apparent mass filter, the effect of the phase of the input stimulus was not as great as with the W_b and W_k frequency-weighted stimuli. In Figures 6(b) and 6(d), it is observed that, for unweighted input stimuli, the maximum r.m.s. and VDV were obtained when the peaks of two frequency components occurred at the same time (i.e., a normalized delay of 0.5). This phase relation was altered by the phase of the frequency weightings (Figures 6(b) and 6(d)). Differences in the dependence on the frequency of the base component of the normalized delay for the maximum value were observed between the r.m.s. and VDV.

When the frequency ratio between two input components was altered and the frequency of the base component was fixed, the maximum difference in the r.m.s. or in the VDV caused by the phase between the two input components and the normalized delay for the maximum r.m.s. or VDV varied, as shown in Figure 7 for the base component frequency of 3 Hz. For the unweighted acceleration, the maximum difference caused by the phase of the input

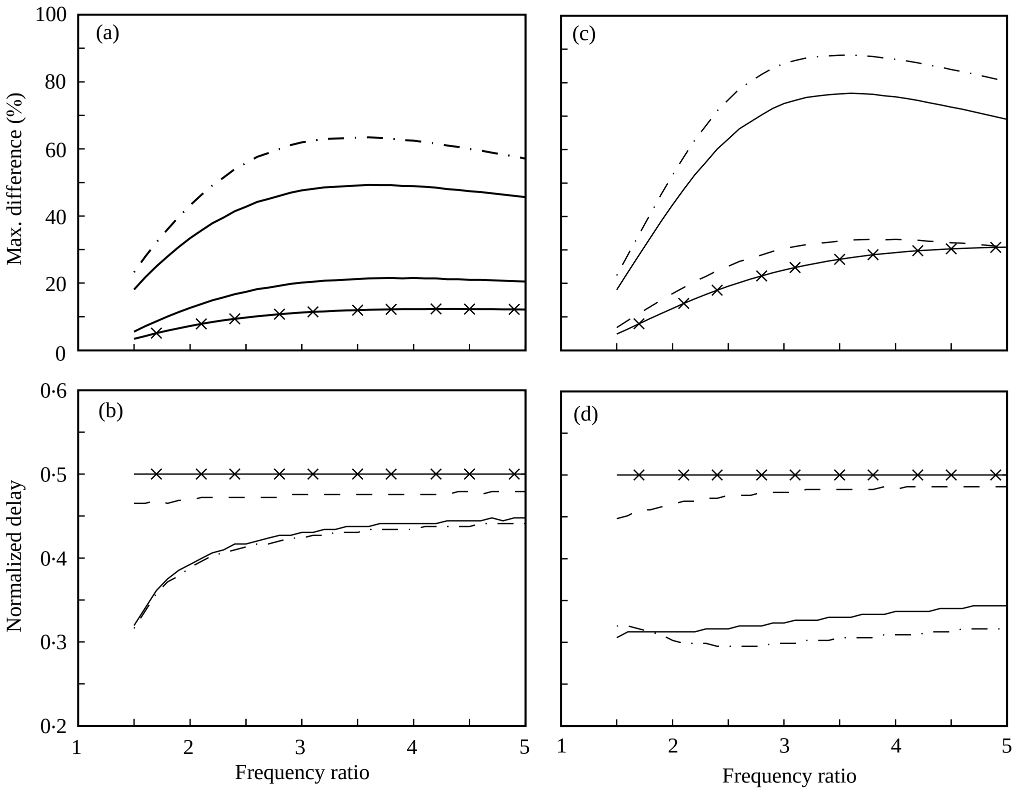

Figure 7. Effect of the frequency ratio of the second input component to the base component on the dependency of the r.m.s. and the VDV on the phase difference between two frequency components in half-sine stimuli: the percentage at which the maximum value is greater than the minimum value for (a) the r.m.s. and (c) the VDV, and the phase of the second component with respect to the base component at which the value is highest for (b) the r.m.s. and (d) the VDV. For the frequency ratio between 1.5 and 5 with the base component frequency fixed at 3 Hz: —×—, no weighting;——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

stimulus (i.e., normalized delay) increased with increases in the frequency ratio between two input components. When the frequency ratio was 3–3·5, the maximum difference caused by the phase of the input stimulus appeared to be greatest for the three frequency-weighted accelerations (Figures 7(a) and 7(c)). The greatest maximum difference was about 60% for the r.m.s. and about 90% for the VDV; both occurred for the W_b frequency-weighted acceleration. The normalized delay for the maximum r.m.s. or VDV appeared not to be significantly affected by the frequency ratio between the two input components, except for the r.m.s. of the W_b and W_k frequency-weighted accelerations with lower frequency ratios (Figures 7(b) and 7(d)).

3.3. SHOCK BASED ON ONE-AND-A-HALF-SINE ACCELERATION

Residual velocity and displacement will be caused by the half-sine acceleration discussed in the previous section (see Figure 4). These waveforms may not, therefore, be realistic. An idealized simple shock that does not cause residual velocity and displacement is shown in Figure 8 and was used to calculate the r.m.s. and the VDV. The acceleration waveform was produced by one-and-a-half cycles of sinusoidal acceleration modulated by a half-cycle

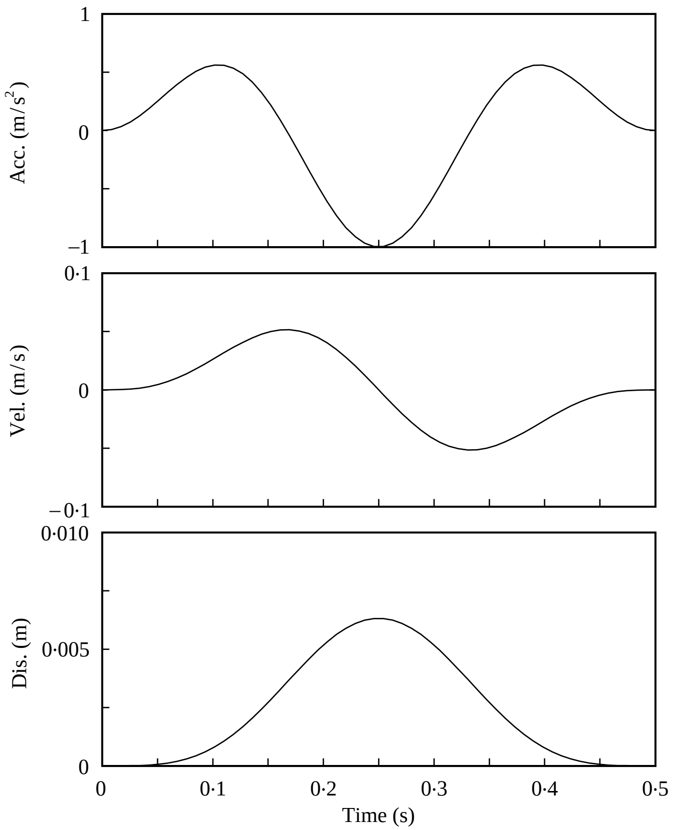

Figure 8. Example of shock based on one-and-a-half cycles of sinusoidal acceleration and corresponding velocity and displacement obtained by the integration. Frequency is 3 Hz. No residual velocity and displacement.

sinusoid at unit amplitude with a period three times longer than the period of the sinusoidal acceleration. Pairs of these accelerations were superimposed to investigate the effect of phase differences between the two components. The r.m.s. and the VDV were calculated as

$$r.m.s. = \left[\frac{1}{t_3}\left\{\int_0^{t_1}\{\sin(\omega t/3)A\sin(\omega t)\}^2\,\mathrm{d}t + \int_{t_1}^{t_2}\{\sin(\omega t/3)A\sin(\omega t) + \sin((r\omega t+\phi)/3)RA\sin(r\omega t+\phi)\}^2\,\mathrm{d}t + \int_{t_2}^{t_3}\{\sin(\omega t/3)A\sin(\omega t)\}^2\,\mathrm{d}t\right\}\right]^{1/2}, \tag{14}$$

$$VDV = \left[\int_0^{t_1}[\sin(\omega t/3)A\sin(\omega t)\}^4\,\mathrm{d}t + \int_{t_1}^{t_2}\{\sin(\omega t/3)A\sin(\omega t) + \sin((r\omega t+\phi)/3)RA\sin(r\omega t+\phi)\}^4\,\mathrm{d}t + \int_{t_2}^{t_3}\{\sin(\omega t/3)A\sin(\omega t)\}^4\,\mathrm{d}t\right\}\right]^{1/4}, \tag{15}$$

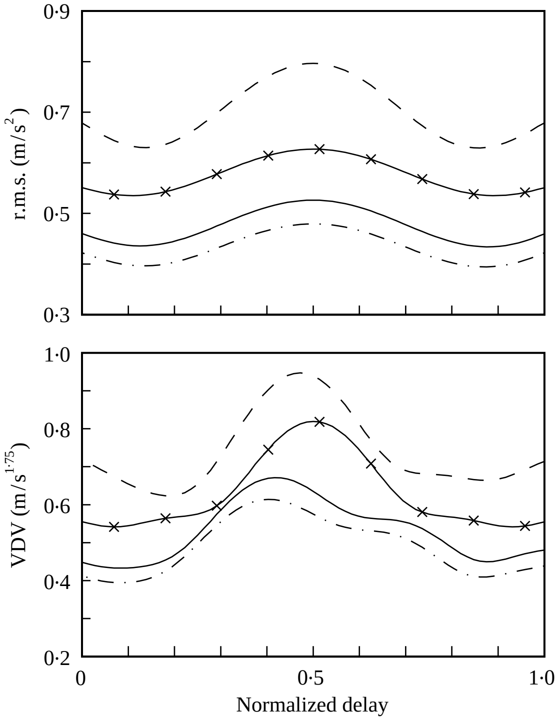

Figure 9. Effect of the phase of the second component with respect to the base component on the r.m.s. and the VDV calculated for shock stimuli with different frequency weightings. Frequency of the base component: 3 Hz; frequency ratio of the second component to the base component: 3; amplitude of the two components: 1 m/s^2: —×—, no weighting; ——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

where $t_1 = -\phi/(r\omega)$ is the time when second shock begins (time delay); $t_2 = (-\phi + 3\pi)/(r\omega)$ is the time when second shock ends (delay + duration of second shock); $t_3 = 3\pi/\omega$ is the time when base shock ends (duration of base shock).

The definitions of ω, ϕ, r, A, and R are the same as those in equations (1) and (2), except that the range of phase was limited to $3\pi(1 - r) \leqslant \phi \leqslant 0$ (i.e., the second shock always occurred within the duration of the base shock). When the integrals in equations (14) and (15) were calculated, the r.m.s. and the VDV were dependent on the phase ϕ, irrespective of the frequency ratio, r, and the other parameters, as for a half-sine shock discussed above.

Figure 9 shows the r.m.s. and the VDV obtained numerically for combined shock-type accelerations with $\omega/2\pi = 3$ Hz, $r = 3$, $A = 1$ m/s^2, and $R = 1$. The phase of the second component with respect to the base component was represented by a normalized delay in Figure 9, as in Figure 6. In Figure 9, however, the time delay was divided by the maximum time delay which was obtained when $\phi = 3\pi(1 - r)$. The interpretation of a normalized delay of zero and of unity is the same as that for the half-sine stimuli. The MTVV was not obtained because of the short duration of the signals. It can be seen in Figure 9 that the phase difference between the two shock components influenced both the r.m.s. and the VDV irrespective of frequency weighting. The effect of the phase in the input stimulus on the VDV was clearer than the effect on the r.m.s. For the unweighted acceleration, the highest value of the r.m.s. was about 17% greater than the lowest r.m.s., the highest VDV was about 51% greater than the lowest VDV. The highest values of r.m.s. and VDV for the unweighted

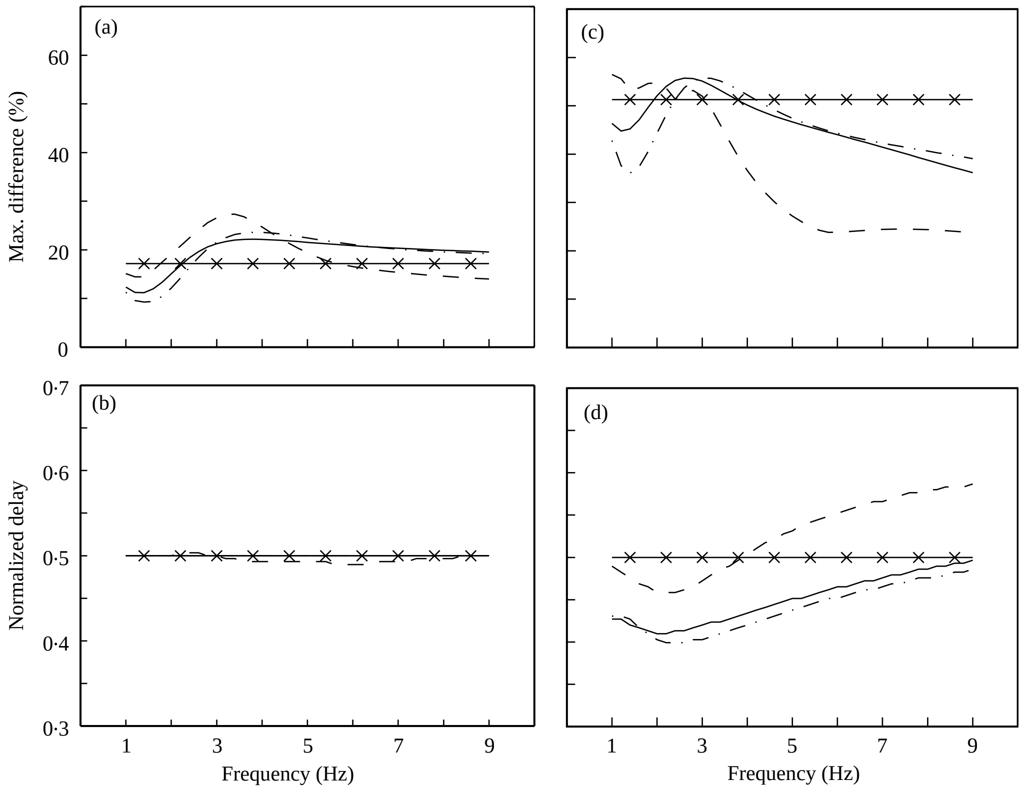

Figure 10. Effect of the base acceleration frequency on the dependency of the r.m.s. and the VDV on the phase difference between two frequency components in one-and-a-half-sine shock stimuli: the percentage at which the maximum value is greater than the minimum value for (a) the r.m.s. and (c) the VDV, and the phase of the second component with respect to the base component at which the value is highest for (b) the r.m.s. and (d) the VDV. For the base acceleration frequency between 1 and 9 Hz with the frequency ratio between two components fixed at 3: —×—, no weighting; ——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

acceleration were obtained when the peaks of the two components occurred at the same time, as in the case of half-sine accelerations described above. Each frequency weighting influenced the magnitude of the r.m.s. and the VDV and the dependency of the magnitude of the VDV on the normalized delay.

In Figure 10, the effect of the base component frequency on the dependency of the r.m.s. and the VDV on the phase difference between the two input components is shown. In the figure, the frequency ratio between the two components is fixed at three (i.e., $r = 3$), as in Figure 9. The maximum difference in the r.m.s. caused by the phase difference between the two input components varied between 9 and 28% for base component frequencies of 1–9 Hz (Figure 10(a)). At frequencies of the base component between 3 and 4 Hz, the maximum difference in the r.m.s. of the weighted accelerations appeared to be greatest in the frequency range investigated. The maximum r.m.s. was obtained at a normalized delay of 0.5 irrespective of frequency weighting (Figure 10(b)). The effect of the base component frequency on the VDV of the weighted accelerations was more significant than that on the r.m.s. The maximum difference in the VDV varied between 22 and 57% (Figure 10(c)). The maximum difference in the VDV was greatest at a base component frequency of 2–3 Hz for the W_b and W_k frequency-weighted shocks. The maximum difference in the VDV decreased

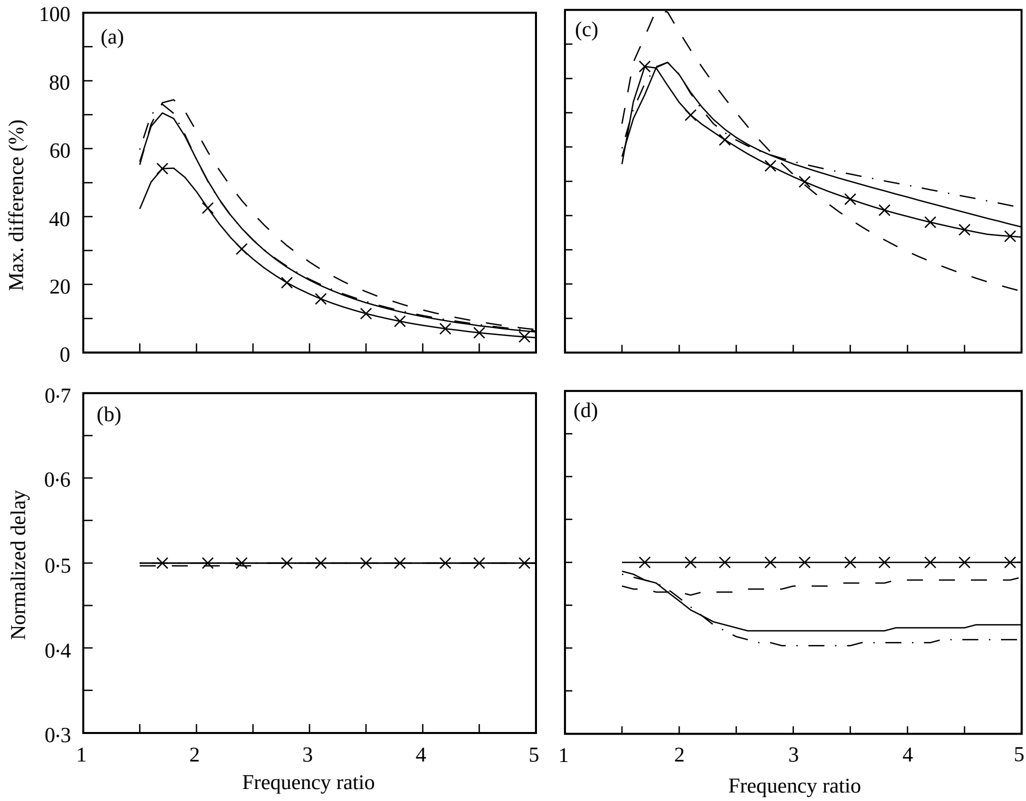

Figure 11. Effect of the frequency ratio of the second input component to the base component on the dependency of the r.m.s. and the VDV on the phase difference between two frequency components in one-and-a-half-sine shock stimuli: the percentage at which the maximum value is greater than the minimum value for (a) the r.m.s. and (c) the VDV, and the phase of the second component with respect to the base component at which the value is highest for (b) the r.m.s. and (d) the VDV. For the frequency ratio between 1.5 and 5 with the base component frequency fixed at 3 Hz: —×—, no weighting; ——, W_k; –·–·–·–, W_b; – – – –, apparent mass filter.

with increases in the base component frequency for the shock weighted by the apparent mass filter. The normalized delay for the maximum VDV was most affected by the W_b and W_k frequency weighting when the base component frequency was about 2–3 Hz (Figure 10(d)).

The effect of the phase difference between the two components in the shock stimuli on the r.m.s. and VDV was dependent on the frequency ratio between the two components, irrespective of frequency weighting (see Figure 11). The frequency ratio between the two components was altered between 1·5 and 5 while the base component frequency was fixed at 3 Hz in Figure 11. Both for the r.m.s. and for the VDV, the maximum difference caused by the phase in the input stimulus was greatest when the frequency ratio was about 1·7–1.8, irrespective of frequency weighting (Figures 11(a) and 11(c)). The normalized delay for the maximum r.m.s. was affected neither by the frequency ratio nor by the frequency weighting (Figure 11(b)). For the VDV, however, the W_b and W_k frequency weighting changed the normalized delay for the maximum VDV to 0.4 for a frequency ratio of 3–5, compared to a normalized delay of 0.5 for unweighted shock (Figure 11(d)). The effect of the apparent mass filter on the normalized delay for the maximum VDV was not as significant as that of the W_b and W_k frequency weighting.

4. DISCUSSION

Morioka and Griffin [17] reported that human subjects exposed to vertical sinusoidal whole-body vibration when seated on a rigid seat just detected a difference in the vibration magnitude if the magnitude difference was more than 10% in the r.m.s. acceleration. Mansfield and Griffin [18] measured the difference threshold when subjects were seated on an automotive seat in a laboratory and exposed to vertical seat vibrations recorded in a car. The difference threshold obtained was approximately 13% in the W_b weighted r.m.s. acceleration. The differences in the r.m.s. and the VDV obtained in this investigation may be compared with those difference thresholds for whole-body vibration obtained experimentally, although the waveforms of the input stimuli are different.

The maximum difference in the r.m.s. caused by the phase relation between two frequency components for the one-and-a-half-sine shock was about 10% or less when the frequency ratio between the two components was greater than four, irrespective of frequency weighting (Figure 11(a)). These differences may not be detected by human subjects according to the results of the previous experimental studies. However, for the same input accelerations, the maximum difference in the VDV caused by the phase of the input stimulus was greater than 15% for all frequency weightings and may be detected by subjects. Therefore, with the one-and-a-half-sine shock with the frequency ratio greater than four, the r.m.s. evaluation and the VDV evaluation yield different conclusions as to the effect of the phase of the input stimulus on subjective response. In the other conditions (i.e., other combinations of input acceleration, base component frequency, frequency ratio) investigated here, the differences in the r.m.s. and the VDV caused by the phase relation between two frequency components were greater than 10% with at least one frequency weighting (including without a frequency weighting). It may be concluded, therefore, that differences caused by the phase relation between two components may be detectable by human subjects.

In the results described above, the effects of two types of "phase" on the r.m.s. and the VDV were investigated; the phase between two frequency components in the input stimulus and the phase of the frequency weighting. The effect of the phase difference between two frequency components in the input stimulus on the r.m.s. and the VDV represented how the phase relation between two components altered an input stimulus in terms of average (i.e., r.m.s.) and accumulated (i.e., VDV) values. The effect of the phase of the frequency weighting on the r.m.s. and the VDV may be thought of as the effect of the phase response of the body system on, in this study, subjective reactions to the input stimuli. For this interpretation of the frequency weightings, it is assumed that the r.m.s. or the VDV of a weighted acceleration may represent the subjective responses to the input stimuli. This is implied in the standards (ISO 2631-1 [9], BS 6841 [8]). The r.m.s. and the VDV of unweighted accelerations are affected only by the phase difference between two input components, whereas the r.m.s. and VDV of weighted accelerations are affected by the phase difference between the input components and the phase response of the filter.

The effect of the phase relation between the two input frequency components was observed on both the r.m.s. and the VDV of unweighted accelerations for the two shock-type accelerations used here (e.g., Figures 5 and 9). However, the r.m.s. and the VDV of unweighted accelerations for the continuous vibration were not affected by the phase relation between the two components, except when the frequencies of the two components coincided ($r = 1$), when $r = 3$ for the VDV, and when the duration of signal was short and was not an integer multiple of the base component period. A difference between the shock-type accelerations and the continuous accelerations used in the present study was that the durations of the two frequency components were different in the shocks but the same in the continuous vibrations. Another difference between the shocks and the continuous vibrations was that the continuous vibrations had the same peak absolute accelerations in the positive and negative directions whereas the shocks had different peak magnitudes in the two directions (see Figures 4 and 8); each frequency component had an incomplete sinusoidal cycle. In the shocks, the second component, with a shorter duration, moderated the effect of a part of the base component with a longer duration, particularly in the direction of the maximum peak of the second component. The r.m.s. and the VDV tended to increase when the second component moderated a part of the base component around its maximum peak, compared to the effect with other phase relations between the two components. With the continuous vibrations, the second component moderated the base component evenly over the stimulus duration in the positive and negative directions. This resulted in no effect, or only a small effect, on the phase difference between the two components on the r.m.s. and the VDV.

For the shock-type accelerations, the most evident effect of the phase relation between the two components on the input acceleration time history and on the resulting r.m.s. and VDV may be the variation of its peak value. Peak acceleration values will have more effect on the VDV than in the r.m.s. because the VDV is calculated from the fourth power of accelerations while the r.m.s. is obtained from squared accelerations. It is, therefore, reasonable that the effect of the phase relation between the two components is more significant in the VDV than in the r.m.s., as seen in Figures 6(a) and 6(c), 7(a) and 7(c), 10(a) and 10(c) and 11(a) and 11(c).

The VDV of unweighted continuous accelerations when the frequency ratio between the two components was three (i.e., $r = 3$) was affected by the phase relation between the two components, although the VDV of unweighted continuous accelerations for other conditions showed no effect, except for the obvious case of $r = 1$. This arose mathematically from the integration of the fourth power of sinusoidal components. The term $(r - 3)$ in the denominator of equation (7) was obtained from the integration of the fourth power of the sum of trigonometric functions of ωt and $r\omega t$. The peak acceleration varied between 1·5 and 2·0 m/s^2 when the phase relation between two components, ϕ, was altered while $A = 1$ m/s^2 and $R = 1$. The VDV was lowest when the peak acceleration was 1·5 m/s^2 and the VDV was highest when the peak was 2·0 m/s^2. Although it seems reasonable that the VDV increases with an increase of peak acceleration, this dependency of the VDV on the peak acceleration was not observed for continuous signals with other frequency ratios. Whether the dependency of the VDV on the phase difference between two continuous accelerations when $r = 3$ is reflected in human responses has not been previously investigated. This has been investigated in the accompanying experimental study [15].

There were clear effects of the phases of the frequency weightings on the r.m.s. and the VDV. In Figure 9, for example, the phase difference between two shock components, represented by the normalized delay, with which the VDV was maximum was 0.5 for the unweighted shock, about 0.4 for the W_k and W_b weighted shock, and about 0·47 for the shock weighted by the apparent mass filter. The maximum VDVs for the shocks were obtained when the peaks of the two components occurred simultaneously. This corresponded to a normalized delay of 0·5 for the unweighted shock. For the weighted shocks, the normalized delay producing the maximum VDV was not 0·5 because each frequency component was differently affected by the phase of the frequency weighting. For the frequency weightings used here, the phase at the second component frequency, 12 Hz in Figure 9, was more delayed than the phase of the base component frequency, 3 Hz. This resulted in the normalized delay of the second component to be less than 0.5 for the maximum VDV of the frequency-weighted shocks. If a similar frequency-dependent delay mechanism were to occur in the human responses to vibration and shock, the phase of the frequency weighting filter may be considered to be a model of the delay mechanism, or phase response, of the body system. The results described above imply that the phase response of the body system affects subjective reactions. However, an effect of the phase of the frequency weighting was small in the r.m.s. value of the one-and-a-half-sine shock, as in Figures 9(a), 10(b) and 11(b). This arises because the r.m.s. of this type of shock is not as sensitive to peak accelerations as the VDV.

The phases of the frequency weightings investigated here might be assumed to represent the relative phase of the subjective reactions to vibration and shock between different frequencies. Payne *et al.* [13] implied that the causality of a frequency weighting filter (i.e., the absolute phase response) was important on the assumption that predictions of instantaneous subjective evaluations of vibration or shock are required. However, it is doubtful whether subjects make "instantaneous" subjective reactions. Accumulative effects, such as the time dependency reviewed by Howarth [19], may be important factors influencing subjective reactions but are not included in the calculation of frequency-weighted acceleration time history. The acceleration time history, weighted by the frequency weightings defined in the standards, by the apparent mass filter used in this investigation, or by any other existing filters, cannot represent the "time history" of subjective reactions. The evaluation methods of vibration and shock defined in ISO 2631-1 [9] and BS 6841 [8] are not intended to assess the instantaneous subjective reactions but the total response. The record of acceleration experienced by a subject is truncated at the end of the motion and there is no future information: the current and future acceleration is zero but the subjective impression from past events remains. The non-causality of the frequency weightings in ISO 2631-1 [9] and BS 6841 [8] may therefore not cause any problem as long as the evaluation method is intended to predict a cumulative or average response. The relative phase response of the filters between different frequencies, as discussed in this study, remains important when predicting such subjective responses; it is not clear whether the absolute phase response is significant.

5. CONCLUSIONS

The effects of the relative phase between input stimuli consisting of two frequency components on the evaluation of vibration with respect to subjective reactions have been investigated analytically. The evaluation has been performed using the r.m.s. and the VDV with the frequency weightings defined in the standards and with a frequency weighting developed from the biodynamic response of the body. The effect of the phase of the frequency weightings, which may represent the phase response of the body system, on the evaluation of vibration and shock has also been investigated.

It has been found that there is no effect of the relative phase between the input stimuli on the evaluation of continuous vibrations, other than a small effect on the VDV when the ratio between the frequencies of two components is three. For the input stimuli used in this investigation, the difference in the VDV was about 12% when no frequency weighting was used, and varied between 2 and 18% when the frequency weightings were used. According to previous experimental studies of the difference thresholds, human subjects may detect a difference of 12% in the unweighted vibration magnitude. The effects of the phase of the W_k and W_b frequency weightings were greatest when the base component frequency was about 3 Hz.

The effect of the relative phase between two input components on the r.m.s. and the VDV has been observed for all shock-type stimuli used in the study. The difference in the r.m.s. or the VDV caused by the relative phase between two components varied from 3 to 100%, depending on the nature of stimulus and the frequency weighting. The effect of the relative phase between two input components was most significant for frequency-weighted shocks when the base component frequency was 1–3 Hz and the frequency ratio between the two components was 2–4. The effect of the phase of the frequency weighting appeared to be different for the two evaluation methods (i.e., the r.m.s. and the VDV). For the one-and-a-half-sine shocks used here, an effect of the phase of the frequency weighting was not found in the r.m.s., while it was found in the VDV, and was most significant for a base component frequency of 2–3 Hz and for a frequency ratio of 3–5. The predictions arising from this study are examined in a separate experimental study of subjective responses to vibration and shocks [15].

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REFERENCES

1. T. Miwa 1967 *Industrial Health* **5**, 183–205. Evaluation methods for vibration effect. 1. Measurements of threshold and equal sensation contours of whole-body for vertical and horizontal vibrations.
2. R. Shoenberger and C. S. Harris 1971 *Human Factors* **13**, 41–50. Psychophysical assessment of whole-body vibration.
3. A. J. Jones and D. J. Saunders 1972 *Journal of Sound Vibration* **23**, 1–14. Equal comfort contours for whole-body vertical pulsed sinusoidal vibration.
4. M. J. Griffin, E. M. Whitham and K. C. Parsons 1982 *Ergonomics* **25**, 603–630. Vibration and comfort: I. Translational seat vibration.
5. M. J. Griffin, K. C. Parsons and E. M. Whitham 1982 *Ergonomics* **25**, 721–739. Vibration and comfort: IV. Application of experimental results.
6. C. Corbridge and M. J. Griffin 1986 Ergonomics **29**, 249–272. Vibration and comfort: vertical and lateral motion in the range 0·5 to 5·0 Hz.
7. M. J. Griffin 1990 *Handbook of Human Vibration*. London: Academic Press. Chapter 3, pp. 53–71.
8. British Standards Institution 1987 *BS* 6841, London. Guide to measurement and evaluation of human exposure to whole-body mechanical vibration and repeated shock.
9. International Organization for Standardization 1997 *ISO* 2631-1, Geneva. Mechanical vibration and shock—evaluation of human exposure to whole-body vibration—Part 1: general requirements.
10. M. J. Griffin 1998 *Journal of Sound and Vibration* **215**, 883–914. A comparison of standardized methods for predicting the hazards of whole-body vibration and repeated shocks.
11. M. J. Griffin and E. M. Whitham 1980 *Journal of Acoustical Society of America* **68**, 1277–1284. Discomfort produced by impulsive whole-body vibration.
12. H. V. C. Howarth and M. J. Griffin 1991 *Journal of Sound and Vibration* **147**, 395–408. Subjective reaction to vertical mechanical shocks of various waveforms.
13. P. R. Payne, J. W. Brinkley and J. Sandover 1994 *Proceedings of the United Kingdom Informal Group Meeting on Human Response to Vibration, Institute of Naval and Medicine, Gosport*. Shock discomfort—a comparison of approaches.
14. P. R. Payne 1996 *Proceedings of the United Kingdom Informal Group Meeting on Human Response to Vibration, Motor Industry Research Association, Nuneaton*. Comparative ride comfort studies.
15. Y. Matsumoto and M. J. Griffin 2002. Effect of phase on discomfort caused by vertical whole-body vibration and shock—experimental investigation (submitted).
16. Y. Matsumoto and M. J. Griffin 1998 *Journal of Sound and Vibration* **215**, 743–762. Movement of the upper-body of seated subjects exposed to vertical whole-body vibration at the principal resonance frequency.

17. M. MORIOKA and M. J. GRIFFIN 2000 *Journal of Acoustical Society of America* **107**, 620–624. Difference thresholds for intensity perception of whole-body vertical vibration: effect of frequency and magnitude.
18. N. J. MANSFIELD and M. J. GRIFFIN 2000 *Applied Ergonomics* **31**, 255–261. Difference thresholds for automotive seat vibration.
19. H. V. C. HOWARTH 1986 *Proceedings of the United Kingdom Informal Group Meeting on Human Response to Vibration, University of Technology, Loughborough*. A review of experimental investigations of the time dependency of subjective reaction to whole-body vibration.

APPENDIX A: MATHEMATICAL DEFINITION OF THE FREQUENCY WEIGHTINGS

The frequency weighting W_k is defined in ISO 2631-1 [9] as a series of transfer functions:

$$H(s) = H_h(s)H_l(s)H_t(s)H_s(s) \quad (s = \mathrm{i}2\pi f, \text{ the Laplace operator}). \tag{A1}$$

Here,

$$H_h(s) = \frac{1}{1 + \sqrt{2}2\pi f_1/\mathrm{s} + (2\pi f_1/\mathrm{s})^2} \qquad \text{(high pass filter)}, \tag{A2}$$

$$H_l(s) = \frac{1}{1 + \sqrt{2}s/(2\pi f_2) + (s/(2\pi f_2))^2} \qquad \text{(low pass filter)}, \tag{A3}$$

$$H_t(s) = \frac{1 + s/(2\pi f_3)}{1 + s/(Q_4 2\pi f_4) + (s/(2\pi f_4))^2} \qquad \text{(acceleration-velocity transition)}, \tag{A4}$$

$$H_s(s) = \frac{1 + s/(Q_5 2\pi f_5) + (s/(2\pi f_5))^2}{1 + s/(Q_6 2\pi f_6) + (s/(2\pi f_6))^2} \frac{f_5^2}{f_6^2} \qquad \text{(upward step)}. \tag{A5}$$

The frequency weighting W_b is defined in BS 6841 [8] also as a series of transfer functions:

$$H(s) = H_b(s)H_w(s). \tag{A6}$$

Here,

$$H_b(s) = \frac{(2\pi f_2 s)^2}{(s^2 + 2\pi f_1 s/Q_1 + (2\pi f_1)^2)(s^2 + 2\pi f_2 s/Q_1 + (2\pi f_2)^2)}$$

$$= \frac{1}{(1 + 2\pi f_1/(Q_1 s) + (2\pi f_1/s)^2)(1 + s/(Q_1 2\pi f_2) + (s/(2\pi f_2))^2)} \qquad \text{(band-limiting)}, \tag{A7}$$

$$H_w(s) = \frac{(s + 2\pi f_3)(s^2 + 2\pi f_5 s/Q_3 + (2\pi f_5)^2)}{(s^2 + 2\pi f_4 s/Q_2 + (2\pi f_4)^2)(s^2 + 2\pi f_6 s/Q_4 + (2\pi f_6)^2)} \frac{2\pi K f_4^2 f_6^2}{f_3 f_5^2}$$

$$= \frac{K(1 + s/(2\pi f_3))(1 + s/(Q_3 2\pi f_5) + (s/(2\pi f_5))^2)}{(1 + s/(Q_2 2\pi f_4) + (s/(2\pi f_4))^2)(1 + s/(Q_4 2\pi f_6) + (s/(2\pi f_6))^2)} \qquad \text{(frequency weighting)}. \tag{A8}$$

The parameters of the transfer functions of the frequency weightings are tabulated in Table A1.

TABLE A1

Parameters of the transfer functions of the W_k and W_b frequency weightings (ISO 2631-1 [9], BS 6841 [8])

Weighting	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	f_5 [Hz]	f_6 [Hz]	Q_1	Q_4 (W_k) Q_2 (W_b)	Q_5 (W_k) Q_3 (W_b)	Q_6 (W_k) Q_4 (W_b)	K
W_k	0·4	100	12·5	12·5	2·37	3·35	—	0·63	0·91	0·91	—
W_b	0·4	100	16	16	2·5	4	0·71	0·55	0·90	0·95	0·4

APPENDIX B: FILTER REPRESENTING THE APPARENT MASS OF THE SEATED HUMAN BODY

A filter that represented the median normalized apparent mass measured by Matsumoto and Griffin [16] was obtained, based on a two-degree-of-freedom lumped parameter model. The transfer function of the filter was derived from the normalized apparent mass of the model:

$$H_m(s) = \frac{(c_1 s + k_1)\{m_1 m_2 s^2 + (m_1 + m_2)(c_2 s + k_2)\}}{(m_1 + m_2)[(m_2 s^2 + c_2 s + k_2)\{(m_1 s^2 + (c_1 + c_2)s + (k_1 + k_2)\} - (c_2 s + k_2)^2]}. \tag{B1}$$

The parameters of the transfer function are tabulated in Table B1. The total mass of the model was set at unity to be consistent with the normalized apparent mass (i.e., $m_1 + m_2 = 1$).

TABLE B1

Parameters of the transfer functions of the filter based on the apparent mass

m_1	m_2	k_1	k_2	c_1	c_2
0·788	0·212	1720	439	36·1	4·34

APPENDIX C: FORMS OF $N_{r.m.s.}$ AND N_{vdv} IN EQUATIONS (6), (7) AND (13)

$N_{r.m.s.}$ in equation (6):

$$\begin{aligned} N_{r.m.s.}(\omega, \phi, r, R, T) &= 2(r-1)r(r+1)(R^2+1)T\omega - (r-1)r(r+1)\sin(2T\omega) - 8rR\sin(\phi) \\ &\quad - 4r(r+1)R\sin((1-r)T\omega - \phi) - 4r(r-1)R\sin((1+r)T\omega + \phi) \\ &\quad + (r-1)(r+1)R^2\{\sin(2\phi) - \sin(2rT\omega + 2\phi)\}. \end{aligned} \tag{C1}$$

N_{vdv} in equation (7):

$$\begin{aligned} N_{vdv}(\omega, \phi, \mathrm{r}, \mathrm{R}, \mathrm{T}) \\ = 12(r-3)(r-1)r(r+1)(r+3)(3r-1)(3r+1)(R^4+4R^2+1)T\omega \end{aligned}$$

$$
\begin{aligned}
&- 8(r-3)(r-1)r(r+1)(r+3)(3r-1)(3r+1)(3R^2+1)\sin(2T\omega)\\
&+ (r-3)(r-1)r(r+1)(r+3)(3r-1)(3r+1)\sin(4T\omega)\\
&+ 16(r-3)(r-1)r(r+1)(r+3)(3r+1)R^3\sin((1-3r)T\omega-3\phi)\\
&- 12(r-3)r(r+1)(r+3)(3r-1)(3r+1)R^2\sin(2(1-r)T\omega-2\phi)\\
&- 48(r-3)r(r+1)(r+3)(3r-1)(3r+1)R(R^2+1)\sin((1-r)T\omega-\phi)\\
&+ 16(r-1)r(r+1)(r+3)(3r-1)(3r+1)R\sin((3-r)T\omega-\phi)\\
&- 96r(3r-1)(3r+1)R(r^2R^2-9R^2-8)\sin(\phi)\\
&+ 8(r-3)(r+3)(3r-1)(3r+1)R^2(r^2R^2-R^2-3)\sin(2\phi)\\
&+ 32(r-3)(r-1)r(r+1)(r+3)R^3\sin(3\phi)\\
&- (r-3)(r-1)(r+1)(r+3)(3r-1)(3r+1)R^4\sin(4\phi)\\
&- 48(r-3)(r-1)r(r+3)(3r-1)(3r+1)R(R^2+1)\sin((1+r)T\omega+\phi)\\
&+ 16(r-3)(r-1)r(r+1)(3r-1)(3r+1)R\sin((3+r)T\omega+\phi)\\
&- 8(r-3)(r-1)(r+1)(r+3)(3r-1)(3r+1)R^2(R^2+3)\sin(2rT\omega+2\phi)\\
&+ 12(r-3)(r-1)r(r+3)(3r-1)(3r+1)R^2\sin(2(1+r)T\omega+2\phi)\\
&+ 16(r-3)(r-1)r(r+1)(r+3)(3r-1)R^3\sin((1+3r)T\omega+3\phi)\\
&+ (r-3)(r-1)(r+1)(r+3)(3r-1)(3r+1)R^4\sin(4rT\omega+4\phi). \qquad \text{(C2)}
\end{aligned}
$$

N_{vdv} in equation (13):

$$
\begin{aligned}
&N_{vdv}(\phi, r, R)\\
&\quad = 3(r-3)(r-1)(r+1)(r+3)(3r-1)(3r+1)(R^4+4R^2+r)\\
&\qquad + 24(r-3)r^2(r+3)R(8r^2R^2+9r^2-1)\{\sin(\phi/r)+\sin(\phi/r+\pi/r)\}\\
&\qquad + 6(r-3)r^3(r+3)(3r-1)(3r+1)R^2\{\sin(2\phi/r)-\sin(2\phi/r+2\pi/r)\}\\
&\qquad - 8(r-1)r^2(r+1)(3r-1)(3r+1)R\{\sin(3\phi/r)+\sin(3\phi/r+3\pi/r)\}. \qquad \text{(C3)}
\end{aligned}
$$