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NON-LINEAR DYNAMIC MODEL OF AN INEXTENSIBLE ROTATING FLEXIBLE ARM SUPPORTED ON A FLEXIBLE BASE

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A mathematical model for a flexible arm undergoing large planar flexural deformations, continuously rotating under the effect of a hub torque and supported by a flexible base is developed. The position of a typical material point along the span of the arm is described using the inertial reference frame via a transformation matrix from the body co-ordinate system, which is attached to the flexible root of the rotating arm. The condition of inextensibility is employed to relate the axial and transverse deflections of the material point, within the beam body co-ordinate system. The position and velocity vectors obtained, after imposing the inextensibility conditions, are used in the kinetic energy expression while the exact curvature is used in the potential energy. Lagrangian dynamics in conjunction with the assumed modes method is utilized to derive, directly, the non-linear equivalent temporal equations of motion. The resulting non-linear model, which is composed of four coupled non-linear ordinary differential equations, is discussed, simulated and the results of this simulation are presented. The effects of the base flexibility are explored by comparing the resulting simulation results, for various flexibility coefficients, with previously published works of the authors. Moreover, the numerical results show that the base flexibility has a very important effect on the stability of rotating flexible arms that should be accounted for when simulating such systems.

1. INTRODUCTION

The use of lightweight structural elements in space applications as well as in robotic manipulators under the requirement of precise positioning has increased the interest in having a precise model that closely represents such mechanical systems and accounts for all their conditions. Based on the fact that such structures have inherent lateral flexibility as well as axial rigidity, the problem of the axial displacement due to bending deformations has been identified as a major contributor to what is known as geometric stiffening. Geometric stiffening, due to axial shortening, was shown to have a significant effect on the stability of such rotating flexible structures and on their positioning control. It became evident that a model of a rotating long-slender flexible arm that accounts for the effect of axial displacement, due to bending deformations, based on physically justified geometrical considerations which also considers the inherent flexibility of the supporting base is highly demanded.

The effect of rotation on the natural frequencies and mode shapes of a rotating beam was reported earlier by Shilhansi [1] and Prudli [2]. These studies have shown that the rotation speed strengthens the beam and produces high natural frequencies. Likins [3] reported a study on the mathematical modelling of spinning elastic bodies. In the same direction, Kaza and Kavternik [4] reported the results of a study on the non-linear flap–lag–axial equations of a rotating beam. They addressed the problem of axial rigidity and the shortening due to transverse deflection. Kaza and Kavternik [4] summarized the four methods accounting for the beam axial rigidity. Stephens and Wang [5] studied the effect of uniform high-speed rotation on the stretching and bending of a rotating beam. They accounted for the beam rotation dynamics in terms of a tensile force that produced axial stress on the beam. In the aforementioned studies, the effect of rotation was taken as a kinematic variable in the form of angular velocity and angular acceleration to be given to the elastic equations which in turn are solved for the natural frequencies and mode shapes. Kane *et al.* [6] studied the dynamic behaviour of a cantilever beam that is attached to rigid base and performing specified motion of rotation and translation. In their work, the elastic degrees of freedom included beam axial extension, bending in two planes, torsion, shear displacement and wrapping. The model is a general three-dimensional elastic beam model. However, the two-way coupling between the rigid body motion and elastic deflections was not accounted for because only specified rigid body motions were considered.

The multibody dynamic approach, in which the rigid motion and flexible deformation are modelled in their coupled form, has attracted many researchers. Baruh and Tadikonda [7] reported some issues in the dynamics and control of flexible robot manipulators. They addressed the problem of axial shortening due to bending deformations by considering the shortening in their kinetic energy expression. Their results have shown that the flexibility of the rotating arm has changed the desired final rigid body position. Tadikonda and Chang [8] reported the effect of end load, due to chain connections on the geometric stiffening. Yigit *et al.* [9] studied the dynamics of a radially rotating beam with impact. They modelled the rigid body motion and the beam elastic co-ordinates using a partial differential equation and Galerkin's method of approximation. The effect of beam axial shortening due to bending deformation and the resulting beam stiffening was considered in their equations. However, the model showed linear inertial coupling between the beam rigid body rotation and its elastic deflections and the effect of shortening appeared as a function of square of beam rigid body rotating speed in the stiffness term. Pan *et al.* [10] reported a dynamic model and simulation results of a flexible robot arm with prismatic joint. They accounted for the effect of axial shortening using a virtual work term added to the elastic potential energy. El-Absy and Shabana [11] studied the geometric stiffness for a rotating beam using different approaches. They introduced the effect of longitudinal deformation due to bending, in the equations of motion, using the principle of virtual work. Al-Bedoor [12] studied the effects of shaft torsional flexibility on the dynamics of rotating blades. The effect of axial shortening was accounted for by using the virtual work in the form of added potential energy due to the centrifugal forces. Numerical simulations have shown that the flexibility and the stiffening effect contribute to the rigid body inertia by quadratic terms. The effect of inextensibility condition on the dynamics of rotating flexible arm and its associated dynamic non-linearity was recently reported by Al-Bedoor and Hamdan [13]. Their numerical results showed that the non-linear terms can improve the stability of the system's dynamic behaviour.

To this end, one can conclude that the available dynamic models of rotating beams that included the effect of axial shortening and its resulting geometrical stiffening did not consider the more general case where the base is flexible. Such a general case produces a higher order model with more dynamic interactions and non-linearities, thus closely simulating the actual dynamic behaviour of the system.

This paper presents a mathematical model and simulation results for a rotating flexible slender arm supported by a laterally flexible base. The multibody dynamic approach is

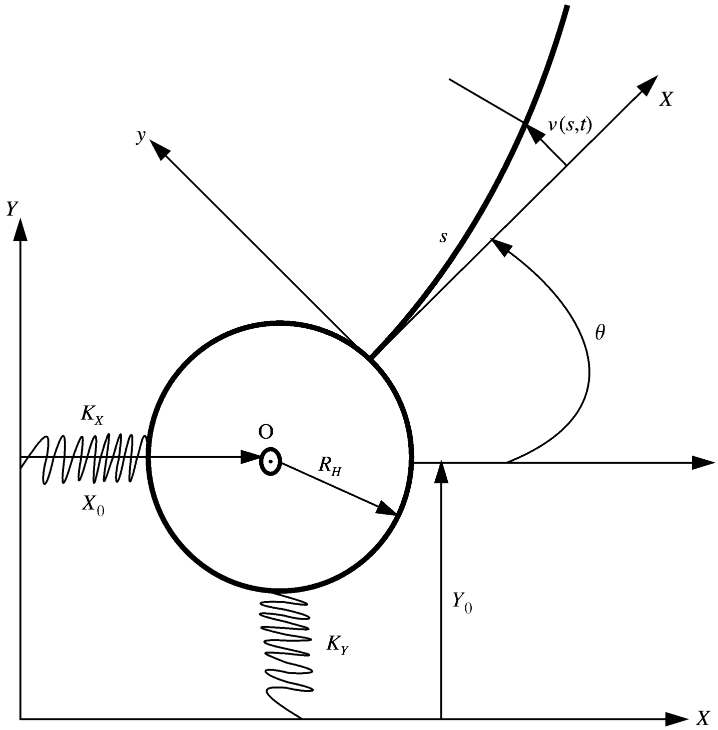

Figure 1. Schematic diagram of the arm–hub–flexible base system.

followed in developing the model by attaching body co-ordinate system to the hub at the root of the arm. The position vector of a typical material point is used in deriving the kinetic energy expression which includes the base 2-d.o.f. motion, the rigid body rotation of the hub–arm system, the arm transverse deflection and its associated axial shortening, that are included in the system kinetic energy. The geometrically exact curvature is employed in expressing the arm elastic potential energy. The system Lagrangian in conjunction with the assumed modes method, and after imposing the beam inextensibility constraint, is used to develop a 4-d.o.f. system. The 4-d.o.f. being the horizontal and vertical position of the hub–arm assembly, the rigid body rotation of the hub, and the transverse deflection of the beam in the modal domain.

2. THE ELASTODYNAMIC MODEL

2.1. SYSTEM DESCRIPTION AND ASSUMPTIONS

Figure 1 shows a schematic of the arm–hub system under consideration. XYZ denotes the inertial reference frame that is fixed in space, while the xyz is a system of orthogonal axes rotating with the hub with its origin fixed to the root of the beam and the x-axis is oriented along the neutral axis of the beam in the undeflected configuration. The hub is assumed to be flexible with linear stiffnesses K_X and K_Y. It has a radius R_H and is rotating about an axis passing through its moving centre O. The beam is assumed to be initially straight, cantilevered at the base having uniform cross-section A, flexural rigidity EI, constant length l and mass per unit length ρ. The thickness of the beam is assumed to be small compared to the beam length so that the effects of shear deformations and rotary inertia can be neglected. The beam motion is assumed to be confined to the x–y plane (i.e., only in-plane flexural motion is allowed). Furthermore, it is assumed that the first mode tip peak amplitude in this planar flexural vibrations may reach relatively large values (can be of the order of the beam length), but the slope of the elastica may not have tangents perpendicular to the neutral axis. The effect of shortening due to beam transverse deformation, determined using the inextensibility condition [13–15], and its time derivative is used to eliminate the dependence of the beam Lagrangian on the axial displacement and the axial velocity. In the following sub-sections, the governing temporal equations of motion are formulated via a combined Lagrangian-assumed mode method.

2.2. THE KINETIC ENERGY EXPRESSION

To develop the kinetic energy expression for the rotating arm–hub system, the deformed configuration of the arm, shown in Figure 1, is used. The global position vector of a material point P, located on the arm, can be written as

$$\mathbf{R}_P = \mathbf{R}_0 + \mathbf{R}_H + [\mathbf{A}(\theta)]\mathbf{r}_P, \tag{1}$$

where $\mathbf{r}_P$ is the position vector of point P in the hub co-ordinate system xy, $[\mathbf{A}(\theta)]$ is the rotational transformation matrix from the hub co-ordinate system to the inertial reference frame, XY, $\mathbf{R}_H$ is the position vector of the origin of the hub co-ordinate system xy in the inertial reference frame. $\mathbf{R}_0$ is the position vector of the hub centre in the inertial reference frame. $\mathbf{R}_H$ and $\mathbf{R}_0$ can be represented, respectively, as follows:

$$\begin{aligned} \mathbf{R}_H &= R_H \cos\theta \mathbf{I} + R_H \sin\theta \mathbf{J}, \\ \mathbf{R}_0 &= X_0 \mathbf{I} + Y_0 \mathbf{J} \end{aligned} \tag{2}$$

and the position vector $\mathbf{r}_P$ of the material point P in the xy co-ordinate system can be written in the form

$$\mathbf{r}_P = (s - u(s,t))\mathbf{i} + v(s,t)\mathbf{j}, \tag{3}$$

where s is the undeflected position, $u(s, t)$ is the axial shortening due to bending deformation and $v(s, t)$ is transverse deflection of the material point P measured with respect to the hub co-ordinate system, xy, which has the unit vectors $\mathbf{i}$ and $\mathbf{j}$. The rotational transformation matrix $[\mathbf{A}(\theta)]$ can be represented as

$$[\mathbf{A}(\theta)] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{4}$$

where θ represents rigid body rotation of the arm.

The velocity vector of the material point P in the inertial reference frame can be obtained by differentiating equation (1) as follows:

$$\dot{\mathbf{R}}_P = \dot{\mathbf{R}}_0 + \dot{\mathbf{R}}_H + [\mathbf{A}(\theta)]\dot{\mathbf{r}}_p + \dot{\theta}[\mathbf{A}_\theta(\theta)]\mathbf{r}_P, \tag{5}$$

where $[\mathbf{A}_\theta]$ is the derivative $[\mathrm{d}\mathbf{A}/\mathrm{d}\theta]$.

Upon substituting for $\mathbf{R}_H$, $\mathbf{R}_0$ $[\mathbf{A}_\theta]$, $\mathbf{r}_P$ and $\dot{\mathbf{r}}_P$ into equation (5), the velocity vector of the material point P in the inertial reference frame can be obtained.

The kinetic energy of the beam can be found from

$$U_B = \frac{1}{2}\int_0^l \rho \dot{\mathbf{R}}_P^T \dot{\mathbf{R}}_P \cdot \mathrm{d}s, \tag{6}$$

where ρ is the beam mass per unit length and l is the beam length.

The kinetic energy of the hub which is assumed to be a uniform disk with radius R_H and mass m_H rotating at angular velocity $\dot{\theta}$ and translating against two springs in the vertical and horizontal directions, can be represented as follows:

$$U_H = \tfrac{1}{2} m_H \dot{X}_0^2 + \tfrac{1}{2} m_H \dot{Y}^2 + \tfrac{1}{4} m_H R_H^2 \dot{\theta}^2. \tag{7}$$

Now, the total kinetic energy expression of the system can be written as follows:

$$U = U_H + U_B. \tag{8}$$

2.3. POTENTIAL ENERGY EXPRESSION

The system potential energy is constituted of the beam elastic strain energy and the base elastic potential energy. The arm is assumed to be rotating in the horizontal plane that results in no gravitational potential energy. The elastic beam strain energy with flexural rigidity $EI(x)$ is given by

$$V_B = \frac{1}{2}\int_0^l EI(s)\mathbf{K}^2\,\mathrm{d}s, \tag{9}$$

where $\mathbf{K}$ is the curvature of the beam centre-line at point s, which will be evaluated in the following sub-sections based on the inextensibility condition.

2.4. THE INEXTENSIBILITY CONDITION

For the present two-dimensional beam problem, Figure 2, the inextensibility condition dictates that total axial shortening $u(s, t)$ is given by [15]

$$\lambda u(\xi, t) = \xi - \int_0^\xi \cos\phi(\eta, t)\,\mathrm{d}\eta, \tag{10}$$

where $\xi = s/l$ and $\lambda = 1/l$. Noting that $\cos\phi = \sqrt{1 - \sin^2\phi}$, $\sin\phi = \mathrm{d}v/\mathrm{d}s$ and expanding the term $[1 - (\lambda v'^2)]^{1/2}$ in the power series, assuming that $(\lambda v')^2 \ll 1$, and retaining the terms up to the fourth order, the axial position of material point can be represented as follows:

$$u = \frac{1}{2}\int_0^\xi \left[\lambda v'^2 + \frac{1}{4}\lambda^3 v'^4\right]\mathrm{d}\eta, \tag{11}$$

where the prime is the derivative with respect to the dimensionless parameter, ξ.

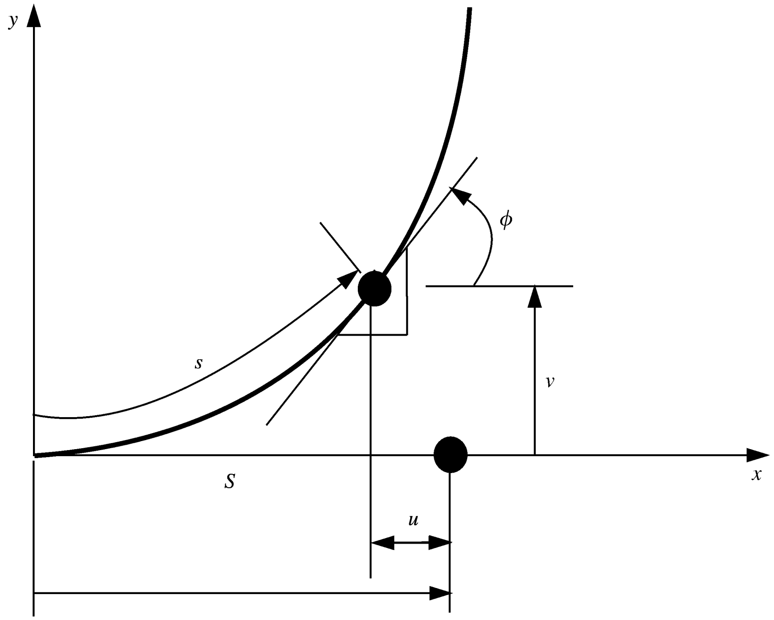

Figure 2. Deflected configuration of the beam segment.

Differentiating equation (11) with respect to time yields

$$\dot{u} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_0^{\xi}\left[\lambda v'^2 + \frac{1}{4}\lambda^3 v'^4\right]\mathrm{d}\eta\right]. \tag{12}$$

In order to express the exact curvature in terms of the transverse deflection v only, the analysis presented in reference [15] is employed. Accordingly, one notes that the curvature is

$$K = \phi', \tag{13}$$

where

$$\sin\phi = \lambda v'. \tag{14}$$

Differentiating equation (14) and noting, as before, that $\cos\phi = \sqrt{1-\sin^2\phi}$, $\sin\phi = \mathrm{d}v/\mathrm{d}s$ and expanding the term $[1-(\lambda v'^2)]^{1/2}$ in the power series, assuming that $(\lambda v')^2 \ll 1$, and retaining the terms up to the fourth order leads to

$$K^2 = \lambda^4 v''^2 + \lambda^6 v'^2 v''^2. \tag{15}$$

Upon substituting equation (11) for the axial position and its time derivative, equation (12) and the curvature equation (15) into the kinetic and potential and energy expressions, the Lagrangian of the system can be obtained.

In addition to the Lagrangian, the virtual work done by the external torque applied at the hub can be represented in the form

$$\delta W = T\delta\theta. \tag{16}$$

2.5. THE ASSUMED MODES (AMM)

The assumed modes method is used in discretizing the beam elastic deformation, $v(s, t)$ used in the Lagrangian expression relative to the hub co-ordinate system, as follows:

$$v(s,t) = \sum_{i=1}^{N} \phi_i(s) q_i(t), \tag{17}$$

where N is the number of modes, q_i is the vector of modal co-ordinates, which is time dependent, and ϕ_i is the vector of the assumed modes.

Upon substituting the assumed modes approximation, AMM, for the beam deformation, equation (17), and forming the Lagrangian expression, defining $b_0 = [C^2(1 + 1/2\mu) + C/2 + 1/3]$ as a dimensionless inertia coefficient with $C = R_H/l$ as the ratio of the hub radius to the beam length and $\mu = m_H/m_B$ as the mass ratio of the hub to the beam gives the coefficients b_i as follows:

$$b_1 = \int_0^1 \phi^2 \,\mathrm{d}\xi, \qquad b_2 = \int_0^1 \phi^2 \,\mathrm{d}\xi = 1, \qquad b_3 = \frac{1}{4}\left[\int_0^1 \left[\int_0^\xi \phi'^2 \,\mathrm{d}\eta\right]\right]\mathrm{d}\xi,$$

$$b_4 = \int_0^1 \xi\left(\int_0^\xi \phi'^2 \,\mathrm{d}\eta\right)\mathrm{d}\xi, \qquad b_5 = \int_0^1 \left(\int_0^\xi \phi'^4 \,\mathrm{d}\eta\right)\mathrm{d}\xi, \qquad b_6 = \int_0^1 \left[\xi \int_0^\xi \phi'^4 \,\mathrm{d}\eta\right)\mathrm{d}\xi,$$

$$b_7 = \int_0^1 \phi\left[\int_0^\xi \phi'^2 \,\mathrm{d}\eta\right]\mathrm{d}\xi, \qquad b_8 = \int_0^1 \left[\int_0^\xi \phi'^2 \,\mathrm{d}\eta\right]^2 \mathrm{d}\xi, \qquad b_9 = \int_0^1 \phi''^2 \,\mathrm{d}\xi,$$

$$b_{10} = \int_0^1 (\phi'\phi'')^2 \,\mathrm{d}\xi, \qquad b_{11} = \int_0^1 \xi\phi \,\mathrm{d}\xi, \qquad b_{12} = b_2 - Cb_3 - b_4,$$

$$b_{13} = \frac{1}{4}(b_8 - b_6 - Cb_5), \qquad b_{14} = 2Cb_1 + 2b_{11}, \qquad \beta^2 = \frac{EI}{l^3 m_b}, \tag{18}$$

where ϕ_i is the mode shape of the cantilever beam.

2.6. THE EQUATIONS OF MOTION

By applying the Euler–Lagrange's equation to the system Lagrangian, the system equations of motion are obtained as follows:

$$\begin{aligned}(1 + \mu)\ddot{X}_0 &+ \tfrac{1}{2}[(1 + 2C + b_3 q^2)\sin\theta - 2b_1 q\cos\theta]\ddot{\theta} + \tfrac{1}{2}[(1 + 2C + b_3 q^2)\cos\theta \\ &+ 2b_1 q\sin\theta]\dot{\theta}^2 + 2[b_3 q\sin\theta - b_1\cos\theta]\dot{q}\dot{\theta} - (b_3\cos\theta)\dot{q}^2 \\ &- [b_1\sin\theta + b_3 q\cos\theta]\ddot{q} + \beta s_1^2 X_0 = F_x,\end{aligned} \tag{19}$$

$$\begin{aligned}(1 + \mu)\ddot{Y}_0 &+ \tfrac{1}{2}[(1 + 2C - b_3 q^2)\cos\theta - 2b_1 q\sin\theta]\ddot{\theta} - \tfrac{1}{2}[(1 + 2C - b_3 q^2)\sin\theta \\ &+ 2b_1 q\cos\theta]\dot{\theta}^2 - 2[b_3 q\cos\theta + b_1\sin\theta]\dot{q}\dot{\theta} - (b_3\sin\theta)\dot{q}^2 \\ &+ [b_1\cos\theta - b_3 q\sin\theta]\ddot{q} + \beta s_2^2 Y_0 = F_y,\end{aligned} \tag{20}$$

TABLE 1

Arm–hub data

Property	Value
Arm length l	3·0 m
Arm mass per unit length, ρ	4·015 kg/m
Arm flexural rigidity, EI	756·0 N/m^2
Hub radius, R_D	0·2 m
Basic hub mass m_H	50 kg

$$
\begin{aligned}
&[b_0 + b_{12}q^2]\ddot{\theta} + 2b_{12}q\dot{q}\dot{\theta} + \tfrac{1}{2}[(1 + 2C + b_3q^2)\sin\theta - 2b_1q\cos\theta]\ddot{X}_0 \\
&\quad + \tfrac{1}{2}[(1 + 2C - b_3q^2)\cos\theta - 2b_1q\sin\theta]\,\ddot{Y}_0 \\
&\quad + \tfrac{1}{2}[b_{14} + b_7q^2]\ddot{q} + b_7q\dot{q}^2 = \frac{T}{m_Bl^2},
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
&(b_2 + b_8q^2)\ddot{q} + \tfrac{1}{2}(b_{14} + b_7q^2)\ddot{\theta} + b_8q\dot{q}^2 - (b_1\sin\theta + b_3q\cos\theta)\ddot{X}_0 \\
&\quad + (b_1\cos\theta - b_3q\sin\theta)\,\ddot{Y}_0 - (2b_{13}q^3 + b_{12}q)\dot{\theta}^2 + \beta^2b_9q + 2\beta^2b_1q^3 = 0.
\end{aligned} \tag{22}
$$

Equations (19)–(22) represent the equations of motion of an inextensible beam, rotating around its hub centre which is, simultaneously, moving against a flexible base. The system is four-coupled non-linear differential equations for the system degrees of freedom X_0, Y_0 as the two directional motions of the base, θ as the rigid body rotation and q as the beam with modal degree of freedom. The base flexibility is defined as a ratio to the beam flexibility using the parameters $s_1 = K_Xl^3/EI$ and $s_2 = K_Yl^3/EI$.

3. NUMERICAL SIMULATION

The computational process started with the evaluation of the coefficients, equations (18), for the cantilever beam mode shapes. The system of non-linear second order equations, equations (19)–(22), is simulated using the MATLAB software. The dimension and material properties of the arm–hub system are given in Table 1.

The inverse dynamic procedure is used to design an open-loop positioning torque, which accounts for the rigid body inertia that is known. The sinusoidal torque profile that is employed to rotate the system, with 3 m long flexible arm, an angle of $\pi/8$ in 5 s is shown in Figure 3. The system is first simulated with a relatively rigid base with spring coefficients $K_X = 1 \times 10^6$ N/m and $K_Y = 1 \times 10^6$ N/m. The resulting arm angular velocity is shown in Figure 4 and the arm equivalent angular position is shown in Figure 5. The associated hub horizontal and vertical deflections are shown in Figures 6 and 7, respectively, wherein, both hub deflections are shown to be affected by the applied torque period. The non-dimensional first mode deflection of the rotating arm is shown in Figure 8. In Figures 6–8, the dynamic system shows damping effects, although no artificial damping is added in this simulation, to

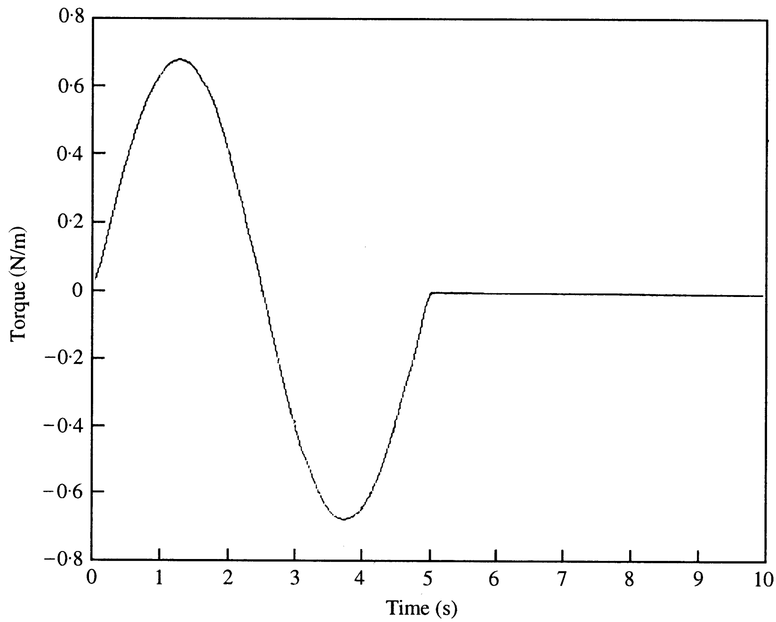

Figure 3. Torque profile.

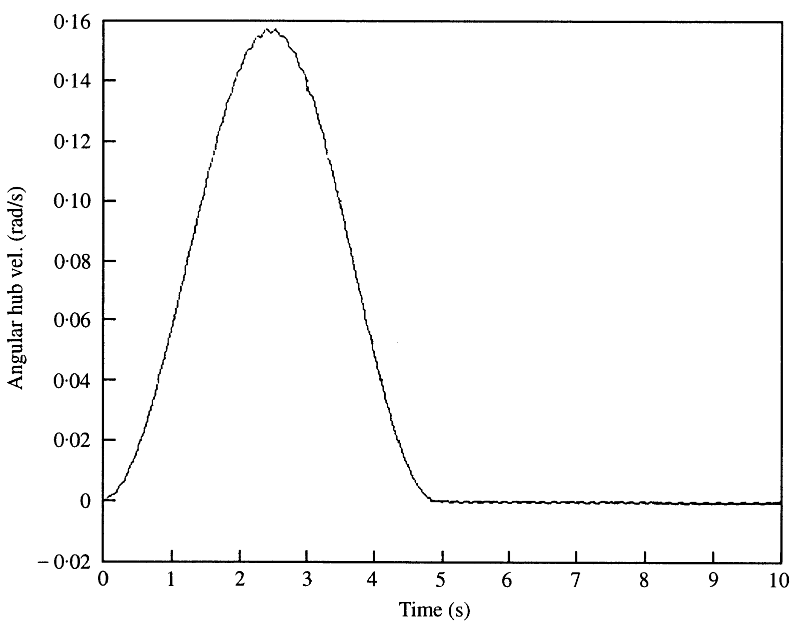

Figure 4. Angular hub velocity.

the system. The effect of damping is most likely developed as a result of non-linear dynamic interaction between different degrees of freedom and thus the energy exchange.

The system is then simulated for a soft base, $K_X = 100\,N/m$ and $K_Y = 100$ N/m by applying the same torque profile shown in Figure 3. The resulting arm angular velocity is

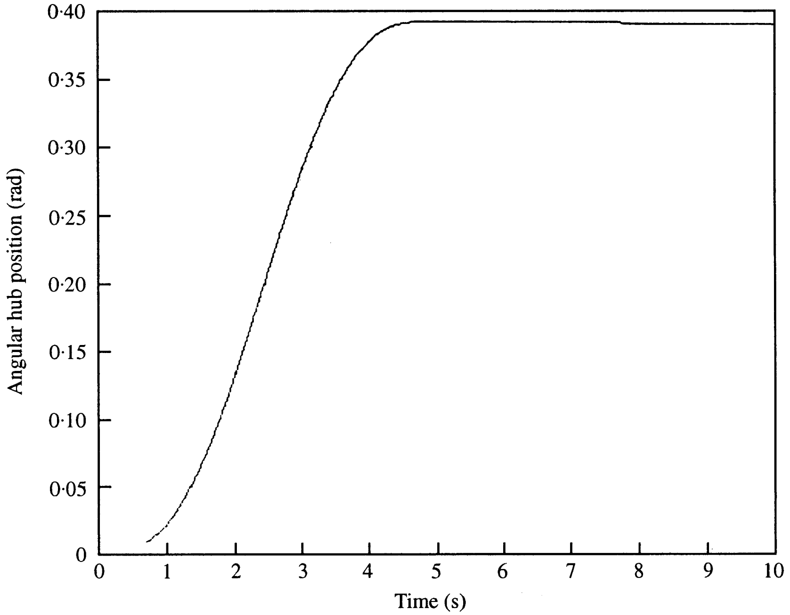

Figure 5. Angular hub position.

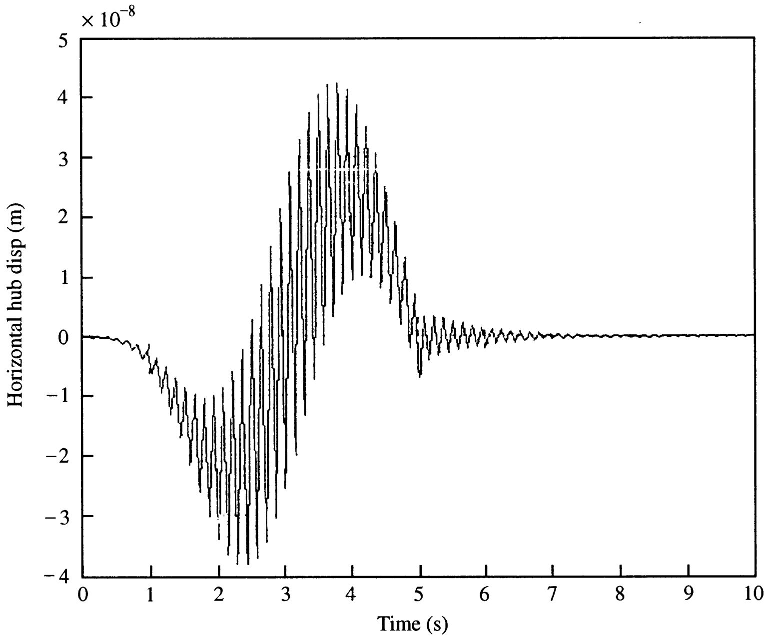

Figure 6. Horizontal hub displacement.

shown in Figure 9, which shows that at the end of the torque period, the arm is rigidly oscillating with increasing amplitude. This angular velocity behaviour does not show when the base stiffness is high. The arm equivalent rigid body angular position, Figure 10, shows little effects due to low base flexibility. The hub horizontal and vertical deflections are

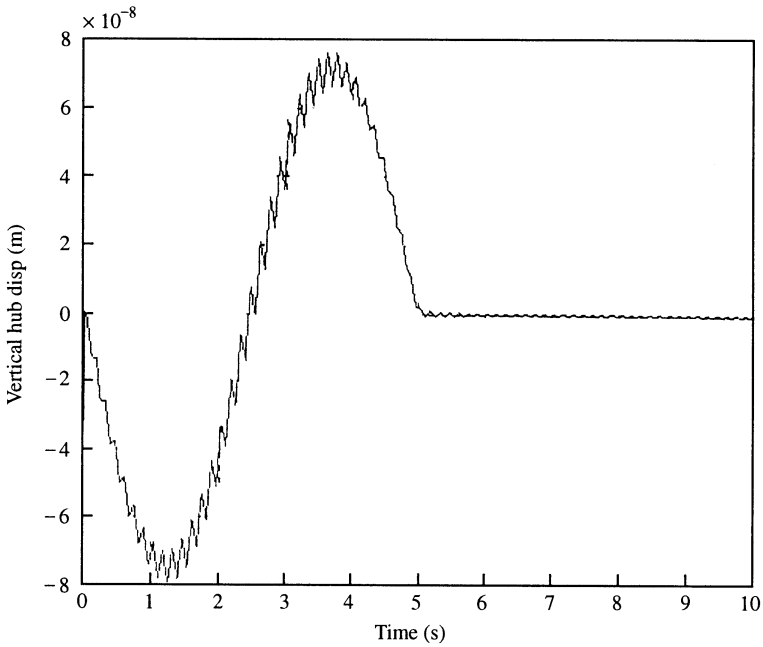

Figure 7. Vertical hub displacement.

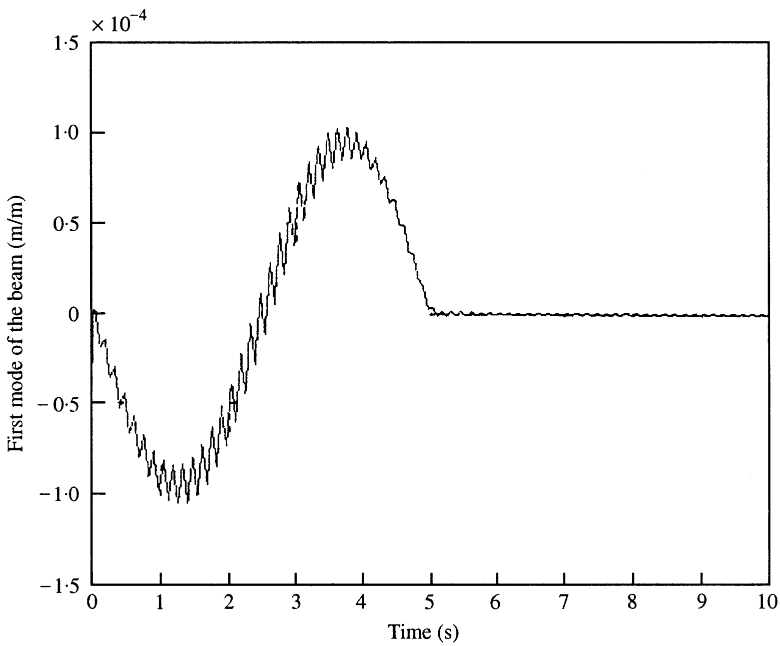

Figure 8. Tip deflection.

shown in Figures 11 and 12 respectively. The hub horizontal and vertical motion occurs at its own natural frequency as shown in Figures 11 and 12, however, with increasing amplitude. The more interesting feature is the arm first mode of vibration shown in Figure 13. The first mode arm deflection is shown to exhibit an increasing amplitude

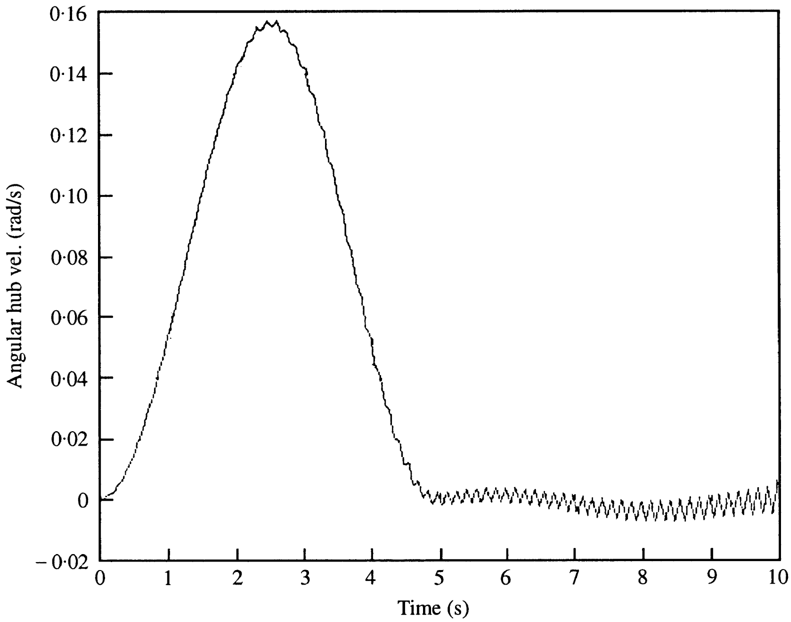

Figure 9. Angular hub velocity (low hub stiffness).

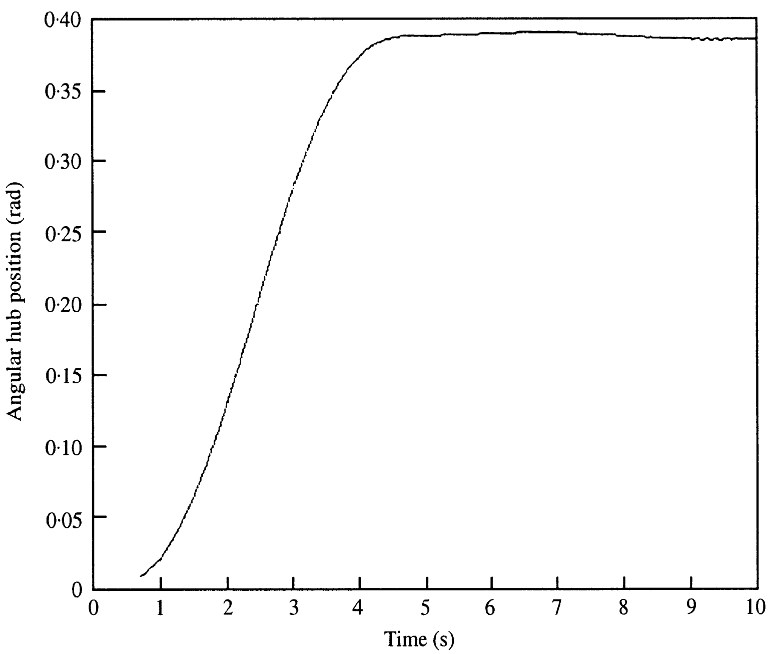

Figure 10. Angular hub velocity (low hub stiffness).

vibration when using a very flexible base at its own natural frequency. The system is simulated for many base stiffness coefficients reaching zero stiffness, i.e., hub without any support. The results have shown that the stiffness of the base plays a very important role in the dynamic behaviour of rotating flexible arms and should be considered when modelling and controlling such systems.

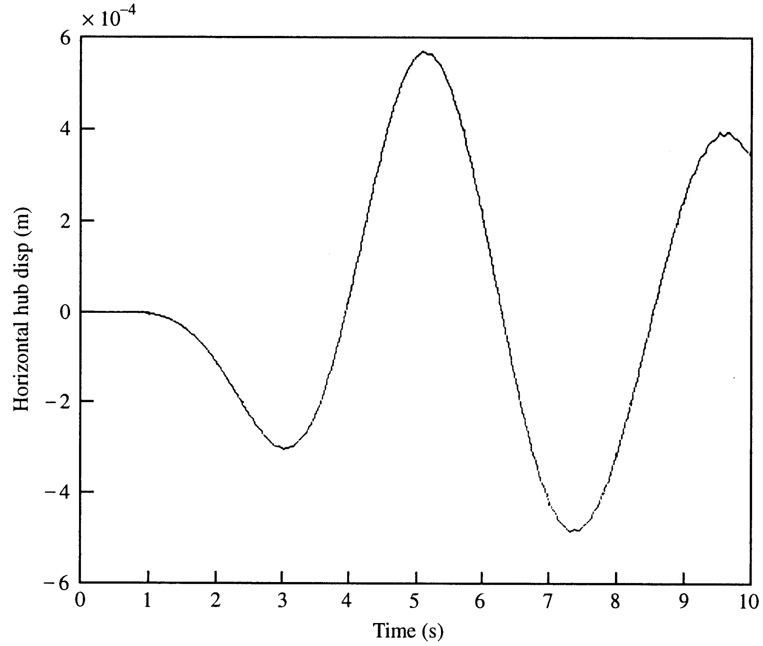

Figure 11. Horizontal hub displacement (low hub stiffness).

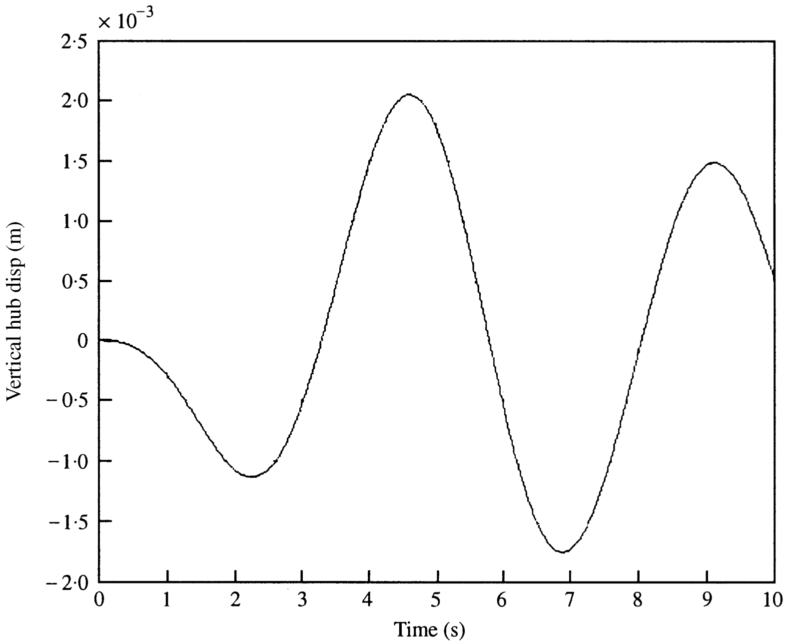

Figure 12. Vertical hub displacement (low hub stiffness).

4. CONCLUSIONS

In this paper, a mechanical model for a rotating flexible arm undergoing large planar flexural deformations, continuously rotating under the effect of hub torque and supported

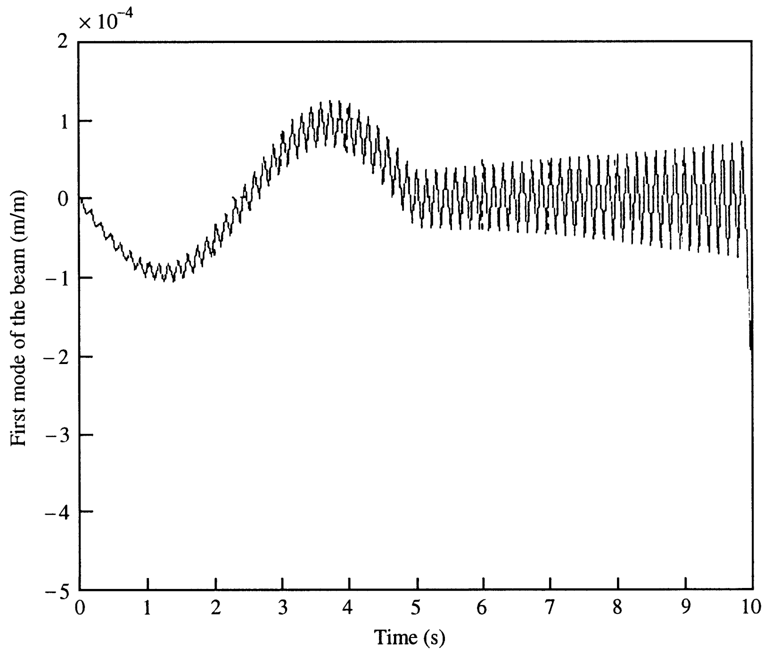

Figure 13. Tip deflection (low hub stiffness).

by a flexible base is developed. The position of a typical material point along the span of the arm is described using the inertial reference frame via a transformation matrix from the body co-ordinate system, which is attached to the flexible root of the rotating arm. The condition of inextensibility is employed to relate the axial and transverse deflections of the material point, within the beam body co-ordinate system. The position and velocity vectors obtained, after imposing the inextensibility conditions, are used in the kinetic energy expression while the exact curvature is used in the potential energy. The Lagrangian dynamics in conjunction with the assumed modes method is utilized to derive directly the non-linear equivalent temporal equations of motion. The resulting non-linear model, which is composed of four coupled non-linear ordinary differential equations, is discussed, simulated and the results of the simulation are presented. The effects of the base flexibility are explored by comparing the resulting simulation results, for various flexibility coefficients, reaching a non-supported case. The numerical results showed that the base flexibility has a significant effect on the dynamics of rotating flexible arms. There exists a stiffness for the base, below which the base has a destabilizing effect on the arm flexural vibrations, which in turn is also reflected in the arm–hub system rigid-body angular velocity.

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