Journal of Sound and Vibration (2002) **251**(2), 377–382
doi:10.1006/jsvi.2001.3822, available online at http://www.idealibrary.com on IDEAL®

VIBRATION OF RIGHT-ANGLED TRIANGULAR PLATES PARTIALLY CLAMPED ON ONE SIDE

R. Singhal

David Florida Laboratory, Canadian Space Agency, Ottawa K2H 8S2, Canada

AND

D. Redekop

Department of Mechanical Engineering, University of Ottawa, Ottawa K1N 6N5, Canada.
E-mail: redekop@eng.uottawa.ca

(*Received* 27 *June* 2001)

1. INTRODUCTION

Triangular plates with partially supported edges serve to model aircraft wings with cracks at the supporting side. A finite element method (FEM) vibration study for this plate geometry was recently carried out by Mirza and Alizadeh [1]. There appears to be no experimental work on this geometry, aside from a study of a cantilever triangular plate with support along the full length of the supported edge [2].

In the present study, the natural vibration characteristics of right-angled triangular plates partially clamped on one side are determined experimentally. A solution using the finite element method is also presented. Three series of plates were studied, having different values for the ratio of the lengths of the cantilevered side to the supported side. For each plate, the first five resonant frequencies were obtained, corresponding to different values of the crack/supported side ratio. The paper ends with a comparison of the experimental and FEM results, and a statement of relevant conclusions.

2. EXPERIMENTAL PROGRAM

The length of the supported side of the plate (Figure 1), denoted by "a", was taken as 10 in (254 mm). The length of the side at right angles, denoted as "b", was taken as either 10, 15, or 20 in (254, 381, or 508 mm). Thus, the three series of plates had aspect ratios $\phi = b/a$ of 1·0, 1·5, and 2·0. The free (crack) length along the supported edge in each series was denoted as "c", and was varied from 0 to 8 in (0–203·2 mm) in steps of 2 in (50·8 mm).

The objective of the experimental study was to establish resonant frequencies and the associated mode shapes for the three series of plates described earlier. For this purpose, the method of impact testing was adopted. Twenty-two measurement point locations were chosen for the 10 in plates, and 41 locations for the 15 and 20 in plates. The impact location was fixed at one point. Only one accelerometer was used, and it was moved from one measurement point to another. For each measurement point, a frequency response function

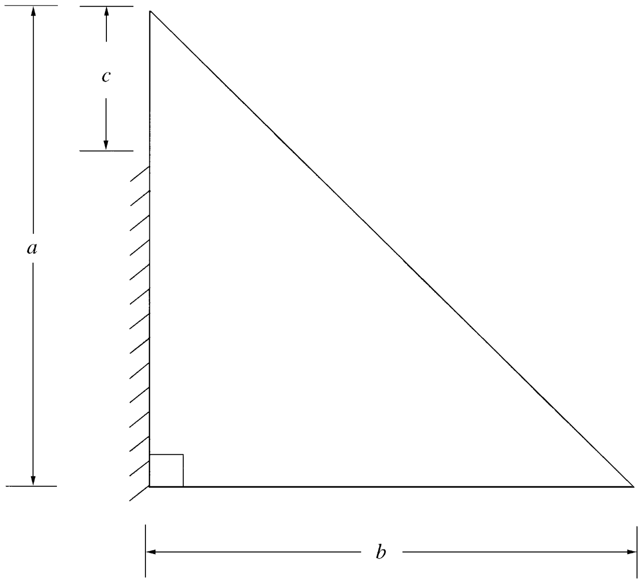

Figure 1. Geometry.

was obtained from the average of five impacts. Figure 2 shows some typical experimental and analytical mode shapes for the model TP10-6 ($b = 10$ in, $c = 6$ in).

3. FEM APPROACH

Numerical results were obtained using the NE-NASTRAN finite element program [3]. A flat plate element with three or four nodes is available for vibration analysis. To validate the approach, numerical results were compiled for a triangular plate with the following values of parameters: $a = 254$ mm, $b = 254$ mm, $h = 1{\cdot}55$ mm, $E = 2{\cdot}0685$ M Pa, $\nu = 0{\cdot}3$, $\rho = 7840$ kg/m^3, where h is the plate thickness, and E, ν, ρ are, respectively, the elastic modulus, the Poisson ratio, and mass density. For this plate results are available in the literature.

Table 1 provides a comparison of the FEM results with the previous work for this validation case. The natural frequencies for the first five modes are given. Results from previous work stem from experimental [2] and analytical studies [4, 5]. Two finite element analyses are represented, based, respectively, on a High Precision Element (HPE) [4], and on the ADINA code [5]. Results of the present work obtained using NE-Nastran [3] correspond to three FEM models representing, respectively, 129, 426, and 1628 triangular elements.

It is observed that there is close agreement in the various sets of results for all five modes. The current results for the finest mesh in particular are very close to the results from the HPE analysis. In general, the analytical results are somewhat higher than the experimental ones.

4. RESULTS AND DISCUSSION

The triangular plates of the present study had the following values of parameters: $a = 10$ in (254 mm), $b = 10$, 15, 20 in (254, 381, 508 mm), $h = 0{\cdot}0625$ in (1·59 mm), $E = 10{\cdot}4 \times 10^6$ psi (71·7 GPa), $\nu = 0{\cdot}32$, $\rho = 0{\cdot}259 \times 10^{-3}$ lb/in^3 (7169 kg/m^3).

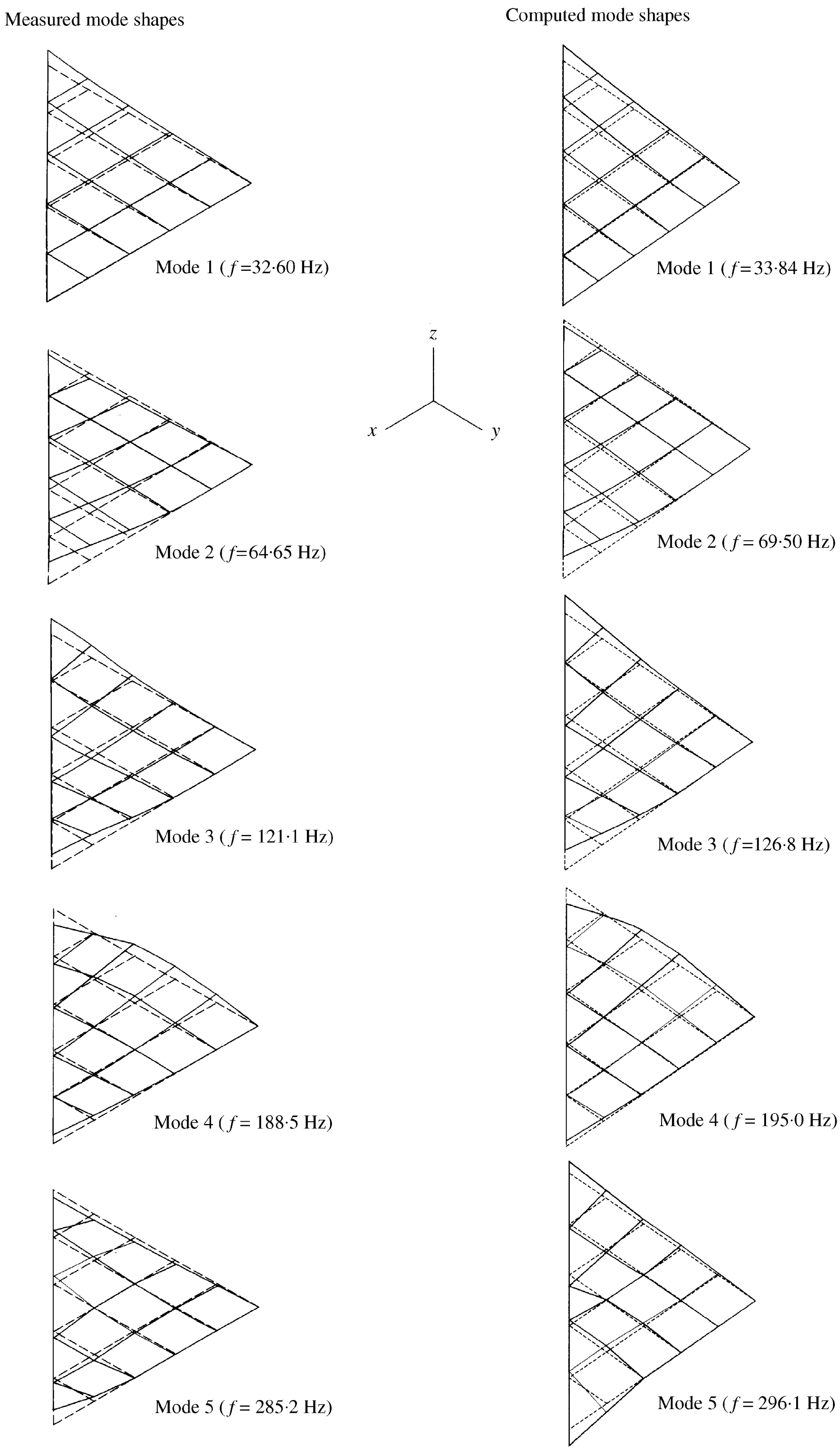

Figure 2. Mode shapes for plate TP10-6: (a) measured shapes; (b) computed shapes.

TABLE 1

Comparison of frequencies for validation case (Hz)

Method Source	Elements	Mode 1	2	3	4	5
Expt[2]		34·5	136·0	190·0	325·0	441·0
HPE[4]	16	36·6	139·3	194·1	333·8	455·4
ADINA[5]	72	36·6	140·3	197·2	341·7	471·4
NASTRAN1	129	36·65	140·0	195·5	338·2	463·3
NASTRAN2	426	36·63	139·5	194·3	334·6	456·4
NASTRAN3	1628	36·62	139·4	193·9	333·6	454·2

TABLE 2

Experimental and FEM frequencies (Hz) $- \phi = 1$

Model	Method	Mode 1	2	3	4	5
TP10-0	Expt	36·02	136·8	192·7	326·9	444·2
	FEM	37·26	141·4	196·6	338·4	461·9
TP10-2	Expt	35·96	134·8	189·5	319·2	434·4
	FEM	37·23	140·5	196·2	332·9	454·4
TP10-4	Expt	35·05	118·0	156·8	191·0	303·1
	FEM	36·60	125·6	169·9	198·8	315·0
TP10-6	Expt	32·60	64·65	121·1	188·5	285·2
	FEM	33·84	69·50	126·8	195·0	296·1
TP10-8	Expt	24·18	35·76	113·9	187·7	246·0
	FEM	26·91	38·64	117·7	193·5	259·9

The results for the three series of plates are given in Tables 2–4. Results from both the experimental program and the NASTRAN FEM analysis are given in each case for the first five modes of vibration.

A review of Tables 2–4 indicates very good agreement between the experimental and FEM results. In general, discrepancies do not exceed 5%. It must be recognized that there are numerous factors which can cause the actual plate and its supports to differ slightly from the ideal plate modelled in the finite element analysis. The plates can have small residual stresses and may differ in geometry from the idealized flat configuration utilized in the calculations. Despite precautions taken in the design of the holding (clamping) fixtures, they may not provide idealized rigidity. Furthermore, the actual plate properties—thickness, density, Young's modulus and the Poisson ratio—may differ slightly from those used in the FEM calculations. In the experience of the authors, these factors could easily account for a difference of about 3%.

Table 3

Experimental and FEM frequencies (Hz) — $\phi = 1{\cdot}5$

Model	Method	Mode 1	2	3	4	5
TP15-0	Expt	16·83	70·34	105·2	170·8	252·8
	FEM	17·37	73·10	106·9	179·1	257·2
TP15-2	Expt	16·93	69·96	103·9	170·6	245·4
	FEM	17·34	72·57	105·7	177·7	250·6
TP15-4	Expt	16·19	63·26	90·34	133·9	172·4
	FEM	16·92	67·81	97·42	147·5	182·8
TP15-6	Expt	14·75	45·66	74·13	108·7	164·7
	FEM	15·62	50·35	77·98	113·2	172·4
TP15-8	Expt	12·35	28·69	67·13	104·5	161·0
	FEM	12·95	30·14	69·89	109·4	166·3

Table 4

Experimental and FEM frequencies (Hz) — $\phi = 2$

Model	Method	Mode 1	2	3	4	5
TP20-0	Expt	9·472	41·45	71·26	100·7	171·3
	FEM	10·00	42·95	74·06	105·2	175·7
TP20-2	Expt	10·10	40·70	70·22	99·45	164·1
	FEM	9·974	42·69	72·84	104·7	170·8
TP20-4	Expt	8·596	38·87	62·93	96·86	124·9
	FEM	9·717	40·94	66·40	101·7	132·0
TP20-6	Expt	8·394	33·08	49·56	85·43	99·57
	FEM	9·025	35·46	52·73	88·29	104·9
TP20-8	Expt	6·664	22·18	41·86	78·42	95·97
	FEM	7·673	24·43	44·12	81·08	100·8

5. CONCLUSION

Experimental and finite element results have been represented for the natural frequencies of a right-angled triangular plate partially clamped on one side. There is a close agreement in the two sets of results. The experimental results provided herein represent useful information for other researchers studying the vibration of triangular plates.

ACKNOWLEDGMENTS

This work was supported in part by a grant from the Natural Sciences and Engineering Research Council of Canada. Experimental work was carried out at the David Florida Laboratory of the Canadian Space Agency in Ottawa, Canada.

REFERENCES

1. S. MIRZA and Y. ALIZADEH 1994 *Computers and Structures* **51**, 143–151. Free vibration of partially supported triangular plates.
2. P. N. GUSTAFSON, W. F. STOKEY and C. F. ZOROWSKI 1953 *Journal of the Aeronautical Sciences* **20**, 331–337. Experimental study of natural vibrations cantilevered plates.
3. Noran Engineering Inc. 1996 *NE/Nastran, User's Manual V*2.3. Garden Grove, CA.
4. G. R. COWPER, E. KOSKO, G. M. LINDBERG and M. D. OLSON 1969 *American Institute of Aeronautics and Astronautics Journal* **7**, 1957–1965. Static and dynamic applications of a high-precision triangular plate bending element.
5. ADINA R & D Inc. 1997 *ADINA Verification Manual*, *ARD* 97-9. Watertown, MA.