Journal of Sound and Vibration (2002) **251**(2), 367–372
doi:10.1006/jsvi.2001.3821, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

WAVE PROPAGATION CHARACTERISTICS OF THIN SHELLS OF REVOLUTION BY FREQUENCY–WAVE NUMBER SPECTRUM METHOD

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(*Received* 6 *November* 2000, *and in final form* 27 *March* 2001)

1. INTRODUCTION

The frequency–wave number (FWN) spectrum method developed during the past 20 years has been conceived as a powerful solution procedure, by which both structural vibrations and wave propagations may be analyzed [1–3]. Shear, extensional and flexural wave propagations in shells and framework structures were studied by Nilson, developing mathematical models [4]. Wave absorption and vibration modal control in a flexible girder and travelling waves control of plate-beam mixed and large space structures were investigated by Flotow, applying wave motion theory [5]. 2-D stress wave propagations in solids with various boundary conditions were analyzed by Rizzi and Doyle, using the frequency spectrum method [6]. Periodicity and sound amplitude–frequency characteristics of wave propagation in periodically stiffened structures were investigated by Chapman, performing real experiments for the first time [7]. In view of the current research activities, however, wave propagations, sound radiation and vibration control of thin shells of revolution often used in industries has not been fully investigated by 2-D wave number domain approach, maximum entropy spectrum principle and related experiments. A method that combined the frequency domain method with the wave number analysis is presented and applied for the analysis of wave propagations in thin shells of revolution. Dynamic responses of the shells in certain excitations are analyzed by 1- and 2-D FWN spectra methods to obtain some key information on wave propagation in the shell.

2. EXPERIMENTAL ANALYSIS

Suppose that M acceleration sensors are arranged in a 1-D base array to measure vibration signals in the wave field. Provided that the number of sampling points is taken as N, acceleration signals measured by the sensors can be expressed in discrete form as

$$\{a(i,n)\} \quad (i=0,\dots,M-1, n=0,\dots,N-1). \tag{1}$$

By the Fourier transform of equation (1) first, the acceleration spectrum may be written as

$$A(i,f)=\sum_{n=0}^{N-1} a(i,n)\mathrm{e}^{-\mathrm{j}2\pi Fn/N} \quad (i=0,\dots,M-1, F=0,1,2,\dots,N-1) \tag{2}$$

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in which $f = F/N\Delta t$ and Δt is sampling time interval. Thus, the estimation of the space series $A(i,f)$ by maximum entropy spectrum yields the FWN spectrum of the form

$$S(k_x,f) = \frac{\mathrm{d}P_m}{|1 + \sum_{r=1}^{m} \varphi_r \mathrm{e}^{-\mathrm{j}(2\pi K_x/2M')r}|^2} \quad (K_x = 0, 1, 2, \dots, M' - 1) \tag{3}$$

in which $k_x = 2\pi K_x/2M'd$, M' is the total number of spectra, d is sampling space interval in the x direction, P_m and φ_r are, respectively, power and filtering coefficient of prediction errors. Formulas for calculating those parameters may be found in reference [4]. The angle of incidence of the waves may be rewritten in terms of wave velocity c as

$$\theta = \cos^{-1}\left(\frac{k_x}{k}\right) = \cos^{-1}\left(\frac{K_x}{2M'}\frac{c}{fd}\right). \tag{4}$$

In order to determine the direction of wave propagation, an $M_x \times M_y$ matrix of acceleration sensors in the x and y directions is fixed on the shell. Similarly, by applying Fourier transform to the time domain responses of acceleration $a(x, y, t)$ measured at those sensing points, the frequency domain responses of acceleration can be written as

$$A(i, l, f_0) = \sum_{n=0}^{N-1} a(i, l, n)\mathrm{e}^{-\mathrm{j}2\pi F n/N} \quad (i = 1, 2, \dots, M_x,\ l = 1, 2, \dots, M_y,$$

$$F = 0, 1, 2, \dots, N - 1). \tag{5}$$

A series of space spectra $A(i, l, f_0)$ which forms as $M_x \times M_y$ matrix is obtained. This matrix of acceleration spectra is estimated by the 2-D maximum entropy spectrum principle and then the 2-D FWN spectrum $S(k_x, k_y, f)$ at f_0 can be derived and plotted. According to components of the wave number k_x and k_y corresponding to the spectrum apexes, the direction of wave propagation θ and wave number k at the spectrum apex can be determined and, further, the wave velocity c and wave type can be obtained.

The same method developed above may be applied for the solution of excited response problems of structures. Suppose that there is an acceleration response $a_Q(x, y, t)$ of the shell under the specified excitation $p(t)$ acting on the shell at point Q and the corresponding powers $P(f)$ and $A_Q(x, y, f)$ to the excitation and acceleration signals, respectively, may be derived by the Fourier transform. Accordingly, the transfer function of the shell may be directly expressed in terms of $A_Q(x, y, f)$ and $P(f)$ as follows:

$$H_Q(x, y, f) = \frac{A_Q(x, y, f)}{P(f)}. \tag{6}$$

Further, by applying equation (5), the transfer function in equation (6) can be rewritten in the discrete form

$$H_Q(i, l, f) = \frac{A_Q(i, l, f)}{P(f)}, \quad (i = 1, \dots, M_x, l = 1, \dots, M_y). \tag{7}$$

By estimation of the maximum entropy spectrum again for the transfer functions $H_Q(i, l, f)$, the 2-D FWN spectrum $H_Q(k_x, k_y, f)$ is then derived, which represents information of acceleration travelling propagation of the measured system from the excitation point Q at frequency f in the direction θ.

3. NUMERICAL EXAMPLES AND DISCUSSION

As shown in Figure 1, the tail part of the shell of revolution attached to an underwater vehicle maybe treated as one section of a slightly conic shell, having total length $L = 600$ mm, the big end of diameter ϕ_1 and the small end of diameter ϕ_2 and is applied by an excited force P on point Q 100 mm from the small end.

A matrix of 8×8 sensing points with intervals of 25 mm in both the axial and circumferential directions is distributed symmetrically along both right and left sides from the top generatrix and closed at a distance of $B = 50$ mm to the big end of the shell for measuring impact and excited vibrations. Co-ordinates of circumferential wave number K_x and circumferential wave number k_x are related to each other in the following form:

$$k_x = \frac{2\pi K_x}{2M_x d_x}, \tag{8}$$

in which M_x is the total number of circumferential 1-D FWN spectra and d_x denotes the distance between sampling points in the circumferential direction. Co-ordinates of axial wave number K_y and axial wave number k_y are related to each other in the following form:

$$k_y = \frac{2\pi K_y}{2M_y d_y}, \tag{9}$$

where M_y is the total number of axial 1-D FWN spectra and d_y denotes a distance between the measuring points on the shell in the axial direction. Numerical results for the 1-D frequency–wave number spectrum of the shell are plotted in Figure 2.

2-D FWN spectra of the measured vibration signals are shown in Figure 3 for the shell with an input of vibration energy. The first four natural frequencies of the shell are 962·5, 1400·0, 1912·5, and 4812·5 Hz respectively. Components of wave number in the axial and circumferential directions, k_x and k_y, are associated with frequency f and wave velocity c to obey a quadratic equation of circle of the form

$$k_x^2 + k_y^2 = \left(\frac{2\pi f}{c}\right)^2. \tag{10}$$

As seen from equation (10), for a given frequency and identical type of waves, the wave numbers and then the velocities of the same type of waves propagating in an isotropic solid

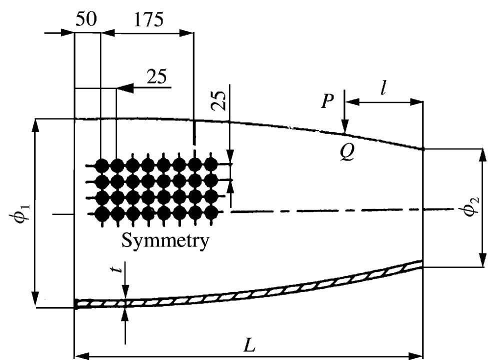

Figure 1. Geometry and test arrangement of the shell of revolution.

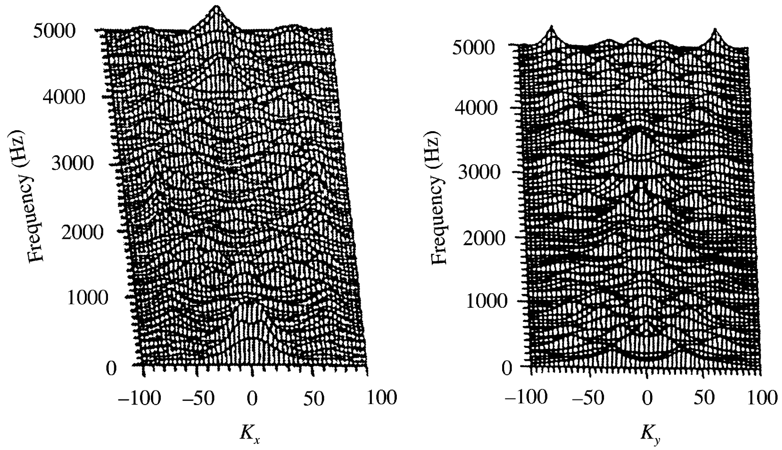

Figure 2. 1-D numerical results of FWNS for the shell of revolution.

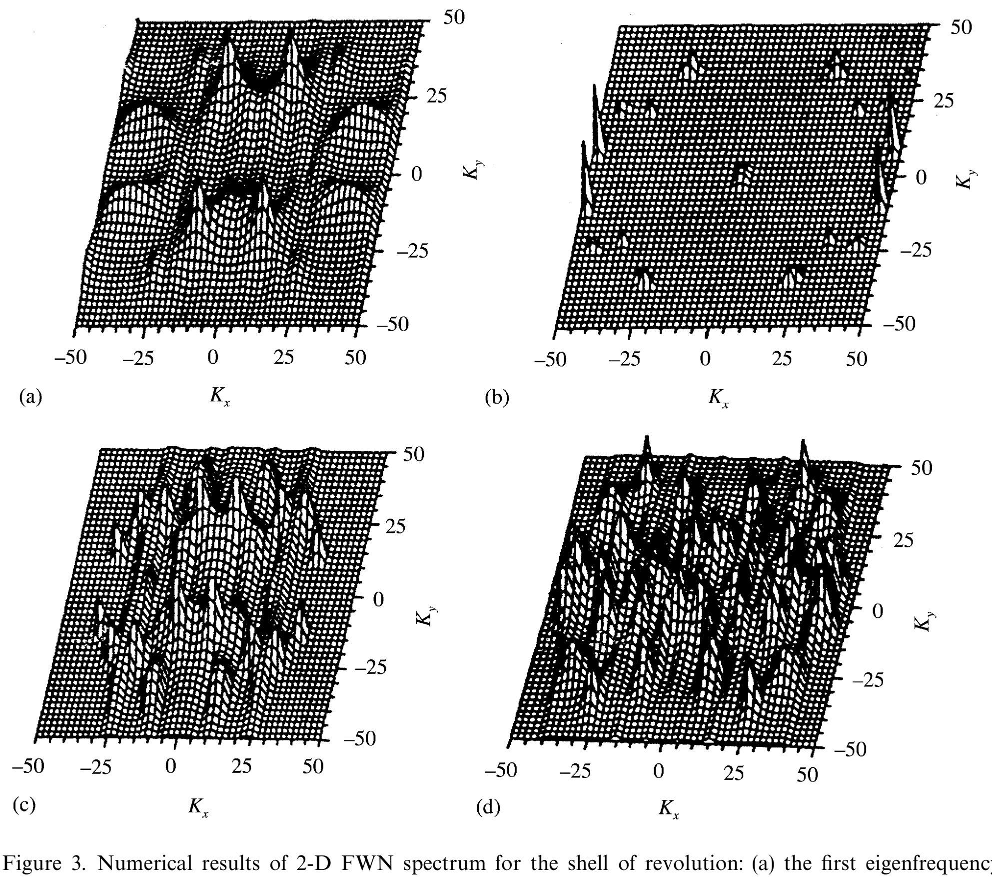

Figure 3. Numerical results of 2-D FWN spectrum for the shell of revolution: (a) the first eigenfrequency 962·5 Hz; (b) the second eigenfrequency 1400·0 Hz; (c) the third eigenfrequency 1912·5 Hz; (d) the fourth eigenfrequency 4812·5 Hz.

media are identical. Also, apexes of the same type of waves are distributed on a circle in the 2-D FWN spectrum. When the shell is excited, three kinds of travelling waves appear in the structure, which are, respectively, flexible, shear and extensional waves. The number of

TABLE 1

Wave propagation characteristics for the shell of revolution

Eigenfrequency (Hz)	Wave numbers (rad/m)			Wave velocities (m/s)			Travelling wave direction (deg)			Maximum spectrum value (mm/s^2)
	k_1	k_2	k_3	c_1	c_2	c_3	θ_1	θ_2	θ_3	
962·5	101·3	64·1		59·9	94·3			64·4		24·12
1400·0	124·0	99·7		70·9	88·2		11·8			16·07
1912·5	93·2	74·9	47·0	128·9	160·4	255·7			74·5	22·31
4812·5	112·7	75·6	26·2	268·3	400·0	1154·7	56·7			23·57

flexible waves is the largest but its velocity is the lowest; the number of extensional waves is the smallest and its velocity is the fastest; and the number and velocity of shear waves are intermediate in comparison with those of flexible and extensional waves. In the 2-D FWN spectrum, therefore, wave apexes distributed in the external area correspond to flexible waves, the ones located in the internal area are extensional waves and those appearing in the middle are shear waves. As shown in Figure 3, flexible and shear waves can be easily recognized except extensional waves, because those acceleration sensors are placed on the shell in the radial direction so that the out-of-plane vibration signals can be tested appropriately. Actually, the sensors cannot work to measure transient responses of the in-plane vibration measures of the shell. Numerical results are given in Table 1 in average for the number k, velocity c and propagation direction θ of the main travelling waves in the tail part of the shell.

At the frequency of 962·5 Hz, the principal travelling waves are shear and the maximum spectrum value is 24·12 originally in Table 1. For the shell there exist originally three types of travelling waves at 1400·0 Hz, shear and flexible waves of which are distributed on an inconspicuous ellipse of a little dispersion. At the frequency of 1912·5 Hz, three types of waves in the shell are found to be of great energy with the maximum spectrum value 22·31 in Table 1. At the frequency of 4812·5 Hz, a number of spectrum apexes are distributed obviously on a circle in Figure 3.

4. CONCLUDING REMARKS

Based on the frequency–wave number (FWN) spectrum method, the 1-D and 2-D experimental investigations have been made on the transient responses and wave propagations in the tail part of the shell of revolution attached to an underwater vehicle. Much information on wave propagation characteristics such as number, vehicle, type and direction of the waves in the shell is obtained. From the 1-D FWN spectrum results computed by the present data process treatment, four apexes of spectrum can be found, corresponding to the four eigenfrequencies of the shell structure, and from the 2-D FWN spectrum results, three types of wave, flexural, shear and extensional waves are obviously observed, propagating in the shell at the four eigenfrequencies. According to the characteristics of these wave propagations, flexural, shear and extensional waves appear in an order of frequency for the wave velocity, but in a descending order of frequency for the wave number. The testing results and experimental treatment presented herein are of practical importance for the design of underwater vehicle structures with fairly good passive control and depression of wave and sound radiations. Further investigation on the pattern of reinforcement and its effect on the eigenfrequencies and vibration depression of the shells of revolution is required for an optimal arrangement of axial stiffeners and circumferential stringers.

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