Journal of Sound and Vibration (2002) **251**(5), 927–935
doi:10.1006/jsvi.2001.3907, available online at http://www.idealibrary.com on IDEAL®

A THEORETICAL INVESTIGATION OF OPTIMAL POWER ABSORPTION AS A NOISE CONTROL TECHNIQUE

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(*Received* 17 *July* 2001)

1. INTRODUCTION

A method is presented that is aimed at reducing the sound power radiated by a vibrating structure using a feedback control system based on optimal structural power absorption. This control strategy is intended for applications where a coherent measurement of the disturbance is not readily available and where the use of feedback control is necessary. The method would apply, for example, in the case of noise inside the aircraft that is generated by the turbulent boundary layer on the outer skin. Current feedback control strategies that are designed to reduce radiated sound power include active structural acoustic control [1–4] and radiation modal control [5, 6]. Such techniques are designed using *a priori* knowledge of the acoustic radiation characteristics of the structure together with structural sensors as control inputs. While the analytical and experimental results obtained to date for both techniques appear promising, a considerable drawback in both cases is the complexity of the controller design that results from the necessity of knowing the acoustic radiation characteristics of the structure. This may be difficult to obtain in practice.

In work described here, an optimal power absorbing control system is proposed which relies on only local vibration information. Such a system can be effective in reducing a structure's radiated sound power. The optimal controller provides a causally constrained impedance match between the control system and structure, maximizing the power flow between the two systems [7, 8]. The result of the impedance match is a reduction in the total energy in the structure and corresponding reduction in sound power radiation. The controller described is a single-input–single-output (SISO) system that requires only local vibration information at the position of the co-located control sensor and actuator. The form of the controller is derived from the solution of the Wiener–Hopf equation.

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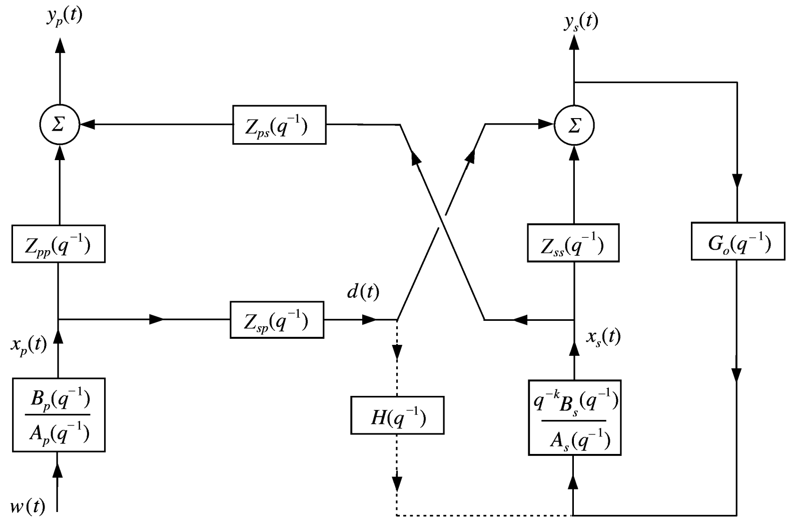

Figure 1. Diagrammatic representation of feedback control of power input.

2. CONTROLLER PARAMETERIZATION

The block diagram used to design the optimal power absorbing control system is shown in Figure 1. The optimal feedback controller, $G_o(q^{-1})$, is designed to be driven by a structural velocity signal, $y_s(t)$, and to feedback a control signal to an actuator, $W_s(q^{-1})$, which applies a force to the structure, denoted $x_s(t)$. Similarly, $x_p(t)$ denotes the primary (or disturbance) force and $y_p(t)$ as the total velocity at the point of application of the primary source. The mobilities relating these variables are denoted by $Z_{pp}(q^{-1})$, $Z_{ss}(q^{-1})$, $Z_{ps}(q^{-1})$ and $Z_{sp}(q^{-1})$. Here we use t to denote the discrete time index and q^{-1} to denote the delay operator. The primary force is assumed to be generated by passing white noise, $w(t)$, through a minimum phase shaping filter $W_p(q^{-1}) = B_P(q^{-1})/A_p(q^{-1})$. We will assume that the controller parameterization is that given by Nelson and Thomas [9],

$$G_o(q^{-1}) = \frac{H(q^{-1})}{1 + H(q^{-1})W_s(q^{-1})Z_{ss}(q^{-1})}, \tag{1}$$

where $H(q^{-1})$ is shown by the dashed line in Figure 1. It can be shown that the controller parameterization given in equation (1) allows the feedback system to be modelled as an equivalent feedforward system, as sketched in Figure 2. This technique is termed "internal model control" [10]. The optimal feedforward filter, $H_o(q^{-1})$, can be obtained by using the well-established filter design techniques of feedforward control. Thus, the optimal controller, $G_o(q^{-1})$, can be obtained using feedforward techniques.

3. THEORETICAL DEVELOPMENT

The design of the optimal power absorbing controller is accomplished by deriving a Wiener–Hopf equation that defines the condition for the optimality of the feedback controller. Referring to Figures 1 and 2, the appropriate cost function to maximize power absorption is given by

$$J = E[x_s(t)y_s(t) + \beta u^2(t)], \tag{2}$$

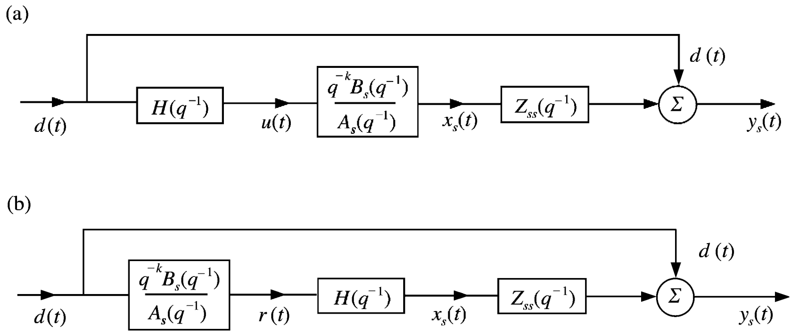

Figure 2. Diagram of equivalent filter design.

where $x_s(t)y_s(t)$ is the power at the secondary source, μ is the control input and β is a weighting factor on the control input. To find the filter $H_o(q^{-1})$ that minimizes this cost function, we assume that $H(q^{-1}) = H_o(q^{-1}) + \varepsilon H_\varepsilon(q^{-1})$, where $H_\varepsilon(q^{-1})$ is a realizable (causal and stable) departure from the optimal filter, $H_o(q^{-1})$, and ε is a small parameter. The Wiener–Hopf equation that results is given by [11]‡

$$R_{rd}(\tau) + H_o(q^{-1})[(Z_{ss}(q) + Z_{ss}(q^{-1}))R_{rr}(\tau) + 2\beta R_{dd}(\tau)] = 0, \quad \tau \geqslant 0. \tag{3}$$

The solution for $H_o(q^{-1})$ that satisfies this equation can be found using spectral factorization techniques and is given by

$$H_o(z^{-1}) = -\frac{1}{S_{z\beta}(z^{-1})}\left[\frac{S_{rd}(z^{-1})}{S_{z\beta}(z)}\right]_+, \tag{4}$$

where $[\,]_+$ denotes the positive-time portion of the function inside the bracket (i.e., the causal portion) and the spectral factors $S_{z\beta}(z^{-1})$ and $S_{z\beta}(z)$ are defined by

$$S_{z\beta}(z^{-1})S_{z\beta}(z) = (Z_{ss}(z^{-1}) + Z_{ss}(z))S_{rr}(z^{-1}) + 2\beta S_{dd}(z^{-1}). \tag{5}$$

An analytical solution to equation (4) is only practical for simple systems due to the complexity of extracting the positive-time portion of the function inside the brackets. A numerical solution to obtain the positive-time portion can be accomplished through the use of a Diophantine equation in the form of

$$A(x)X(x) + B(x)Y(x) = C(x), \tag{6}$$

which may be written as

$$\frac{X(x)}{B(x)} + \frac{Y(x)}{A(x)} = \frac{C(x)}{A(x)B(x)}, \tag{7}$$

‡It is noted that with the removal of the causality constraint and letting $\beta = 0$, the substitution of equation (3) into equation (1) results in the optimal controller being equal to the complex conjugate of the structural impedance at the position of the control system; this is the condition needed for maximum power flow between the two systems. The inclusion of the causality constraint as shown in equation (3), results in a causally constrained impedance match and provides maximal power flow for a feedback system.

where $A(x)$, $B(x)$ and $C(x)$ are known polynomials and $X(x)$ and $Y(x)$ are polynomials to be determined. Thus, we want to transform the function inside the brackets of equation (4) into the form of the Diophantine equation given in equation (7).

We begin the transformation by defining the disturbance and secondary plants as a ratio of polynomials such that

$$W_d(z^{-1}) = \frac{B_d(z^{-1})}{A_d(z^{-1})}, \qquad Z_{ss}(z^{-1}) = \frac{B_z(z^{-1})}{A_z(z^{-1})} \tag{8, 9}$$

and similarly define the actuator polynomial as

$$W_s(z^{-1}) = \frac{z^{-k} B_s(z^{-1})}{A_s(z^{-1})}, \tag{10}$$

where z^{-k} represents the delay in the control system. It can be shown that by defining

$$A(z^{-1})A(z) = A_z(z^{-1})A_z(z)A_s(z^{-1})A_s(z)A_d(z^{-1})A_d(z), \tag{11}$$

$$B(z^{-1})B(z) = (B_z(z^{-1})A_z(z) + B_z(z)A_z(z^{-1}))B_s(z^{-1})B_s(z)A_d(z^{-1})A_d(z), \tag{12}$$

$$C(z^{-1})C(z) = A_z(z^{-1})A_z(z)A_s(z^{-1})A_s(z)B_d(z^{-1})B_d(z), \tag{13}$$

and further

$$D_c(z^{-1})D_c(z) = B(z^{-1})B(z) + 2\beta A(z^{-1})A(z), \tag{14}$$

$$D_f(z^{-1})D_f(z) = Q_w C(z^{-1})C(z) + Q_v A(z^{-1})A(z), \tag{15}$$

that equation (5) can be written as

$$S_{z\beta}(z^{-1})S_{z\beta}(z) = \frac{D_c(z^{-1})D_c(z)D_f(z^{-1})D_f(z)}{A(z^{-1})A(z)A(z^{-1})A(z)}. \tag{16}$$

If we further define

$$\bar{D}_c(z) = A_s(z)D_c(z), \qquad \bar{B}(z) = B_s(z)A(z), \tag{17, 18}$$

the substitution of equations (16)–(18) into equation (4) results in

$$\left[\frac{S_{rd}(z^{-1})}{S_{z\beta}(z)}\right]_+ = \frac{1}{Q_w}\left[\frac{z^k D_f(z^{-1})\bar{B}(z)}{A(z^{-1})\,\bar{D}_c(z)}\right]_+. \tag{19}$$

Using the Diophantine equation defined in equation (7), it may be written as

$$\frac{z^k D_f(z^{-1})\bar{B}(z)}{A(z^{-1})\bar{D}_c(z)} = \frac{G(z^{-1})}{A(z^{-1})} + \frac{z^g F(z^{-1})}{\bar{D}_c(z)}, \tag{20}$$

where $G(z^{-1})$ and $z^g F(z^{-1})$ are unknown functions. While the first term on the right-hand side of equation (20) is entirely causal, the second term consists of both causal and non-causal terms. By properly assigning the value of g, the second term becomes anti-causal and equation (20) may be written

$$\left[\frac{S_{rd}(z^{-1})}{S_{z\beta}(z)}\right]_+ = \frac{1}{Q_w}\frac{G(z^{-1})}{A(z^{-1})}. \tag{21}$$

TABLE 1

Beam, acoustic and signal processing parameters

Beam parameters	
Young's modulus:	$7\cdot1\times10^{10}$ Pa
Density:	2700 kg/m^3
Length:	0·7 m
Thickness:	0·002 m
Width:	0·01 m
Damping ratio:	0·02
Acoustic fluid parameters	
Sound speed:	343 m/s
Density:	1·21 kg/m^3
Signal processing parameters	
Sampling frequency:	1024 Hz

The optimal feedback compensator is then written

$$G_o(z^{-1}) = \frac{A(z^{-1})G(z^{-1})}{z^{-k}B(z^{-1})G(z^{-1}) - D_c(z^{-1})D_f(z^{-1})}. \tag{22}$$

It should be noted that the transition from equation (20) to equation (21) is beyond the scope of this article and will appear in a future publication.

As discussed in the introduction, the optimal power absorbing controller requires only local vibration information. It is seen by the examination of equation (3) that the design of the control system requires models of the disturbance at the secondary control position, $W_d(z^{-1})$, the mobility relating the output force of the controller and the co-located velocity, $Z_{ss}(z^{-1})$, and the actuator plant, $W_s(z^{-1})$. A model of each transfer function in the form of a ratio of polynomials in the z-domain is obtained using a least squares curve fit routine of the actual transfer functions obtained by either analytical derivation or experimental measurements. The curve fit routine used in this work is performed using the INVFREQZ command in the MATLAB Signal Processing Toolbox.

It is important to note that previous work [7, 8, 12] in power absorbing controllers has determined that the use of power absorption as a cost function can lead to an increase in the energy in the system. The occurrence of this phenomenon is due to the secondary source driving the primary source to generate additional power in order to achieve greater power dissipation. Sharp [11] and Nelson [13], however, argue that as the predictability of the disturbance decreases (which, in the limit, results in a white-noise disturbance input), the possibility of a change in the disturbance power input also decreases. In this work, we will assume the disturbance to be a white-noise input which guarantees not to alter the disturbance power input as long as the disturbance and control system are not co-located. A future publication will present a more thorough analysis of this aspect of optimal power absorption as applied to acoustic enclosures and vibrating systems.

4. OPTIMAL POWER ABSORPTION APPLIED TO A SIMPLY SUPPORTED BEAM

We now investigate the ability of optimal power absorption to reduce the radiated sound power from a simply supported beam. The beam, acoustic and signal processing parameters

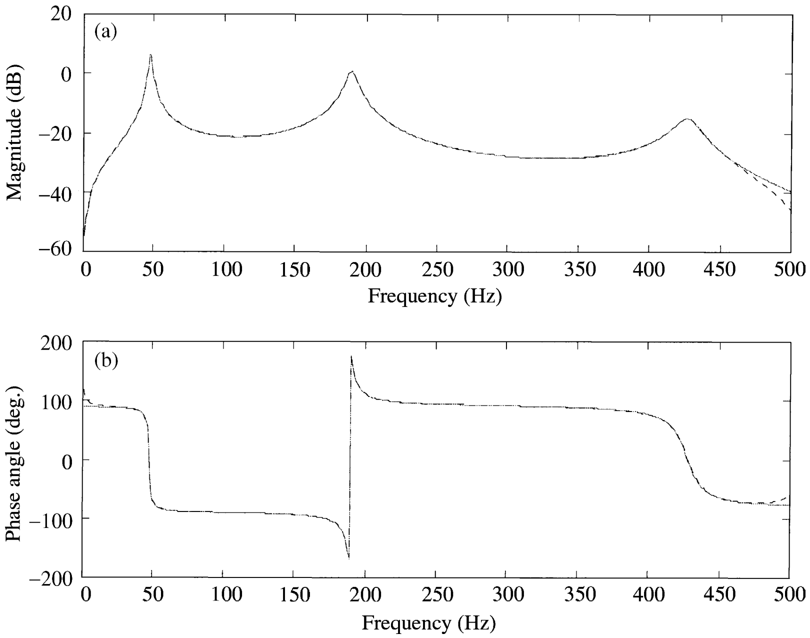

Figure 3. System identification of Z_{sp} transfer function; ——, actual plant; - - -, modelled plant.

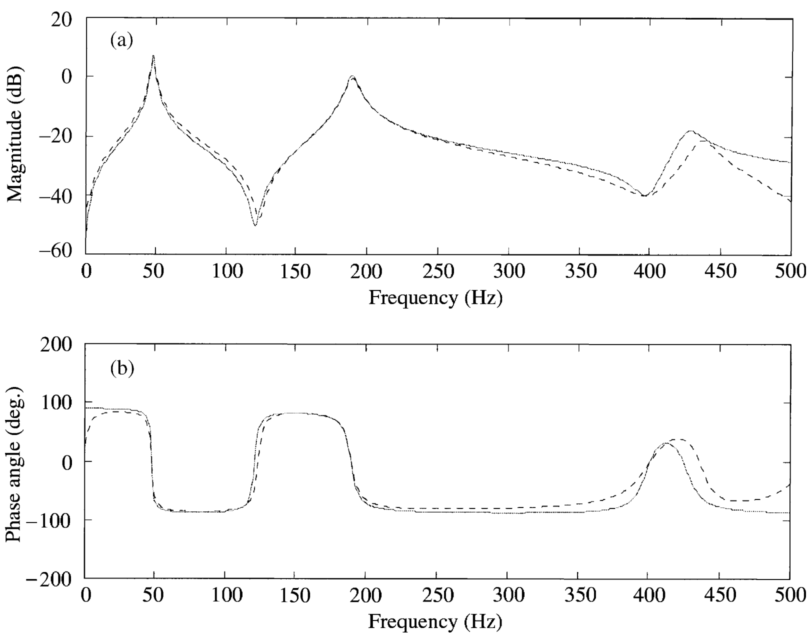

Figure 4. System identification of Z_{ss} transfer function; ——, actual plant; - - -, modelled plant.

for this example are listed in Table 1. The disturbance is a point force located at 0·175 m, the control force and sensor are co-located at 0·5 m and the disturbance input is assumed to be white noise (i.e., the shaping filter, W_p, is unity). A control weighting factor of $\beta = 10^{-15}$ was used in the simulation.

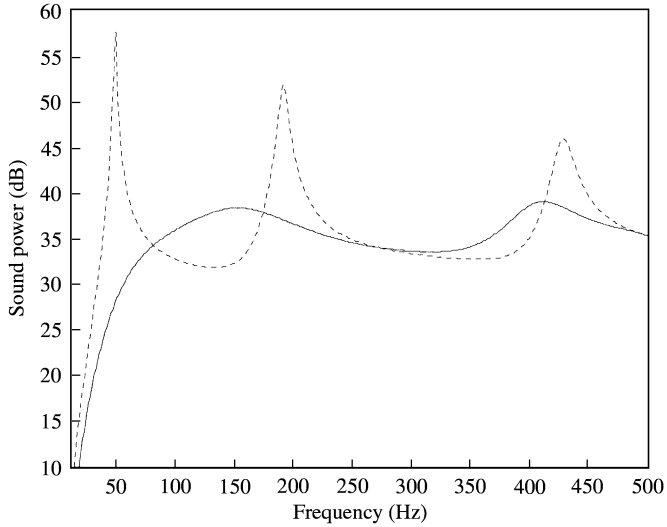

Figure 5. Radiated sound power without (– – –) and with (——) control.

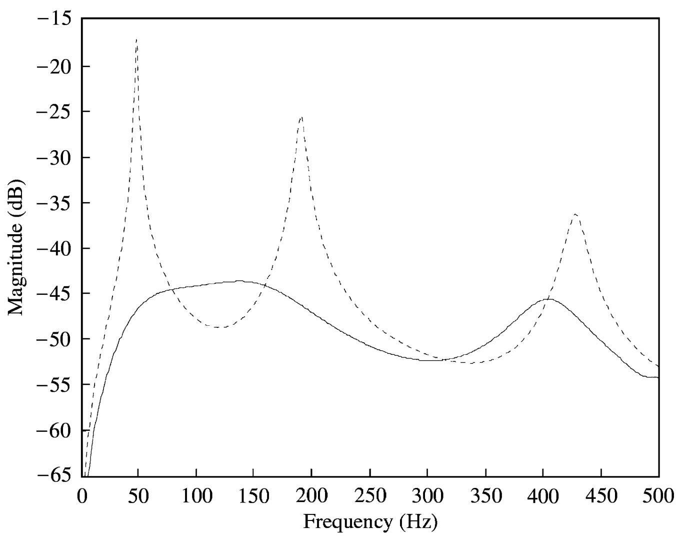

Figure 6. Total kinetic energy in the beam without (– – –) and with (——) control.

In order to obtain a model of the disturbance, $W_d(z^{-1})$, we note that the disturbance is simply the cascade of the shaping filter, $W_p(z^{-1})$ and $Z_{sp}(z^{-1})$. Since the shaping filter is specified by the authors, we only require a model of $Z_{sp}(z^{-1})$. The results of the system identification for $Z_{sp}(z^{-1})$ and $Z_{ss}(z^{-1})$ are shown in Figures 3 and 4, where both models have a sixth order numerator and denominator. The actuator plant is assumed to have unity gain and zero delay.

The radiated sound power from the beam with and without optimal power absorption is shown in Figure 5. The figure indicates that the sound power is reduced at all resonance frequencies and increased between the resonance frequencies (which agrees well with the results presented by Nelson *et al.* [14] for causally constrained control systems). Sound

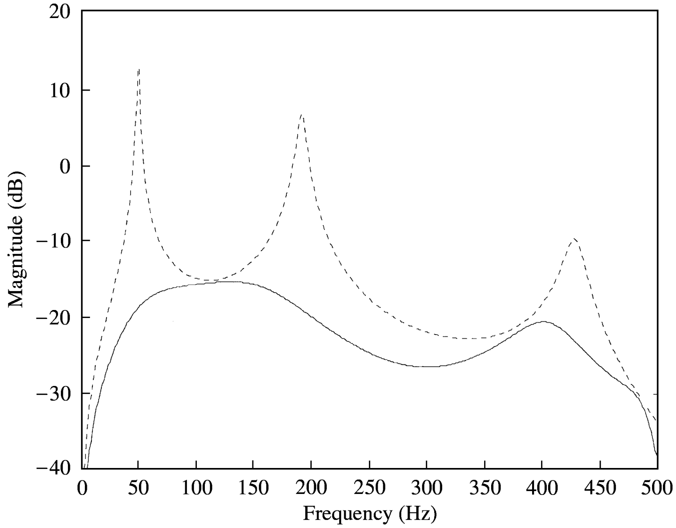

Figure 7. Velocity spectrum at the position of the secondary source without (– – –) and with (——) control.

power reductions of 30·4, 14·9 and 7·7 dB are achieved at the three beam resonance frequencies respectively. The total kinetic energy in the beam is shown in Figure 6, and Figure 7 shows the velocity spectrum at the position of the controller. The effect of the power absorbing controller is to not only reduce the vibration at the position of the control system but provide global vibration attenuation, resulting in a reduction in the radiated sound power. A global vibration attenuation is expected since the optimal power absorbing controller provides the optimal causally constrained impedance match between the control system and the structure, maximizing the power flow between to the two systems.

5. CONCLUSIONS

The use of an optimal power absorbing control system is introduced as a method to reduce the sound power radiated from a vibrating structure. This design approach requires only local vibration information and suggests an alternative to current feedback design approaches which require information about the radiation characteristics of the structure.

Future work by the authors includes a detailed description to the solution of the Diophantine equation for various systems parameters and the application of optimal power absorbing control systems to acoustic enclosures and plate systems including experimental verification.

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