Journal of Sound and Vibration (2002) **252**(4), 729–737
doi:10.1006/jsvi.2001.3943, available online at http://www.idealibrary.com on IDEAL®

SURFACE QUALITY OF A WORK MATERIAL'S INFLUENCE ON THE VIBRATIONS OF THE CUTTING PROCESS

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(*Received* 20 *October* 2000, *and in final form* 3 *April* 2001)

The problem of stability in the machining processes is very important and is strictly connected with the final quality of the product. In this paper, the vibrations of a tool–workpiece system induced by random disturbances in a straight turning process, and their effect on a product surface is considered. Based on experimentally obtained system parameters simulations are provided using a one-degree-of-freedom model. Noise has been introduced into the model by the Langevin equation. The product surface shape and its dependence on the level of noise has also been analyzed.

1. INTRODUCTION

The quality of the final surface of a workpiece due to a cutting process is of natural interest within industry and technology. Grabec [1, 2] and Gradisek *et al.* [3] analyzed a simple orthogonal cutting model and found that chaotic conditions in the tool–workpiece system due to appropriate system parameters are clearly possible. As they have demonstrated the appearance of such chaotic conditions can have a crucial effect on the stability of the cutting process. Chaotic vibrations were also investigated experimentally by Tansel *et al.* [4]. On the other hand instabilities in the cutting process have, for a long time, been known as a chatter phenomenon [5–7]. The mechanism of their appearance includes the effect of non-linear self-excitation during the cutting process, leading to vibrations with a larger value of amplitude often beyond the admissible limit. One of the sources of instability can be identified in the roughness of the initial surface of a material, which introduces a randomness into the material resistance during the dynamic process. Wiercigroch and Cheng [8] have investigated the influence of noise on the orthogonal cutting system. In their analysis, they started from the spectral representation of stochastic process. Nevertheless, the most common treatment of dynamical processes influenced by noise is the Fokker–Plank approach [9–11]. Because of considerable difficulties which are met by solving higher dimensions, as well as for numerical reasons, the problem could be transformed to the corresponding Langevin equations [10, 11]. Here, following references [11–14] we use the stochastic Langevin equations with an additive white noise and then solve the dynamic equations of the system. Previous articles [8, 11, 13] devoted to the cutting process in the presence of noise focussed rather on the problem of the dynamics and the possibilities of bifurcations which can be induced by random disturbances. Interestingly,

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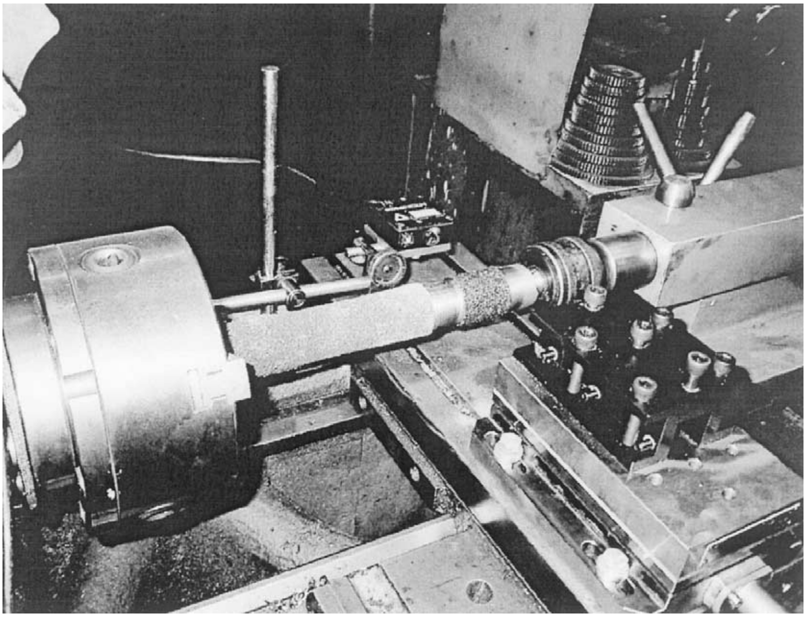

Figure 1. Experimental standing with a cast iron workpiece.

Wiercigroch and Cheng [8], and later Przystupa and Litak [11], investigated the orthogonal cutting process with two degrees of freedom and claimed that in some conditions weak noise can even act to stabilize the chaotic attractor. On the other hand, reference [14] deals with the reconstruction of the system dynamics from the stochastic time series. In that treatment, the cutting process was assumed to be deterministic, but the measured data were influenced by noise coming from the measurement procedure itself.

This new paper is also a contribution to the complicated problem of the dynamics of the cutting process but focusses on the final quality of the product surface, and in this context, on the stability of the process as well. Note that in Figure 1 the shape of an initial surface of a cut workpiece is shown. The shape has numerous imperfections which are modelled by means of random deviations from an ideal cylindrical surface. Adopting a simple one-degree-of-freedom model for regenerative cutting [15] the effect of a previous pass of a straight turning process has been included by means of a time delay term [16].

2. DETERMINISTIC MODEL OF CUTTING PROCESS

The physical model of a straight turning process corresponding to the experimental system used is presented in Figure 2. Here we have introduced the following notations: v_f is the relative velocity between the tool and the workpiece, h_0 is an assumed initial depth whilst h is the actual cutting depth, w is the principal axis of relative vibrations, y indicates the direction normal to the axis of workpiece symmetry, κ is the tool cutting edge angle; k and c are the stiffness and damping coefficients of the system, respectively, n denotes the rotational velocity of the workpiece, f denotes the direction of feed in straight turning, and m is the effective mass of the system.

The main vibration is in the w-direction, perpendicular to the cutting edge (Figure 2) and to be precise one should analyze the vibrations as well as the cutting force in the w direction.

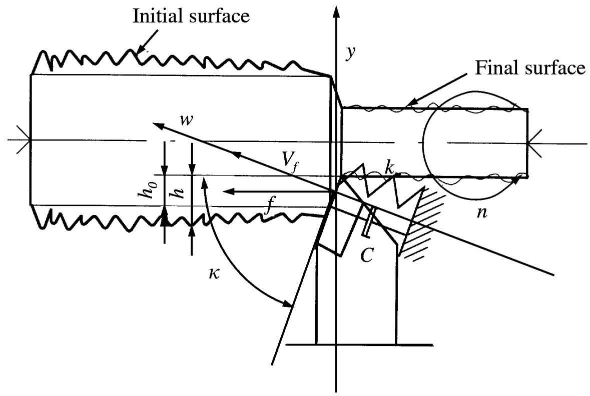

Figure 2. Physical model of a straight turning process.

However, the interest is in the final surface profile given by the time history of y, and not by the actual profile w. To analyze the vibration in the y direction we have simultaneously projected the vibrations and forces into the y direction. Thus the deterministic equation of motion of the dynamical system, projected onto the normal (to final surface) direction y, can be written as follows [15]:

$$\ddot{y} + 2\tilde{n}\dot{y} + p^2 y = \frac{K}{m} g_y(h, v_f), \tag{1}$$

where p is the natural frequency of free vibrations of the workpiece, $p^2 = k/m$, whilst $2\tilde{n} = c/m$. Non-linearities appearing in that system are included in the g_y function [1–3, 8, 15]

$$g_y(h, v_f) = \left[c_2\left(\left|\frac{v_f}{v_0}\right| - 1\right)^2\right]\left[c_3\left(\frac{h}{h_0} - 1\right)^2 + 1\right]\frac{h}{h_0}\Theta(h)\mathrm{sgn}(v_f). \tag{2}$$

The cutting depth h and relative velocity v_f are defined [14, 15] by

$$h = h_0 + y(t') - y(t), \quad v_f = 1 - \frac{\dot{y}}{v_0}. \tag{3}$$

$\Theta(h)$ and $\mathrm{sgn}(v_f)$ correspond to step functions, namely the Heaviside and sign functions, respectively, and v_0 is the linear velocity of the rotational motion of the workpiece during the steady cutting process.

t' is the time of a previous pass

$$t' = t - \Delta t, \tag{4}$$

where Δt is the workpiece revolution time during machining. The shape of the non-linear function g_y (equation (2)), dependent on h and v_f, is presented in Figure 3. Note that the two-dimensional surface $g_y = g_y(h, v_f)$ was plotted only for positively defined h. In the case of negative h, the force on the left side of equation (1), is zero because of the contact loss between the tool and the workpiece. The sudden sign change of the cutting force, as a function of the relative velocity v_f, is due to a frictional phenomenon between the tool and the chip.

The model is equations (1)–(3) with one-degree-of-freedom is a serious simplification of a physical situation, however the aim is not to provide a comprehensive description of the

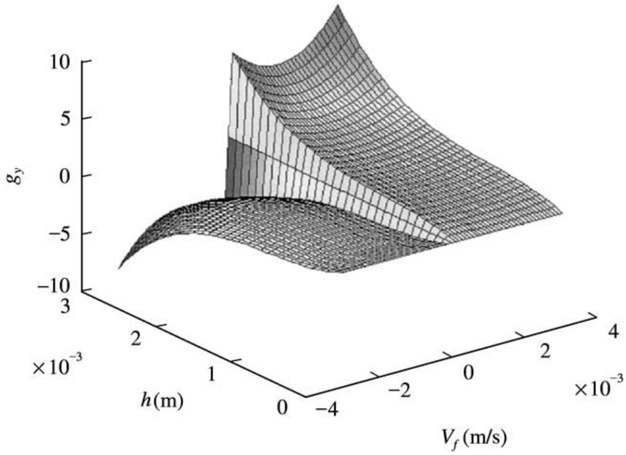

Figure 3. Non-linear function $g_y(h, v_f)$ versus cutting relative velocity.

cutting process but to concentrate on particular aspects of it. In spite of the simplicity of the model the chatter vibrations can still be generated due to the non-linearities in the cutting force $g_y(h, v_f)$ as was shown in reference [15]. In the model in this paper chatter is generated by a combination of the friction phenomenon between the tool and chips, and the impact of the tool after it loses contact with the workpiece. Warmiński *et al.* [15] examined the second pass of the orthogonal cutting process by using a similar model and the results obtained have indicated that such a model can lead to periodic, quasi-periodic, as well as chaotic vibrations, due to the initial harmonic modulation of the machined surface.

For the purpose of numerical calculation the equations (1)–(4) have been written in a discrete way by introducing the constant time step τ:

$$
\begin{aligned}
t_{r+1} &= t_r + \tau, \\
y_{r+1} &= y_r + v_r \tau, \\
v_{r+1} &= v_r + \left(-2\tilde{n} v_r - p^2 y_r + \frac{K}{m} g_{yr} \right) \tau,
\end{aligned} \tag{5}
$$

where t_r is a discrete sampling time after r time steps. The function g_{yr} should be expressed as

$$
\begin{aligned}
g_{yr} &= g_y(h_r, v_{fr}), \\
h_r &= h_0 + y_s - y_r, \\
v_{fr} &= 1 - \frac{v_r}{v_0},
\end{aligned} \tag{6}
$$

where r and s are natural numbers. The time difference between the y_r and y_s co-ordinates; $\Delta t = (r - s)\tau$ relates to the time of the workpiece revolution (equation (4)). The system parameters obtained from the experiment are as follows $p = 785$ rad/s, $m = 12{\cdot}1$ kg, $K = 620$ N, $h_0 = 1{\cdot}5 \times 10^{-3}$ m, $f = 0{\cdot}1 \times 10^{-3}$ m/rev, $2\tilde{n} = 190$ 1/s, $v_0 = 0{\cdot}1$ m/s, $\kappa = 70°$, and $c_2 = 0{\cdot}5$, $c_3 = 1{\cdot}55$ are cutting process constants derived from references [7, 15].

3. INFLUENCE OF NOISE

Most real dynamical processes are disturbed by random signals. In the case of the cutting process they come through the roughness of the initial surface (Figure 1). Other sources of disturbance can be found in the spontaneous breaking of chips and the couplings of the tool and the workpiece to other dynamic parts of the experimental system.

To describe the stochastic system, a random component is introduced to the model by means of an additive white noise of Gaussian distribution [10–14]. Usually, stochastic dynamic systems are investigated by using the Fokker–Planck equation [8–11]. The one-dimensional version of this is

$$\frac{\partial}{\partial t} P(y, t) = \frac{\partial}{\partial y}[v(y)P(y, t)] + D\frac{\partial^2}{\partial y^2} P(y, t), \tag{7}$$

where D denotes the diffusion coefficient, $v(x)$ is, in general, the non-linear drift term (driving force) and $P(x, t)$ is the probability distribution function. Solving the Fokker–Plank equation involves some considerable difficulties [10, 11] and so it has been transformed into the corresponding Langevin equation

$$\dot{y} = z(y) + g\Gamma(t), \tag{8}$$

where $g\Gamma(t)$ is a Gaussian distributed random "force" with strength g, and $\Gamma(t)$ is assumed to satisfy

$$\langle \Gamma(t) \rangle = 0,$$

$$\langle \Gamma(t), \Gamma(t') \rangle = 2\delta(t - t'), \tag{9}$$

where the brackets denote an average over the probability distribution function. Starting with the definitions of the drift v, and diffusion D, coefficients obtained via the Kramers–Moyal expansions in the derivation of the Fokker–Plank equation from the Chapman–Kolmogorov equation [17], one can find the relation between $v(y)$ and D of the Fokker–Plank equation (equation (7)) with $z(y)$ and g of the Langevin equation (equation (8)). Thus the Langevin equation can finally be expressed in terms of the drift and diffusion terms of the initial Fokker–Plank equation as

$$\dot{y} = v(y) + \sqrt{D}\hat{\Gamma}(t). \tag{10}$$

For the actual numerical calculation the discretized forms:

$$y(t + \tau) - y(t) = \int_t^{t+\tau} dt' v(t') + \sqrt{D}\int_t^{t+\tau} dt'\Gamma(t') \approx v(t)\tau + \sqrt{D}\hat{\Gamma}(t), \tag{11}$$

where

$$\hat{\Gamma}(t) = \int_t^{t+\tau} dt'\Gamma(t') \tag{12}$$

is a superposition of Gaussian-distributed random numbers, which again are of a Gaussian form. Namely

$$\hat{\Gamma}(t) = a\omega(t). \tag{13}$$

In the present case, the average value of ω has been chosen to be $\langle\omega\rangle = 0$ while its variance is $\langle\omega^2\rangle = 2$ respectively. From the integration of the Gaussian function of equation (12) the coefficient a (equation (13)) depends on the time integration step τ via $a = \sqrt{\tau}$. Equation (8) can, in general, be solved by higher order algorithm such as Runge–Kutta [18]. However, for simplicity, here we have limited our discussion to the simplest type of Euks algorithm. Thus, the final form of the Langevin equation at the lowest order of perturbation, case as suitable for numerical integration is given by the following expression:

$$y_{r+1} = y_r + v(y_r)\tau + \sqrt{D}\sqrt{\tau}\omega(t_r). \tag{14}$$

Here the one-dimensional version of the Langevin and Fokker–Plank equations has been analyzed, but a similar discussion on the m-dimensional stochastic Langevin equation, and its relation to the corresponding Fokker–Plank equation, can be found in reference [14].

In the model of this paper the equation for y_r has to be supplemented by equations (5), (6) for discrete time t_r and velocity v_r, which is now substituted by the corresponding drift term v_r:

$$v_{r+1} = v_r + \left(-2\tilde{n}v_r - p^2 y_r + \frac{K}{m} g_{yr}\right)\tau. \tag{15}$$

Using the above procedure (equations (4), (5), (14), (15)) the simulations for a constant time step $\tau = 0{\cdot}741 \times 10^{-4}$ s have been completed. These correspond to the workpiece revolution time $\Delta t = 0{\cdot}741 \times 10^{-1}$ s, and a number of diffusion constant values D.

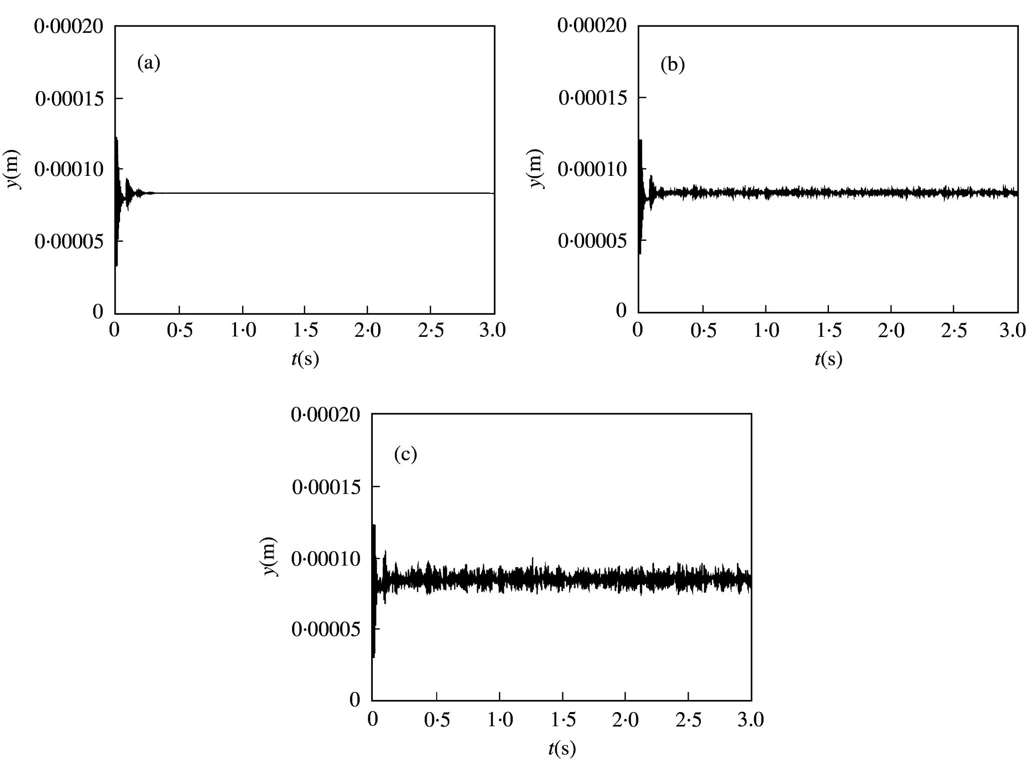

Figure 4. Time histories of y for various values of diffusion constants (a) $D = 0$, (b) $D = 10^{-5}$, (c) $D = 10^{-4}$.

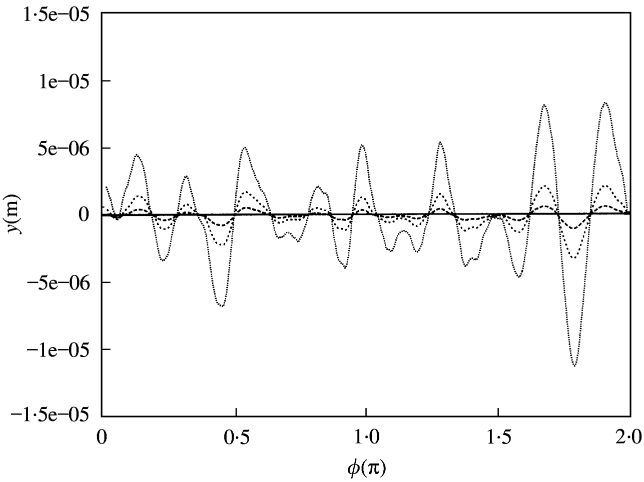

Figure 5. Shape error as a function of workpiece rotation angle after 3 s of cutting. $D = 0$, ——; $D = 10^{-6}$ - - - - -; $D = 10^{-5}$, · · · · · ·; $D = 10^{-4}$, ...

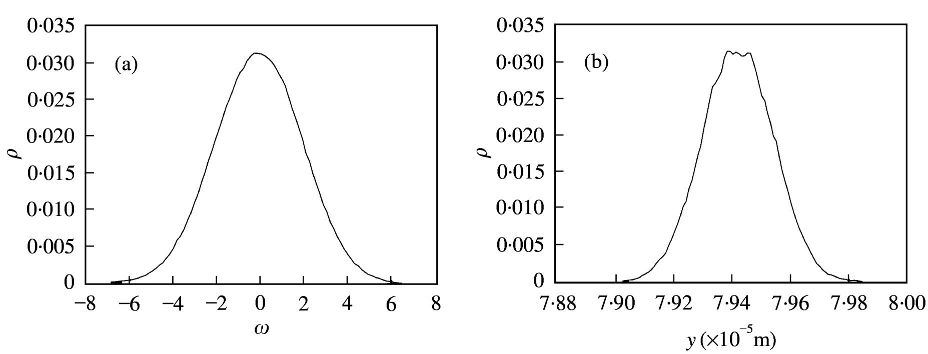

Figure 6. Distribution probabilities of random input (a) and output y (b) signals of the model for $D = 10^{-5}$.

Figures 4(a)–4(d) show the time histories of y for the initial 3 s of cutting work for experimentally identified system parameters (section 2). For a deterministic system ($D = 0$, Figure 4(a)), the stable cutting process is observed with no vibrations. Figures 4(b) and 4(c) relate to the cutting process in the presence of noise. The diffusion constant values for these figures are $D = 10^{-5}$ and 10^{-4} respectively. One can see easily that the presence of small vibrations (Figure 4(b)), which grow with an increase in the noise level (Figure 4(c)). Obviously, such vibrations have a significant effect on the quality of the workpiece surface. It is shown in Figure 5 that the error shape of the surface is plotted as a function of the workpiece rotation angle after 3 s of cutting work.

The modulation of the shape caused by random disturbances depends on the noise level. Both input and output random signals can easily be measured by means of standard deviations. For various values of D such as 10^{-6}, 10^{-5} and 10^{-4} the following values of standard deviations arise $\sigma = 3{\cdot}79 \times 10^{-7}$, $1{\cdot}21 \times 10^{-6}$ and $4{\cdot}16 \times 10^{-6}$ m respectively. In Figures 6(a) and 6(b), the input and output random signals are compared for one of the above cases ($D = 10^{-5}$). Figure 6(a) shows the distribution of Gaussian disturbances of input noise ω (equations (8), (9)) whilst Figure 6(b) corresponds to the errors in the

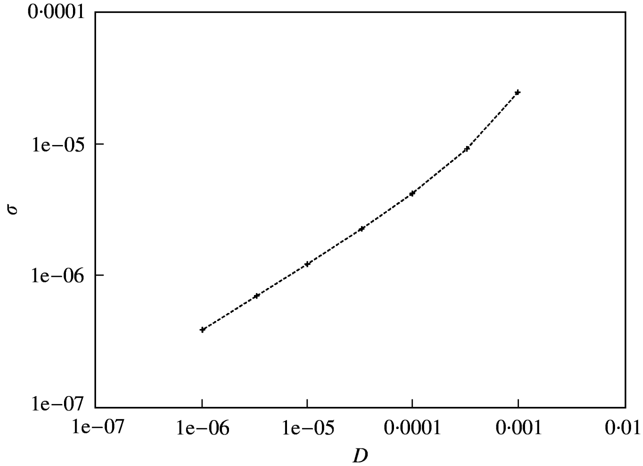

Figure 7. Standard deviation σ as a function of a diffusion constant D.

workpiece shape after cutting. In Figure 6(b) the deviation from the normal probability distribution is caused by the non-linear dynamics of the cutting process. To quantify the system the standard deviations of the output signal σ are used as a function of performance diffusion constant D. This is plotted on a logarithmic scale in Figure 7. It has been checked that $\sigma(D)$ can be scaled as the square root as long as the noise level is low, whilst for a stronger noise the effect on the fluctuations of y is more pronounced.

4. SUMMARY AND CONCLUSIONS

The vibrations of a tool–workpiece system have been considered for a straight turning process induced by random disturbances, and their effect on the product surface. Using a single-degree-of-freedom model the combined effects of the friction non-linearities and tool–workpiece contact less have been considered. It has been noticed that for a large enough level of noise the tool and workpiece start to vibrate due to random forcing. Such excitations can interact with the complex dynamics of the system and lead to process in stability, and, in the end, to a much worse final quality in the machined product. In the case of a relatively small level of noise (a small value of the diffusion constant D), the surface shape error scales as the square root of D. For a higher value of noise level the shape error is proportional to D. Clearly for straight turning the initial surface roughness influences the quality of the final product. This is the principal result of this paper which leads to the conclusion that one has to prepare the workpiece so that the initial surface satisfies the appropriate criteria.

ACKNOWLEDGMENTS

The work has been partially supported by the Polish State Committee for Scientific Research (KBN) under the grant No. 126/E-361/SPUB/COST/T-7/DZ 42/99. The authors would like to thank the organizers of the 3rd International Symposium entitled "Investigation of Nonlinear Dynamic Effects in Production Systems" 26–27 September 2000, in Cottbus (Germany), for giving two of them (J.L. and R.R.) the opportunity to present this work. They would also like to thank the unknown referee for some valuable comments and Dr W. Przystupa for helpful discussions.

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