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A NEW MATRIX METHOD FOR SOLVING VIBRATION AND STABILITY OF CURVED PIPES CONVEYING FLUID

Y. Huang, G. Zeng and F. Wei

Department of Mechanics Huazhong University of Science & Technology, Wuhan 430074, *Hubei, People's Republic of China*

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This paper proposes a new matrix method for calculating critical flow velocity of curved pipes conveying fluid, which have arbitrary centerline shape and spring supports. Its main advantage over other methods is that the corresponding characteristic equation can be reduced to a third order one, no matter how many elements are discretized in calculation. This will lead to saving computer time and obtaining a solution with good precision.

1. INTRODUCTION

Vibration and stability of curved pipes conveying fluid has been the subject of increasing attention in current engineering practice encountered often in hydroelectric and nuclear power plants, suction and pressure pipes, and fuel feeding lines in aerospace. Research work on this problem originated in the early 1950s (Ashley and Haviland, 1950; Housner, 1952) when the dynamics of an elastic pipe carrying fluid was a topic of intense interest. The investigation carried out at that time were concerned mainly with the analyses of straight pipes. Studies on curved pipes have been undertaken in recent years due to their greater complexity relative to the straight ones. Misra [1] calculated a curved pipe with complex shape by finite element method in 1988. Next, Aithal and Steven Gipson [2] studied a semi-circular pipe conveying fluid by an analytical method. All the methods have proved to be very effective in the analysis of certain curved pipes conveying fluid, but the analytical method proved not suitable for curved pipes with complex shape, as the finite element method requires a great deal of computer time as the number of discrete nodes becomes relatively large. Furthermore, the two methods are difficult for solving curved pipes which have intermediate spring supports. Unfortunately, such a case is frequently found in practice.

In order to overcome these disadvantages, a new matrix method is presented in this paper for solving vibration and stability problems of curved pipes with intermediate spring supports and complex shape. Based on the approach of the initial parameter method [3], first, a set of displacement and force relations between the two nodes of a segment of curved pipe is derived. Then using the boundary condition and the compatible conditions at the interfaces between two adjacent segments, the characteristic equation of the curved pipe conveying fluid is further obtained from which we can determine the corresponding critical velocity of the fluid flow. Compared to the analytical method and finite element method, the proposed method has better precision and saves computer time in the solving of the curved pipes with variable rigidity, complex shape and intermediate spring supports.

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2. GOVERNING EQUATION AND BOUNDARY CONDITIONS

According to reference [2], in the absence of damping and extensibility of its centerline, the dimensionless governing equation of a circular pipe conveying fluid can be described as

$$\frac{\partial^6 \xi}{\partial \theta^6} + (2 + v^2)\frac{\partial^4 \xi}{\partial \theta^4} + 2\beta^{1/2} v \frac{\partial}{\partial \tau}\left(\frac{\partial^3 \xi}{\partial \theta^3}\right) + \left(1 + 2v^2 + \frac{\partial^2}{\partial \tau^2}\right)\frac{\partial^2 \xi}{\partial \theta^2}$$

$$+ 2\beta^{1/2} v \frac{\partial}{\partial \tau}\left(\frac{\partial \xi}{\partial \theta}\right) + \left(v^2 - \frac{\partial^2}{\partial \tau^2}\right)\xi = 0, \tag{1}$$

where

$$\xi = w/R, \quad \beta = M_f/(m_t + M_f), \quad v = (M_f/EI)^{1/2} RV, \quad \tau = [EI/(m_t + M_f)]^{1/2} t/R^2 \tag{2}$$

represent the dimensionless tangential displacement, mass ratio, flow velocity and time respectively. In equation (2), w denotes the displacement along the tangential direction, R is the radius of curved pipe, EI is the flexural rigidity, m_t and M_f are the mass per unit length of tube and fluid, respectively, t is the time, θ is the co-ordinate and V denotes the flow velocity with a constant-magnitude as shown in Figure 1.

Referring to Figure 1 and the classical first order theory of bending beam analysis, strain–displacement and stress–strain relations are as follows:

$$\varepsilon = \frac{1}{R}\left(\frac{\partial w}{\partial \theta} - u\right), \qquad \chi = \frac{1}{R^2}\left(\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial w}{\partial \theta}\right), \tag{3}$$

$$N = EA\varepsilon = \frac{EA}{R}\left(\frac{\partial w}{\partial \theta} - u\right),$$

$$M = EI\chi = \frac{EI}{R^2}\left(\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial w}{\partial \theta}\right), \tag{4}$$

$$Q = \frac{\partial M}{R\partial \theta} = \frac{EI}{R^3}\left(\frac{\partial^3 u}{\partial \theta^3} + \frac{\partial^2 w}{\partial \theta^2}\right),$$

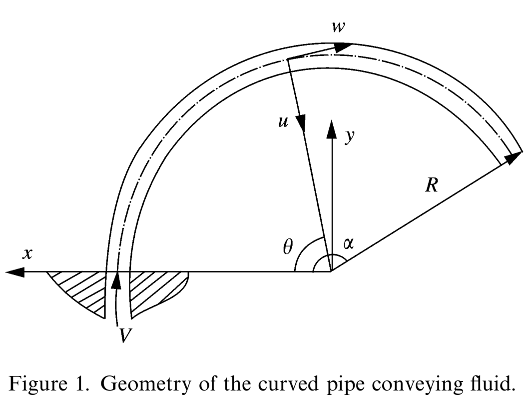

Figure 1. Geometry of the curved pipe conveying fluid.

where u is the displacement along the radial direction, ε the axial strain, χ the variation of curvature, N the axial force, M the bending moment, Q the transverse shear force and A the cross-sectional area of the pipe.

The boundary conditions of a cantilevered pipe is

$$\text{At the fixed end } (\theta = 0): \quad u = w = 0, \psi = \frac{\partial u}{R\partial\theta} = 0, \tag{5a}$$

$$\text{At the free end } (\theta = \alpha): \quad M = Q = 0. \tag{5b}$$

In addition, the natural boundary condition ($\theta = \alpha$) is [2]

$$\begin{aligned} N_1 = {} & \frac{EI}{R^4}\left(\frac{\partial^5 w}{\partial\theta^5} + 2\frac{\partial^3 w}{\partial\theta^3} + \frac{\partial w}{\partial\theta}\right) + \frac{M_f V^2}{R^2}\left(\frac{\partial^3 w}{\partial\theta^3} + \frac{\partial w}{\partial\theta}\right) + \frac{2M_f V}{R}\frac{\partial^3 w}{\partial\theta^2\partial t} \\ & + \frac{M_f V}{R}\frac{\partial w}{\partial t} + (M_f + m_t)\frac{\partial^3 w}{\partial\theta\,\partial t^2} + \frac{M_f V}{R}\left(\frac{\partial w}{\partial t} + V\right) = 0, \end{aligned} \tag{5c}$$

which means the generalized axial force.

3. SOLUTION FOR A SINGLE CIRCULAR PIPE

For a self-excitation vibration, we may consider the undetermined function $\xi(\theta, \tau)$ to be separable into two parts in the following form:

$$\xi(\theta, \tau) = \Phi(\theta)\exp(\mathrm{i}\Omega\tau), \tag{6}$$

where $\mathrm{i} = \sqrt{-1}$, $\Phi(\theta)$ is the amplitude of vibration, τ the dimensionless time (see equation (2)), and Ω the non-dimensional natural frequency defined by

$$\Omega = [(M_f + m_t)/EI]^{1/2}R^2\omega, \tag{7}$$

with ω being the natural frequency of the pipe. In general, Ω is a plural number. Substituting equation (6) into equation (1), one obtains a sixth order ordinary differential equation of $\Phi(\theta)$, namely,

$$\begin{aligned} & \frac{\mathrm{d}^6\Phi}{\mathrm{d}\theta^6} + (2 + v^2)\frac{\mathrm{d}^4\Phi}{\mathrm{d}\theta^4} + 2\mathrm{i}\Omega\beta^{1/2}v\frac{\mathrm{d}^3\Phi}{\mathrm{d}\theta^3} + (1 + 2v^2 - \Omega^2)\frac{\mathrm{d}^2\Phi}{\mathrm{d}\theta^2} \\ & + 2\mathrm{i}\Omega\beta^{1/2}v\frac{\mathrm{d}\Phi}{\mathrm{d}\theta} + (v^2 + \Omega^2)\Phi = 0. \end{aligned} \tag{8}$$

Obviously, $\Phi \equiv 0$ is a trivial solution of this equation, but it is meaningless.

According to the initial parameter method [3], the solution of equation (8) can be written as

$$\Phi(\theta) = \Phi_0 f_1(\theta) + \Phi_0^{(1)} f_2(\theta) + \Phi_0^{(2)} f_3(\theta) + \Phi_0^{(3)} f_4(\theta) + \Phi_0^{(4)} f_5(\theta) + \Phi_0^{(5)} f_6(\theta), \tag{9}$$

where $\Phi_0, \Phi_0^{(1)}, \ldots, \Phi_0^{(5)}$ are the six initial parameters to be determined, which denote the values of Φ and its 1–5 order derivatives at $\theta = 0$, respectively (Figure 1); $f_1(\theta)$, $f_2(\theta), \ldots, f_6(\theta)$ are solutions of equation (8), respectively, under the following conditions:

$$\begin{bmatrix} f_1(0) & f_1^{(1)}(0) & f_1^{(2)}(0) & f_1^{(3)}(0) & f_1^{(4)}(0) & f_1^{(5)}(0) \\ f_2(0) & f_2^{(1)}(0) & f_2^{(2)}(0) & f_2^{(3)}(0) & f_2^{(4)}(0) & f_2^{(5)}(0) \\ f_3(0) & f_3^{(1)}(0) & f_3^{(2)}(0) & f_3^{(3)}(0) & f_3^{(4)}(0) & f_3^{(5)}(0) \\ f_4(0) & f_4^{(1)}(0) & f_4^{(2)}(0) & f_4^{(3)}(0) & f_4^{(4)}(0) & f_4^{(5)}(0) \\ f_5(0) & f_5^{(1)}(0) & f_5^{(2)}(0) & f_5^{(3)}(0) & f_5^{(4)}(0) & f_5^{(5)}(0) \\ f_6(0) & f_6^{(1)}(0) & f_6^{(2)}(0) & f_6^{(3)}(0) & f_6^{(4)}(0) & f_6^{(5)}(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (10)$$

In the following, attention will be focused on seeking the solutions $f_1(\theta) - f_6(\theta)$, which is called "standard set of solutions" to equation (8). As it is well known, the solution to equation (8) can also be written as

$$\Phi(\theta) = \sum_{n=1}^{6} C_n \exp(\mathrm{i}\lambda_n \theta). \quad (11)$$

Using equations (10) and (11) leads to

$$f_i(\theta) = \sum_{n=1}^{6} C_{in} \exp(\mathrm{i}\lambda_n \theta), \quad i = 1, 2, \ldots, 6, \quad (12)$$

where λ_n are the roots of the characteristic equation (8), which is

$$\lambda^6 - (2 + v^2)\lambda^4 - 2\beta^{1/2} v \Omega \lambda^3 + (1 + 2v^2 - \Omega^2)\lambda^2 + 2\beta^{1/2} v \Omega \lambda - (v^2 + \Omega^2) = 0. \quad (13)$$

Substituting equation (12) into equation (10), one obtains

$$[C][D] = [I], \quad [C] = [D]^{-1}, \quad (14)$$

the elements (C_{ij}) of $[C]$ are just the coefficient of equation (12), the elements of $[D]$ $D_{ij} = (\mathrm{i}\lambda_i)^{j-1}$, $i, j = 1, 2, \ldots, 6$, $[I]$ is a sixth order unit matrix. It is seen from equation (14) that C_{ij} will be determined once the six roots λ_i are obtained by solving equation (13). Referring back to equation (12), we may immediately get $f_i(\theta)$.

4. A NEW MATRIX METHOD

In this section, it is necessary to derive a transfer matrix for any circular pipe segment and the characteristic equation for a curved pipe with variable curvature. Use of equation (9) leads to the following relation:

$$\begin{Bmatrix} \Phi(\theta) \\ \Phi^{(1)}(\theta) \\ \Phi^{(2)}(\theta) \\ \Phi^{(3)}(\theta) \\ \Phi^{(4)}(\theta) \\ \Phi^{(5)}(\theta) \end{Bmatrix} = \begin{bmatrix} f_1(\theta), & f_2(\theta), & f_3(\theta), & f_4(\theta), & f_5(\theta), & f_6(\theta), \\ f_1^{(1)}(\theta), & f_2^{(1)}(\theta), & f_3^{(1)}(\theta), & f_4^{(1)}(\theta), & f_5^{(1)}(\theta), & f_6^{(1)}(\theta), \\ f_1^{(2)}(\theta), & f_2^{(2)}(\theta), & f_3^{(2)}(\theta), & f_4^{(2)}(\theta), & f_5^{(2)}(\theta), & f_6^{(2)}(\theta), \\ f_1^{(3)}(\theta), & f_2^{(3)}(\theta), & f_3^{(3)}(\theta), & f_4^{(3)}(\theta), & f_5^{(3)}(\theta), & f_6^{(3)}(\theta), \\ f_1^{(4)}(\theta), & f_2^{(4)}(\theta), & f_3^{(4)}(\theta), & f_4^{(4)}(\theta), & f_5^{(4)}(\theta), & f_6^{(4)}(\theta), \\ f_1^{(5)}(\theta), & f_2^{(5)}(\theta), & f_3^{(5)}(\theta), & f_4^{(5)}(\theta), & f_5^{(5)}(\theta), & f_6^{(5)}(\theta), \end{bmatrix} \begin{Bmatrix} \Phi(0) \\ \Phi^{(1)}(0) \\ \Phi^{(2)}(0) \\ \Phi^{(3)}(0) \\ \Phi^{(4)}(0) \\ \Phi^{(5)}(0) \end{Bmatrix}. \quad (15)$$

This formula can be rewritten briefly as

$$\{\delta\} = [T]\{\delta_0\},$$

where $\{\delta\}$ and $\{\delta_0\}$ represent the left and right vector of equation (15) respectively.

The elements of matrix $[T]$, $T_{ij} = f_j^{(i-1)}(\theta) = \sum_{n-1}^{6} C_{jn}(\mathrm{i}\lambda_n)^{i-1} \exp(\mathrm{i}\lambda_n\theta)$, $i, j = 1, 2, \ldots, 6$.

Up to now, we have obtained the relation between the two amplitude vectors $\{\delta\}$ and $\{\delta_0\}$ of the two ends of a circular pipe segment.

From equation (2), one obtains

$$w = R\xi. \quad (16)$$

If the pipe is assumed to be inextensible, then

$$\varepsilon = 0, \quad \therefore\ u = \partial w/\partial\theta = R\xi^{(1)}. \quad (17)$$

Substitution of equation (17) into equations (3), (4) and (5c), respectively, yields the angle of rotation, bending moment, transverse shear force and generalized axial force

$$\psi = \partial u/(R\partial\theta) = R\xi^{(2)}, \quad (18)$$

$$M = \frac{EI}{R^2}\left(\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial w}{\partial\theta}\right) = \frac{EI}{R}(\xi^{(3)} + \xi^{(1)}), \quad (19)$$

$$Q = \frac{\partial M}{R\partial\theta} = \frac{EI}{R^2}(\xi^{(4)} + \xi^{(2)}), \quad (20)$$

$$N_1 = \frac{EI}{R^3}\left[(\xi^{(5)} + \xi^{(3)}) + 2\beta^{1/2} v \frac{\partial}{\partial\tau}(\xi^{(2)} + \xi) + \frac{\partial^2}{\partial\tau^2}(\xi^{(1)}) + v^2\right]. \quad (21)$$

Equations (16–21) denote the displacements and forces of a particle on the pipe respectively. Substituting equation (6) into the above equations yields the relation between the displacement and force vector, $\{q\}$, and the vector of dimensionless displacement and force, $\{\delta\}$, as

$$\{q\} = [H]\{\delta\} + \{\Delta\}, \quad (22a)$$

where $\{q\} = [w, u, \psi, M, Q, N_1]^{\mathrm{T}}$; the elements of matrix $[H]$ are

$$H_{11} = H_{22} = R, \quad H_{33} = 1, \quad H_{42} = H_{44} = EI/R, \quad H_{53} = H_{55} = EI/R^2,$$

$$H_{61} = H_{63} = 2\mathrm{i}\Omega\beta^{1/2} vEI/R^3, \quad H_{62} = -\Omega^2 EI/R^3, \quad H_{64} = H_{66} = EI/R^3,$$

and the remainder is zero; the elements of vector $\{\Delta\}$ are $\Delta_6 = v^2 EI/R^3$ and the remainder is zero.

By virtue of equation (22a), one finds that

$$\{\delta\} = [H]^{-1}\{q\} - [H]^{-1}\{\Delta\}. \tag{22b}$$

It is worthwhile to point out that all the above equations are suitable only for the circular pipes. As for the curved pipes with variable curvature, we need to divide it into a series of pipe segments and each one of the segments has a constant curvature. As long as the length of each pipe segment is small enough, a satisfactory accuracy can be achieved by this procedure. If R and EI in equations (1–22) are replaced by R_k and EI_k, respectively, all the above equations can be used for each pipe segment. Therefore, the kth pipe segment has the following relations according to equations (15), (22b) and (22a):

$$\{\delta\}_k = [T]_k\{\delta\}_{k-1}, \qquad \{\delta\}_{k-1} = [H]_k^{-1}\{q\}_{k-1} - [H]_k^{-1}\{\Delta\}_k, \tag{22c, d}$$

$$\{q\}_k = [H]_k\{\delta\}_k + \{\Delta\}_k. \tag{22e}$$

Substituting equation (22d) into equation (22c), and then substituting equation (22c) into equation (22e), one can get the following relation about the displacement and force vector of the two end nodes of the kth segment, k and $k-1$:

$$\{q\}_k^l = [S_k]\{q\}_{k-1}^r + [P_k]\{\Delta\}_k, \tag{23}$$

where

$$[S_k] = [H_k][T_k][H_k]^{-1}, \quad [P_k] = [I] - [S_k],$$

the superscripts l and r represent the left and right cross-section's value of $\{q\}$ (Figure 2); the subscript k of $\{q\}_k$ denotes the number of nodes, while the subscript k of $[S_k]$, $[T_k]$, $[H_k]$ and $[P_k]$ denotes the number of elements.

In order to discuss the relation between $\{q\}_{k-1}^r$ and $\{q\}_{k-1}^l$, we consider two cases:

(1) The $(k-1)$th node having no spring supports (Figure 2(a)), the relation is $\{q\}_{k-1}^r = \{q\}_{k-1}^l$, because of the continuity of displacement and force at interface $k-1$.
(2) The $(k-1)$th node having spring supports (Figure 2(b)) and letting K_1 and K_2 denote the elastic anti-shifting coefficient along the normal direction and the elastic anti-rotating coefficient of the spring, the balance of forces and moments acting on the $(k-1)$th node requires

$$Q_{k-1}^r = Q_{k-1}^l - K_1 u_{k-1}, \qquad M_{k-1}^r = M_{k-1}^l + K_2 u_{k-1}^{(1)}.$$

So

$$\{q\}_{k-1}^r = [F_{k-1}]\{q\}_{k-1}^l, \tag{24}$$

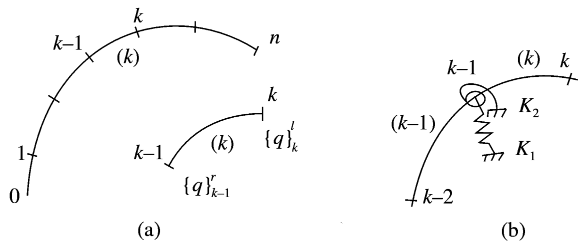

Figure 2. Discretization of a curved pipe (a) without spring supports at the $(k-1)$th node; and (b) with spring supports at the $(k-1)$th node.

where the elements of matrix $[F_{k-1}]$ are $F_{ii}=1, (i=1,2,\ldots,6)$, $F_{43}=K_2$, $F_{52}=-K_1$ and the remainder is zero. It is seen from equation (24) that if the $(k-1)$th node has no spring supports, $[F_{k-1}]$ is a sixth order unit matrix.

Substitution of equation (24) into equation (23) leads to

$$\{q\}^l_k=[S_k]\{q\}^r_{k-1}+[P_k]\{\Delta\}_k=[S_k][F_{k-1}]\{q\}^l_{k-1}+[P_k]\{\Delta\}_k.$$

Similarly,

$$\{q\}^l_{k-1}=[S_{k-1}][F_{k-2}]\{q\}^l_{k-2}+[P_{k-1}]\{\Delta\}_{k-1}.$$

Therefore,

$$\{q\}^l_k=[S_k][F_{k-1}][S_{k-1}][F_{k-2}]\{q\}^l_{k-2}+[S_k][F_{k-1}][P_{k-1}]\{\Delta\}_{k-1}+[P_k]\{\Delta\}_k.$$

In the same way, one obtains

$$\begin{aligned}\{q\}^r_n=[F_n]\{q\}^l_n=&[[F_n][S_n][F_{n-1}][S_{n-1}]\cdots[S_1][F_0]]\{q\}^l_0\\&+\{[F_n][S_n][F_{n-1}][S_{n-1}]\cdots[F_2][S_2][F_1][P_1]\{\Delta\}_1\\&+[F_n][S_n][F_{n-1}][S_{n-1}]\cdots[F_3][S_3][F_2][P_2]\{\Delta\}_2\\&+\cdots+[F_n][S_n][F_{n-1}][S_{n-1}][F_{n-2}][P_{n-2}]\{\Delta\}_{n-2}\\&+[F_n][S_n][F_{n-1}][P_{n-1}]\{\Delta\}_{n-1}\\&+[F_n][P_n]\{\Delta\}_n\}.\end{aligned}$$

This formula can be written in a brief form:

$$\{q\}^r_n=[U]\{q\}^l_0+\{\tilde{\Delta}\}. \tag{25}$$

Substituting the boundary conditions (5a, b) into equation (25) yields:

$$\begin{Bmatrix} w_n \\ u_n \\ \psi_n \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{16} \\ u_{21} & u_{22} & \cdots & u_{26} \\ \vdots & \vdots & & \vdots \\ u_{61} & u_{62} & \cdots & u_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ M_0 \\ Q_0 \\ N_{10} \end{Bmatrix} + \begin{Bmatrix} \tilde{\Delta}_1 \\ \tilde{\Delta}_2 \\ \vdots \\ \tilde{\Delta}_6 \end{Bmatrix}.$$

So

$$\begin{Bmatrix} -\tilde{\Delta}_4 \\ -\tilde{\Delta}_5 \\ -\tilde{\Delta}_6 \end{Bmatrix} = \begin{bmatrix} u_{44} & u_{45} & u_{46} \\ u_{54} & u_{55} & u_{56} \\ u_{64} & u_{65} & u_{66} \end{bmatrix} \begin{Bmatrix} M_0 \\ Q_0 \\ N_{10} \end{Bmatrix}, \tag{26}$$

where $u_{ij}(i, j = 4, 5, 6)$ is a function of flow velocity v, mass ratio β and frequency Ω.

In most circumstances, equation (26) may be solved for a unique set of $[M_0, Q_0, N_{10}]^{\mathrm{T}}$. However, the onset of instability corresponds mathematically to the circumstance that equation (26) does not have a unique solution set. Therefore, the requirement for a non-trivial solution is that the determinant of the coefficient matrix must vanish, namely,

$$\det[A] = 0, \tag{27}$$

where $[A]$ is the coefficient matrix of equation (26). Equation (27) is the characteristic equation of curved pipes with intermediate spring supports, variable rigidity and complex shape.

For given values of β and v, the natural frequency Ω may be computed numerically from equation (27). If $\mathrm{Im}(\Omega)$ is positive, the pipe performs damped oscillatory motion. When $\mathrm{Im}(\Omega)$ is negative, the pipe undergoes unstable oscillations. The threshold of instability is characterized by $\mathrm{Im}(\Omega) = 0$. Thus, the equation above may also be used for determining the critical flow velocity at which the pipe undergoes instability. If $\mathrm{Re}(\Omega) = 0$, the instability is of a divergent type or of a simple bucking mode. If $\mathrm{Re}(\Omega) \neq 0$, then the instability will be of a flutter type.

5. RESULTS AND DISCUSSION

Example 1. In order to compare with the existing work, consider a semi-circular curved pipe fixed at its left end with radius $R = 200\,\mathrm{cm}$, flexural rigidity $EI = 2 \times 10^8\,\mathrm{kgf\,cm^2}$ and mass ratio $\beta = 0{\cdot}5$. And there are two spring supports placed at the right end of the pipe. The elastic coefficients K_1 and K_2 are so large that the boundary condition may be considered as a clamped–clamped case, which has been studied by Chen [4].

Table 1 shows the critical velocities obtained by reference [4] and this paper, where the values of K_1 and K_2 are $K_1 = 2 \times 10^8\,\mathrm{kgf/cm}$, $K_2 = 2 \times 10^8\,\mathrm{kgf\,cm/rad}$. It is seen from Table 1 that the authors' results are in good agreement with those by Chen.

TABLE 1

Comparison of critical flow velocity

Mode	Critical flow velocity (v_{cr})	
	Result of this paper	Result of reference [4]
First	3·8414	3·9338
Second	5·4318	5·0003

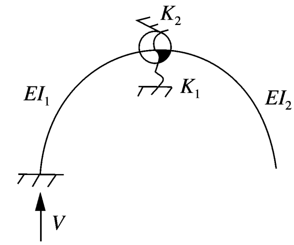

Figure 3. A stepped semi-circular pipe with two spring supports.

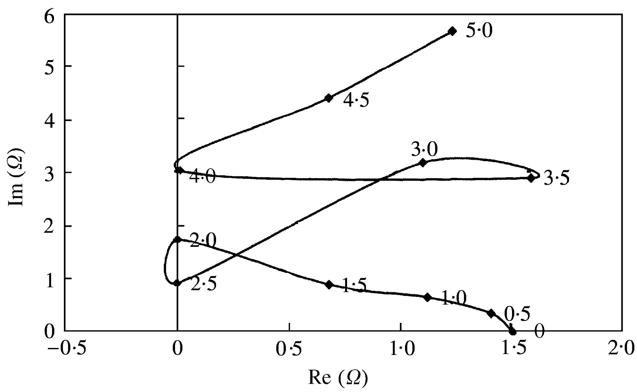

Figure 4. Argand diagram of the first mode.

Example 2. Consider a stepped semi-circular pipe with two spring supports at the middle of the pipe (Figure 3). Its parameters are:

$EI_1 = 3{\cdot}2 \times 10^8$ kgf cm^2, $EI_2 = 2{\cdot}0 \times 10^8$ kgf cm^2, $R = 200$ cm, $K_1 = 200$ kgf/cm, $K_2 = 200$ kgf cm/rad, $\beta = 0{\cdot}5$.

Figures 4–6 are the first three modes' Argand diagrams of the stepped semi-circular pipe. It should be noted that the second mode will undergo flutter when the flow velocity is 4·2110, while the other two modes will not. Similar to reference [1], the small flow velocities may reduce the natural frequency of the pipe, Re(Ω).

Figure 7 shows the critical dimensionless velocities for various values of K_1 and K_2 (they are assigned the same magnitude but different dimension). From it we can see that the critical dimensionless velocities increase when the values of K_1 and K_2 increase.

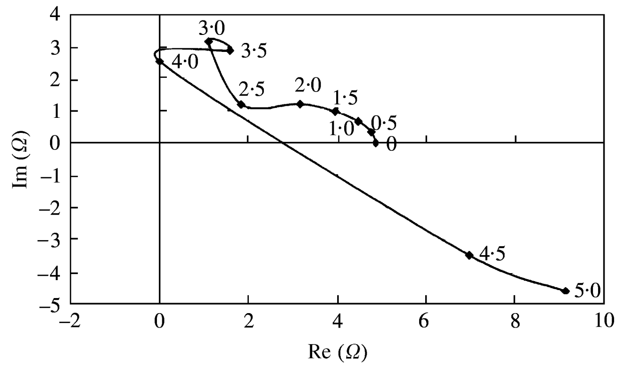

Figure 5. Argand diagram of the second mode.

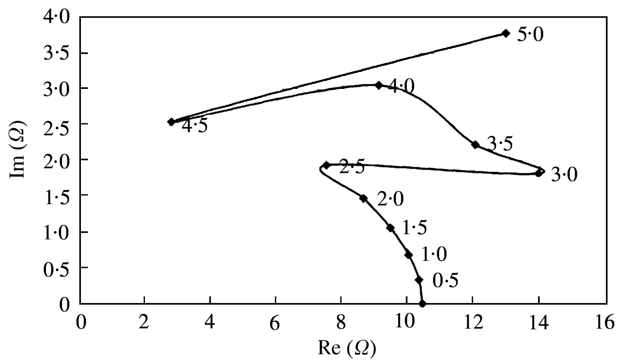

Figure 6. Argand diagram of the third mode.

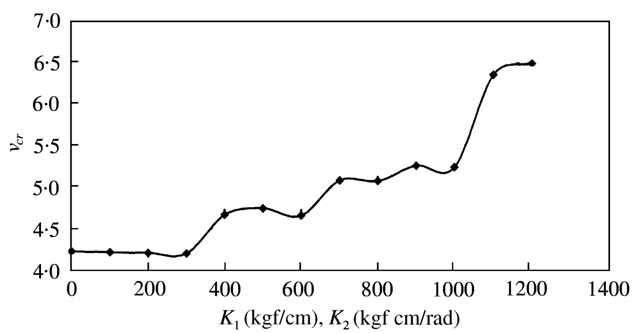

Figure 7. Critical dimensionless velocities for various values of the elastic coefficient of spring supports.

Figure 8 shows the critical dimensionless velocities for various values of the mass ratio β, from which it is seen that the curve has a maximum value at the point $\beta = 0{\cdot}3$. The conclusion is similar to reference [2].

6. CONCLUSIONS

The new matrix method proposed in this paper is well-suited to analyze the vibration and stability of curved pipe conveying fluid with complex shape. Compared to existing

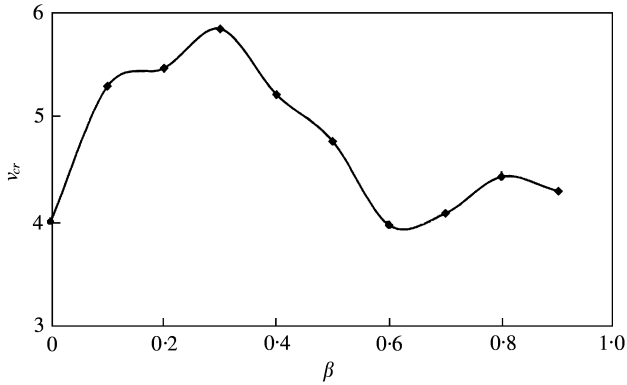

Figure 8. Critical dimensionless velocities for various values of the mass ratio.

analytical methods, our method is easy to deal with the curved pipes that have spring supports and variable curvature. Compared to the finite element method the proposed method can reduce the characteristic equation to a third-order one, no matter how many elements are discretized in calculation, which will lead to saving computer time.

In addition, it should be pointed out that, although the results are presented only for the case of circular pipe, pipes with arbitrary shapes that may be approximated as an assemblage of circular pipe segments having different radii can easily be handled with equal facility.

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REFERENCES

1. A. K. Misra, M. P. Paidoussis and K. S. Van 1988 *Journal of Fluids and Structures* **2**, 221–244. On the dynamics of curved pipes transporting fluid. Part I: Inextensible theory.
2. R. Aithal and G. Steven Gipson 1990 *Journal of Engineering Mechanics, American Society of Civil Engineers* **116**, 77–90. Instability of damped curved pipes.
3. Y. Huang 1987–88 *Mechanics of Structures and Machines* **15**, 463–479. A new matrix method for solving bucking and free vibration problems of circular arches with variable rigidity.
4. S. S. Chen 1973 *Nuclear Engineering and Design* **23**, 29–38. Flow induced in-plane stability of curved pipes.