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LETTERS TO THE EDITOR

PREDICTION OF VIBROACOUSTICS ENERGY USING A DISCRETIZED TRANSIENT LOCAL ENERGY APPROACH AND COMPARISON WITH TSEA

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1. INTRODUCTION

The predictions of transient dynamics of structures are often demanded in many important areas of engineering application. The transient behavior, such as shock and impact, can cause structural failure or unwanted noise, especially in high- and mid-frequency bands. The time-varying energy, rather than other conventional quantities, may be a better measurement to evaluate the structural transient dynamics. In fact, in the early development of the well-known statistical energy analysis (SEA), Manning and Lee [1] proposed a method based on a steady state power balance equation to deal with the mechanical shock transmission. After that, some work has been done on the study of transient conditions in the context of transient statistical energy analysis (TSEA), and it has been reported to be successful in predicting the time-varying envelope of the shock excitation [2].

However, as was pointed out by Manning and Lee [1], TSEA was not developed formally because the definition of coupling loss factors in transient conditions was just "transplanted" from steady state SEA. It does not seem to be appropriate and reliable all the time. In references [3, 4], Lai and Soom discovered the deficiency of TSEA and proposed the concept of time-varying coupling loss factors in an attempt to make TSEA more practicable. Of course, it is limited by the modification of the parameters such as coupling loss factor, while TSEA really needs more theoretical investigations.

As for the alternative methods to TSEA, Nefske and Sung proposed an energy equation in the time domain [5]. It is based on the assumption of "vibrational conductivity", as the energy flow vector is proportional to the gradient of energy density. Actually, the vibrational conductivity approach should be considered as a generalization of the SEA technique [6], which can be easily applied to distributed structures. More recently, Pinnington and Lednik [7, 8], published a comparative study between TSEA and exact results for a two-degree-freedom (d.o.f.) model, the conclusions of which showed that TSEA is not always satisfactory to predict the transmitted energy precisely except for the cases of weak coupling.

This paper presents a theoretical study of transient dynamics by using a new transient local energy approach (TLEA) and its discretized format. The two-oscillator system and the corresponding two coupling subsystems are studied. A wide range of examples is considered in an effort to judge the validity and robustness of TLEA by comparing TLEA solutions with the exact results and TSEA solutions published by Pinnington and Lednik [7, 8], and to this end the comparative study in this paper will be partly based on the same conditions used in reference [7].

2. TLEA EQUATION AND ITS DISCRETIZED FORMAT

In this section, the derivation of equations of TLEA based on some mathematical manipulation will be given and the discretized format of TLEA equation is followed.

2.1. THE GENERAL TLEA EQUATION

Some basic definitions and assumptions used to derive the TLEA equation are generally summarized as

$$e(s,t) = e^{+}(s,t) + e^{-}(s,t), \tag{1}$$

$$I(s,t) = I^{+}(s,t) + I^{-}(s,t), \tag{2}$$

$$I^{\pm}(s,t) = \pm\, c \cdot e^{\pm}(s,t), \tag{3}$$

where $e^{+}(s,t)$ and $e^{-}(s,t)$ are the energy density associated with the right and left train waves, while $I^{+}(s,t)$ and $I^{-}(s,t)$ are the incident and reflected power flows, the right train wave is considered separate from the left one. c is the energy velocity, the same as the group velocity of waves in a slight damping medium. These assumptions emphasize that the energy transmitted in the structure exhibits the characteristics of wave propagation rather than other physical features like thermal conductivity. The fact that only the incoherent energy part is of interest within the local energy way of thinking should be addressed. This is an approximation which is often used in the high-frequency asymptotic behavior and which can be justified using a statistical wave theory as can be found in references [9, 10]. The local power balance considerations lead to the result

$$\frac{\partial e(s,t)}{\partial t} + \pi_{diss} + \nabla \cdot \vec{I} = 0, \tag{4}$$

where the damping model is the same as in SEA:

$$\pi_{diss} = \eta\omega e(s,t). \tag{5}$$

By substituting equations (1–3, 5) into the power balance equations (4), it yields

$$I(s,t) = -\frac{c^2}{\eta\omega}\nabla e(s,t) - \frac{1}{\eta\omega}\frac{\partial I(s,t)}{\partial t}. \tag{6}$$

Differentiating equation (6) and equation (4) with respect to space and time, respectively, then adding them, leads to the expression

$$\frac{1}{\eta\omega}\frac{\partial^2 e(s,t)}{\partial t^2} - \frac{c^2}{\eta\omega}\nabla^2 e(s,t) - \frac{\partial I(s,t)}{\partial s} + \frac{\partial e(s,t)}{\partial t} = 0. \tag{7}$$

Finally, the energy equation is obtained from equation (7) by substituting the expression of $\partial I/\partial s$ from the energy balance (4)

$$\frac{\partial^2 e(s,t)}{\partial t^2} - c^2 \frac{\partial^2 e(s,t)}{\partial s^2} + 2\eta\omega \frac{\partial e(s,t)}{\partial t} + (\eta\omega)^2 e(s,t) = 0. \tag{8}$$

Then, formula (8) is called the TLEA equation. It can be written in another form as

$$\frac{\partial e(s,t)}{\partial t} + \eta\omega e(s,t) + \left[\frac{1}{\eta\omega}\frac{\partial^2 e(s,t)}{\partial t^2} + \frac{\partial e(s,t)}{\partial t} - \frac{c^2}{\eta\omega}\nabla^2 e(s,t)\right] = 0. \tag{9}$$

Comparing TLEA equation (9) with power balance equation (4), one can clearly see that the energy flow term $\nabla\cdot\vec{I}$ actually consists of the time-varying part $[1/\eta\omega(\partial^2 e(s,t)/\partial t^2) + (\partial e(s,t)/\partial t)]$ and the spatial-varying part $(c^2/\eta\omega)\nabla^2 e(s,t)$. The latter is the same as that appearing in the vibrational conductivity equation, which has been previously been derived by Nefske and Sung [5]:

$$\frac{\partial e(s,t)}{\partial t} + \eta\omega e(s,t) - \frac{c^2}{\eta\omega}\nabla^2 e(s,t) = 0. \tag{10}$$

Unfortunately, equation (10) and the classic TSEA do not consider the time-varying part of the energy flow term, and this omission results in some inevitable errors which will be discussed in the following section. It should also be noted that the properties and characteristics of TLEA lead to an interesting physical meaning which is very different from the transient vibrational conductivity equation (10). The detailed discussions can be seen in references [11–13].

2.2. THE DISCRETIZED FORMAT OF THE TLEA EQUATION

The TLEA equation will be discretized, so that it can be used in interconnected subsystems or multi-d.o.f. oscillators. In this section, the TLEA is written in its discretized format by discretizing its spatial-varying part of the energy flow term, $(c^2/\eta\omega)\nabla^2 e(s,t)$. Some work has been done on how to manipulate the term $(c^2/\eta\omega)\nabla^2 e(s,t)$, because it has been adopted widely in the vibrational conductivity equation. For example, Galerkin's method can be used to develop a finite-element approximation. The related zero order discretizing process can be seen in reference [14]. For the sake of brevity, another method is used here by analyzing the boundary condition between elements. It is somewhat similar to the method shown in reference [6].

The distributed structure can be divided into some discretized element. Figure 1 shows the schematic of two adjacent one-dimensional subsystems. Suppose the nodal value is of zero order in the finite elements shown in Figure 1, and the concept of *total energy* rather than *energy density* turns the TLEA equation into the form

$$\int_V \left[\frac{1}{\eta\omega}\frac{\partial^2 e(s,t)}{\partial t^2} - \frac{c^2}{\eta\omega}\nabla^2 e(s,t) + 2\frac{\partial e(s,t)}{\partial t} + \eta\omega e(s,t)\right]\mathrm{d}V = \Pi_{in}. \tag{11}$$

The *total* energy flow term in equation (11) can be expressed as

$$\int_V \left(-\frac{c^2}{\eta\omega}\nabla^2 e(s,t)\right)\mathrm{d}V = -\frac{c^2}{\eta\omega}\int_{A_{12}} (\mathbf{n}\cdot\vec{\nabla}e)\,\mathrm{d}A = \frac{c^2}{\eta\omega}A_{12}\frac{(e_1 - e_2)}{\Delta_{12}}, \tag{12}$$

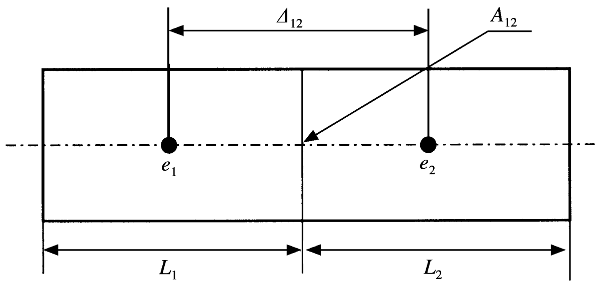

Figure 1. Finite-element model.

where A_{12} is the cross-sectional area between two elements, V is the volume of the element. Substituting equation (12) into equation (11) and using the total energy in the element, $E_i = A_i L_i e_i$, the zero order transient elemental energy for element 1 will be

$$\frac{1}{\eta_1 \omega}\frac{\mathrm{d}^2 E_1}{\mathrm{d}t^2} + 2\frac{\mathrm{d}E_1}{\mathrm{d}t} + \eta_1 \omega E_1 + \frac{c^2}{\eta_1 \omega \Delta_{12} L_1} E_1 - \frac{c^2}{\eta_1 \omega \Delta_{12} L_2} E_2 = \Pi_{in1}. \tag{13}$$

Its simplified form will be

$$\frac{1}{\eta_1 \omega}\frac{\mathrm{d}^2 E_1}{\mathrm{d}t^2} + 2\frac{\mathrm{d}E_1}{\mathrm{d}t} + \eta_1 \omega E_1 + \eta_{e12}\omega E_1 - \eta_{e21}\omega E_2 = \Pi_{in1}, \tag{14}$$

where some coefficients are defined as

$$\eta_{e12} = \frac{c^2}{\eta_1 \omega^2 \Delta_{12} L_1}, \qquad \eta_{e21} = \frac{c^2}{\eta_1 \omega^2 \Delta_{12} L_2}. \tag{15}$$

The analogous result will obviously be obtained if the same process is used on element 2.

3. ENERGY RESULTS OF THREE METHODS

3.1. THE EXACT SOLUTION

A two-oscillator system is shown in Figure 2. The exact energy results are investigated on the two coupled oscillators subjected to an impulse excitation.

Consider that an initial unit impulse is applied to mass m_1, then the initial energy is obtained as

$$E_1(0) = \frac{1}{2m_1}. \tag{16}$$

The total energy of the oscillator can be expressed as the sum of kinetic energy and potential energy (see Appendix A),

$$E_i(t) = \tfrac{1}{2} m_i (\dot{x}_i)^2 + \tfrac{1}{2}(k_i + k)(x_i)^2 \qquad (i = 1, 2), \tag{17}$$

where x_i and $\dot{x}_i$ are the displacement and velocity of each oscillator which are obtained from the simultaneous differential equations of motion. In order to compare the exact energy

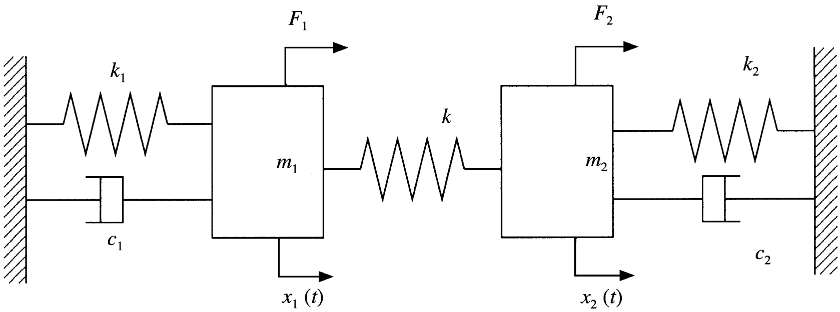

Figure 2. Two-degree-of-freedom model.

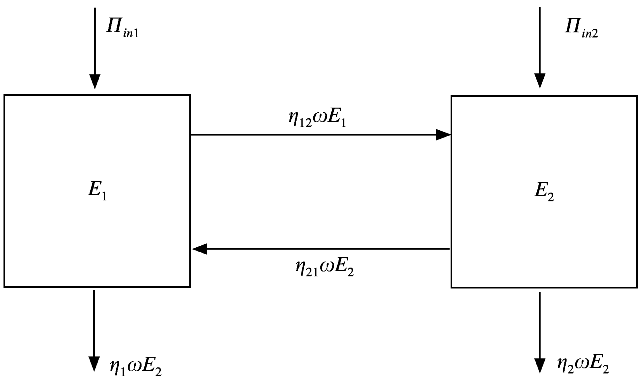

Figure 3. Model for a two-oscillator system used in TLEA and TSEA.

results with the solution of TLEA and TSEA, some simplifications adopted in reference [7] are also taken into account here, for example, the constant bandwidth, equal inherent loss factors, etc.

3.2. ENERGY SOLUTION OF TLEA

The studied model of two subsystems is presented in Figure 3, in which the input powers Π_{in1}, Π_{in2} and subsystem energies E_1, E_2 are time-varying quantities: η_1, η_2 are the inherent loss factors, η_{12}, η_{21} are the coupling loss factors, and ω is the average frequency in an excitation band.

The discretized TLEA equation applied to the model gives

$$\Pi_{in1} = \frac{1}{\eta_1\omega_1}\frac{\mathrm{d}^2E_1}{\mathrm{d}t^2} + 2\frac{\mathrm{d}E_1}{\mathrm{d}t} + \eta_1\omega E_1 + \eta_{12}\omega E_1 - \eta_{21}\omega E_2, \tag{18}$$

$$\Pi_{in2} = \frac{1}{\eta_2\omega_2}\frac{\mathrm{d}^2E_2}{\mathrm{d}t^2} + 2\frac{\mathrm{d}E_2}{\mathrm{d}t} + \eta_2\omega E_2 + \eta_{21}\omega E_2 - \eta_{12}\omega E_1. \tag{19}$$

To solve the TLEA equations, the initial energies $E_1(0), E_2(0)$ and the initial energy derivatives $\mathrm{d}E_1(0)/\mathrm{d}t$, $\mathrm{d}E_2(0)/\mathrm{d}t$ are required. Suppose an impulse excitation is applied to the system, the input powers, Π_{in1}, Π_{in2} are zero, and $E_2(0)$, $\mathrm{d}E_2(0)/\mathrm{d}t$ are also zero. This is the same case as that adopted by Pinnington and Lednik [7], where $\eta_1 = \eta_2$. By applying the Laplace transform method, the two second order differential equations (18) and (19) are converted into two linear algebraic equations, the solutions of which are

$$E_1(t) = \frac{E_1(0)\mathrm{e}^{-\eta_1 \omega t}}{1 + r_1}\left[r + \cos\left(\sqrt{\eta_1 \eta_{12}(1 + r_1)}\omega t\right)\right], \tag{20}$$

$$E_2(t) = \frac{E_1(0)\mathrm{e}^{-\eta_1 \omega t}}{1 + r_1}\left[1 - \cos\left(\sqrt{\eta_1 \eta_{12}(1 + r_1)}\omega t\right)\right], \tag{21}$$

where $r_1 = \eta_{21}/\eta_{12}$, $r = \eta_{12}/\eta_1$ and $E_1(0)$ can be found in formula (16). The coupling loss factor used in TLEA and TSEA is the same definition as in steady state conditions. For the purpose of comparing the solutions of TLEA and TSEA with the exact results, the coupling loss factor described in the model of a two equal oscillator system is used here [2]:

$$\eta_{12} = \frac{k^2}{2\eta_1(k + k_1)^2}. \tag{22}$$

Furthermore, the coupling ratio is defined as

$$r = \frac{\eta_{12}}{\eta_1} = \frac{k^2}{2\eta_1^2(k + k_1)^2}. \tag{23}$$

This coupling ratio will be used in the next section in view of a parametric study. Note that the computation of the different oscillator case is given in depth in Appendix B.

3.3. TRANSIENT STATISTICAL ENERGY SOLUTION

The model in Figure 3 is also used for the TSEA study. The energy balance is

$$\Pi_{in1} = \frac{\mathrm{d}E_1}{\mathrm{d}t} + \eta_1 \omega E_1 + \eta_{12}\omega E_1 - \eta_{21}\omega E_2, \tag{24}$$

$$\Pi_{in2} = \frac{\mathrm{d}E_2}{\mathrm{d}t} + \eta_2 \omega E_2 + \eta_{21}\omega E_2 - \eta_{12}\omega E_1. \tag{25}$$

By introducing a similar method to solve equations (24), (25), the TSEA energy results are given by

$$E_1(t) = \frac{E_1(0)}{2b}\mathrm{e}^{-a\omega t}\left[\left(\frac{D_1}{\omega} + \eta_b\right)\mathrm{e}^{b\omega t} - \left(\frac{D_2}{\omega} + \eta_b\right)\mathrm{e}^{-b\omega t}\right], \tag{26}$$

$$E_2(t) = \frac{E_1(0)}{2b}\eta_{12}\mathrm{e}^{-a\omega t}[\mathrm{e}^{b\omega t} - \mathrm{e}^{-b\omega t}], \tag{27}$$

where $a = (\eta_a + \eta_b)/2$, $b = \frac{1}{2}\sqrt{(\eta_a - \eta_b)^2 + 4\eta_{12}\eta_{21}}$, $\eta_a = \eta_1 + \eta_{12}$, $\eta_b = \eta_2 + \eta_{21}$, and $D_1, D_2 = -\omega(a \mp b)$. The TSEA energy expressions and the parameters definition are almost the same as those obtained by Pinnington and Lednik [7]. Therefore, a comparison can be made easily between TLEA, TSEA and the exact solution. A generalization, for two different oscillators, can be found in Appendix C.

4. NUMERICAL SIMULATION AND ANALYSIS

4.1. THE COMPARISON OF ENERGY RESULTS FROM THREE METHODS: TWO EQUAL OSCILLATORS

In this section, the time-varying energy results of three methods are compared numerically over a wide range of coupling conditions. In the exact two-d.o.f. model shown in Figure 2, $m_1 = m_2 = 2$ kg, $\eta_1 = \eta_2 = 0{\cdot}1$ and the bandwidth was maintained constant at 100 rad/s. Therefore, the coupling loss factor η_{12} in equation (22) and the coupling ratio r were changed by altering the stiffness values of k and k_1 while $(k + k_1) = 2 \times 10^6$ N/m. These parameters will also be used in the numerical simulation of TLEA and TSEA solutions.

The following examples describe the comparison of results from three methods plotted in Figures 4–8 according to the different coupling conditions.

The first example is the case of very strong coupling ($r = 2$) shown in Figure 4. It can be seen that the solution of TLEA is a precise and smooth curve along the exact energy data. The perfect similarity between the TLEA solution and exact results will be shown in other

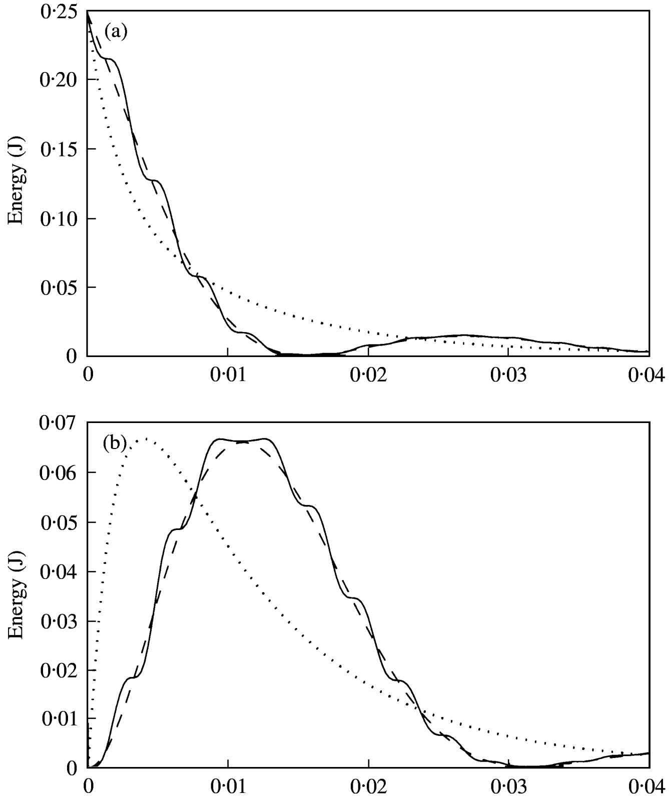

Figure 4. Comparison of three results with the coupling ratio $r = 2$: (a) Input energy; (b) transmitted energy. ——, Exact results; ····, TSEA solutions; – – –, TLEA solutions.

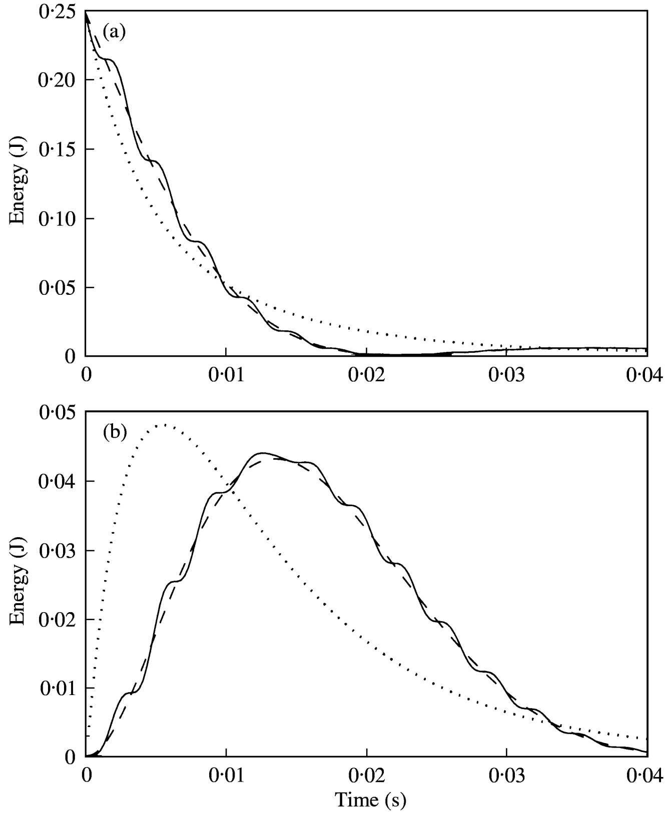

Figure 5. Comparison of three results with the coupling ratio $r = 1$: (a) Input energy; (b) transmitted energy. ——, Exact results; $\cdots\cdot$, TSEA solutions; – – –, TLEA solutions.

figures. For strong coupling, the energy is exchanged much more rapidly between the two oscillators than it is dissipated; hence, both the input energy (energy of mass 1) and transmitted energy (energy of mass 2) exhibit an oscillatory character by sharing the total energy. The TSEA solutions, however, will never show the oscillatory character even though there exists a strong coupling ratio. This can be understood easily by the nature of TSEA solutions in equations (26, 27).

Figures 5 and 6 show the same similarity of TLEA solutions and the exact results; however, the oscillatory trend of energy is not obvious due to a slightly weaker coupling ($r = 1$ and 0·5).

The transient SEA solutions, however, predict the energies in different decay rates compared with TLEA solutions and exact results. Figures 7 and 8 give the logarithmic displays for very weak coupling ($r = 0{\cdot}1$ and 0·005). The input energy results of the three methods seem to be in a very good agreement, while the transmitted energy results show some difference. In fact, when the coupling is very weak, the energy exchanged between two oscillators is a very small quantity. The oscillator of mass 1 will almost behave like a single-d.o.f. one, and the oscillator of mass 2 will be at a very small energy level. These phenomena are clearly illustrated in Figures 7 and 8.

It can be explained why the TLEA solutions approach the TSEA ones in cases of weak coupling. On an extremity condition, when the coupling loss factor η_{12} and η_{21} in TLEA solution (20) and TSEA solution (26) are set to be zero, they both yield the same energy

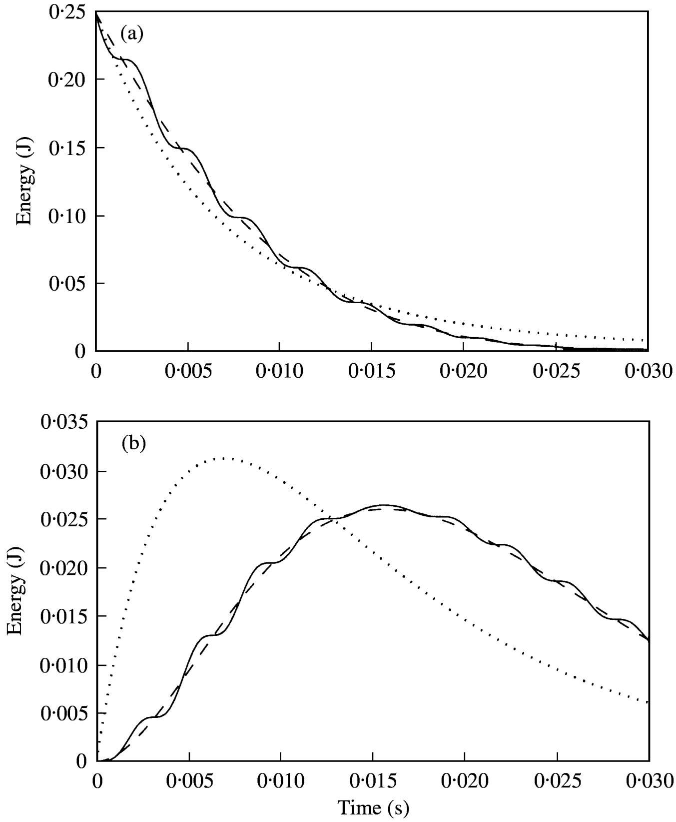

Figure 6. Comparison of three results with the coupling ratio $r = 0{\cdot}5$: (a) Input energy; (b) transmitted energy. ——, Exact results; ····, TSEA solutions; – – –, TLEA solutions.

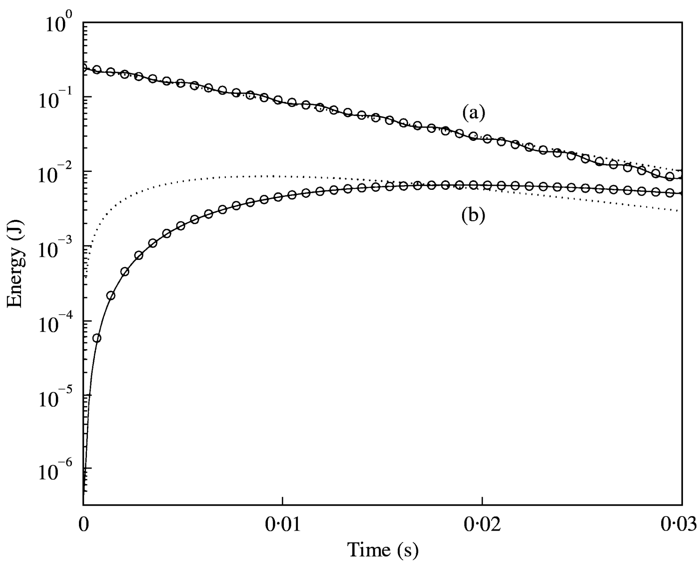

Figure 7. Comparison of three results with the coupling ratio $r = 0{\cdot}1$: (a) Input energy; (b) transmitted energy. ——, Exact results; ····, TSEA solutions; ○, TLEA solutions.

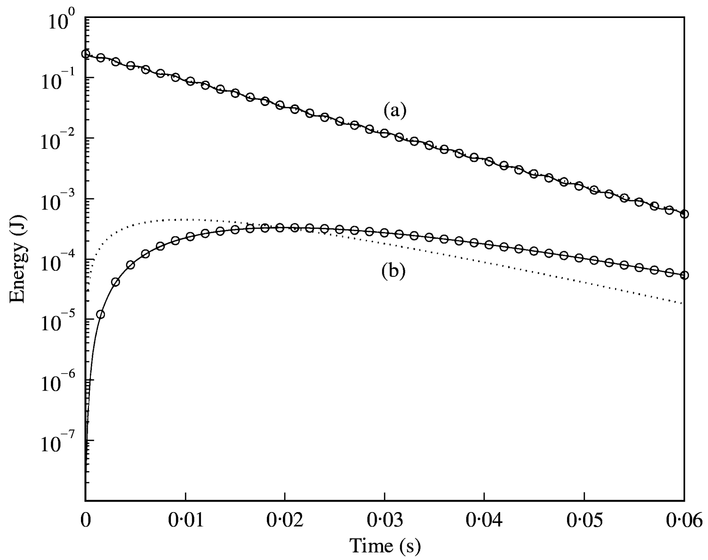

Figure 8. Comparison of three results with the coupling ratio $r = 0{\cdot}005$: (a) Input energy; (b) transmitted energy. ——, Exact results; ····, TSEA solutions; ○, TLEA solutions.

expressions for single-d.o.f. oscillator.

$$E_{\text{TLEA}} = E_{\text{TSEA}} = E(0)\mathrm{e}^{-\eta_1 \omega t}. \tag{28}$$

When the coupling subsystems exist, however, the TSEA solutions are no longer accurate as shown in Figures (4–6), while the TLEA solutions are in good agreement with the exact results.

4.2. THE COMPARISON OF ENERGY RESULTS FROM THREE METHODS: TWO DIFFERENT OSCILLATORS

It should be stated again that the foregoing numerical simulations have been carried out just in order to compare three energy results under the same condition as used in reference [7]. However, the two identical oscillator system in the previous section does not accurately represent many of the structures that occur in practice. A two completely different oscillator system is studied here to illustrate a more general case, and the numerical parameters are listed in Table 1.

The exact energy results are still obtained from equation (17). The TSEA and TLEA solutions of two different oscillators which are somewhat more complex, can be found in the Appendix.

There are two different coupling loss factors (η_{12}, η_{21}) and inherent loss factors (η_1, η_2) in the different oscillators, and the coupling rates are defined as

$$r_1 = \eta_{12}/\eta_1, \qquad r_2 = \eta_{21}/\eta_2. \tag{29}$$

Figure 9 gives the comparison of energy results from the three methods for two different oscillators associated with strong coupling. It is similar to the figures shown in the previous

TABLE 1

Parameters of two different oscillators

Test	Oscillators	Mass (kg)	Inherent loss factors	Coupling stiffness (N/m)	Block frequencies (rad/s)	Coupling loss factors† η_{ij}, η_{ji}	Coupling rate† η_{ij}/η_i
A	1	2·5	0·08	5×10^5	1000	0·26	3·25
	2	2	0·093		1072·4	0·243	2·60
B	1	3	0·08	1×10^5	836·66	0·0187	0·23
	2	1·5	0·156		856·35	0·0182	0·12

† $i, j = 1, 2$; $i \neq j$.

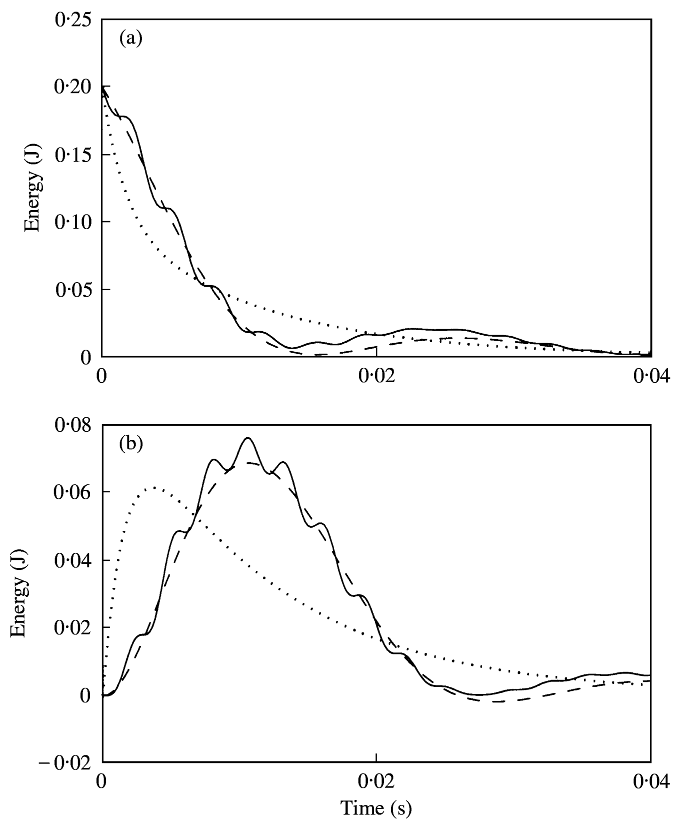

Figure 9. Comparison of energy results of two different oscillators. Test A: (a) Input energy; (b) transmitted energy. ——, Exact results; ····, TSEA solutions; – – –, TLEA solutions.

section. Figure 10 displays the comparison in the case of very weak coupling. Figure 10(b) gives the comparison of *transmitted* energies in logarithmic scale to emphasize the difference among the three methods, because it is very difficult to distinguish them, respectively, in figure (a) due to the very weak coupling.

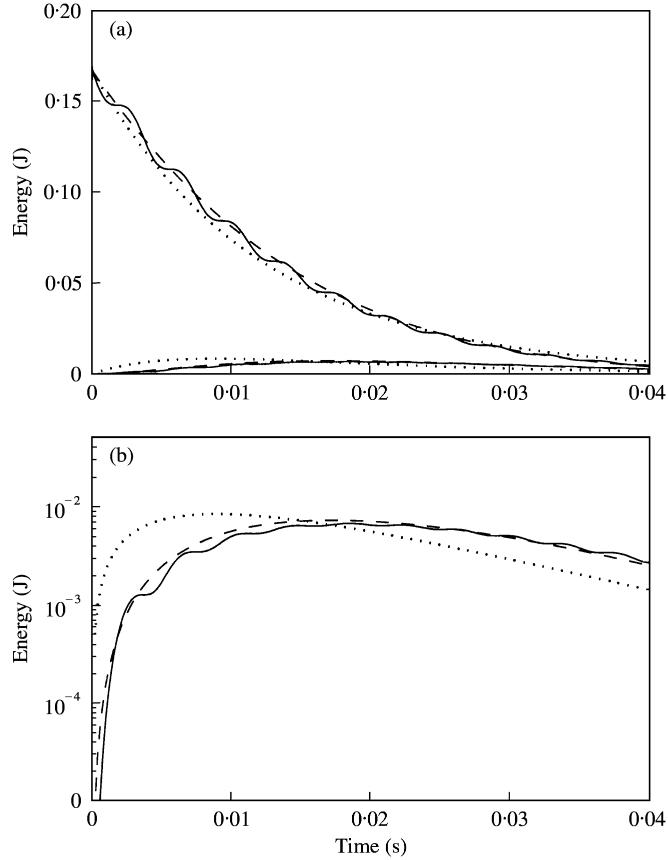

Figure 10. Comparison of energy results of two different oscillators. Test B: (a) Input energy and transmitted energy; (b) transmitted energy in logarithmic scale. ——, Exact results; ····, TSEA solutions; – – –, TLEA solutions.

4.3. ANALYSIS ON COMPARISON RESULTS

In the study of transient dynamics, it is necessary to determine the energy transfer rate, the peak value of transmitted energy, and the rise time taken to reach this value. These important quantities can be obtained from the TLEA solutions because the previous section has shown that the TLEA solution is a perfect alternative to exact results while it is very difficult to get explicit expressions from the exact energy results.

Returning to the TLEA solution in equation (21), one obtains the time derivative of E_2(t) as the energy transfer rate

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \frac{2\eta_1\omega E_0\sqrt{1+r(1+r_1)}}{1+r_1}\sin\left(\tfrac{1}{2}\eta_1\omega t\sqrt{1+r(1+r_1)}\right)\sin\left(\Phi - \tfrac{1}{2}\eta_1\omega t\sqrt{1+r(1+r_1)}\right), \tag{30}$$

where $\Phi = \arcsin((\sqrt{\eta_{12}+\eta_{21}})/(\sqrt{\eta_1+\eta_{12}+\eta_{21}})$, $r = \eta_{12}/\eta_1$. By setting equation (30) to zero, it yields the rise time, t_r. Especially, when $\eta_{12} = \eta_{21}$, namely, $r_1 = 1$ occurs in the equal

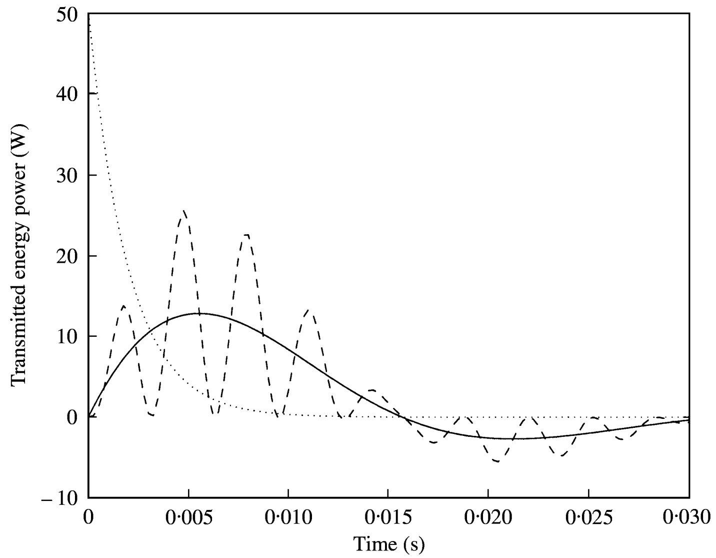

Figure 11. Comparison of transmitted energy flow (from element 1 to 2) with the coupling ratio $r = 2$. ——, TLEA solutions; ····, TSEA solutions; – – –, exact results.

oscillators, the rise time is

$$t_r = \frac{\sqrt{2\Phi}}{\eta_1 \omega \sqrt{r}}. \tag{31}$$

Then the transmitted energy will reach its maximum value when the time $t = t_r$,

$$E_{2max} = \frac{2rE_0 \mathrm{e}^{-\sqrt{2\Phi}/\sqrt{r}}}{1 + 2r}. \tag{32}$$

The expressions for the peak value of transmitted energy and rising time obtained from TLEA are rather concise. It can be seen in Figures 4–8 that the TLEA solutions give precise prediction of these quantities, while the TSEA solutions always give earlier rise time because of its different decay rate and non-oscillatory character. The expression of the rise time of TSEA can be found in reference [7].

For deeper insight into the difference between TLEA and TSEA, an examination of energy flow is taken here. The energy flow term in TLEA is

$$\left[\frac{1}{\eta\omega}\frac{\partial^2 e(s,t)}{\partial t^2} + \frac{\partial e(s,t)}{\partial t} - \frac{c^2}{\eta\omega}\nabla^2 e(s,t)\right],$$

the one in TSEA is $-(c^2/\eta\omega)\nabla^2 e(s,t)$, and the energy flow term from exact results can be expressed as $kx_1(t)\dot{x}_2(t)$ [3], where k is the stiffness of the coupling spring in Figure 2. The comparisons among them are shown in Figures 11 and 12 with different coupling ratios. They illustrate that the energy flow term (from element 1 to 2) in TLEA can be viewed as the mean value of the exact ones, while the energy flow terms in TSEA have remarkably different decay rate and initial value. For example, at the initial time, the transmitted energy flow from both TLEA and exact results are zero; however, the TSEA one gives out a very large quantity.

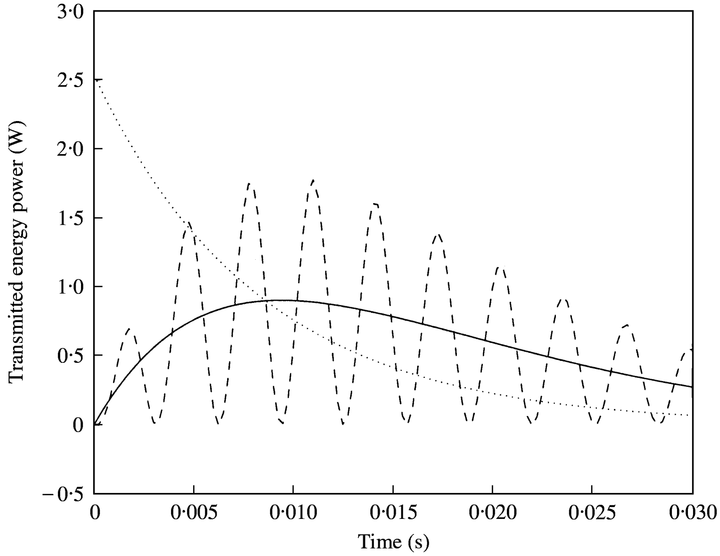

Figure 12. Comparison of transmitted energy flow (from element 1 to 2) with the coupling ratio $r = 0{\cdot}1$. ——, TLEA solutions; ····, TSEA solutions; – – –, exact results.

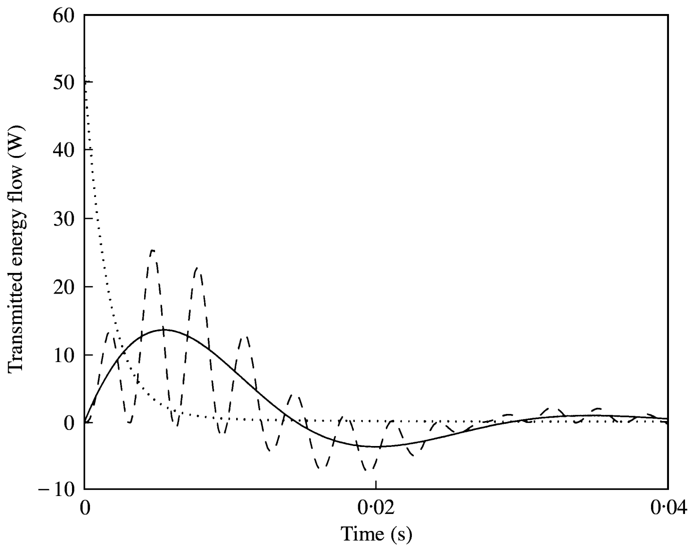

Figure 13. Comparison of transmitted energy flow (from element 1 to 2). Test A. ——, TLEA solutions; ····, TSEA solutions; – – –, exact results.

Finally, Figure 13 displays the comparison of transmitted energy flow for two different oscillators (test A).

5. CONCLUSIONS

A (TLEA) is proposed as a new method for prediction of transient energy in this paper. The discretized format of the TLEA equation is developed so that it can be applied to the multi-subsystem case and to compare it with the TSEA one.

The transient dynamics of two-degree-of-freedom oscillators subject to an impulse and a two-subsystem model are studied by three kinds of methods. In order to compare the TLEA solution with the published results [7], a similar model of two identical oscillators is adopted at first, and then a more general example, with two different oscillators, is given. The comparisons and contrast show that the TLEA solution is the precise time-varying energy, because it gives more detailed information and description of transmitted energy flow between subsystems.

The transient energy flow analysis of the two oscillators is the basis of the steady state condition in SEA. In the previous study for transient conditions such as TSEA, some concepts and definitions, like the coupling loss factors, etc., are cited from steady state condition. The comparison of results in this paper shows that the coupling loss factor has an important role in the oscillatory trend of transmitted energy. It does not influence the decay rate of energy in damping the structure. However, in TSEA, the coupling loss factors partly serve as the coefficient of the energy decay rate. In fact, a transient condition is very different from the steady state condition because the energy flow between subsystems is periodically time-varying. Furthermore, the local relation between the energy flow vector and energy density in equation (6) is different from the vibrational conductivity assumption, which states that the energy flow vector is proportional to the gradient of energy density. Actually, TLEA gives a smooth description of the time-varying characteristic of energy exchange between subsystems.

TLEA can be applied to both distributed and discrete structures because the variable employed in TLEA is energy density. Some work has been done on distributed structure [11, 12]. Moreover, the equation of TLEA can be generalized to multi-dimensional systems, and it implies other applications in the sphere of vibroacoustics. Such work is under progress.

Generally, the TLEA solutions show the definite advantages and accuracy to describe the characteristics of transient condition, such as peak value of transmitted energy and the rise time. So TLEA can be considered as a suitable predictive tool in some specific engineering areas.

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APPENDIX A: THE ENERGY RESULTS FOR TWO COUPLED OSCILLATORS

The equations of motion for the two coupled oscillators subject to an impulse shown in Figure 2 are

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + (k_1 + k)x_1 - kx_2 = 0, \qquad m_2\ddot{x}_2 + c_2\dot{x}_2 + (k_2 + k)x_2 - kx_1 = 0. \quad \text{(A.1)}$$

They can be rearranged as

$$\ddot{x}_1 = -\Delta_1\dot{x}_1 - \omega_1^2 x_1 + {p_1}^2 x_2, \qquad \ddot{x}_2 = -\Delta_2\dot{x}_2 - \omega_2^2 x_2 + {p_2}^2 x_1, \quad \text{(A.2, A.3)}$$

where $\omega_1^2 = (k_1 + k)/m_1$, $\omega_2^2 = (k_2 + k)/m_2$, $\Delta_1 = \eta_1\omega_1 = c_1/m_1$, $\Delta_2 = \eta_2\omega_2 = c_2/m_2$, ${p_1}^2 = k/m_1$, and ${p_2}^2 = k/m_2$.

Let $\mathbb{X} = \{x_1 \;\; x_2 \;\; \dot{x}_1 \;\; \dot{x}_2\}^{\mathrm{T}}$, then the equations can be written in the form of the matrix

$$\frac{\mathrm{d}\mathbb{X}}{\mathrm{d}t} = \mathbb{A}\mathbb{X}, \quad \text{(A.4)}$$

where

$$\mathbb{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -\omega_1^2 & {p_1}^2 & -\Delta_1 & 0 \\ {p_2}^2 & -\omega_2^2 & 0 & -\Delta_2 \end{bmatrix}. \quad \text{(A.5)}$$

The initial value of matrix equation (A.4) is $\mathbb{X}_{t=0} = \{0 \;\; 0 \;\; 1/m_1 \;\; 0\}^{\mathrm{T}}$.

Suppose $\mathbb{D} = \{d_1 \;\; d_2 \;\; d_3 \;\; d_4\}^{\mathrm{T}}$ and $\mathbb{V} = \{\mathbf{V}_1 \;\; \mathbf{V}_2 \;\; \mathbf{V}_3 \;\; \mathbf{V}_4\}^{\mathrm{T}}$ are the eigenvalues and eigenvectors of matrix $\mathbb{A}$, then the solution of equation (A.4) is

$$\mathbb{X} = \sum_{i=1}^{4} c_i \mathbf{V}_i \mathrm{e}^{d_i t}, \quad i = 1, \ldots, 4, \quad \text{(A.6)}$$

where the coefficient c_i can be obtained from

$$\mathbb{C} = \{c_1 \;\; c_2 \;\; c_3 \;\; c_4\}^{\mathrm{T}} = \mathbb{V}^{-1}\mathbb{X}_{t=0}. \quad \text{(A.7)}$$

Finally, the exact energy results are obtained as

$$E_i(t) = \tfrac{1}{2} m_i (\dot{x}_i)^2 + \tfrac{1}{2}(k_i + k)(x_i)^2 \quad (i = 1, 2). \tag{A.8}$$

APPENDIX B: THE TLEA SOLUTION OF TWO DIFFERENT SUBSYSTEMS

The TLEA solution of two different subsystems shown in Figure 3 can be obtained in a similar way. The TLEA equations (18, 19) can be rearranged as

$$\frac{\mathrm{d}^2 E_1}{\mathrm{d}t^2} = -2\Delta_1 \frac{\mathrm{d}E_1}{\mathrm{d}t} - ({\Delta_1}^2 + \Delta_1 \eta_{12}\omega_1)E_1 + \Delta_1 \eta_{21}\omega_2 E_2, \tag{B.1}$$

$$\frac{\mathrm{d}^2 E_2}{\mathrm{d}t^2} = -2\Delta_2 \frac{\mathrm{d}E_2}{\mathrm{d}t} - ({\Delta_2}^2 + \Delta_2 \eta_{21}\omega_2)E_2 + \Delta_2 \eta_{12}\omega_1 E_1. \tag{B.2}$$

Similarly, let $\mathbb{E} = \{E_1 \;\; E_2 \;\; \mathrm{d}E_1/\mathrm{d}t \;\; \mathrm{d}E_2/\mathrm{d}t\}^{\mathrm{T}}$, then the equations will be

$$\frac{\mathrm{d}\mathbb{E}}{\mathrm{d}t} = \mathbb{B}\mathbb{E}, \tag{B.3}$$

where

$$\mathbb{B} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -(\Delta_1^2 + \Delta_1\eta_{12}\omega_1) & \Delta_1\eta_{21}\omega_2 & -2\Delta_1 & 0 \\ \Delta_2\eta_{12}\omega_1 & -({\Delta_2}^2 + \Delta_2\eta_{21}\omega_2) & 0 & -2\Delta_2 \end{bmatrix}. \tag{B.4}$$

The initial values of matrix equation (43) is $\mathbb{E}_{t=0} = \{E(0) \;\; 0 \;\; -\Delta_1 E(0) \;\; 0\}^{\mathrm{T}}$.

If the eigenvalues and eigenvectors of matrix $\mathbb{B}$ are $\mathbb{S} = \{s_1 \;\; s_2 \;\; s_3 \;\; s_4\}^{\mathrm{T}}$ and $\mathbb{U} = \{\mathbf{U}_1 \;\; \mathbf{U}_2 \;\; \mathbf{U}_3 \;\; \mathbf{U}_4\}^{\mathrm{T}}$, then the solution of equation (B.3) is

$$\mathbb{E} = \sum_{i=1}^{4} f_i \mathbb{U}_{\mathrm{i}} \mathrm{e}^{s_i t}, \quad i = 1, \ldots, 4, \tag{B.5}$$

where the coefficient f_i can be obtained from

$$\mathbb{F} = \{f_1 \; f_2 \; f_3 \; f_4\}^{\mathrm{T}} = \mathbb{U}^{-1}\mathbb{E}_{t=0}. \tag{B.6}$$

APPENDIX C: THE TSEA SOLUTION OF TWO DIFFERENT SUBSYSTEMS

The TSEA equations for two general subsystems are

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \eta_1\omega_1 E_1 + \eta_{12}\omega_1 E_1 - \eta_{21}\omega_2 E_2 = 0, \tag{C.1}$$

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \eta_2\omega_2 E_2 + \eta_{21}\omega_2 E_2 - \eta_{12}\omega_1 E_1 = 0. \tag{C.2}$$

The solution for an initial energy, $E(0)$, in the source and zero initial energy of receiver subsystem is

$$E_1(t) = \frac{E(0)}{2b}\,\mathrm{e}^{-at}[(a + b - \eta_a\omega_1)\mathrm{e}^{bt} + (b - a + \eta_a\omega_1)\mathrm{e}^{-bt}], \tag{C.3}$$

$$E_2(t) = \frac{E(0)}{2b}\,\eta_{12}\omega_1\mathrm{e}^{-at}[\mathrm{e}^{bt} - \mathrm{e}^{-bt}], \tag{C.4}$$

where

$$\eta_a = \eta_1 + \eta_{12}, \quad \eta_b = \eta_2 + \eta_{21}, \quad a = (\eta_a\omega_1 + \eta_b\omega_2)/2,$$

$$b = (\sqrt{\eta_a\omega_1 - \eta_b\omega_2)^2 + 4\eta_{12}\eta_{12}\omega_1\omega_2})/2.$$