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VIBRATION CHARACTERISTICS OF SLOTTED SHAFTS

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The dynamics of asymmetric rotors has been investigated in the literature. However, considering the importance in the practical fields, the vibration characteristics of slotted shafts are not studied in detail. It needs more attention particularly with the emerging of new materials, such as composites for shafts. The present study aims at the analysis of the slotted shafts including the compensatory inertia slots. Also, the slotted composite shaft has been modelled based on first order shear deformation theory using finite element method with shell elements. Different materials such as, Boron epoxy, Carbon epoxy and Graphite epoxy have been tried for various stacking sequences. The slot parameters, stacking sequences and material properties have found to influence to great extent on the vibrational characteristics of rotors. The results are compared with those of isotropic slotted shaft.

1. INTRODUCTION

Rotors with longitudinal slots such as two-pole generators or electrical motors give dual flexural rigidity. The dynamics of asymmetric rotors has been the subject of interest for several decades and many papers were published on this, to name a few by Brosens and Crandall [1], Yamamoto and Ota [2, 3], and Ardayfio and Frohrib [4]. Yuan Kang *et al.* [5] utilizing finite element formulation generalized the previous works to accommodate the asymmetry effects.

However, in spite of their importance in practical fields, the vibration characteristics of slotted shafts have not been studied in detail. The dynamics of slotted shaft needs more attention particularly with the emergence of new materials such as composites. Sakata *et al.* [6], summarized second order (or twice per revolution) forced vibrations excited by the weight of the rotor. Unstable whirl in the region of first order critical vibrations was dealt with previously in references [7, 8]. Sakata *et al.* [6] discussed and analyzed the slots both experimentally and theoretically using both transfer matrix and three-dimensional finite element method. However, the model rotors treated in reference [6] are simplified ones in which the reduction of flexural rigidity due to the longitudinal slots for the electric windings has been represented by parallel flats on the model rotors. Goodwin [9] points out that it is important for the designer to be aware of the potential region of instability defined by the eigenfrequencies associated with the different lateral stiffnesses of the rotor in two orthogonal directions.

In the present work through FEM (NASTRAN), a parametric study has been carried out to find the effects of the slot dimensions (slot width and depth) on the changes of eigenfrequencies in orthogonal directions. An attempt to compensate through inertia slots to reduce the asymmetry of the rotor shaft has been made. In addition, the vibration characteristics of slotted composite shafts for different slot parameters with various stacking

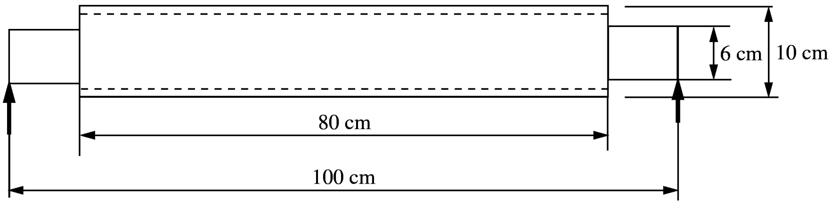

Figure 1. Model of the slotted shaft.

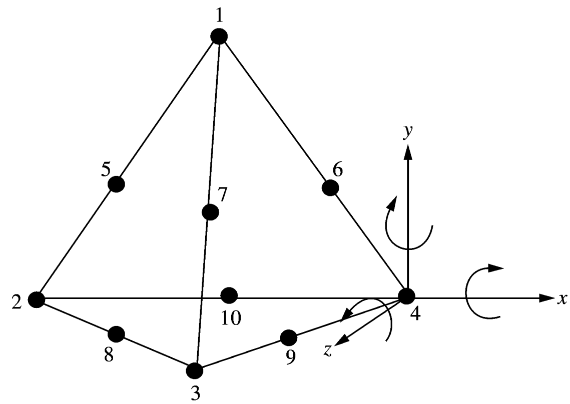

Figure 2. Tetrahedral element.

sequences and materials have also been considered in the present study. The results are compared with those of the isotropic slotted shaft.

2. MODELLING OF ISOTROPIC SLOTTED SHAFT

The present analysis of slotted shaft has been done using FEMAP 97 as pre- and post-processor and NASTRAN as the solver. The slotted steel shaft as shown in Figure 1 is analyzed using tetrahedral element, CTETRA (Figure 2) with 10 nodes. The PSOLID and MAT1 cards are used to input the element and the material properties respectively. In NASTRAN, eigenvalue analysis is performed using SOL3 and SOL103.

Figure 3(a–c) shows the solid model of an isotropic slotted shaft. The slots in the shaft are generated by using bullion operations. Six longitudinal slots are considered for this model with 30° angle between the two adjacent slots. Repeating bullion operations with circular disk generates inertia slots, which are also shown in Figure 3(a–c). Four numbers of inertia slots (compensatory) are considered in this model. As shown in Figure 4, the rotor has been discretized by using 1600 ten-noded tetrahedral elements. Simply supported boundary conditions are assumed for the shaft ends.

3. MODELLING OF COMPOSITE SLOTTED SHAFT

Composite materials have many advantages, which made them applicable to a wide variety of products and attempts have been made to replace isotropic shafts by composite

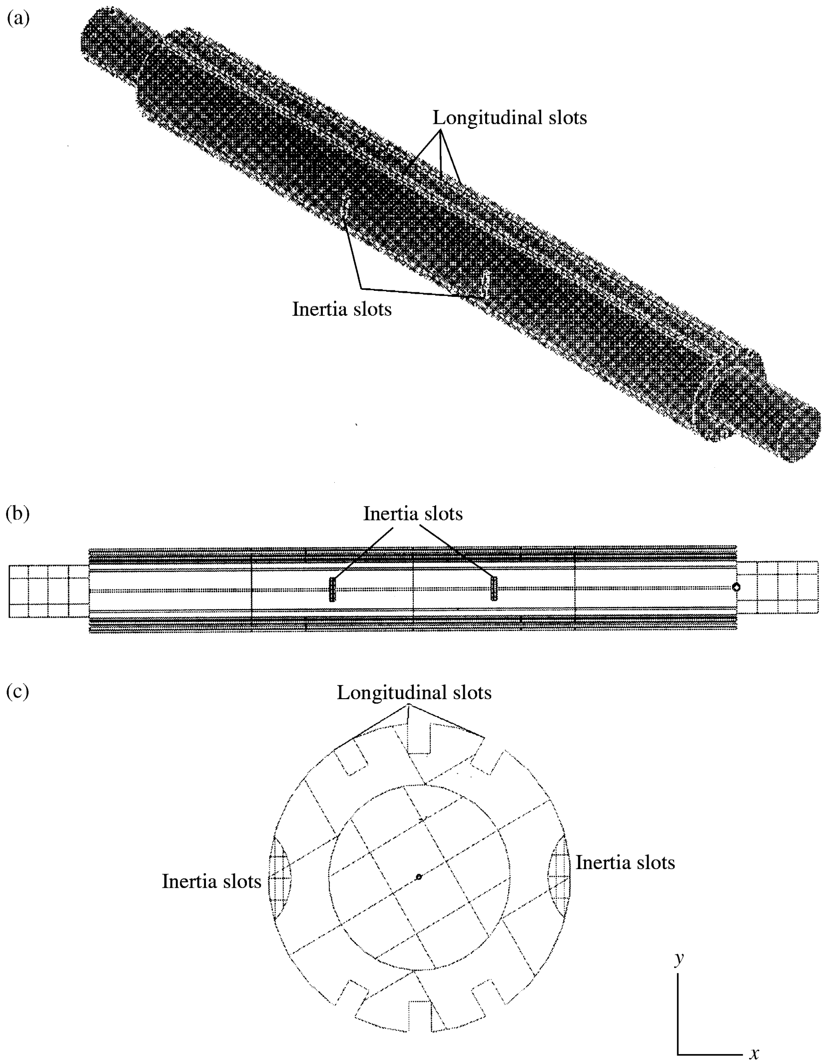

Figure 3. (a) Slotted isotropic shaft with inertia slots: (b) front view; (c) side view.

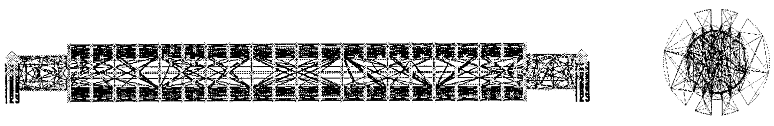

Figure 4. Slotted isotropic shaft model from NASTRAN.

shafts. There have been very few works reported on composite shafts even though, Zinberg and Symmonds [10] studied these way back. Some more works include those of Henrique dos *et al.* [11], Singh and Gupta [12, 13], Wettergren [14] and Lim and Darlow [15]. Very recently, Sekhar and Ravi Kumar [16] based on the first order shear deformation theory

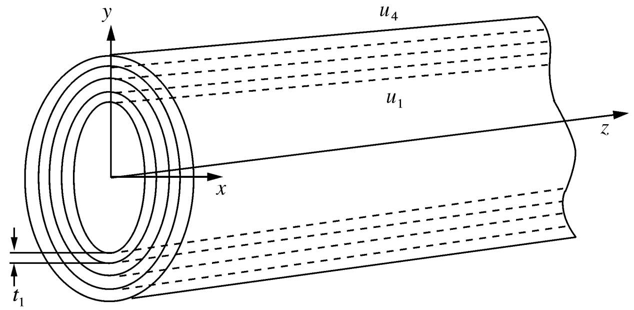

Figure 5. Composite shaft model.

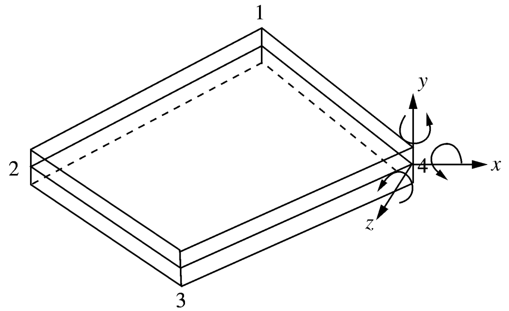

Figure 6. Four-noded shell element.

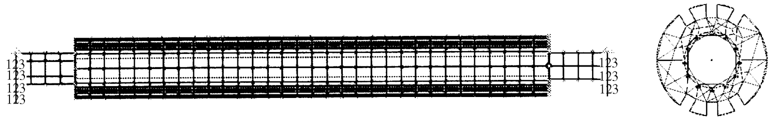

Figure 7. Slotted composite shaft model.

using finite element method analyzed the dynamic aspects of composite shafts for different materials and fibre orientations. Using the same theory, analysis has been carried out for the slotted composite shafts in the present study.

Figure 5 shows a composite shaft created using a four-noded shell element (CQUAD4) which is shown in Figure 6. The CQUAD4 element is a four-noded, bilinear, isoparametric element capable of representing membrane, bending (with transverse shear effects) and membrane-bending coupling behaviour. The property of the laminae is supplied using PCOMP property cards. A two-dimensional orthotropic material is used for the analysis using MAT8 card.

Figure 7 shows the discretized model of the slotted composite shaft with 1100 elements. Four layers of equal thickness (0·004 m) with different stacking sequence are considered for this problem. Dimensions of the shaft are the same as shown in Figure 1. Simply supported boundary conditions are considered for the shaft ends. The eigenvalue analysis is carried out in NASTRAN in a similar way as was done for the isotropic shaft.

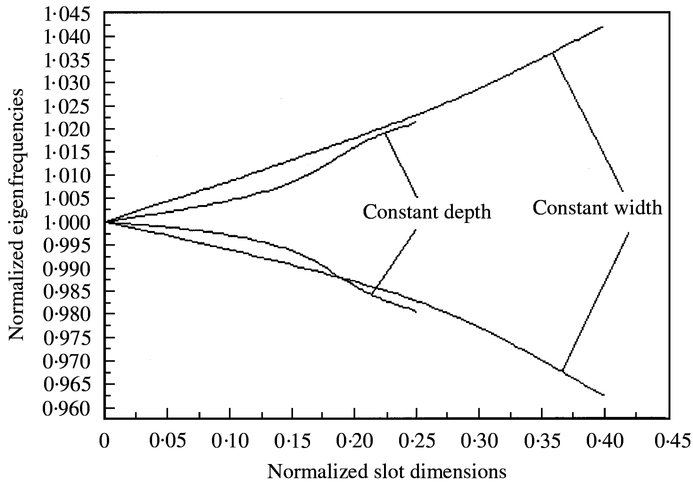

Figure 8. Effect of slot dimensions on eigenfrequencies for isotropic shaft.

TABLE 1

Effect of longitudinal slots and inertia slots on eigenfrequencies of slotted isotropic shaft

Mode no.	Eigenfreq. without slots	Eigenfreq. at slot dimensions $w = 0{\cdot}0075$ m, $D = 0{\cdot}02$ m	Eigenfreq. with inertia slots (Hz)
I	274·3	272·6	274·2
	274·5	281·4	275·4
II	874·1	859·9	821·6
	874·7	893·1	884·1
III	1688·6	1672·3	1608·7
	1689·4	1716·2	1698·7

4. RESULTS AND DISCUSSION

4.1. ISOTROPIC SLOTTED SHAFT

The dimensions of slot width and depth are varied and each time, the eigenvalue analysis is carried out. The results of which are shown in Figure 8 where the normalization of each eigenfrequency of the slotted shaft associated with dual flexural rigidity has been done with that of unslotted shaft, and the normalization of slot dimensions is done with the outer radius of the shaft.

4.1.1. *Effect of slots on eigenfrequency*

As expected from Table 1, it can be noted that when slots are introduced rotor has different eigenfrequencies in both x and y directions. As discussed previously [9], this asymmetry sometimes leads to unstable whirl in the regime between the split eigenfrequencies. By split eigenfrequencies is meant, throughout this paper, the eigenfrequencies in flexure associated with the direction of maximum and minimum

TABLE 2

Comparison of NASTRAN's solution and previous results

Method	I Eigenfreq. (Hz)	Reference
Equivalent modulus beam theory (EMBT)	96·33	Zinberg and Symmonds [10]
FEM with beam elements from Donnell's shell theory	82·50	Henrique dos Reis *et al.* [11]
Layerwise beam theory (LBT)	93·67	Singh and Gupta [12]
EMBT	95·77	Singh and Gupta [12]
Present FEM solution	98·56	

TABLE 3

Properties of different composite materials

Material	E_1 (GPa)	E_2 (GPa)	ν_{12}	$G_{12} = G_{13}$ (GPa)	G_{23} (GPa)	ρ (kg/m^3)
Boron/epoxy	211	24·1	0·36	6·9	6·9	1967
Graphite/epoxy	139	11	0·313	6·05	3·78	1578
Carbon/epoxy	130	10	0·25	7·0	7·0	1500

rigidities. It can be seen from Figure 8 that the influence of slot depth to increase the regime between the split eigenfrequencies is more compared to that of the width.

4.1.2. *Inertia slots*

Four inertia slots are created on pole face as explained in section 2 (see Figure 3), there by reducing the bending stiffness in the x–x plane to a value closer to that for the y–y plane. The results with the inertia slots are shown in Table 1. From Table 1 it is clear that due to these slots the rotor is made almost symmetric.

4.2. COMPOSITE SLOTTED SHAFT

For the purpose of comparing the results with the previous ones, a composite hollow shaft of the following data [12] was considered: length of the shaft = 2·47 m, radius of the rotor = 0·0635 m, thickness = 0·132E-3 m, density = 1965 kg/m^3, $E1$ = 211 GPa, $E2$ = 24·1 GPa, G_{12} = 6·9 GPa, ν = 0·36, bearing stiffness = 10E5 N/m, 10 layers of equal thickness with stacking sequence of [90, 45, − 45, $[0]_6$, 90°].

The results of the eigenvalue analyses as shown in Table 2 match reasonably well with the previous models. As mentioned in Table 2 of reference [12], the results of EMBT and LBT indicate that the difference is not large. The present FEM results are closer to that given in reference [12]. Further, the discussion on the variations in the values of the eigenfrequencies are discussed in detail in reference [12].

In the present study, four layers of equal thickness (0·004 m) with different stacking sequences are considered for the analysis of slotted composite shaft. The other dimensions of the shaft are the same as shown in Figure 1. The properties of the different materials are

TABLE 4

Effect of material and stacking sequence on eigenfrequencies of slotted composite shaft: width = 0·0075 *m*; *Depth* = 0·01 *m*

Material	Stacking sequence	Eigenfrequency (Hz)
Isotropic		273·3
		276·1
Carbon/epoxy	0/0/90/90°	316·3
		352·4
	90/90/0/0°	334·2
		358·3
	0/90/90/0°	332·9
		364·2
	90/0/90/0°	339·7
		367·5
Boron/epoxy	0/0/90/90°	316·6
		350·2
	90/90/0/0°	335·3
		358·9
	0/90/90/0°	329·5
		358·8
	0/90/90/0°	337·2
		363·4
Graphite/epoxy	0/0/90/90°	303·8
		337·6
	90/90/0/0°	320·8
		343·7
	0/90/90/0°	319·07
		348·3
	90/0/90/0°	325·8
		351·7

shown in Table 3. A parametric study has been carried out to determine the influence of stacking sequence and material on the eigenvalues of the composite shaft.

4.2.1. *Comparison between isotropic and composite shafts*

Table 4 reveals that the composite shafts have higher eigenfrequencies as compared to that of the isotropic shaft of the same dimensions. However, the considered composite shaft is a hollow one. From Figures 9 and 10 it is observed that the influence of longitudinal slots to split eigenfrequency in the case of the composite shaft is more as compared to that of the isotropic shaft. Thus, possible asymmetry due to longitudinal slots in the case of composite shafts is slightly higher as compared to that in the isotropic shafts.

4.2.2. *Effect of material and stacking sequence on eigenfrequency*

The effects of material and stacking sequence on the eigenfrequency are shown in Table 4. As can be seen from the table, the split eigenfrequencies of the shaft made of carbon/epoxy are slightly higher as compared to shafts made of boron/epoxy and graphite/epoxy. It can also be seen that the eigenfrequencies depend significantly on the

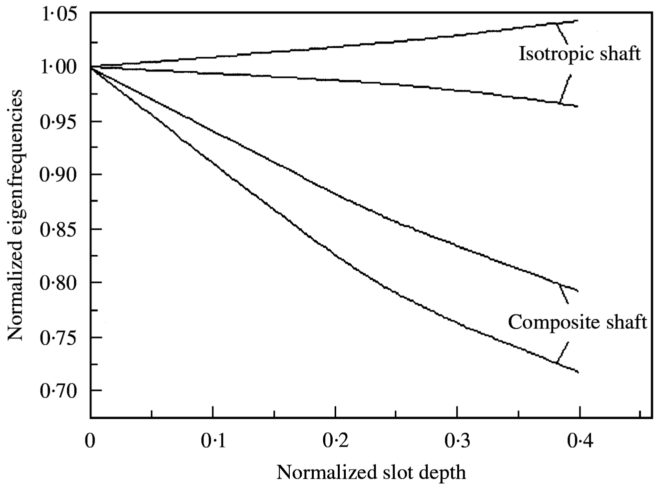

Figure 9. Effect of slot depth on eigenfrequencies for slotted carbon epoxy and isotropic shafts.

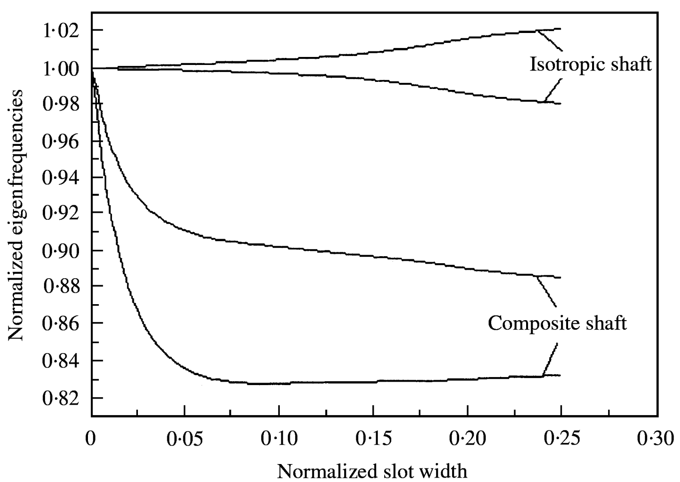

Figure 10. Effect of slot width on eigenfrequencies for slotted carbon epoxy and isotropic shafts.

orientation of the fibres. The stacking sequences play a predominant role on the dynamics of composite shafts.

4.2.3. *Effect of slot dimensions on eigenfrequencies*

Eigenfrequencies have been found by varying one of the slot dimensions (width and depth), while keeping the other constant. It can be observed from Figure 11 that the eigenfrequencies decrease rapidly and the split zone increases with slot depth for constant width. Whereas, in the case of constant slot depth, the eigenfrequency splits largely in the beginning, but split regime remains almost constant with increase in the width of the slot. Thus, it can be seen that the effect of slot depth on eigenfrequencies is higher as compared to slot width.

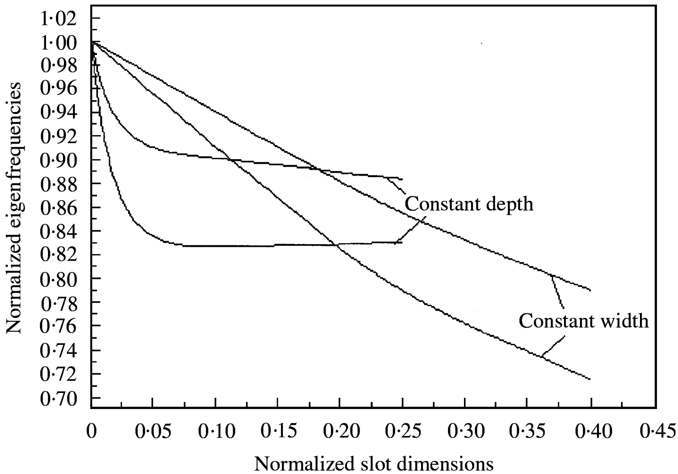

Figure 11. Effect of slot dimensions on eigenfrequencies for carbon epoxy at the stacking sequence 90/0/90/0.

5. CONCLUSIONS

From the results it can be seen that the difference between the eigenfrequencies of a shaft in two orthogonal directions increases due to the presence of slots. This difference between the eigenfrequencies is greatly influenced by the depth of the longitudinal slot as compared to its width. Hence, if more winding space is required, it is better to increase the width of the slot, so that the asymmetry of the shaft will not be much. Also, it can be seen that by introducing compensating slots the asymmetry of the shaft can be reduced.

The stacking sequence has a significant influence on the eigenfrequencies of composite shaft. It has been observed that if the fibre orientations are layered properly, then the rotor can be made stiffer. In the case of slotted composite shafts also similar vibration behaviour to isotropic shafts is observed. However, the influence of longitudinal slots to split eigenfrequency in the case of the composite shaft is more as compared to that of the isotropic shaft. Thus, possible asymmetry due to longitudinal slots in the case of composite shafts is slightly higher as compared to that in the case of isotropic shafts.

REFERENCES

1. P. J. Brosens and S. H. Crandall 1961 *American Society of Mechanical Engineers Journal of Applied Mechanics* **28**, 355–362. Whirling of unsymmetrical rotor.
2. T. Yamamoto and H. Ota 1964 *American Society of Mechanical Engineers Journal of Applied Mechanics* **31**, 515–522. On unstable vibration of a shaft carrying an unsymmetrical rotor.
3. T. Yamamoto, H. Ota and K. Kono 1968 *American Society of Mechanical Engineers Journal of Applied Mechanics* **35**, 313–321. On unstable vibration of a shaft with unsymmetrical stiffness carrying an unsymmetrical rotor.
4. D. Ardayfio and D. A. Frohrib 1976 *American Society of Mechanical Engineers Journal of engineering for Industry* **98**, 1161–1165. Instabilities of an asymmetric rotor with asymmetric shaft mounted on symmetric elastic supports.
5. Yuan Kang, Yuan-Pin Shih and An-Chen Lee 1992 *American Society of Mechanical Engineers Journal of Vibrations and Acoustics* **114**, 194–208. Investigation on the steady state responses of asymmetric rotors.
6. M. Sakata, M. Endo and K. Kishimoto 1983 *Journal of Sound and vibration* **87**, 61–70. Secondary critical speed flexible rotors with inertia slots.

7. R. E. D. BISHOP and A. G. PARKINSON 1965 *Philosophical Transactions A* **1095**, 1–31. Second order vibrations of flexible shafts.
8. A. TONDL 1965. *Some Problems in Rotor Dynamics*. London: Chapman and Hall.
9. M. J. GOODWIN 1989. *Dynamics of Rotor-Bearing systems*. London: Unwin-Hyman.
10. H. ZINBERG and M. F. SYMMONDS 1970 *Presented at the 26th Annual National Forum of American Helicopter Society*. The development of an advanced composite tail rotor drive shaft.
11. L. M. HENRIQUE DOS REIS, R. B. GOLDMAN and P. H. VERSTRATE 1987 *Journal of Composite Technology and Research* **9**, 58–62. Thin walled laminate composite cylindrical tubes — part 3, critical speed analysis.
12. S. P. SINGH and K. GUPTA 1996 *Journal of Sound and Vibration*. **191**, 739–756. Composite shaft rotor dynamic analysis using layerwise theory.
13. K. GUPTA and S. P. SINGH 1998 *Journal of Sound and Vibration* **211**, 513–320. Damping measurements in fibre reinforced composite rotors.
14. H. L. WETTERGREN 1997 *Journal of Sound and Vibration* **206**, 99–116. The influence of imperfections on the eigenfrequencies of a rotating composite shaft.
15. J. W. LIM and M. S. DARLOW 1986 *Journal of the American Helicopter Society* **31**, 75–83. Optimal sizing of composite power transmission shafting.
16. A. S. SEKHAR and N. RAVI KUMAR 1999 *Proceedings of* 17*th Biennial ASME Vibrations Conference*, *Las Vegas*, *Nevada*, *September* 1999. Paper No. DETC99/VIB-8277. Vibrational characteristics of composite shafts.