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FREE VIBRATION ANALYSIS OF COMPOSITE RECTANGULAR MEMBRANES WITH AN OBLIQUE INTERFACE

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The natural frequencies and mode shapes of a composite rectangular membrane with no exact solutions are found by using an analytical method appropriate for the geometric feature of the title problem membrane presented here. The method has a key feature in which the theoretical development is very simple and only a small amount of numerical calculation is required. Example studies show that the natural frequencies and their associated modes obtained from the method are found to be very accurate compared with the results by the *FEM* (*SYSNOISE*) or exact solutions. Furthermore, the natural frequencies converge rapidly and accurately to the exact values or the numerical results obtained from the finite element model using meshes sufficient to yield already converging natural frequencies, even when a small number of series functions are used in the proposed method.

1. INTRODUCTION

In the free vibration analysis of membranes, there have been many examples attempted in solving a non-homogeneous membrane of simple geometry. For example, Laura dealt with a great variety of membranes of simple geometry: a rectangular membrane with the density linearly varying in the direction of one of x- and y-axis [1], a composite annular membrane with density linearly varying in the radial direction using an approximate variational approach or with the stepped radial density using an exact method [2, 3], and a composite circular membrane with a central point support [4]. Bambill performed the free vibration analysis of a composite, doubly connected square membrane using the conformal mapping approach [5], and Masad studied the analytical method for obtaining the approximate solution of a rectangular membrane with smoothly varying inhomogeneity [6]. Later, Wang showed that the membrane with linear density variation among the non-homogeneous membranes studied by Masad has a closed form exact solution [7]. Furthermore, Cortinetz revealed that there exists an exact solution for a composite rectangular membrane with the stepped density varying in the direction of the x-axis [8] (the concerned results were compared with those obtained using the Kantorovich method [9, 10]).

Although many engineering applications have dealt with various non-homogeneous membranes as reviewed above, there commonly exists a limitation where the boundaries of each homogeneous region in the membrane of interest have to be paralleled with one of the axes of the co-ordinate system selected. Similarly, this paper deals with a composite rectangular membrane of which the four edges are paralleled with one of the axes, but the interface between two homogeneous regions with different surface densities is not paralleled with any axis and is oblique. Note that this geometric feature differs from that of the title problem studied by Cortinez [8].

The composite membrane considered in this paper has no exact solution and thus a suitable analytical method for the geometric feature of the composite membrane is proposed in this study. Case studies for the verification of the present method reveal that the method gives very accurate and rapidly converging natural frequencies, and also that the mode shapes found using this method are in good agreement with those obtained by the FEM, in spite of the amount of numerical calculation being much smaller than with the FEM.

2. THEORETICAL FORMULATION

2.1. ASSUMPTION OF SOLUTIONS

As shown in Figure 1, consider a composite rectangular membrane composed of two homogeneous regions D_{I} and D_{II}, of which the common boundary Γ_c is oblique against the Y-axis. As the initial stage of theoretical development for the free vibration analysis of the composite membrane, two semi-infinite membranes are imagined as shown in Figure 2 where each semi-infinite membrane has fixed boundary condition at the corresponding three edges. In Figure 2, the first semi-infinite membrane fixed at $X = Y = 0$ and $Y = b$ is considered to depict the transverse vibration of the region D_{I}. Similarly, the second fixed at $X = a$ and $Y = 0 = b$ is considered for the region D_{II}.

The eigenfield of the composite membrane for transverse vibration is assumed by employing the transverse displacement functions $W_{\text{I}}(X, Y)$ and $W_{\text{II}}(X, Y)$, respectively, for the two homogeneous regions. Using the method of variable separation

$$W_{\text{I}}(X, Y) = f_{\text{I}}(X)g_{\text{I}}(Y), \quad W_{\text{II}}(X, Y) = f_{\text{II}}(X)g_{\text{II}}(Y) \tag{1,2}$$

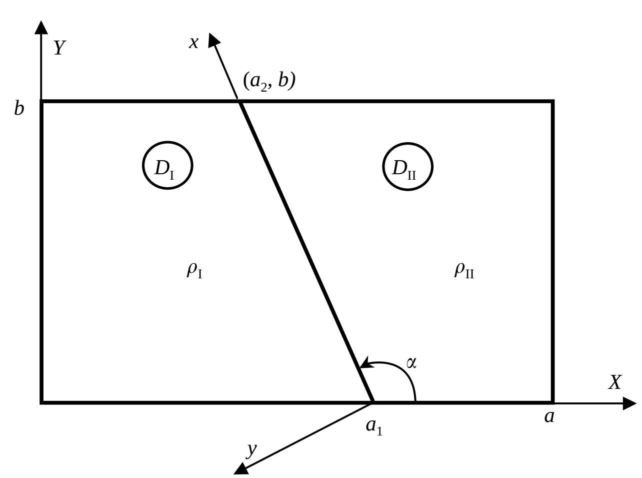
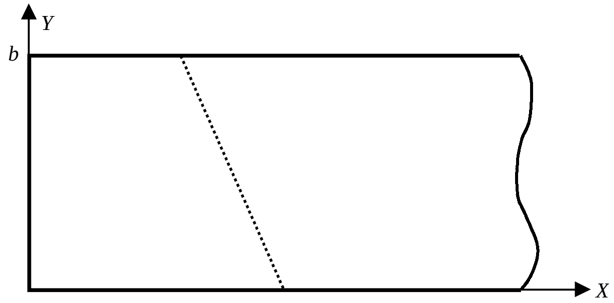


Figure 1. Composite rectangular membrane with the oblique interface between the subdomains D_{I} and D_{II}.



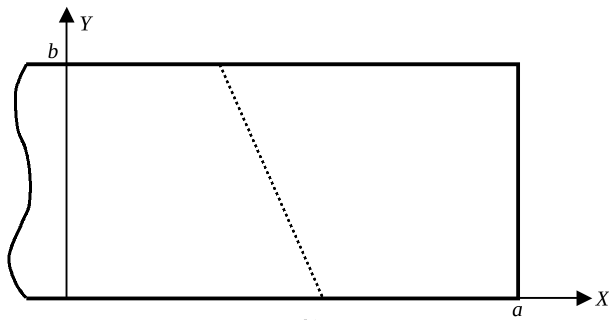


Figure 2. Two semi-infinite membranes with fixed edges at (a) $X = 0$, $Y = 0$ and $Y = b$; (b) $X = a$, $Y = 0$ and $Y = b$.

and substituting each of them into the governing differential equation

$$\nabla^2 W + k^2 W = 0, \tag{3}$$

where k denotes the wavenumber, one can obtain the general solutions

$$W_{\mathrm{I}}(X, Y) = (A^{(\mathrm{I})} \sin k_x^{(\mathrm{I})} X + B^{(\mathrm{I})} \cos k_x^{(\mathrm{I})} X)(C^{(\mathrm{I})} \sin k_y^{(\mathrm{I})} Y + D^{(\mathrm{I})} \cos k_y^{(\mathrm{I})} Y), \tag{4}$$

$$W_{\mathrm{II}}(X, Y) = (A^{(\mathrm{II})} \sin k_x^{(\mathrm{II})} X + B^{(\mathrm{II})} \cos k_x^{(\mathrm{II})} X)(C^{(\mathrm{II})} \sin k_y^{(\mathrm{II})} Y + D^{(\mathrm{II})} \cos k_y^{(\mathrm{II})} Y), \tag{5}$$

where $(k_x^{(\mathrm{I})})^2 + (k_y^{(\mathrm{I})})^2 = (k^{(\mathrm{I})})^2$ and $(k_x^{(\mathrm{II})})^2 + (k_y^{(\mathrm{II})})^2 = (k^{(\mathrm{II})})^2$.

If the boundary conditions for each semi-infinite membrane described in the above

$$W_{\mathrm{I}}(X = 0) = W_{\mathrm{I}}(Y = 0) = W_{\mathrm{I}}(Y = b) = 0, \tag{6}$$

$$W_{\mathrm{II}}(X = a) = W_{\mathrm{II}}(Y = 0) = W_{\mathrm{II}}(Y = b) = 0 \tag{7}$$

are applied to the general solutions, equations (4, 5) lead to

$$W_{\mathrm{I}} = \sum_{m=1}^{N} A_m^{(\mathrm{I})} \sin[k^{(\mathrm{I})} X] \sin[m\pi Y/b], \tag{8}$$

$$W_{\mathrm{II}} = \sum_{n=1}^{N} A_n^{(\mathrm{II})} \sin[k^{(\mathrm{II})}(a - X)] \sin[n\pi Y/b], \tag{9}$$

where $k^{(\mathrm{I})} = \sqrt{(\omega/c_{\mathrm{I}})^2 - (m\pi/b)^2}$ and $k^{(\mathrm{II})} = \sqrt{(\omega/c_{\mathrm{II}})^2 - (n\pi/b)^2}$ denote the wavenumbers for each semi-infinite membrane, expressed by the angular frequency $\omega = 2\pi f$, and the speed of wave propagation, $c_i = \sqrt{T/\rho_i}$, using the tension per unit length T and the surface density given by ρ_{I} or ρ_{II}. It should be noted that the assumed functions satisfy the Helmholtz equation (3) and the fixed boundary conditions (6, 7) at all edges except the common boundary.

In order for the assumed functions to become eigensolutions for the free vibration of the composite membrane, it is required that $W_{\mathrm{I}}(x, y)$ and $W_{\mathrm{II}}(x, y)$ satisfy the compatibility condition, that is the two functions must satisfy the conditions of continuity in displacement and slope at the common boundary given between the co-ordinates $(a_1, 0)$ and (a_2, b): i.e.,

$$W_{\mathrm{I}}|_{\Gamma_c} = W_{\mathrm{II}}|_{\Gamma_c}, \quad \partial W_{\mathrm{I}}/\partial n|_{\Gamma_c} = \partial W_{\mathrm{II}}/\partial n|_{\Gamma_c}, \tag{10, 11}$$

where n represents the normal direction from the common boundary.

2.2. SYSTEM MATRIX FROM THE COMPATIBILITY CONDITIONS

It has been well known that there is no exact solution in the case of the composite rectangular membrane with the oblique common boundary as shown in Figure 1. If the common boundary is paralleled with the Y-axis, this membrane will be solved by the exact method using the compatibility conditions at the common boundary or by the Kantorovich method using the variation technique [6]. However, there has not been any research published so far that has analytically solved composite rectangular membranes with such an oblique common boundary as in the title problem. The reason is that, when the global co-ordinates (X, Y) are used, the geometric variables X and Y are both varied along the oblique common boundary.

In this section, a co-ordinate transformation is first performed to overcome the above problem. Correspondingly, the relationship of the local co-ordinate system (x, y) to the global co-ordinate system (X, Y), shown in Figure 1, is given by

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{bmatrix} p & -q \\ p & q \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} a_1 \\ 0 \end{Bmatrix}, \tag{12}$$

where $p = \cos\alpha$ and $q = \sin\alpha$ (the angle α is indicated in Figure 1). Substituting the co-ordinate relationship,

$$X = px - qy + a_1, \qquad Y = qx + py, \tag{13, 14}$$

which are obtained from equation (12), into the equations (8, 9) leads to

$$W_{\mathrm{I}}(x, y) = \sum_{m=1}^{N} A_m^{(\mathrm{I})} \sin[k^{(\mathrm{I})}(px - qy + a_1)] \sin[m\pi(qx + py)/b], \tag{15}$$

$$W_{\mathrm{II}}(x, y) = \sum_{n=1}^{N} A_n^{(\mathrm{II})} \sin[k^{(\mathrm{II})}(a - px + qy - a_1)] \sin[n\pi(qx + py)/b]. \tag{16}$$

2.2.1. *Condition of continuity in displacement*

Prior to considering the conditions of continuity at the common boundary, it is important to note the physical concept associated with the harmonic transverse vibration of the composite membrane. Concretely speaking, since the composite membrane *harmonically* vibrates at the entire region including the common boundary at any excitation frequency, the transverse displacement shape $U(x, y=0)$ created along the common boundary may be expressed by linearly superposing the bases $\sin \pi x/L$, $\sin 2\pi x/L, \ldots, \sin N\pi x/L$ where L is the length of the common boundary: i.e.,

$$U(x, y=0) = \sum_{i=1}^{N} B_i \sin i\pi x/L. \tag{17}$$

Using equation (17), equation (10) may be expressed as

$$U(x, y=0) = W_{\mathrm{I}}(x, y=0), \qquad U(x, y=0) = W_{\mathrm{II}}(x, y=0). \tag{18, 19}$$

Substituting equations (15, 17) into equations (18, 19) gives the following explicit results expressed in terms of the single local co-ordinate variable x:

$$\sum_{i=1}^{N} B_i \sin i\pi x/L = \sum_{m=1}^{N} A_m^{(\mathrm{I})} \sin[k^{(\mathrm{I})}(px + a_1)] \sin[m\pi x/L], \tag{20}$$

$$\sum_{i=1}^{N} B_i \sin i\pi x/L = \sum_{n=1}^{\mathrm{N}} A_{\mathrm{n}}^{(\mathrm{II})} \sin[k^{(\mathrm{II})}(a - px - a_1)] \sin[n\pi x/L]. \tag{21}$$

It should be noted in the current step that equations (20, 21) do not give complete forms because the geometric variable x is included in the equations.

In order to eliminate x from the equations, the sth basis $\sin s\pi x/L$ is multiplied to both sides of the equations and the integration procedure from 0 to L is performed along the common boundary. Then, equations (20, 21) lead to, respectively,

$$B_s = L/2 \sum_{m=1}^{N} SM_{sm}^{(\mathrm{I})} A_m^{(\mathrm{I})}, \quad s = 1, 2, \ldots, N, \tag{22}$$

$$B_s = L/2 \sum_{n=1}^{N} SM_{sn}^{(\mathrm{II})} A_n^{(\mathrm{II})}, \quad s = 1, 2, \ldots, N, \tag{23}$$

where $SM_{sm}^{(\mathrm{I})}$ and $SM_{sn}^{(\mathrm{II})}$ are given by

$$SM_{sm}^{(\mathrm{I})} = \int_0^L \sin[k^{(\mathrm{I})}(px + a_1)] \sin[m\pi x/L] \sin[s\pi x/L] \, \mathrm{d}x, \tag{24}$$

$$SM_{sn}^{(\mathrm{II})} = \int_0^L \sin[k^{(\mathrm{II})}(px + a_1)] \sin[n\pi x/L] \sin[s\pi x/L] \, \mathrm{d}x. \tag{25}$$

Since equations (22, 23) are identical for all s, N linear equations are obtained as

$$\sum_{m=1} SM_{sm}^{(\mathrm{I})} A_m^{(\mathrm{I})} = \sum_{n=1} SM_{sn}^{(\mathrm{II})} A_n^{(\mathrm{II})}, \quad s = 1, 2, \ldots, N, \tag{26}$$

which may be finally rewritten in the simple matrix form

$$\mathbf{SM}^{(\mathrm{I})} \mathbf{A}^{(\mathrm{I})} = \mathbf{SM}^{(\mathrm{II})} \mathbf{A}^{(\mathrm{II})}. \tag{27}$$

2.2.2. *Condition of continuity in slope*

In this section, the condition of continuity in slope at the common boundary is considered in the same manner as in section 2.2.1. First, a slope variation function U_n along the common boundary is assumed as

$$U_n(x, y = 0) = \sum_{i=1}^{N} C_i \sin i\pi x/L. \tag{28}$$

Using equation (28), equation (11) may be expressed as

$$U_n(x, y = 0) = \partial W_{\mathrm{I}}(x, y = 0)/\partial n, \qquad U_n(x, y = 0) = \partial W_{\mathrm{II}}(x, y = 0)/\partial n. \tag{29, 30}$$

Substituting equations (15, 16) into equations (29, 30) gives

$$\sum_{i=1}^{N} C_i \sin i\pi x/L = \frac{\partial}{\partial y}\left(\sum_{m=1}^{N} A_{\mathrm{m}}^{(\mathrm{I})} \sin[k^{(\mathrm{I})}(px - qy + a_1)] \sin[m\pi(qx + py)/b]\right)_{y=0}, \tag{31}$$

$$\sum_{i=1}^{N} C_i \sin i\pi x/L = \frac{\partial}{\partial y}\left(\sum_{n=1}^{N} A_n^{(\mathrm{II})} \sin[k^{(\mathrm{II})}(a - px + qy - a_1)] \sin[n\pi(qx + py)/b]\right)_{y=0}. \tag{32}$$

In order to eliminate the geometric variable, the sth basis $\sin s\pi x/L$ is multiplied to both sides of equations (31, 32), and the integration procedure along the common boundary is carried out. Then, one obtains

$$C_s = L/2 \sum_{m=1}^{N} VM_{sm}^{(\mathrm{I})} A_m^{(\mathrm{I})}, \quad s = 1, 2, \dots, N, \tag{33}$$

$$C_s = L/2 \sum_{n=1}^{N} VM_{sn}^{(\mathrm{II})} A_n^{(\mathrm{II})}, \quad s = 1, 2, \dots, N, \tag{34}$$

which leads to the N linear equations

$$\sum_{m=1}^{N} VM_{sm}^{(\mathrm{I})} A_m^{(\mathrm{I})} = \sum_{n=1}^{N} VM_{sn}^{(\mathrm{II})} A_n^{(\mathrm{II})}, \quad s = 1, 2, \dots, N, \tag{35}$$

where $VM_{sm}^{(\mathrm{I})}$ and $VM_{sn}^{(\mathrm{II})}$ are given by

$$VM_{sm}^{(\mathrm{I})} = \int_0^L \{-qk^{(\mathrm{I})} \cos[k^{(\mathrm{I})}(px + a_1)] \sin[m\pi x/L] + (m\pi p/b) \sin[k^{(\mathrm{I})}(px + a_1)] \cos[m\pi x/L]\} \sin s\pi x \,\mathrm{d}x, \tag{36}$$

$$VM_{sn}^{(\mathrm{II})} = \int_0^L \{qk^{(\mathrm{II})} \cos[k^{(\mathrm{II})}(a - px - l)] \sin[n\pi x/L] + (n\pi p/b) \sin[k^{(\mathrm{II})}(a - px - a_1)] \cos[n\pi x/L]\} \sin s\pi x \,\mathrm{d}x. \tag{37}$$

Finally, one obtains the simple matrix form from equation (35) as follows.

$$\mathbf{VM}^{(\mathrm{I})}\mathbf{A}^{(\mathrm{I})} = \mathbf{VM}^{(\mathrm{II})}\mathbf{A}^{(\mathrm{II})}. \tag{38}$$

Matrix equations (27, 38) developed from the compatibility conditions may take the form of a single matrix equation:

$$\mathbf{SM}(f)\mathbf{A} = \mathbf{0}, \tag{39}$$

where the square system matrix $\mathbf{SM}(f)$ of the order $2N$ and the unknown coefficient vector $\mathbf{A}$ are, respectively, given by

$$\mathbf{SM}(f) = \begin{bmatrix} \mathbf{SM}^{(\mathrm{I})} & -\mathbf{SM}^{(\mathrm{II})} \\ \mathbf{VM}^{(\mathrm{I})} & -\mathbf{VM}^{(\mathrm{II})} \end{bmatrix}, \quad \mathbf{A} = \begin{Bmatrix} \mathbf{A}^{(\mathrm{I})} \\ \mathbf{A}^{(\mathrm{II})} \end{Bmatrix}. \tag{40, 41}$$

On the other hand, the natural frequencies of the composite membrane may be found from the non-trivial condition that the solution of equation (39) should be non-trivial: i.e., the roots of the determinant equation $\det[\mathbf{SM}(f)] = 0$ correspond to the natural frequencies. In addition, the ith mode shape for the ith natural frequency, f_i, may be plotted from equations (8, 9). For this purpose, the ith eigenvector, which corresponds to the unknown coefficient vector obtained from equation (39) when $f = f_i$, is substituted into equations (8, 9).

3. CASE STUDIES

The validity of the proposed method is verified through numerical tests of composite rectangular membranes, of which the dimensions are given by $a = 1{\cdot}8$, $b = 1{\cdot}0$, $a_1 = 1{\cdot}0$ and $a_2 = 0{\cdot}7$ m (see Figure 1). The values of surface density of the two homogeneous regions D_{I} and D_{II} are given by $\rho_{\mathrm{I}} = 1{\cdot}293 \times 10^{-5}$ kg/m^2 and $\rho_{\mathrm{II}} = 2\rho_{\mathrm{I}}$. Prior to the verification of the composite membrane of the above feature, a homogeneous membrane with the same dimensions as the composite membrane but with the homogeneous surface density $\rho_{\mathrm{I}} = \rho_{\mathrm{II}} = 1{\cdot}293 \times 10^{-5}$ kg/m^2 is first solved for a comparison of the result obtained from the proposed method with the exact solution.

3.1. HOMOGENEOUS RECTANGULAR MEMBRANE

A simple case where ρ_{II} is identical to ρ_{I} ($\rho = \rho_{\mathrm{I}} = \rho_{\mathrm{II}} = 1{\cdot}293 \times 10^{-5}$ kg/m^2) is first considered in the section. Since the exact natural frequencies of the homogeneous rectangular membrane are easily obtained in this case, the natural frequencies calculated by the proposed method can be compared with the exact natural frequencies. The accuracy of the proposed method may be verified from the comparison. On the other hand, since the current rectangular membrane has the uniform surface density, $\rho = 1{\cdot}293 \times 10^{-5}$ kg/m^2, the wavenumber throughout the entire region of the membrane is defined as $k = \sqrt{(\omega/c)^2 - (m\pi/b)^2}$ where $c = \sqrt{T/\rho}$ and $\omega = 2\pi f$.

Logarithmic values of $\det[\mathbf{SM}(f)]$ for $N = 2$, 3, 4, and 5 are plotted as a function of the frequency f in Figure 3 where the values of f corresponding to the troughs represent the singular values of the system matrix $\mathbf{SM}(f)$. The singular values are labelled as f_{c1}, f_{c2}, f_1, $f_2, \ldots, f_{10}$ in Figure 3. Of the singular values, $f_1, f_2, \ldots, f_{10}$ correspond to the natural

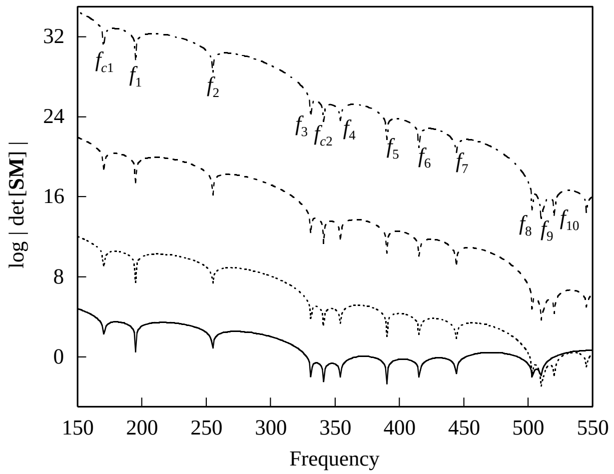


Figure 3. Determinant of the system matrix versus frequency for the homogeneous rectangular membrane when N is increased: ——, $N = 2$; - - - - -, $N = 3$; – – – –, $N = 4$; – · – · –, $N = 5$.

frequencies of the homogeneous membrane, and f_{c1} and f_{c2} represent the cut-off frequencies within the frequency range, 150–550 Hz. Note that the cut-off frequencies can also be calculated from $k = \sqrt{(\omega/c)^2 - (m\pi/b)^2} = 0$.

The reason why the two cut-off frequencies are included in the singular values of $\mathbf{SM}(f)$ in Figure 3 is that the transverse displacements W_{I} and W_{II} have trivial solutions when the two cut-off frequencies are substituted into equations (15, 16): i.e., $W_{\mathrm{I}} = W_{\mathrm{II}} = 0$ due to $k = k^{(\mathrm{I})} = k^{(\mathrm{II})} = 0$. As a result, the system matrix $\mathbf{SM}(f)$ becomes singular at the cut-off frequencies as well as at the natural frequencies. This fact may be considered to be a weak point of the proposed method, as additional work is needed so that troughs corresponding to the cut-off frequencies can be discriminated from troughs corresponding to the natural frequencies. Note, however, that the additional work can be successfully executed by confirming the cut-off frequencies using $k = \sqrt{(\omega/c)^2 - (m\pi/b)^2} = 0$.

Basically, the proposed method gives both correct natural frequencies and incorrect ones (the cut-off frequencies). The incorrect natural frequencies result from the fact that the system matrix becomes singular at particular frequencies, which satisfy the equation $k = \sqrt{(\omega/c)^2 - (m\pi/b)^2} = 0$ (the particular frequencies are called the cut-off frequencies). From this fact, troughs corresponding to the cut-off frequencies in Figure 3 can be discriminated from those corresponding to the natural frequencies of the membrane, by previously confirming the cut-off frequencies from $k = \sqrt{(\omega/c)^2 - (m\pi/b)^2} = 0$.

In Table 1, the natural frequencies obtained by the proposed method are compared with the exact frequencies and FEM results. It may be said that the natural frequencies by the proposed method converge rapidly and accurately to the exact frequencies in the case of only $N = 2$. Furthermore, the proposed method is more accurate than the FEM results using $N_{ele} = 800$ when the present results and the FEM results are compared with the exact natural frequencies.

3.2. NON-HOMOGENEOUS RECTANGULAR MEMBRANE

In this section, we consider the composite rectangular membrane of which the values of surface density of the two subdomains D_{I} and D_{II} are, respectively, given by

TABLE 1

Comparison of the natural frequencies of the homogeneous rectangular membrane obtained by the proposed method, the exact method, and FEM

Natural frequencies	Proposed method				Exact value (Mode shape)	FEM			
	$N = 2$	$N = 3$	$N = 4$	$N = 5$		$N_{ele} = 800$	$N_{ele} = 450$	$N_{ele} = 200$	$N_{ele} = 50$
f_1	195·01	195·01	195·01	195·01	195·01 (1,1)	195·18	195·32	195·70	197·76
f_2	254·83	254·83	254·83	254·83	254·83 (2,1)	255·12	255·35	256·01	259·55
f_3	331·34	331·34	331·34	331·34	331·34 (3,1)	332·07	332·64	334·28	343·15
f_4	353·85	353·85	353·85	353·85	353·85 (1,2)	355·22	356·29	359·35	375·92
f_5	390·02	390·02	390·02	390·02	390·02 (2,2)	391·42	392·51	395·63	412·48
f_6	415·41	415·41	415·41	415·41	415·41 (4,1)	417·03	418·29	421·90	441·29
f_7	443·81	443·81	443·81	443·81	443·81 (3,2)	445·47	446·77	450·46	470·30
f_8	503·28	503·28	503·28	503·28	503·28 (5,1)	506·46	508·95	516·08	546·55
f_9	509·66	509·66	509·66	509·66	509·66 (4,2)	512·02	513·86	519·11	554·74
f_{10}	None	520·11	520·11	520·11	520·11 (1,3)	524·80	528·46	538·95	592·56

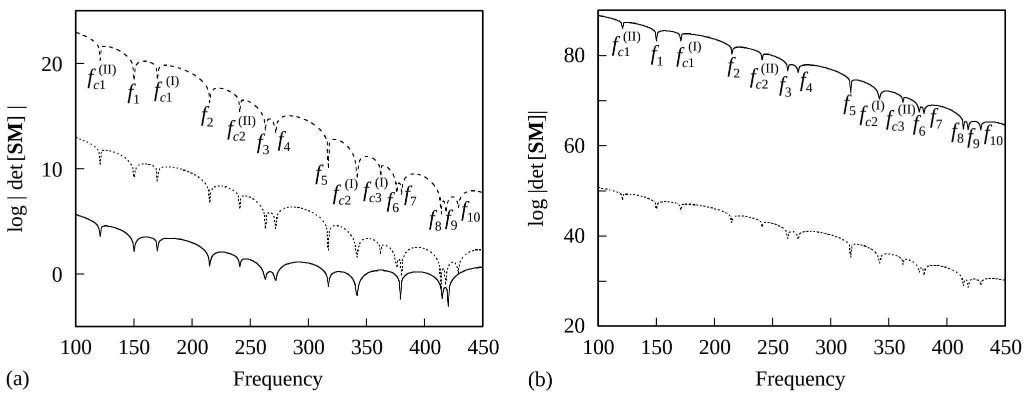


Figure 4. Determinant of the system matrix versus frequency for the composite rectangular membrane when N is increased. (a) $N = 2$, 3, and 4: ——, $N = 2$; - - - - -, $N = 3$; – – – –, $N = 4$. (b) $N = 6$ and 8: - - - - -, $N = 6$; ——, $N = 8$.

TABLE 2

Comparison of the natural frequencies of the composite rectangular membrane obtained by the proposed method and FEM

Natural frequencies	Proposed method					FEM			
	$N = 2$	$N = 3$	$N = 4$	$N = 6$	$N = 8$	$N_{ele} = 800$	$N_{ele} = 450$	$N_{ele} = 200$	$N_{ele} = 50$
f_1	150·2	150·2	150·2	150·2	150·2	150·3	150·4	150·8	152·5
f_2	215·2	215·2	215·2	215·2	215·2	215·5	215·8	216·6	220·4
f_3	262·6	262·9	262·9	262·9	262·9	263·9	264·6	266·8	278·6
f_4	271·6	271·6	271·6	271·6	271·6	272·3	272·8	274·2	282·0
f_5	317·2	317·0	317·0	317·0	317·0	318·3	319·3	322·2	337·9
f_6	342·2	342·1	342·1	342·1	342·1	343·8	345·0	348·7	367·4
f_7	None	375·5	375·7	375·7	375·7	377·9	379·5	383·8	404·0
f_8	379·0	380·0	380·1	380·2	380·2	383·0	385·4	392·4	429·8
f_9	414·9	414·1	414·1	414·1	414·1	416·9	419·1	425·2	454·5
f_{10}	420·0	418·2	418·1	418·1	418·1	421·0	423·0	428·7	458·8

$\rho_{\mathrm{I}} = 1{\cdot}293 \times 10^{-5}\,\mathrm{kg/m^2}$ and $\rho_{\mathrm{II}} = 2\rho_{\mathrm{I}}$. In the same manner as in the previous section, by plotting logarithmic values of $\det[\mathbf{SM}(f)]$with respect to f (See Figure 4), the natural frequencies of the composite membrane are obtained from the troughs as shown in Figure 4. Note that, among the troughs, some troughs are related to the cut-off frequencies and have been formed because $k^{(\mathrm{I})} = 0$ and $k^{(\mathrm{II})} = 0$. The troughs corresponding to the cut-off frequencies are represented by $f_{c1}^{(\mathrm{I})}$, $f_{c2}^{(\mathrm{I})}$, $f_{c1}^{(\mathrm{II})}$, $f_{c2}^{(\mathrm{II})}$ and $f_{c3}^{(\mathrm{II})}$, as shown in Figure 4.

Table 2 shows the natural frequencies obtained from the proposed method and FEM (SYSNOISE). As can be seen in Table 2, the proposed method yields more rapidly converging results in spite of much less computational effort than in the FEM results. Although this membrane has no exact solution, the accuracy and effectiveness of the proposed method may be verified from the comparison between the present results for $N = 8$ and the FEM results for $N_{ele} = 800$. As can be seen in Table 2, the first six natural frequencies obtained by the proposed method with only $N = 3$ have already converged to the corresponding steady values and have an extremely small amount of error compared with the FEM results for $N_{ele} = 800$.

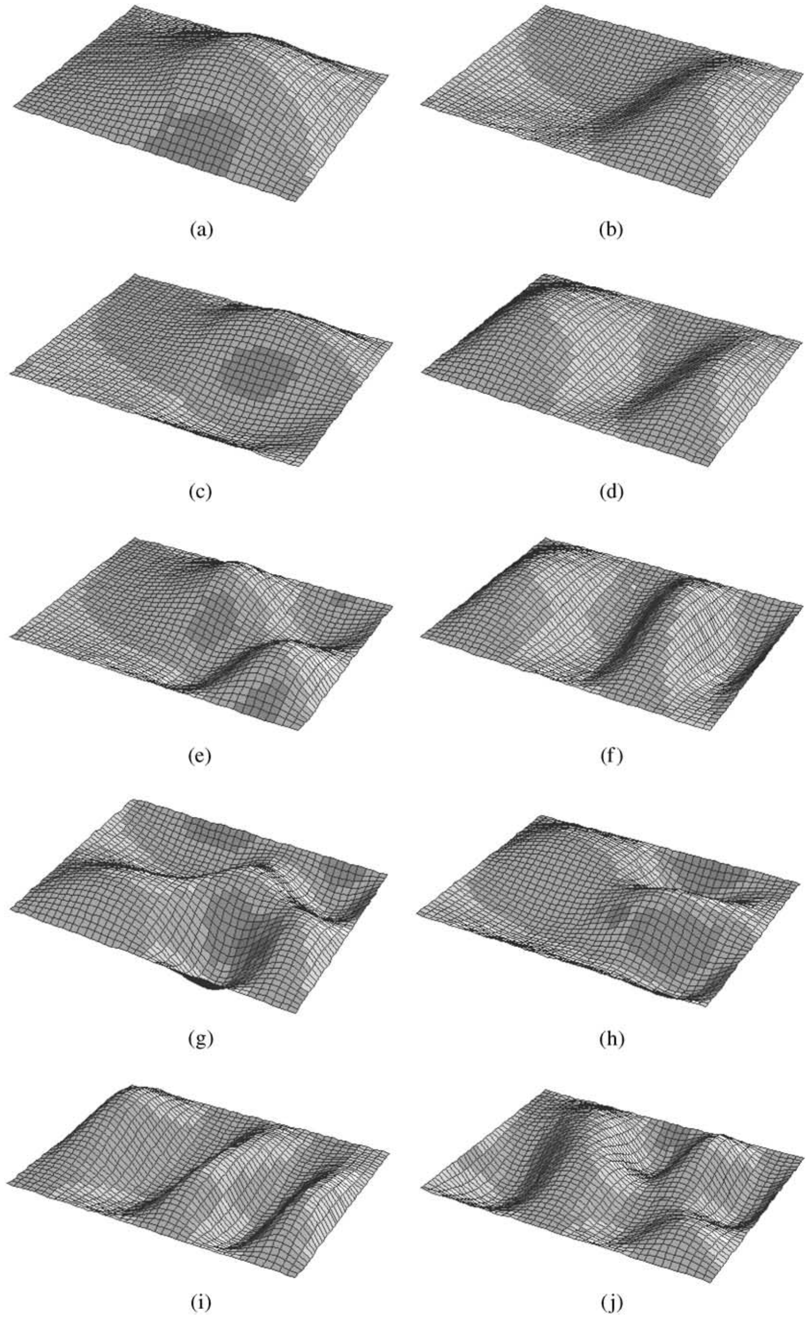


Figure 5. Mode shapes of the composite rectangular membrane obtained by the present method when $N = 3$: (a)–(j) correspond to 1st–10th modes respectively.

Interestingly, the seventh natural frequency cannot be found for $N = 2$. The reason may be due to the fact that the corresponding mode has many nodal points along the common boundary compared with the other modes (See Figure 5(g)). It may, therefore, be imagined that a larger value of N is required to describe displacement variations along the common boundary. The mode shapes of the composite membrane obtained by the proposed method

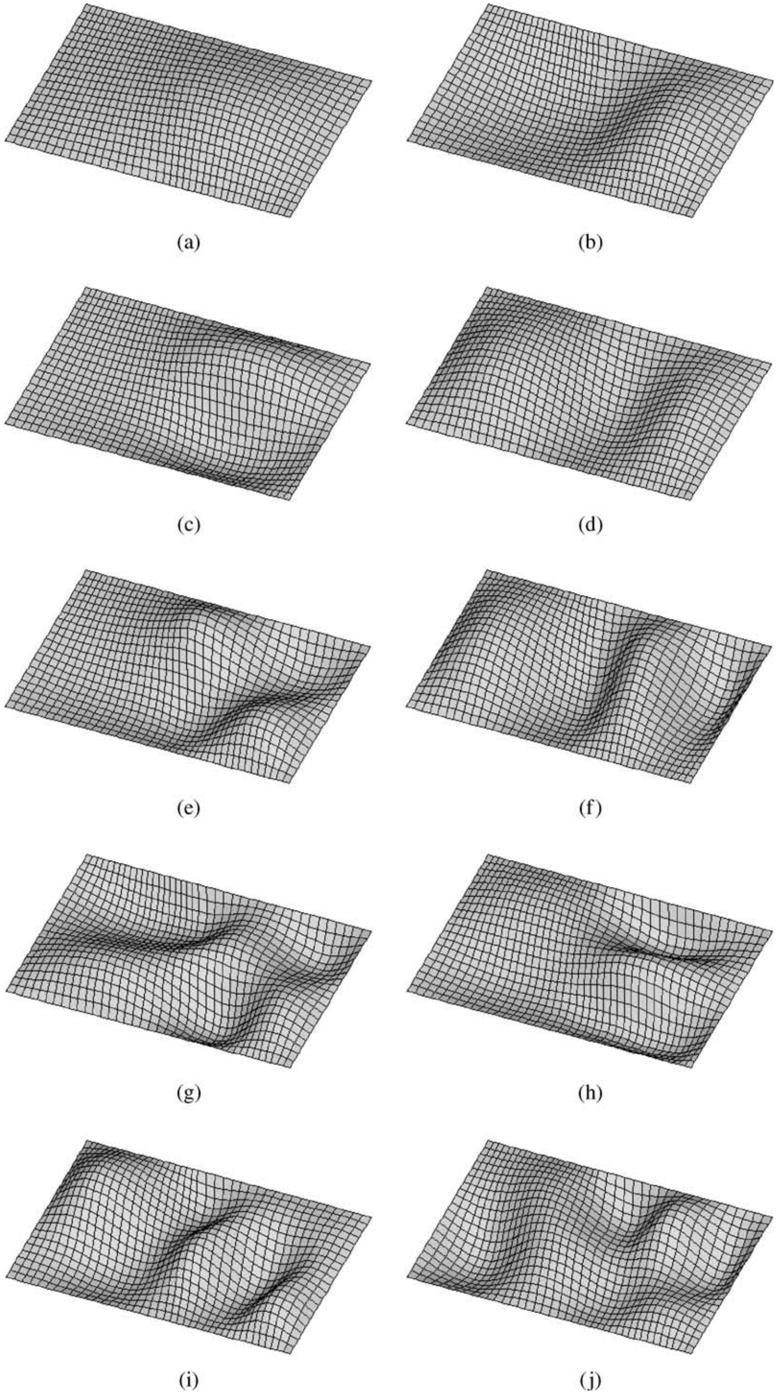


Figure 6. Mode shapes of the composite rectangular membrane obtained by FEM when $N_{ele} = 800$: (a)–(j) correspond to 1st–10th modes respectively.

for $N = 3$ are shown in Figure 5. It may be said that the mode shapes are very similar to those computed by the FEM, which are shown in Figure 6.

4. CONCLUSION

A simple and accurate method for determining the natural frequencies and mode shapes of composite rectangular membranes with the oblique interface has been described. The method yields very accurate and rapidly converging results even when a small number of sine series functions are used. Furthermore, even when a large number of sine series functions are used for predicting higher order modes, the method gives steady natural frequencies and mode shapes over all modes including lower order modes.

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