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DETERMINATION OF ELASTIC CONSTANTS OF ANISOTROPIC LAMINATED PLATES USING ELASTIC WAVES AND A PROGRESSIVE NEURAL NETWORK

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In this paper, a procedure is suggested to inversely determine the elastic constants of anisotropic laminated plates using a progressive neural network (NN). The surface displacement responses are used as the inputs for the NN model. The outputs of the NN are the elastic constants of anisotropic laminated plates. The hybrid numerical method (HNM) is used to calculate the displacement responses of laminated plates to an incident wave for given elastic constants. The NN model is trained using the results from the HNM. A modified back-propagation learning algorithm with a dynamically adjusted learning rate and an additional jump factor is developed to tackle the possible saturation of the sigmoid function and to speed up the training process for the NN model. The concept of orthogonal array was adopted to generate the representative combinations of elastic constants, which reduces significantly the number of training data while maintaining its data completeness. Once trained, the NN model can be used for on-line determination of the elastic constants if the dynamic displacement responses on the surface of the laminated plate can be obtained. The determined elastic constants are then used in the HNM to calculate the displacement responses. The NN model would go through a progressive retraining process until the calculated displacement responses using the determined results are sufficiently close to the actual responses. This procedure is examined for an actual glass/epoxy laminated plate. It is found that the present procedure is very robust and efficient for determining the elastic constants of anisotropic laminated plates.

1. INTRODUCTION

Composites have been widely used in modern industry. From space shuttle, motor vehicle to computer chip, composites have penetrated into every region of the modern technology with their obvious advantages over those made of conventional materials. The effective use of composites relies on a precise knowledge of the material property of the constituent materials; hence, their property evaluation has been one of the focuses of research. It is obviously valuable to develop a reliable method to non-destructively measure the properties of composites.

Advanced non-destructive methods for material characterization of composites utilize the complex relationship between the structure behaviors and the material property. This relationship is often represented by a known mathematical model defining the forward problem, which can be analyzed numerically or otherwise. Thus, if a set of reasonably

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accurate experimentally measured structure behavior data is available, then the material property of the composite may be identified by solving an inverse problem properly formulated. The material property can often be characterized by minimizing the sum of the squares of the deviations between the experimental and the calculated structure behavior data. Ultrasonic wave velocity has been used as the structure behavior data for determination of elastic constants of anisotropic composites [1–3]. Rokhin and co-workers [4–6] proposed several modifications of the immersion ultrasonic technique to determine the elastic constants of composites. In these techniques, Christoffel equation was adopted to establish the relationship between material properties and bulk wave velocity, comparably complicated techniques were needed to measure the phase velocity of ultrasonic bulk waves in anisotropic materials. Balasubramaniam and Rao [7] investigated the reconstruction of material stiffness properties of unidirectional fiber-reinforced composites from obliquely incident ultrasonic bulk wave data. Genetic algorithms (GAs) were used as the inversion technique and detailed discussion on advantages as well as disadvantages of GAs for the identification problem over conventional methods were presented. However, only single-ply materials were considered in their works. Mota Soares *et al.* [8] presented a technique to predict the mechanical properties of composite plate specimens using experimentally measured eigenfrequencies. However, it can be generally concluded that these inverse procedures require too many calls for forward solvers.

Neural network (NN) is a novel tool for information processing. It provides a unique computing architecture, which enjoys a massive parallel processing structure. The parallelism of NN enables it to solve many problems that cannot be handled by analytical approaches. NNs provide an effective approach for engineering applications in a very broad spectrum [9–11]. Furthermore, the NN technique is well known for its ability to model the non-linear and complex relationship between the structure parameters and the dynamic characteristics.

NN techniques have also been applied in solving inverse problems. Examples include the reconstruction of constitutive properties using depth–load responses [12] and using either group velocities, phase velocities or slowness measurements [13], and estimation of contact forces from impact-induced strain [14] and prediction of impact contact forces [15]. There is generally a well-defined forward problem that may have a solution, but the inverse problem is often ill-posed, and conventional approaches often require computationally intensive iterative processes to find a solution. NN approaches can offer the advantages of very high-efficient inversion operation and can avoid the need for thousands of times of calling of forward solvers. However, the relationship between the material property and the wave responses can be extremely complex. It is very difficult, if not impossible, to train an NN model for such a relationship valid in a wide range of parameters.

In this paper, a novel progressive NN procedure is suggested for the determination of elastic constants of anisotropic laminated plates. In the present NN model, the input data are the dynamic displacement responses on the surface of the plate, which can be easily measured using conventional experimental techniques.

There are two key factors governing the success of an NN in practical applications. The first one is that the inputs of the NN models should be carefully chosen so that variation in the outputs can be truthfully reflected by the changes of these inputs. The second one is the training samples, including both the initial training and the retraining, that should be carefully selected in order to describe the inverse characteristics of the problems. In this paper, the combination of response of time history of two displacement components is used as the input for the NN model. The combined displacement response can still be easily and accurately measured and is more sensitive to the change of the elastic constants than the displacement response of only one component. The concept of orthogonal array is adopted to generate some representative combinations of parameter change. The use of orthogonal array can significantly reduce the number of training data while maintaining the data completeness.

In the present procedure, the NN model is first trained off-line using a set of initial training data that contain various assumed elastic constants and their corresponding displacement responses calculated using the hybrid numerical method (HNM) [16] as the forward solver. A modified back-propagation (BP) algorithm is used as the learning process. The NN model is then used to determine the elastic constants of the laminated plate by feeding in displacement responses. The determined elastic constants are then used in the HNM to calculate the displacement responses. The NN model would go through a progressive retraining process if the calculated displacement responses deviate unacceptably from the actual ones.

This proposed procedure is verified using two sets of elastic constants of glass/epoxy laminated plates.

2. STATEMENT OF THE PROBLEM

Consider a laminated plate with any number of anisotropic layers in the thickness direction. The thickness of the plate is denoted by H. An NN model is used for the determination of elastic constants of anisotropic laminated plates. The outputs of the NN model are elastic constants. The inputs of the NN model are the time history of displacement responses on the surface of the laminated plate, which can be easily measured using conventional experimental techniques. In this paper, we utilize computer-generated displacement responses based on actual elastic constants of laminated plates. The incident excitation waves to the plate are assumed to be a vertical line load in the z direction and a shear line load in the y direction both acting at $x = 0$ on the upper surface. The line loads are independent of the y-axis but as a function of t as

$$f(t) = \begin{cases} \sin(2\pi t/t_d), & 0 < t < t_d, \\ 0, & t \leqslant 0 \ \text{and} \ t \geqslant t_d, \end{cases} \tag{1}$$

where t_d is the time duration of the incident wave. Equation (1) implies that the time history of the incident wave is one cycle of the sine function.

Only one receiving point is chosen on the surface of the laminated plates, and the responses in the time domain for displacement components in x and y directions are selected as the inputs for the NN model.

3. AN NN PROCESS FOR DETERMINING ELASTIC CONSTANTS

An NN model is trained using initial training data containing a set of assumed elastic constants, which represents various elastic constants of laminated plates, and their corresponding displacement responses calculated from the HNM solver [16]. The trained NN model is used to determine the elastic constants by feeding in the measured displacement responses. The determined elastic constants are then used in the HNM solver to calculate the dynamic displacement responses. The NN model would go through a retraining process if the calculated displacement responses deviate unacceptably from the actual ones. This progressive NN process for determination of the elastic constants of

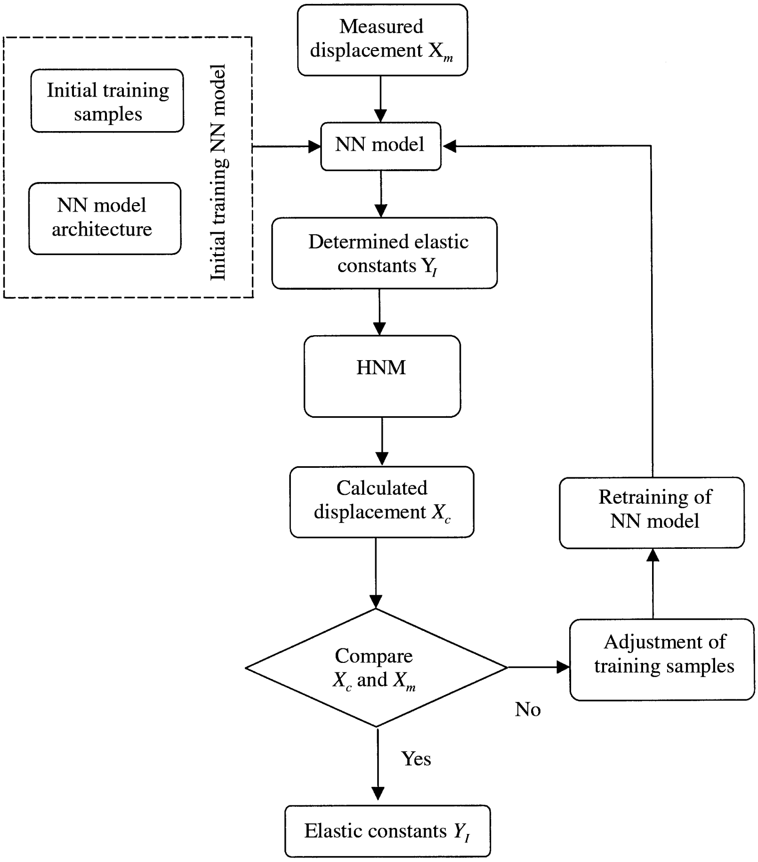

Figure 1. A progressive learning NN model for determination of elastic constants of anisotropic laminated plate.

laminated plates is outlined in Figure 1. The detail of each part of this process will be given in the following sections.

3.1. TEACHER

A neural network requires a large training set to successfully learn and to generalize the characteristic features from input–output pairs. Consequently, it is essential that the calculation of the forward problem is performed as efficiently as possible to generate the training data set. The HNM is employed as the teacher for training the NN model. The HNM has been proven to be very effective in computing the response of a laminated plate subjected to an incident wave. A brief description of the formulation of the HNM is given as follows.

A laminated plate is divided into N-layer elements, as shown in Figure 2. A set of approximate partial differential equations for an element is obtained by using the principle of virtual work. By assembling the matrices of adjacent elements, we obtain the dynamic equilibrium equation of the whole plate [16]

$$\mathbf{F}(x, y, t) = \mathbf{K}_D\mathbf{d}(x, y, t) + \mathbf{M}\ddot{\mathbf{d}}(x, y, t), \tag{2}$$

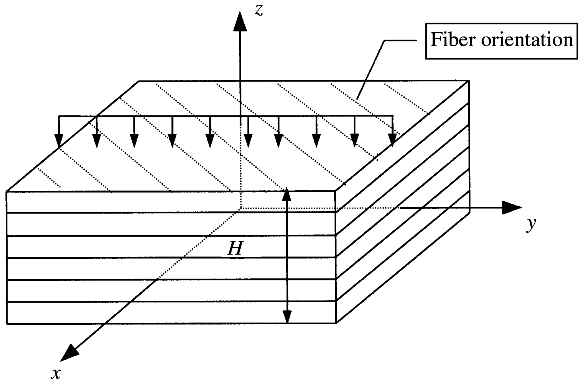

Figure 2. An anisotropic laminated plate subjected to a line load.

where **F** is the external force vector acting on the nodal planes that divide the plate into layer elements and **d** is the displacement vector on the nodal planes. The matrix $\mathbf{K}_D$ is a differential operator matrix for the plate.

The Fourier transformations can be introduced with respect to the horizontal co-ordinates x and y as follows:

$$\tilde{\mathbf{d}}(k_x, k_y, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{d}(x, y, t)\mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{\mathrm{i}k_y y}\,\mathrm{d}x\,\mathrm{d}y, \tag{3}$$

where the real transformation parameters k_x and k_y are the wave numbers corresponding to the horizontal co-ordinates x and y respectively.

The application of equation (3) to equation (2) leads to a set of dynamic equilibrium equations for the whole laminated plate:

$$\tilde{\mathbf{F}} = \mathbf{M}\ddot{\tilde{\mathbf{d}}} + \mathbf{K}\tilde{\mathbf{d}}, \tag{4}$$

where $\tilde{\mathbf{F}}$, $\ddot{\tilde{\mathbf{d}}}$ and $\tilde{\mathbf{d}}$ are the Fourier transformations of **F**, $\ddot{\mathbf{d}}$ and **d** respectively.

Using the model analysis, the displacement vector $\tilde{\mathbf{d}}$ in the Fourier transformation domain can be obtained; finally, the displacement response in the space–time domain can be obtained using the inverse Fourier transformation [17, 18].

3.2. PROGRESSIVE NEURAL NETWORK

An NN model is referred to as a type of computational model that consists of hidden-layer neurons connected between the input and output neurons. The connections between the neurons are described by weights which are to be determined through training. The non-linear hyperbolic functions are usually used as the activation functions to increase the modelling flexibility. The NN is trained with a modified BP training algorithm.

3.2.1. *Topology*

An NN model, which consists of a set of nodes arranged into four layers as shown in Figure 3, is used in this work. There are N inputs representing the displacement responses

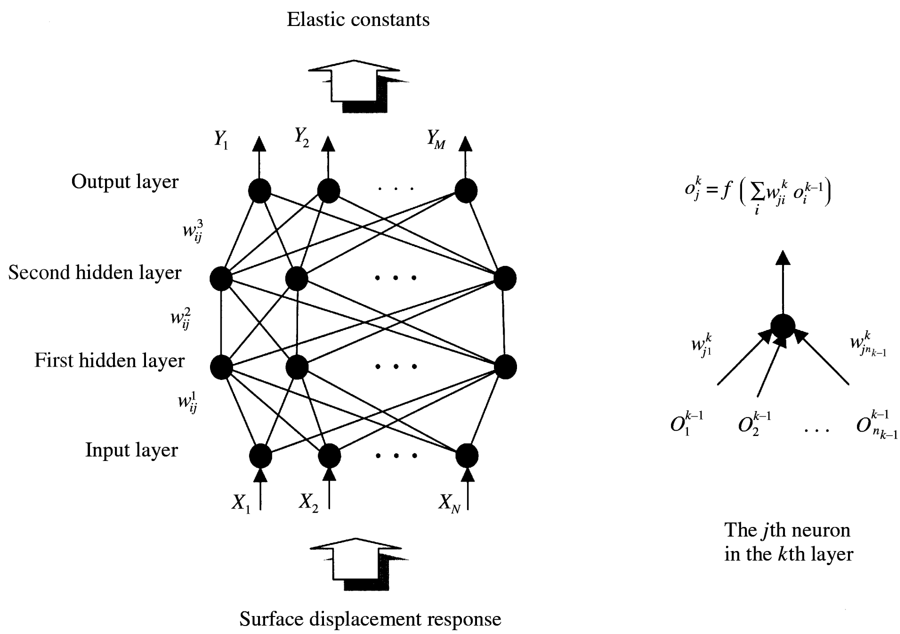

Figure 3. A two-hidden layer NN model for determination of elastic constants of an anisotropic laminated plate.

on the surface and M outputs representing the elastic constants to be determined. As commended by Masri *et al.* [19] that an NN model with two hidden layers is sufficient in most of the structural problems, two hidden layers are used in this work. Mathematically, the NN model represents a non-linear mapping between inputs $\mathbf{X} = \{x_i, i = 1, \dots, N\}$ and outputs $\mathbf{Y} = \{y_i, i = 1, \dots, M\}$ via the following equation:

$$\mathbf{Y} = g(\mathbf{W}, \mathbf{X}), \tag{5}$$

where $\mathbf{W} = \{w_{ij}^k, i = 1, \dots, N_i, j = 1, \dots, N_j; k = 1, 2, 3\}$ is a matrix of weights corresponding to the connections between the layers, and N_i and N_j are the numbers of neurons for the ith and jth layers respectively. Training of the NN model is referred to as the calculation of the weight matrix $\mathbf{W}$ using the training data set. Once the training is complete, the NN calculation is very fast regardless of the complexity of the actual physics of the problem.

3.2.2. *Learning algorithm*

A modified BP learning algorithm with a dynamically adjusted learning rate and an additional jump factor is employed as the learning algorithm. This learning algorithm can overcome the possible saturation of the sigmoid function and speed up the training process of the NN model. A brief description of the formulation of the modified BP is given as follows.

The error norm E between the determined output $\mathbf{Y}_I$ vector and the targeted output vector Y^t can be defined as

$$E(\mathbf{W}) = \| \mathbf{Y}_I - \mathbf{Y}^t \|_2. \tag{6}$$

The operator $\| \|_2$ represents the vector norm. The weight matrix is adjusted iteratively based on the following equations:

$$\mathbf{W}^{I+1} = \mathbf{W}^{I} + \Delta \mathbf{W}^{I}, \tag{7}$$

$$\Delta \mathbf{W}^{I} = -\eta \left.\frac{\partial E}{\partial \mathbf{W}}\right|_{\mathbf{W}=\mathbf{W}^{I}} + \alpha\eta \left.\frac{\partial E}{\partial \mathbf{W}}\right|_{\mathbf{W}=\mathbf{W}^{I-1}}, \tag{8}$$

where η is the learning rate, α is the momentum rate and I is the iterative number.

There are two important features in this modified BP algorithm. Firstly, the learning rate is adjusted once every p iterations instead of every iteration. Vogl *et al.* [20] originally proposed this improvement for fast convergence that the learning rate varies according to whether or not an iteration decreases the error norm. Luo and Hanagud [21] later adopted this improvement using the dynamic learning rate steepest decent method. The dynamic adjustment of the learning rate is used in this paper. The learning rate for the nth iteration is represented as $\eta(n)$, and it can be adjusted at the $(n+p)$th iteration based on the following criterion:

$$\eta(n+p) = c\eta(n), \tag{9}$$

where the range of c could be selected based on the numerical studies [22].

It can be found from equation (8) that the change of $\Delta\mathbf{W}$ is not only dependent on the learning rate η, but also on the partial derivative $\partial E/\partial \mathbf{W}$. Then, another feature of the modified BP is that the jump factor is added in this modified BP algorithm to avoid the stagnation of the value of $\partial E/\partial \mathbf{W}$ as follows.

The derivatives in equation (8) can be written as

$$\frac{\partial E}{\partial w_{ij}^{k}} = -\delta_i^k o_j^{k-1}, \tag{10}$$

where the w_{ij}^k is the weight coefficient between the ith neuron in the kth layer and the jth neuron in the $(k+1)$th layer, the o_j^k is the output from the jth neuron of the kth layer and δ_i^k is expressed as

$$\delta_i^k = \begin{cases} (y_i^t - y_i)\,(f'(z_i^k) + \beta), & k = 3, \\ \left(\sum_j \delta_j^{k+1} w_{ji}^{k+1}\right)(f'(z_i^k) + \beta), & k \neq 3, \end{cases} \tag{11}$$

where

$$z_i^k = \sum_j w_{ij}^k o_j^{k-1} \tag{12}$$

and $f'(z_i^k)$ is the first derivative of the activation function $f()$ with respect to z_i^k. The activation function used in this work is a sigmoid function. In equation (11), β is the additional jump factor for overcoming the stagnation of the weight matrix, the value should be between 0 and 0·15 and can be varied during the training process. The purpose of adding this small positive value to $f'(z_i^k)$ is to maintain a non-zero δ_i^k value and prevent the weight matrix from stagnation.

3.2.3. *Retraining of the NN model*

After the initial training of the NN model, the determination of the elastic constants begins by feeding the measured displacement response data $\mathbf{X}_m$ into the NN model. The outputs of the NN model are the determined elastic constants $\mathbf{Y}_I$. These determined elastic constants are then fed into the HNM to produce a set of calculated displacement response data $\mathbf{X}_c$. A comparison between the calculated displacement response $\mathbf{X}_c$ and measured displacement responses $\mathbf{X}_m$ is made based on a given criterion. If these two sets differ significantly such that the criterion is not satisfied, then the NN model will be retrained on-line using adjusted training samples that contain $\mathbf{X}_c$ and $\mathbf{Y}_I$. The retrained NN model is then used to determine the elastic constants again by feeding in the measured displacement responses $\mathbf{X}_m$. This determination and on-line retraining procedure is repeated until the difference between the calculated and measured displacement responses satisfies the given criterion. At the end of the progression, the final determined elastic constants are guaranteed to produce the displacement responses that are very close to the measured ones when fed into the HNM.

Retraining of the NN model is achieved by adding new samples into the original pool of samples and enforcing a more stringent convergence criterion. It has been pointed out that it could be difficult to achieve the same level of convergence while maintaining the same NN architecture when the number of samples increases. To avoid this problem, a dynamic adjustment method for selecting samples for retraining is proposed. While adding the new sample related to the determined elastic constants by the NN model and the displacement responses from the HNM, one sample from the original sample set should be removed so as to maintain the same number of samples. The sample to be removed is the one that has the largest distance norm from the measured displacement responses $\mathbf{X}_m$. The distance norm of the displacement responses between the ith sample $\mathbf{X}_i$ and the measured displacement responses $\mathbf{X}_m$ is defined as

$$d = \|\mathbf{X}_m - \mathbf{X}_i\|^2. \tag{13}$$

By replacing this remote sample with a new sample, the sample density around the measured displacement responses increases as the process progresses. As a result, the modelling accuracy of the NN model in the neighborhood of the measured displacement responses could be improved.

4. APPLICATIONS

This NN process for determination of the elastic constants of laminated plates is illustrated using one actual laminated plate consisting of 10 glass/epoxy layers. The stacking sequence of the laminated layers is denoted by $[0/+45/-45/60/-60]_s$, where the digital numbers stand for the angles of fiber-orientation of each ply to the x-axis. The subscript of "s" means that the plate is symmetrically stacked. The glass/epoxy material is the transversely isotropic material; there are only five elastic constants as listed in the second column in Table 1 [23]. Hence, there are five parameters, named as $c_{11}, c_{12}, c_{22}, c_{23}$ and c_{55}, that need to be determined.

The NN model used in this paper has two hidden layers, and the neuron numbers of the input, output, first and second hidden layers are 10, 5, 30 and 16 respectively.

4.1. INPUTS OF THE NN MODEL

The displacement responses data on the surface of the laminated plate are selected as the inputs. As a necessary condition for successfully utilizing the NN model, the sought outputs

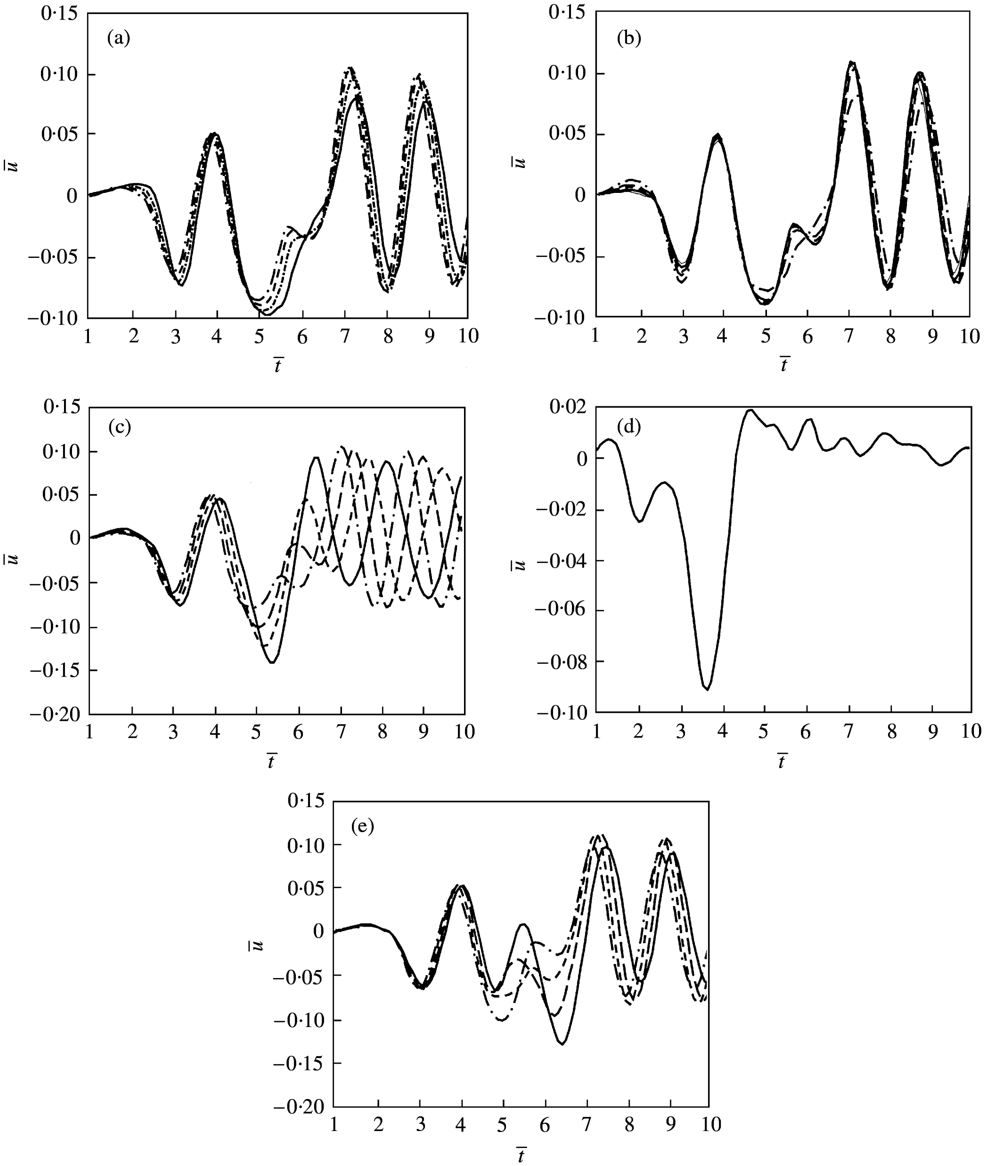

Figure 4. Time history of displacement response in the x direction at $x = 3{\cdot}0H$ on the upper surface of single-ply glass/epoxy plate excited by an incident vertical wave of one cycle sine function at $x = 0{\cdot}0$. (a) ——, 30; – – –, 38; — —, 46; — · ·, 54. (b) ——, 4·0; – – –, 5·33; — —, 6·66; — · ·, 8·0. (c) ——, 10·0; – – –, 12·67; — —, 15·33; — · ·, 18·0. (d) ——, 5·0; – – –, 6·33; — —, 7·66; — · ·, 9·0. (e) ——, 2·0; – – –, 2·67; — —, 3·33; — · ·, 4·0.

should have significant dependence on the input data. Therefore, in the first step, the effect on the displacement responses of varying elastic constants is studied in detail. All the results are based on the HNM, and the dimensionless variables defined by Liu *et al.* [16] are used.

Examples of the displacement response in the x direction at $x = 3{\cdot}0H$ on the upper surface of the single-ply glass/epoxy plate excited by an incident vertical line load of one cycle of sine function at $x = 0{\cdot}0$ are displayed in Figure 4(a)–4(e). Figure 4(a)–4(e) show that each of $c_{11}, c_{12}, c_{22}, c_{23}$ and c_{55} displays an appreciable influence on the response curve when the other elastic constants are set to their actual values. However, the effect of

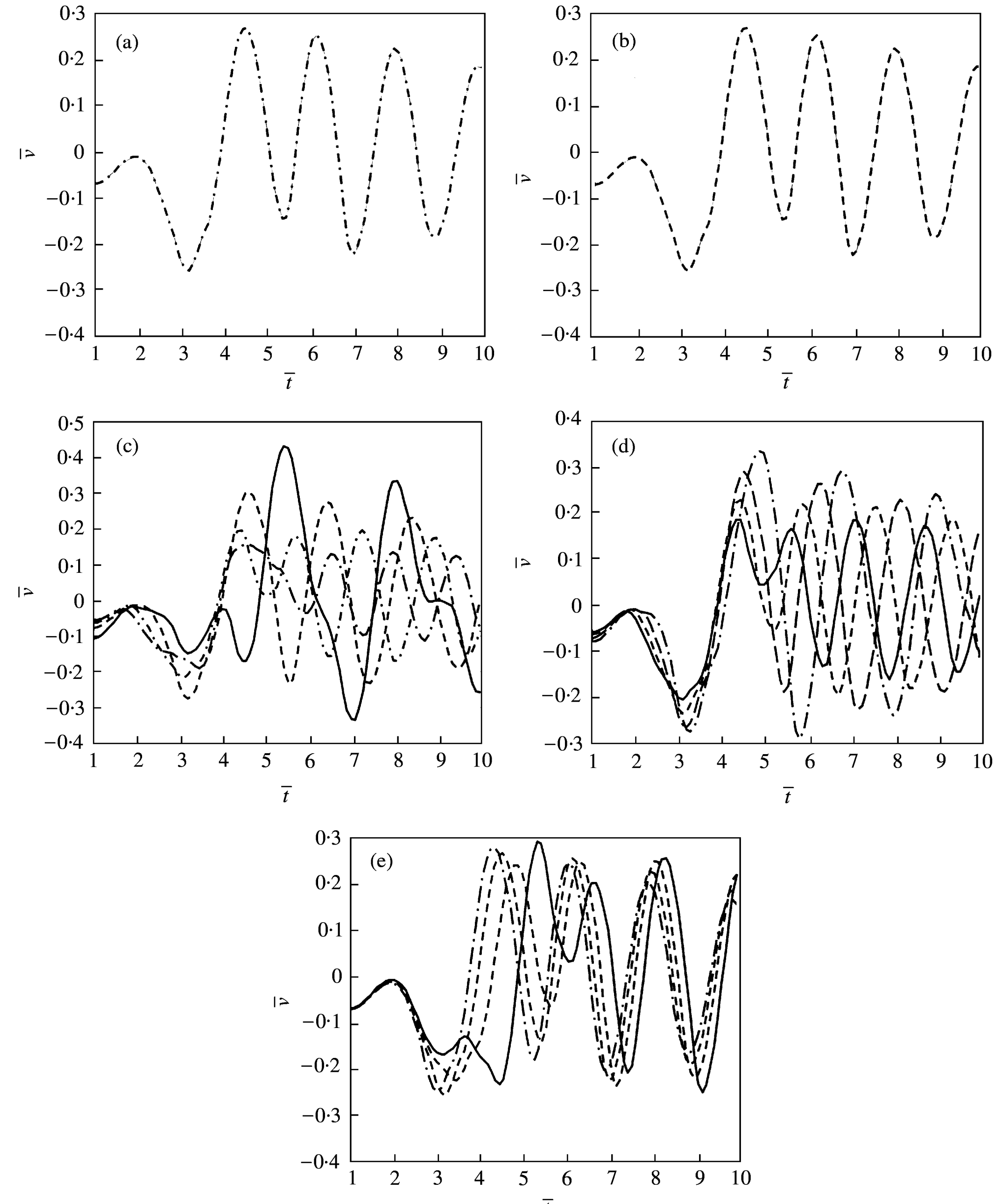

Figure 5. Time history of displacement response in the x direction at $x = 3{\cdot}0H$ on the upper surface of single-ply glass/epoxy plate excited by an incident vertical wave of one cycle sine function at $x = 0{\cdot}0$. (a) ——, 30; – – –, 38; — —, 46; — · ·, 54. (b) ——, 4·0; – – –, 5·33; — —, 6·66; — · ·, 8·0. (c) ——, 10·0; – – –, 12·67; — —, 15·33; — · ·, 18·0. (d) ——, 5·0; – – –, 6·33; — —, 7·66; — · ·, 9·0. (e) ——, 2·0; – – –, 2·67; — —, 3·33; — · ·, 4·0.

c_{23} cannot be seen clearly (even no effect) from the response curve as shown in Figure 4(d). To reflect the effect of c_{23}, we have to consider the component of displacement response in the y direction subjected to a shear load in the y direction acting at $x = 0{\cdot}0$. The influences of the change of the elastic constants on this displacement component are shown in Figure 5(a)–5(e). It can be found that the influence of the change of elastic constant c_{23} can be clearly observed from Figure 5(d), but the effect of the changes of c_{11} and c_{12} cannot be

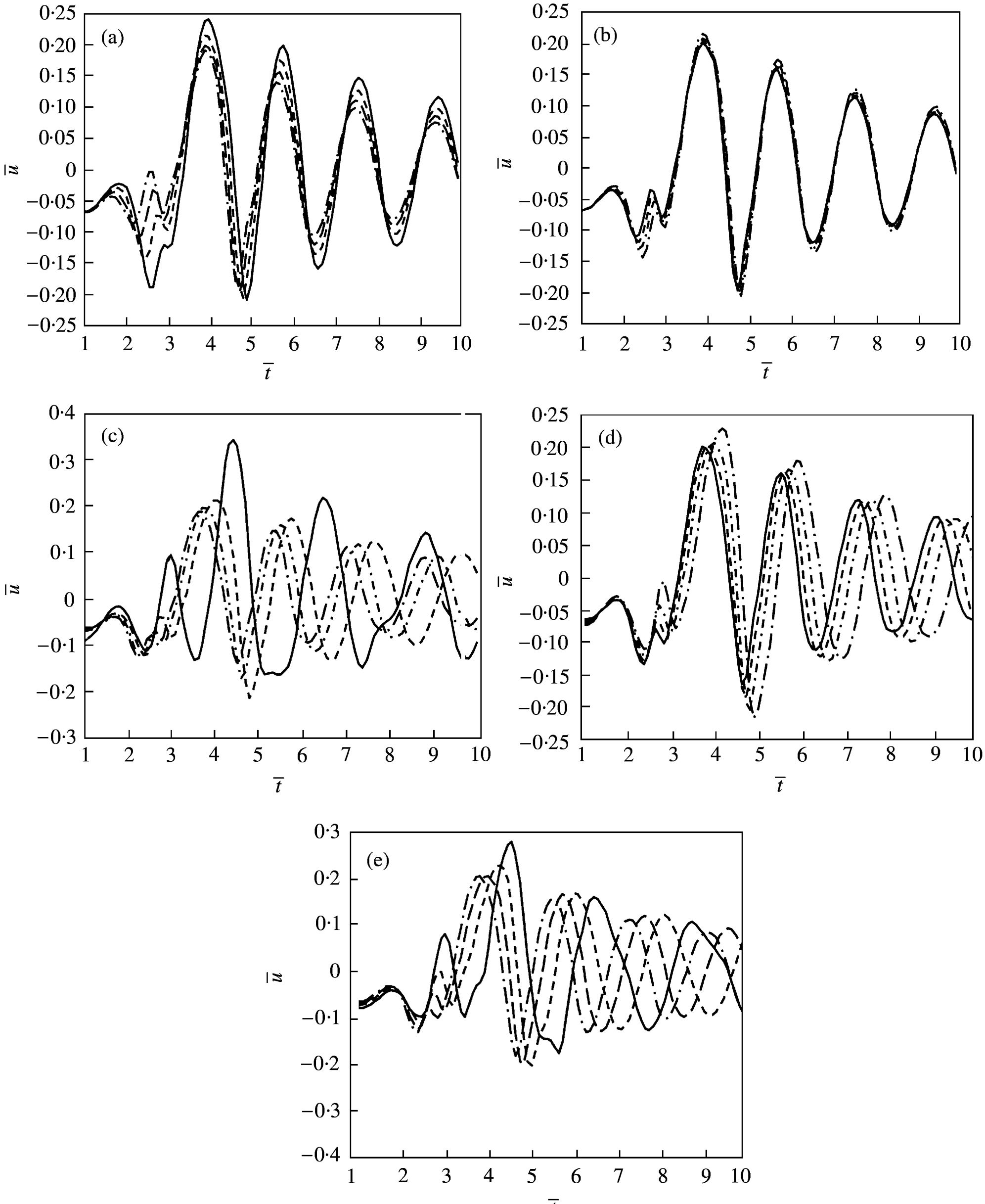

Figure 6. Time history of displacement response in the x direction at $x = 3{\cdot}0H$ on the upper surface of single-ply glass/epoxy plate excited by an incident vertical wave of one cycle sine function at $x = 0{\cdot}0$. (a) ——, 30; – – –, 38; — —, 46; — · ·, 54. (b) ——, 4·0; – – –, 5·33; — —, 6·66; — · ·, 8·0. (c) ——, 10·0; – – –, 12·67; — —, 15·33; — · ·, 18·0. (d) ——, 5·0; – – –, 6·33; — —, 7·66; — · ·, 9·0. (e) ——, 2·0; – – –, 2·67; — —, 3·33; — · ·, 4·0.

reflected from the response of the displacement component in y direction, as shown in Figure 5(a) and 5(b) respectively.

Figures 6 and 7 show the effect of the changes of the elastic constants on the response curves for the 10-layer glass/epoxy $[0/+45/-45/60/-60]_s$ laminated plate. Similar to the single-ply glass/epoxy plate, it can be found from Figure 6 that the response of displacement component in the x direction is sensitive to the change of a number of the

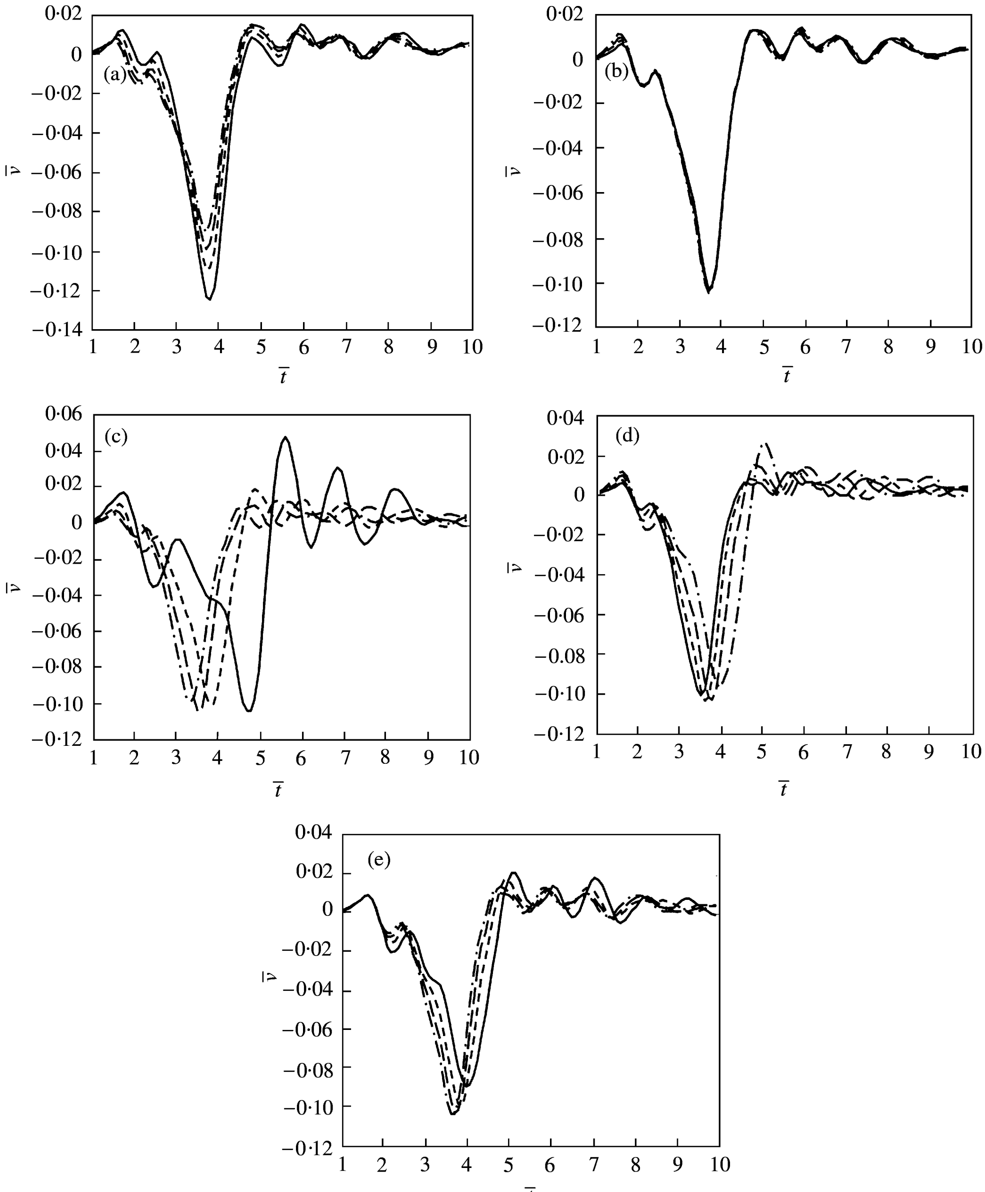

Figure 7. Time history of displacement response in the x direction at $x = 3{\cdot}0H$ on the upper surface of single-ply glass/epoxy plate excited by an incident vertical wave of one cycle sine function at $x = 0{\cdot}0$. (a) ——, 30; – – –, 38; — —, 46; — · ·, 54. (b) ——, 4·0; – – –, 5·33; — —, 6·66; — · ·, 8·0. (c) ——, 10·0; – – –, 12·67; — —, 15·33; — · ·, 18·0. (d) ——, 5·0; – – –, 6·33; — —, 7·66; — · ·, 9·0. (e) ——, 2·0; – – –, 2·67; — —, 3·33; — · ·, 4·0.

elastic constants, but insensitive to the change of the other elastic constants. The same phenomena can be found from Figure 7 for the displacement component in the y direction. It is natural to expect that a combination of these two components of displacement responses may be sensitive to the change of all the elastic constants of the laminated plate. From the combination of Figures 6 and 7, it can be observed that there is a "special region", which results from the change of the elastic constants. The significant change occurs both in amplitude and pattern of the response curve within this region. The effect of the change of the elastic constants on the displacement response is thus obviously reflected. It is now necessary to decide what special information from the displacement responses is to be included in the input training sample. In this paper, two components of displacement responses are used as the input of the NN model, one component is displacement responses in the x direction subjected to a vertical line load in the z direction, another is displacement responses in the y direction subjected to a shear load in the y direction. Considering the features of Figures 6 and 7, the special region is located in the time duration [2·35, 3·25]; thus, we select the combination of responses of two displacement components of five points in the time duration [2·35, 3·25], namely $\bar{t} = 2{\cdot}35, 2{\cdot}89, 2{\cdot}98, 3{\cdot}16, 3{\cdot}25$ as the inputs.

4.2. SIMULATED MEASUREMENT OF DISPLACEMENT RESPONSES

Instead of carrying out the actual experiment, the measured displacement responses are simulated using the HNM with the actual elastic constants. In order to simulate the measured displacement responses, noise-contaminated displacement responses are also used for determination of elastic constants. A Gauss noise is directly added to the computer-generated displacement responses to simulate the noise-contamination. A vector of pseudo-random number is generated from a Gauss distribution with mean a and standard deviation b using Box–Muller method [24]. In this work, the mean a is set to zero, and the standard deviation b is defined as [25]

$$b = 0{\cdot}01 \times \left[1/N \left(\sum_{i=1}^{N} u_i^m \right)^2 \right]^{0 \cdot 5}, \tag{14}$$

where u_i^m is the computer-generated displacement reading at the ith sample point.

4.3. TRAINING SAMPLES

The training samples for the initial training of the NN model consists of a number of sets of inputs and outputs. These training samples should cover all possible values of elastic constants. Obviously, it is impossible to generate all the combinations of elastic constants, and hence a good cross-section of possible alterations is required.

In this paper, we proposed a method combining the orthogonal array with random selection for generating training samples. The orthogonal array is a method developed for the experimentalist to reduce the number of experimental trials normally required in a full factorial experimental design. It allows an experimentalist to analyze the average change in factor levels under different sets of experimental conditions. We adopt this orthogonal array method for the selection of part of the training samples. Assuming that there is no interaction among the q parameters to be identified, the number of samples required based on the orthogonal array [26] for q parameters with p levels is $q(p-1)+1$. This sample number is significantly smaller than the complete sample number of p^q. To further reinforce the sample set, another sample group is created from a random selection of the parameters.

For this problem, a search range of $\pm 30\%$ off from the actual value of elastic constants is used, and shown in Table 1. To formulate the initial training samples, it was assumed that there were four levels of change in the search range for these five elastic constants, which correspond to $c_{11}, c_{12}, c_{22}, c_{23}$ and c_{55} of their discrete values. Based on the orthogonal array method, these five four-level parameters would only require 16 samples to cover the whole domain. In addition, 21 other samples created randomly were added into the training

TABLE 1

The search range for the elastic constants to be reconstructed for glass/epoxy laminated plates

Elastic constants	Actual data (GPa)	Search range (GPa)
c_{11}	42·020	30–54
c_{12}	6·067	4–8
c_{22}	13·500	10–18
c_{23}	7·277	5–9
c_{55}	3·410	2–4

data set. This combined strategy covers a good cross-section of all possible elastic constants variations.

4.4. NORMALIZATION OF TRAINING DATA SETS

The NN model requires the normalization of the input and the output data. As the sigmoid transfer function is used in the BP algorithm, the system cannot actually reach its extreme values of 0 and 1 without infinitely large weights. However, it is found better, in practice, to normalize the input patterns as well as output patterns between 0·1 and 0·9 [27]. The inputs of the training samples are normalized linearly based on the following formulas:

$$\bar{x}_i = \frac{x_i - x_{i\,min} + \varepsilon_1}{x_{i\,max} - x_{i\,min} + \varepsilon_2}, \tag{15}$$

where $x_{i\,min}$ and $x_{i\,max}$ are the minimal and maximal values of the ith input value x_i, respectively, in the sample data set, $\bar{x}_i$ is the normalized value of parameter x ranging between 0·1 and 0·9. The ε_1 and ε_2 are the scaling factors for ensuring that the normalized values would not be close to 0 or 1. The outputs can be normalized in exactly the same way.

4.5. RESULTS AND DISCUSSIONS

Two sets of elastic constants of the $[0/+45/-45/60/-60]_s$ glass/epoxy laminated plates are determined using the present procedure. One set is the actual value listed in the second column in Table 2. Another is an assumed arbitrary set of elastic constants from the search range given for the first set, which is listed in the second column in Table 3. An NN model is built for the determination of elastic constants of the actual $[0/+45/-45/60/-60]_s$ glass/epoxy laminated plate using training samples generated based on the range of actual elastic constants. The NN model is then used to determine both the set of actual elastic constants and the set of arbitrary elastic constants from the search range for the actual elastic constants to validate the stability of the present procedure. The displacement responses of the sample points on the upper surface of these two sets are calculated using the HNM and used as inputs to the NN model. In order to simulate the measured displacement responses, noise-contaminated inputs are used. The two types of displacement responses on the five points in the time history are used as inputs, which are calculated using HNM, and all the values of inputs and outputs in the training samples are normalized according to equation (15).

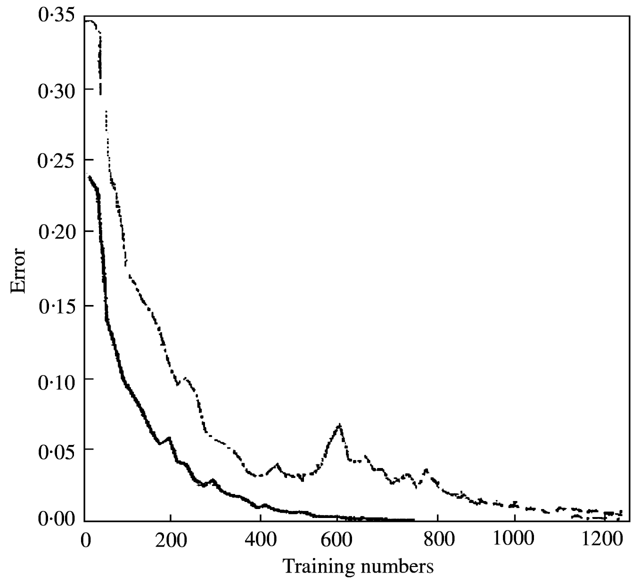

Figure 8. The comparison of convergence of the error norm for the initial training: ——, modified BP; ·—··—, BP.

For comparison and validation of the presented modified BP algorithm, Figure 8 shows the convergence of the error norm for the NN model during the initial training stage. It is found that the convergence of the conventional BP algorithm is not smooth and the convergence is slower than that of the modified BP algorithm.

Table 2 summarizes the reconstructed results of the elastic constants for the first case. The results for six progressions are listed. It can be found that the result at the first progression is not accurate as the maximum deviation is high, and the displacement responses corresponding to these reconstructed elastic constants are quite different from the simulated ones using the actual values of elastic constants. A retraining for the NN model is required. A new sample is created from the first determined result and the corresponding displacement responses calculated from the HNM. The new sample is added into the original sample pool to replace the sample with large distance norm. The retraining process is repeated until the displacement responses corresponding to the reconstructed elastic constants are sufficiently close to the simulated measurements. Figure 9 shows the summary of training or retraining error norm of the progressive NN model for determining the elastic constants. The results at stages of progressive training are also listed in Table 2. It can be seen from Table 2 that the accuracy of the determined results increases as the progression number increases, and the determined result is very accurate after six progressions. The maximum deviation of the sixth progression elastic constants is as low as 5%. It can also be found that the determined result remains stable regardless of the presence of the noise, and the required number of progression is not changed, even when the noise is added.

Another set of elastic constants of glass/epoxy $[0/+45/-45/60/-60]_s$ laminated plate is also reconstructed. The result for this case is shown in Table 3. It can be found that very accurate results can be obtained after six progressions, even though the maximum deviation of the first reconstructed elastic constants is as large as 17·2%. Compared to the first case, the maximum deviation of the first determined elastic constants is bigger but accurate results can still be obtained. This can be explained as follows. The training samples are selected based on a range of elastic constants for the first case; this set of samples is not the

TABLE 2

Reconstructed results of elastic constants of $[0/+45/-45/60/-60]_s$ *glass/epoxy laminated plate* (*case* 1)

Elastic constants	Original value (GPa)	Result (deviation) at progressions					
		1	2	3	4	5	6
(a) *Noise free*							
c_{11}	42·020	41·311 (− 1·7%)	41·642 (− 0·9%)	42·991 (2·3%)	42·990 (2·3%)	42·930 (2·2%)	42·930 (2·2%)
c_{12}	6·067	5·395 (− 11·1%)	6·695 (10·4%)	6·590 (8·6%)	6·520 (7·5%)	6·470 (6·6%)	6·275 (3·4%)
c_{22}	13·500	12·950 (− 6·8%)	14·230 (5·4%)	14·020 (3·9%)	13·600 (0·7%)	13·871 (2·7%)	13·760 (1·9%)
c_{23}	7·277	7·920 (12·3%)	7·871 (8·2%)	7·883 (8·2%)	7·635 (4·9%)	7·655 (5·2%)	7·535 (3·6%)
c_{55}	3·410	3·745 (9·8%)	3·490 (2·4%)	3·535 (3·7%)	3·620 (6·2%)	3·571 (4·6%)	3·573 (4·8%)
(b) *Noise added*							
c_{11}	42·020	40·980 (− 2·5%)	43·564 (3·7%)	42·720 (1·7%)	42·720 (1·7%)	42·510 (1·2%)	42·360 (0·8%)
c_{12}	6·067	5·355 (− 11·7%)	6·985 (15·1%)	6·555 (8·0%)	6·545 (7·9%)	6·440 (6·2%)	6·235 (2·8%)
c_{22}	13·500	12·980 (− 3·9%)	13·930 (3·2%)	14·010 (3·8%)	13·51 (0·0%)	13·910 (3·0%)	13·780 (2·1%)
c_{23}	7·277	7·995 (9·9%)	7·565 (4·0%)	7·943 (9·1%)	7·455 (2·5%)	7·760 (6·6%)	7·640 (4·9%)
c_{55}	3·410	3·750 (10·0%)	3·390 (− 0·6%)	3·550 (4·1%)	3·575 (4·9%)	3·573 (4·9%)	3·578 (4·9%)

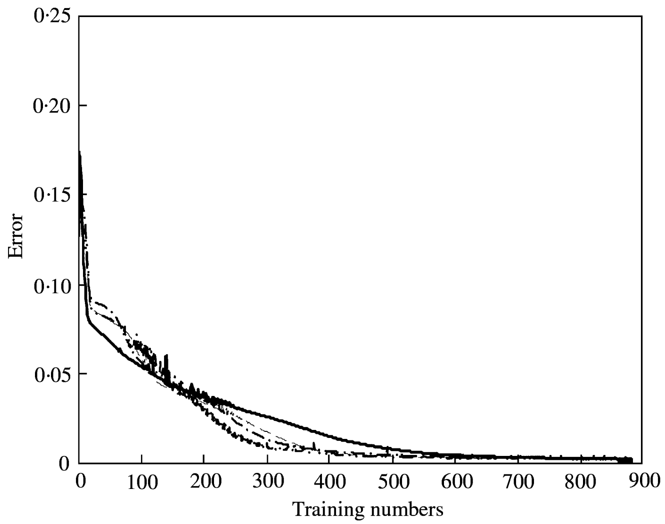

Figure 9. Summary of the first-four training error norm of the progressive NN model for determination of the first set of elastic constants of the $[0/+45/-45/60/-60]_s$ glass/epoxy laminated plate: ——, progression 1; – – –, progression 2; — · ·, progression 3; ·—··—, progression 4.

best suitable sample sets for the second case. However, after several progressive trainings, the sample density around the simulated measurement of displacement responses increases until the desired accuracy.

For a 6-time progressive NN model, the forward HNM solver has only been called for 42 times, compared to about 2500 times using the GA to solve the same problem. It can be clearly concluded that this NN model for the determination of elastic constants is very efficient. This advantage of NN model is extremely significant if the forward solver requires longer CPU time for a single run.

It should be noted that the presented training algorithm is still slow and computationally expensive even though it has been modified. Using modern network architectures and training algorithms may give further improvements in the presented NN procedures. As an example, reference [10] has shown that the radial basis function (RBF) network with the orthogonal least-squares algorithm can be trained significantly faster than a multi-layer network with BP.

5. A MORE COMPLICATED CASE STUDY

Now, we try to extend the presented NN procedure to a more complicated case considering the same plate as described in section 4, but the material is graphite/epoxy [4]. This material is the orthotropic material; there are nine elastic constants as listed in the second column in Table 4. Hence, there are nine parameters, named as $c_{11}, c_{12}, c_{22}, c_{13}, c_{23}, c_{33}, c_{44}, c_{55}$ and c_{66}, which need to be determined. The search range is illustrated in Table 4.

By exactly following the process described in section 4, the inputs of the model, training samples are obtained. The NN model used in this application has two hidden layers, and the neuron numbers of the input, output, first and second hidden layers are 14, 9, 36 and 28 respectively. There are totally 36 training samples.

TABLE 3

Reconstructed results of elastic constants of $[0/+45/-45/60/-60]_s$ *glass/epoxy laminated plate* (*case* 2)

Elastic constants	Original value (GPa)	Result (deviation) at progressions					
		1	2	3	4	5	6
(a) *Noise free*							
c_{11}	50·000	51·48 (3·0%)	50·430 (0·9%)	49·022 (2·0%)	48·990 (2·0%)	49·680 (0·6%)	50·040 (0·0%)
c_{12}	5·00	4·167 (16·7%)	4·995 (−0·1%)	4·560 (8·8%)	4·612 (7·8%)	4·650 (7·0%)	4·843 (3·2%)
c_{22}	12·00	11·370 (−5·3%)	13·000 (8·3%)	12·370 (3·1%)	12·070 (0·6%)	12·011 (0·1%)	12·256 (2·1%)
c_{23}	5·50	5·710 (3·8%)	5·971 (8·9%)	5·810 (5·6%)	5·525 (0·5%)	5·525 (0·5%)	5·710 (3·8%)
c_{55}	2·50	2·803 (12·1%)	2·420 (3·2%)	2·513 (4·0%)	2·440 (2·4%)	2·518 (0·7%)	2·528 (1·1%)
(b) *Noise added*							
c_{11}	50·00	51·300 (2·6%)	50·220 (0·4%)	48·810 (2·4%)	48·810 (2·4%)	49·590 (1·2%)	50·04 (0·0%)
c_{12}	5·00	4·140 (−17·2%)	4·935 (1·3%)	4·510 (9·8%)	4·585 (8·3%)	4·635 (6·2%)	4·805 (3·9%)
c_{22}	12·00	11·390 (−5·1%)	13·050 (8·8%)	12·321 (2·7%)	12·100 (0·8%)	12·070 (3·0%)	12·260 (2·2%)
c_{23}	5·50	5·800 (5·5%)	6·115 (11·2%)	5·880 (6·9%)	5·610 (2·0%)	5·625 (2·3%)	5·780 (5·0%)
c_{55}	2·50	2·815 (12·6%)	2·438 (2·5%)	2·525 (1·0%)	2·453 (1·9%)	2·525 (1·0%)	2·358 (1·5%)

TABLE 4

The search range for the elastic constants to be reconstructed for graphite/epoxy laminated plates

Elastic constants	Actual data (GPa)	Search range (GPa)
c_{11}	144·00	115–172
c_{12}	5·47	4·3–6·5
c_{22}	13·60	10·0–16·0
c_{13}	5·00	4·0–6·0
c_{23}	7·00	5·6–8·4
c_{33}	12·00	9·6–14·4
c_{44}	3·70	2·9–4·4
c_{55}	6·00	4·8–7·2
c_{66}	6·50	5·2–7·8

TABLE 5

Reconstructed results of elastic constants of $[0/+45/-45/+60/-60]_s$ *graphite/epoxy laminated plate*

Elastic constants	Original value (GPa)	Result (deviation) at progressions		
		1	2	3
c_{11}	144·00	149·762 (4·0%)	145·295 (0·9%)	144·10 (0·0%)
c_{12}	5·47	5·934 (8·5%)	5·404 (− 1·2%)	5·306 (− 3·0%)
c_{22}	13·60	12·988 (− 4·5%)	13·288 (− 2·3%)	13·437 (− 1·2%)
c_{13}	5·00	5·3244 (6·5%)	5·078 (1·6%)	5·072 (1·4%)
c_{23}	7·00	6·607 (− 5·6%)	7·302 (4·3%)	6·891 (− 1·6%)
c_{33}	12·00	12·289 (2·4%)	12·384 (3·2%)	12·147 (1·2%)
c_{44}	3·70	3·570 (− 3·5%)	3·715 (0·4%)	3·859 (4·3%)
c_{55}	6·00	5·543 (− 7·6%)	6·396 (6·6%)	6·040 (0·7%)
c_{66}	6·50	6·096 (− 6·2%)	6·227 (− 4·2%)	6·543 (0·7%)

Table 5 summarizes the reconstructed results of the elastic constants for this complicated case. It is found that very accurate results can be obtained after three progressions. However, it should be pointed out here that the following limitations remain in these results. As shown in Table 4, the search range of $\pm 20\%$ off from the actual value is used in this case. This range is smaller than the range of $\pm 30\%$ off from the actual value in section 4. As in our study, we cannot obtain the convergent solution in the search range of $\pm 30\%$, e.g., the accuracy of the characterized results does not increase as the progression number increases. It was found that this phenomenon is more obvious for the elastic constants relating to the shear performance. This is due to the strong anisotropy of the material. In addition, we cannot get accurate results using the noise-contaminated displacement responses even by using the search range of $\pm 20\%$ off from the actual value.

For a successful extension of the presented procedure to more complicated cases, e.g., the wide range of parameters and more parameters, the forward solver is directly applicable. However, this inputs, training samples, the structures of the NN model as well as the learning algorithm should be given individual consideration for individual applications.

6. CONCLUSION

A progressive NN procedure is proposed for the determination of elastic constants of anisotropic laminated plates, using dynamic displacement responses on the surface as the input data. In this procedure, the HNM is employed as a forward solver to calculate the displacement responses on the surface of the laminated plates. The NN model is trained and retrained progressively using the calculated result from the HNM. A modified BP algorithm, which can overcome possible saturation of the sigmoid function and speed up the training process, is adopted as the learning algorithm. The concept of orthogonal array is proposed to generate the representative combinations of elastic constants, which reduces significantly the number of training data while maintaining its data completeness. After several progressions of retraining the NN model, the elastic constants can be determined from the retrained NN model, by feeding in the simulated measurement of dynamic displacement responses. The determined result is stable regardless of the presence of the noise. The accuracy of output from the NN model increases with the increase in the number of retraining cycles, the required accuracy can therefore be obtained by repeating the retraining process.

This paper mainly addressed the computational method of the material characterization using elastic waves and NNs. The practical application of the presented NN procedure relies on the tests of real structures, and the NN procedure should be validated using practical experimental results. However, it should be noted that it is often very difficult to train an NN model for the complex relationship between material property and the dynamic responses to be valid in a wide range of parameters and for more parameters.

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