Journal of Sound and Vibration (2002) **253**(2), 507–517
doi:10.1006/jsvi.2001.4010, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

NATURAL FREQUENCIES OF FLUID CONVEYING TENSIONED PIPES AND CARRYING A STATIONARY MASS UNDER DIFFERENT END CONDITIONS

H. R. Öz

Department of Mechanical Engineering, University of Nevada Reno, Reno NV 89557, *U.S.A.*
E-mail: *oz@unr.edu*

(*Received* 23 *July* 2001, *and in final form* 20 *August* 2001)

1. INTRODUCTION

Some examples of axially moving continua are high-speed magnetic tapes, band-saws, power-transmission chains and belts, textile and composite fibers, aerial cable tramways, flexible robotic manipulators with prismatic joints, flexible appendages on spacecraft, paper sheets during processing, pipes and beams conveying fluid, etc. This subject has been studied widely [1–12]. Some studies related to pipes conveying fluid were represented in references [13–19]. Second and fourth order pipes were investigated by neglecting fluid friction effects, the energetics is considered, and the principal parametric resonances of tensioned pipes conveying fluid with harmonic velocity have been investigated. The studies concerning fluid conveying pipes and travelling string or beam with stationary mass can be found in references [20–29].

In this study, the vibrations of highly tensioned pipes with stationary mass are investigated. The fluid velocity is assumed to be constant. Two different end conditions are considered. The flexural stiffness of the pipe is assumed to be negligible. The natural frequencies are analytically presented. Amplitude variations of vibrations are studied. The effects of fluid velocity, position of stationary mass and ratio of fluid and fluid–pipe masses per unit length are investigated.

2. EQUATIONS OF MOTION AND APPROXIMATE SOLUTIONS

The dimensionless equation of motion for tensioned pipe conveying fluid and carrying a stationary mass in transverse vibrations is [29]

$$\{1 + \alpha\delta(x - x_c)\}\,\ddot{w} + 2\sqrt{\beta}v\dot{w}' + (v^2 - 1)w'' = 0 \tag{1}$$

and the fixed–sliding and sliding–sliding end conditions for this problem are [17, 19]

$$w(0, t) = w'(1, t) = 0, \quad w'(0, t) = w'(1, t) = 0. \tag{2, 3}$$

In equations (1)–(3), w is the transverse displacement, $\ddot{w}$, $2\dot{w}'v$ and v^2w'' denote local, Coriolis and centrifugal acceleration components, respectively, v is the constant fluid velocity. The ratio of stationary mass to total mass of pipe and fluid α, and the ratio of mass of fluid to fluid and pipe mass per unit length β, respectively, are

$$\alpha = \frac{m_c}{L(m_f + m_p)}, \quad \beta = \frac{m_f}{m_f + m_p}, \tag{4}$$

where m_c, m_f, m_p denote stationary mass, masses of fluid and pipe per unit length respectively. The length of the pipe is L. A stationary mass placed at $x = x_c$ is defined by a Dirac delta function. The derivatives with respect to the spatial variable and time are shown by $(\ )'$ and $(\cdot)$ respectively.

Solutions of the approximate eigenvalue problem are restricted to systems in which the mass ratio α, is small. The transition of solutions from those of the tensioned pipe conveying fluid to those of the tensioned pipe conveying fluid and carrying a stationary mass system was studied by the method of strained parameters to determine a first order perturbation solution for small α by Öz and Evrensel [29] for fixed–fixed end conditions. The transverse displacement function was given in terms of the shape function and frequency, and expanded in terms of α. In a similar way the downstream and upstream wave numbers without stationary mass can be obtained [17, 19, 29] as

$$\bar{k}_d = \frac{\bar{\lambda}_n^{(0)}}{\sqrt{\beta}v + \sqrt{1 + (\beta - 1)v^2}}, \quad \bar{k}_u = \frac{\bar{\lambda}_n^{(0)}}{-\sqrt{\beta}v + \sqrt{1 + (\beta - 1)v^2}} \tag{5}$$

and the natural frequency equations are for fixed–sliding and sliding–sliding end conditions are, respectively,

$$\bar{\lambda}_n^{(0)} = \frac{(n - 1/2)\pi(1 - v^2)}{\sqrt{1 + (\beta - 1)v^2}}, \quad n = 1, 2, 3, \ldots, \tag{6}$$

$$\bar{\lambda}_n^{(0)} = \frac{n\pi(1 - v^2)}{\sqrt{1 + (\beta - 1)v^2}}, \quad n = 0, 1, 2, 3, \ldots, \tag{7}$$

where n is the mode number and $\mathrm{i} = \sqrt{-1}$. By following the solutions in references [17, 19, 29], the shape functions for fixed–sliding and sliding–sliding end conditions are obtained, respectively, as

$$Y_n^{(0)}(x) = c(\mathrm{e}^{-\mathrm{i}\bar{k}_d x} - \mathrm{e}^{+\mathrm{i}\bar{k}_u x}), \quad Y_n^{(0)}(x) = c\left(\mathrm{e}^{-\mathrm{i}\bar{k}_d x} + \frac{\bar{k}_d}{\bar{k}_u}\mathrm{e}^{+\mathrm{i}\bar{k}_u x}\right) \tag{8}$$

and the perturbed eigenvalue is

$$\lambda_n = \mathrm{i}\bar{\lambda}_n^{(0)} + \alpha \frac{\bar{\lambda}_n^{(0)^2} | Y_n^{(0)} \bar{Y}_n^{(0)} |_{x = x_c}}{2\left\{\mathrm{i}\bar{\lambda}_n^{(0)} \int_0^1 Y_n^{(0)} \bar{Y}_n^{(0)} \,\mathrm{d}x + \sqrt{\beta}v \int_0^1 Y_n^{(0)\prime} \bar{Y}_n^{(0)} \,\mathrm{d}x\right\}}, \tag{9}$$

where $\bar{Y}_n^{(0)}$ and $\bar{Y}_n^{(0)}$ are the shape function and the complex conjugate of it at the first order of perturbation respectively. The imaginary part of the perturbed eigenvalue corresponds to the frequency of oscillations and the real part is related to the amplitude variation of the system with stationary mass. The second part of equation (9) is related to the stationary mass and the imaginary part of it is correction term for the frequencies.

3. NUMERICAL ANALYSIS

In this section, numerical solutions for the natural frequencies and amplitude variations for different end conditions and parameters will be presented.

In Figures 1(a–d), the natural frequency variation with fluid velocity is given for different α (0·1 and 0·2) and x_c (0·2, 0·5 and 0·8) values for fixed–sliding end conditions for the first two modes for $\beta = 0{\cdot}5$. Natural frequencies decrease with an increase in the flow velocity and at the critical velocity divergence instability occurs. This is the characteristics of the axially moving continua. The mass ratio α decreases the frequencies as expected [25, 27–29]. The frequencies are affected much when the stationary mass is moved towards the sliding end in

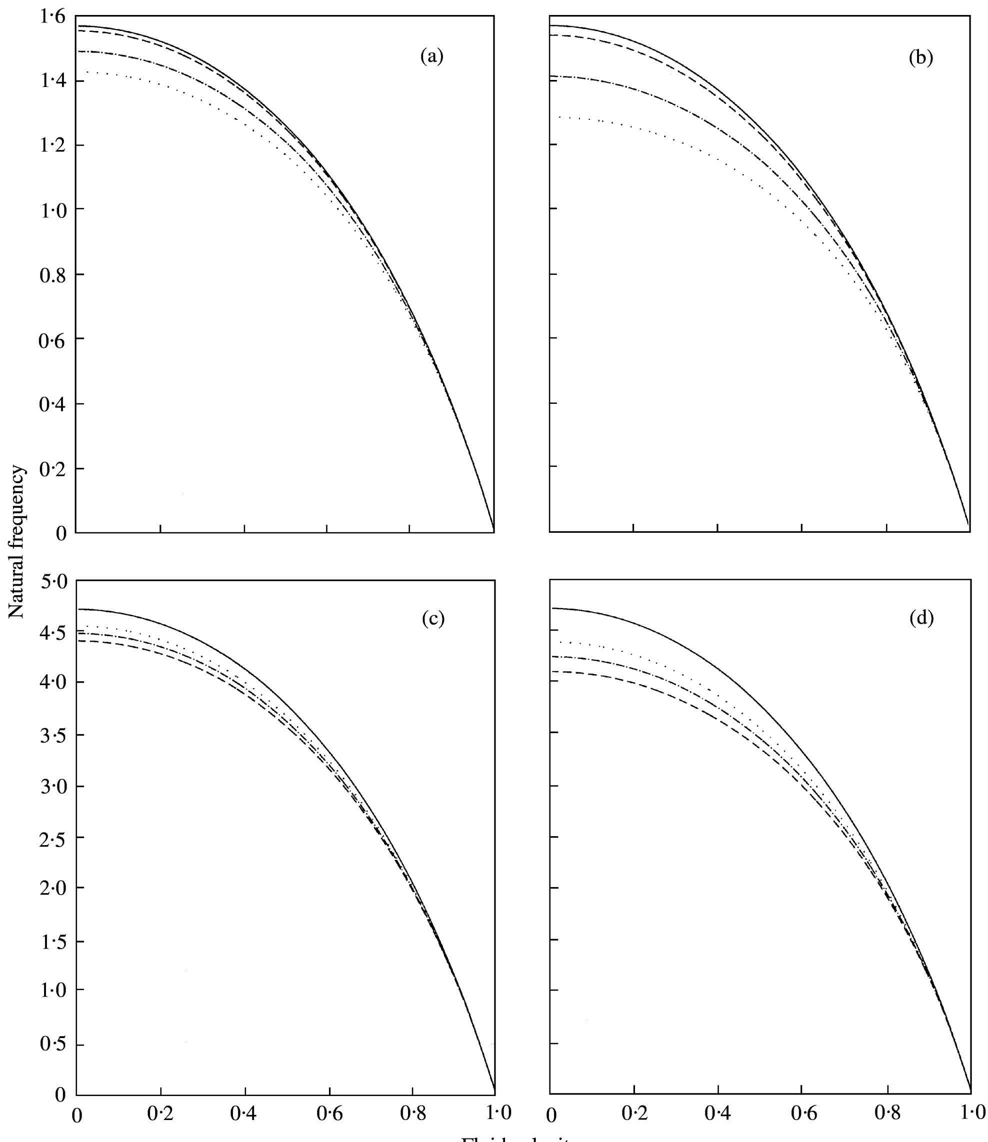

Figure 1. Natural frequency of the system versus fluid velocity for fixed–sliding end conditions, $\beta = 0{\cdot}5$: ——, no mass; - - -, $x_c = 0{\cdot}20$; -·-·-, 0·50; ····, 0·80. (a) $n = 1$, $\alpha = 0{\cdot}1$, (b) $n = 1$, $\alpha = 0{\cdot}2$, (c) $n = 2$, $\alpha = 0{\cdot}1$, (d) $n = 2$, $\alpha = 0{\cdot}2$.

the first mode. In Figures 2(a–d), the variation of frequency with fluid velocity is presented for sliding–sliding end conditions. α (0·1 and 0·2) and x_c (0·2, 0·35 and 0·5) are assumed for these end conditions. The frequency values for the sliding–sliding case are greater than those for the fixed–sliding case. The $n = 0$ for the sliding–sliding case corresponds to the rigid body motion. Moving the mass towards the middle of the sliding–sliding pipe decreases the frequencies further in the first mode. In Figures 3(a–d) and 4(a–d), the natural frequency variation with the position of stationary mass is drawn for the two pipes for the first mode for $\alpha = 0{\cdot}1$. For the fixed–sliding case (Figures 3(a–d)), the frequencies decrease as the mass is moved towards the sliding end for all β ratios. The figures depict the variation for different

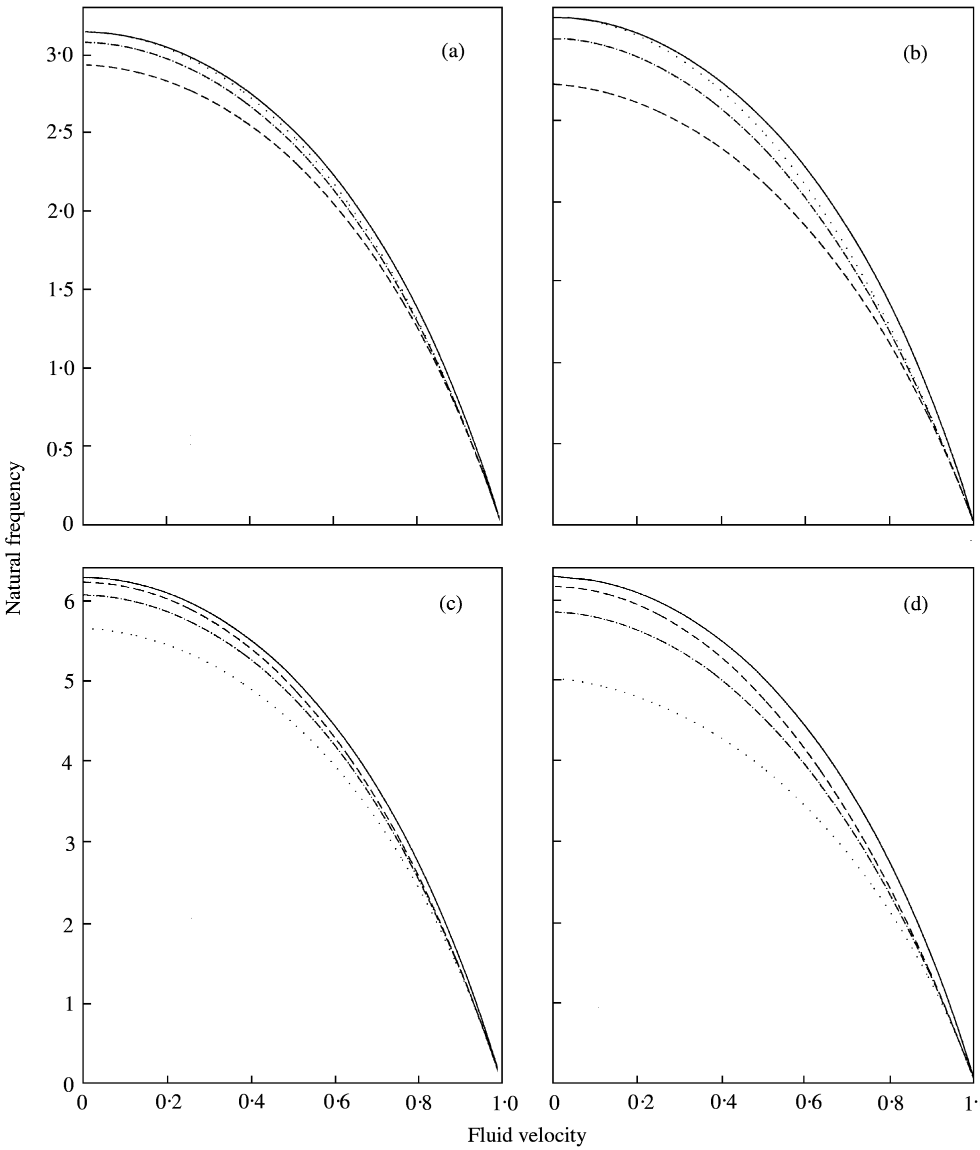

Figure 2. Natural frequency of the system versus fluid velocity for sliding–sliding end conditions, $\beta = 0{\cdot}5$: ——, no mass; - - -, $x_c = 0{\cdot}20$; -·-·-, 0·35; ····, 0·50. (a) $n = 1$, $\alpha = 0{\cdot}1$, (b) $n = 1$, $\alpha = 0{\cdot}2$, (c) $n = 2$, $\alpha = 0{\cdot}1$, (d) $n = 2$, $\alpha = 0{\cdot}2$.

flow velocities (0·2, 0·4, 0·6, 0·8). For sliding–sliding case (Figures 4(a–d)), the frequencies increase when the mass is placed towards the middle of the pipe. The effects of flow velocities (0·2, 0·4, 0·6, 0·8) on the frequencies can be seen in the figures. In Figures 5(a–d) and 6(a–d), the variation of amplitude of oscillations with fluid velocity is presented for the fixed–sliding and sliding–sliding cases for the first two modes. The mass is placed in three different locations, $x_c = 0{\cdot}2$, 0·5, 0·8. The amplitude variation increases when the mass is moved towards the sliding end for the fixed–sliding pipe case as shown in Figures 5(a, b). In the second mode (Figures 5(c, d)), moving mass towards sliding end decreases the amplitude variations for all flow velocities. Increase in mass ratio α results in an increase in

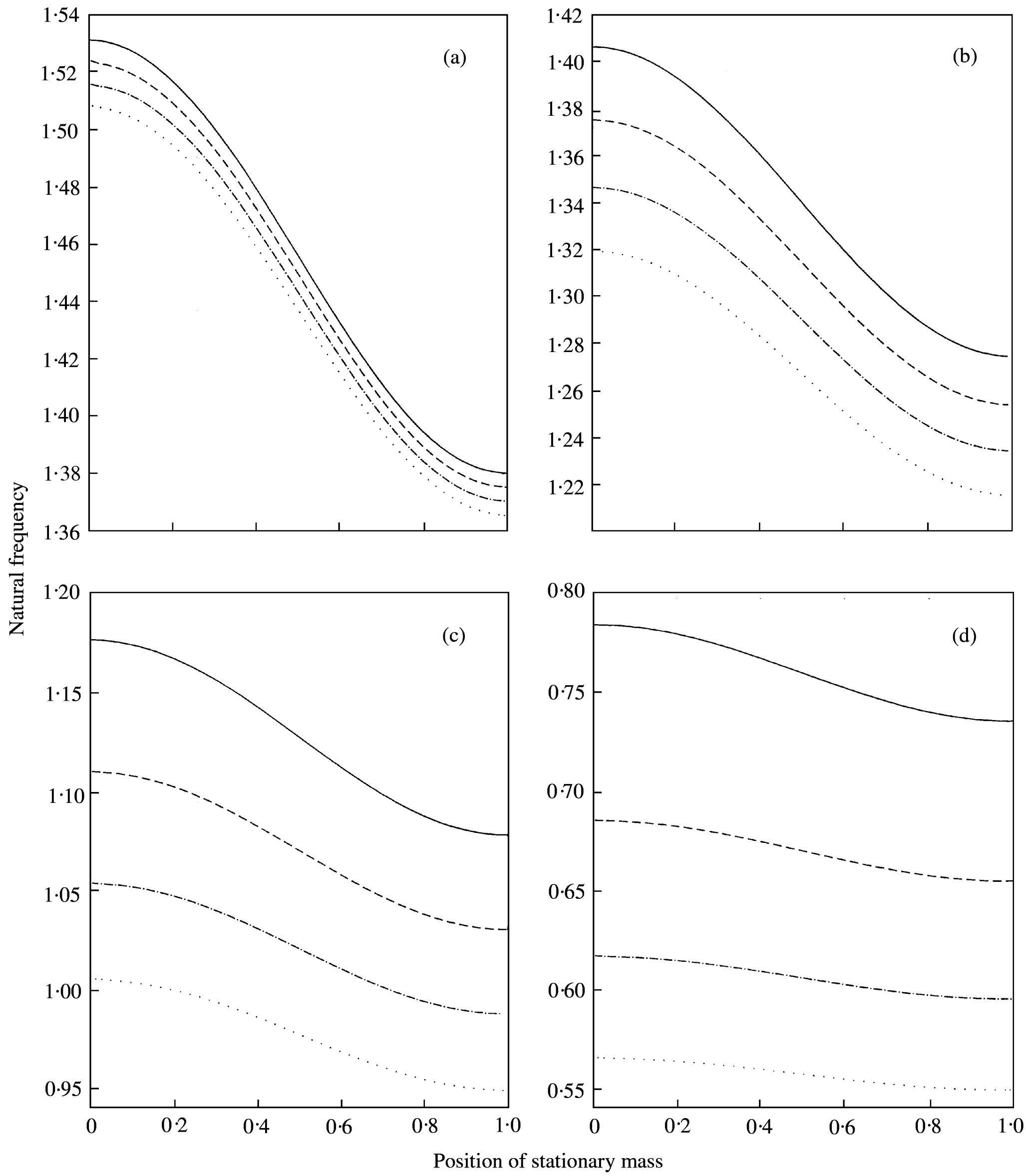

Figure 3. Natural frequency of the system versus position of stationary mass for fixed–sliding end conditions, $n = 1$, $\alpha = 0{\cdot}1$: ——, $\beta = 0{\cdot}25$; - - -, 0·50; - · - · -, 0·75; · · · ·, 1·00. (a) $v = 0{\cdot}2$, (b) $v = 0{\cdot}4$, (c) $v = 0{\cdot}6$, (d) $v = 0{\cdot}8$.

the amplitude variations. For the sliding–sliding case, moving mass towards the middle decreases the amplitude variations for the first mode and the reverse situation can be observed for the second mode as shown in Figures 6(a–d). In Figures 7(a–d) and 8(a–d), the amplitude variation with the position of stationary mass is presented for four different flow velocities and β values in the first mode. For the fixed–sliding case, as the β ratio increases, the amplitude variation increases for all stationary mass positions when flow velocities are $v = 0{\cdot}20$ and $0{\cdot}40$. When the flow velocities are increased ($v = 0{\cdot}60$ and $0{\cdot}80$), amplitude variation curves interchange their sequences and the system with lower β values has higher amplitude variation. For the sliding–sliding case, moving the mass towards the

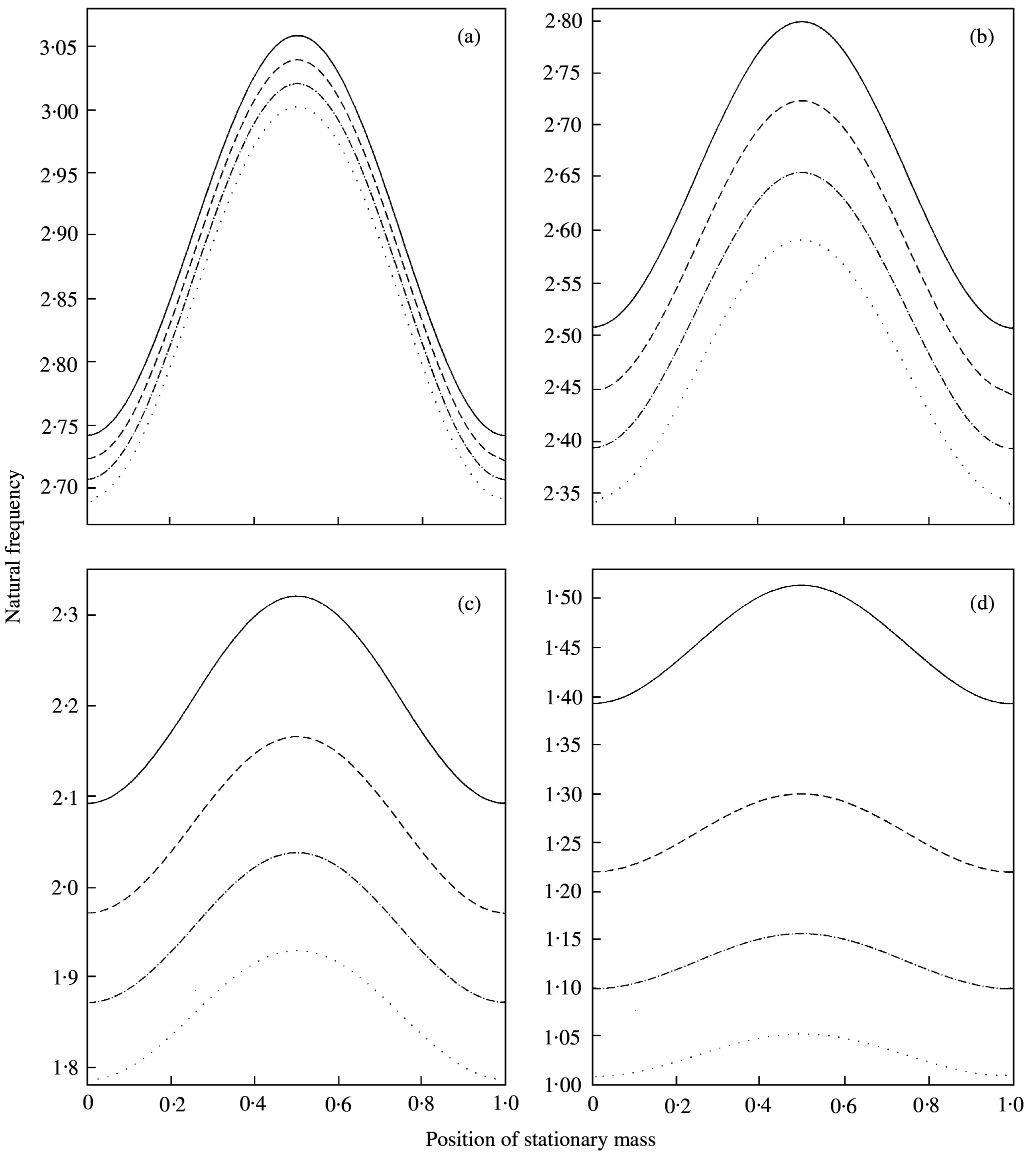

Figure 4. Natural frequency of the system versus position of stationary mass for sliding–sliding end conditions, $n = 1$, $\alpha = 0{\cdot}1$: ——, $\beta = 0{\cdot}25$; - - -, 0·50; -·-·-, 0·75; ····, 1·00. (a) $v = 0{\cdot}2$, (b) $v = 0{\cdot}4$, (c) $v = 0{\cdot}6$, (d) $v = 0{\cdot}8$.

middle of the pipe decreases the amplitude variations for all β values, all positions of stationary mass and for $v = 0{\cdot}20$, $0{\cdot}40$. As the flow velocity is increased ($v = 0{\cdot}60$, $0{\cdot}80$), the flow with lower β values has lower amplitude variation.

4. CONCLUSIONS

The linear transverse vibration of highly tensioned pipes conveying fluid with constant velocity is considered. The pipe has a negligible flexural stiffness and carries a stationary

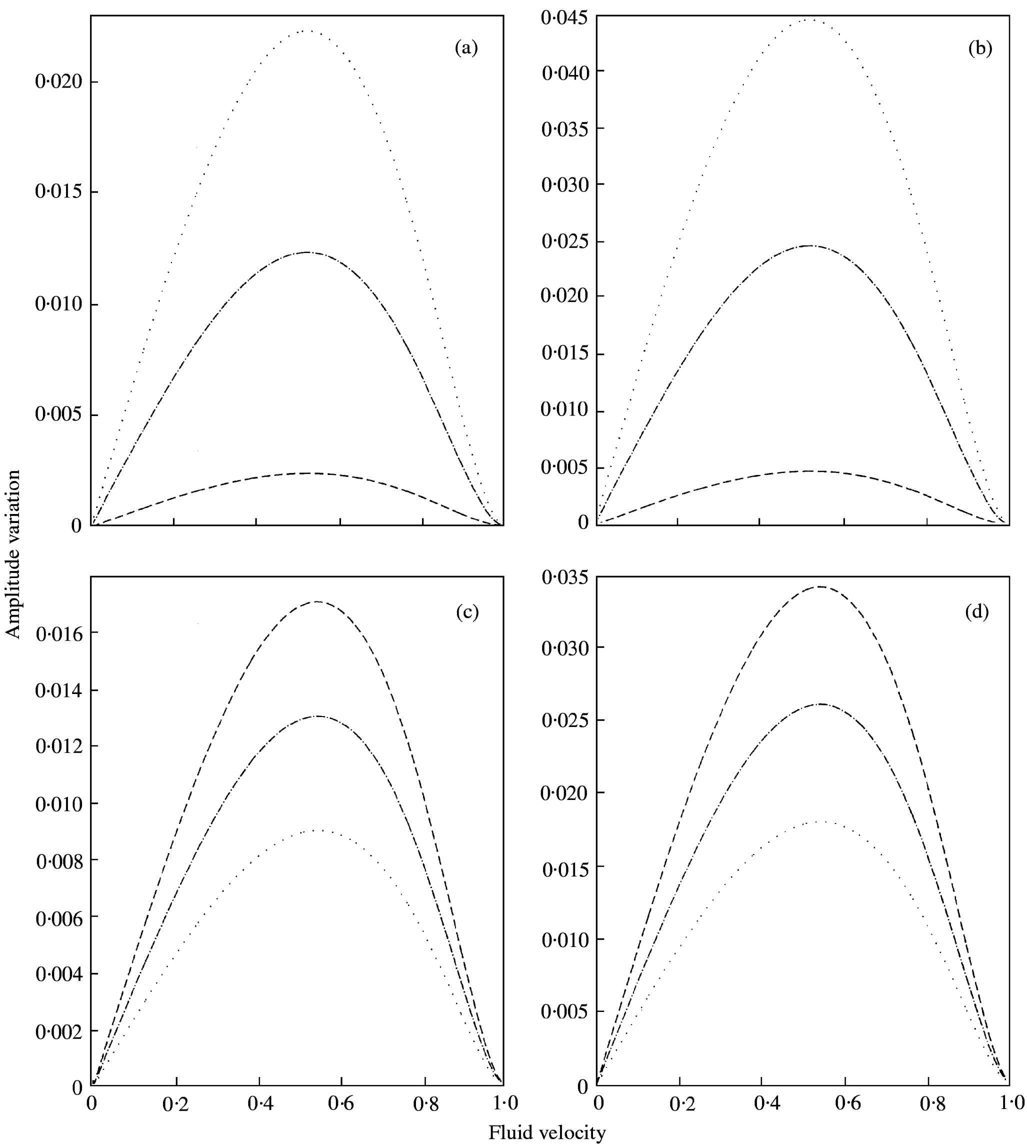

Figure 5. Amplitude variation versus flow velocity for fixed–sliding end conditions, $\beta = 0{\cdot}5$: - - -, $x_c = 0{\cdot}20$; - · - · -, 0·50; · · · ·, 0·80. (a) $n = 1$, $\alpha = 0{\cdot}1$, (b) $n = 1$, $\alpha = 0{\cdot}2$, (c) $n = 2$, $\alpha = 0{\cdot}1$, (d) $n = 2$, $\alpha = 0{\cdot}2$.

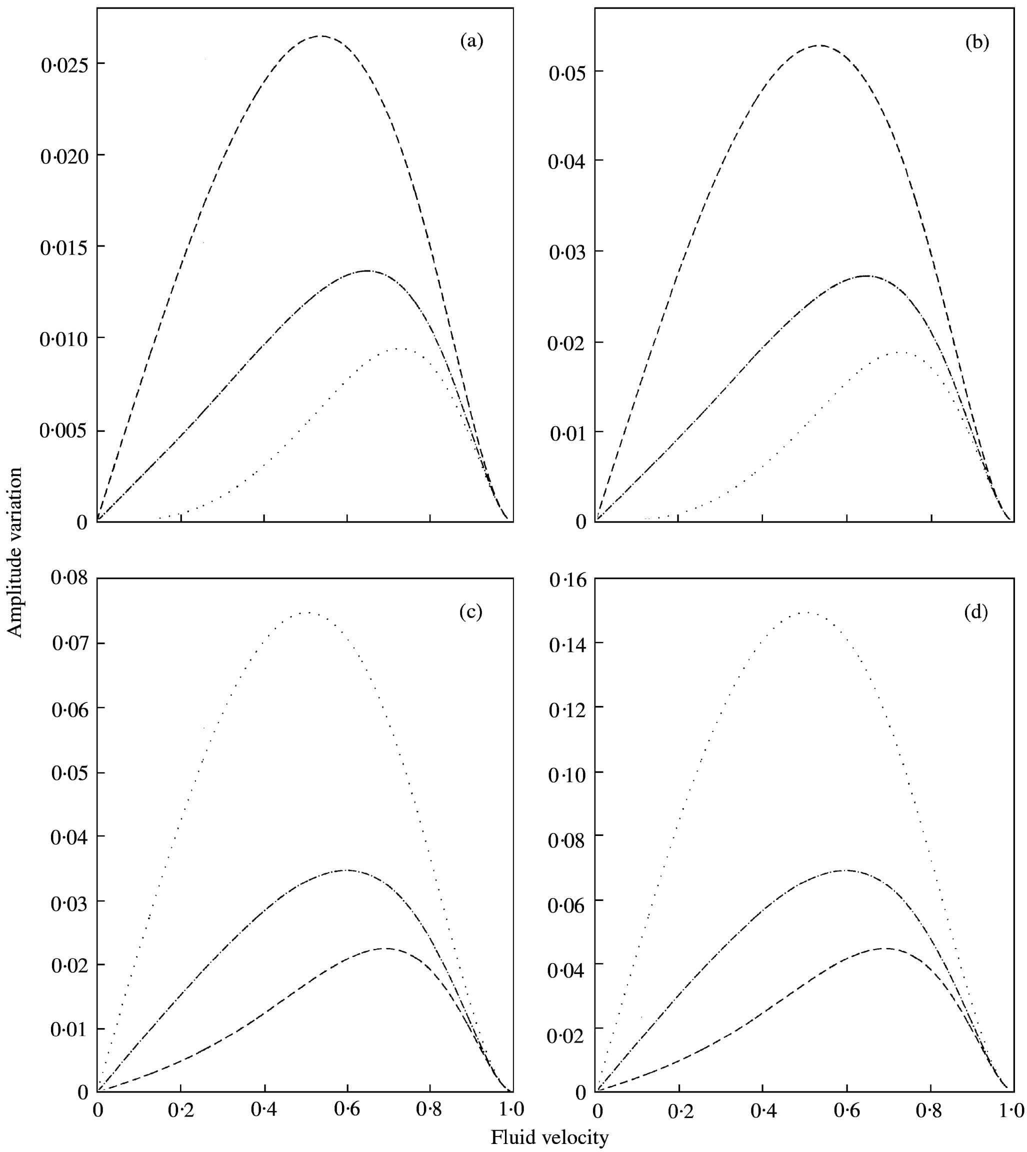

Figure 6. Amplitude variation versus flow velocity for sliding–sliding end conditions, $\beta = 0{\cdot}5$: - - -, $x_c = 0{\cdot}20$; - · - · -, 0·35; · · · ·, 0·50. (a) $n = 1$, $\alpha = 0{\cdot}1$, (b) $n = 1$, $\alpha = 0{\cdot}2$, (c) $n = 2$, $\alpha = 0{\cdot}1$, (d) $n = 2$, $\alpha = 0{\cdot}2$.

mass. The natural frequencies are presented for fixed–sliding and sliding–sliding end conditions. The effect of the value and position of the stationary mass is investigated. The variation of amplitude with the position of stationary mass is analyzed for different flow velocities and mass ratios.

REFERENCES

1. A. G. Ulsoy, C. D. Mote Jr and R. Syzmani 1978 *Holz als Roh-und Werkstoff* **36**, 273–280. Principal developments in band saw vibration and stability research.

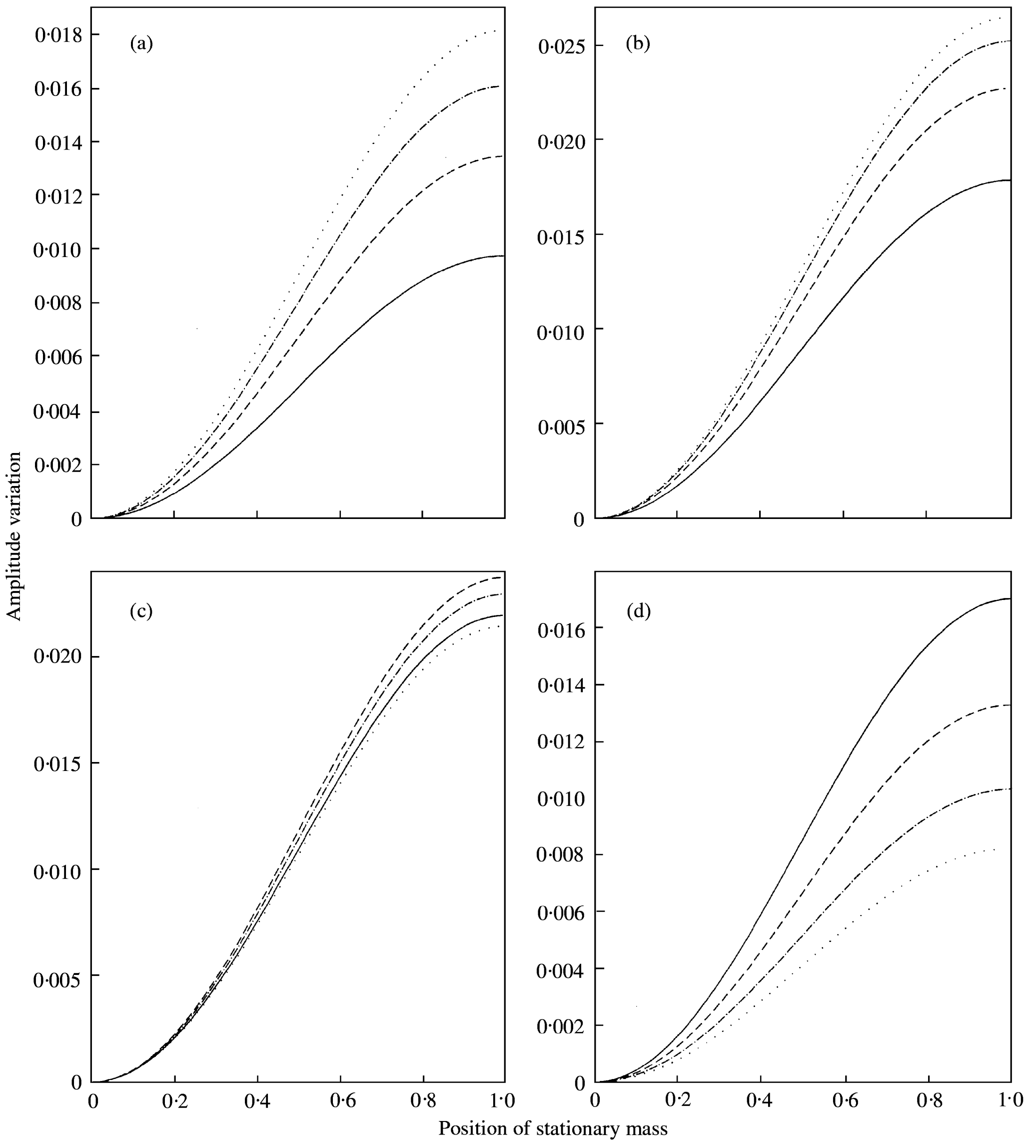

Figure 7. Amplitude variation versus position of stationary mass for fixed–sliding end conditions, $n = 1$, $\alpha = 0{\cdot}1$: ——, $\beta = 0{\cdot}25$; - - -, 0·50; - · - · -, 0·75; · · · ·, 1·00. (a) $v = 0{\cdot}20$, (b) $v = 0{\cdot}40$, (c) $v = 0{\cdot}60$, (d) $v = 0{\cdot}80$.

2. J. A. WICKERT and C. D. MOTE JR 1988 *Shock and Vibration Digest* **20**, 3–13. Current research on the vibration and stability of axially moving materials.
3. J. A. WICKERT and C. D. MOTE JR 1989 *Journal of the Acoustical Society of America* **85**, 1365–1368. On the energetics of axially moving continua.
4. J. A. WICKERT and C. D. MOTE JR 1990 *American Society of Mechanical Engineers Journal of Applied Mechanics* **57**, 738–744. Classical vibration analysis of axially moving continua.
5. M. PAKDEMİRLİ, A. G. ULSOY and A. CERANOĞLU 1994 *Journal of Sound and Vibration* **169**, 179–196. Transverse vibration of an axially accelerating string.
6. M. PAKDEMİRLİ and H. BATAN 1993 *Journal of Sound and Vibration* **168**, 371–378. Dynamic stability of a constantly accelerating strip.

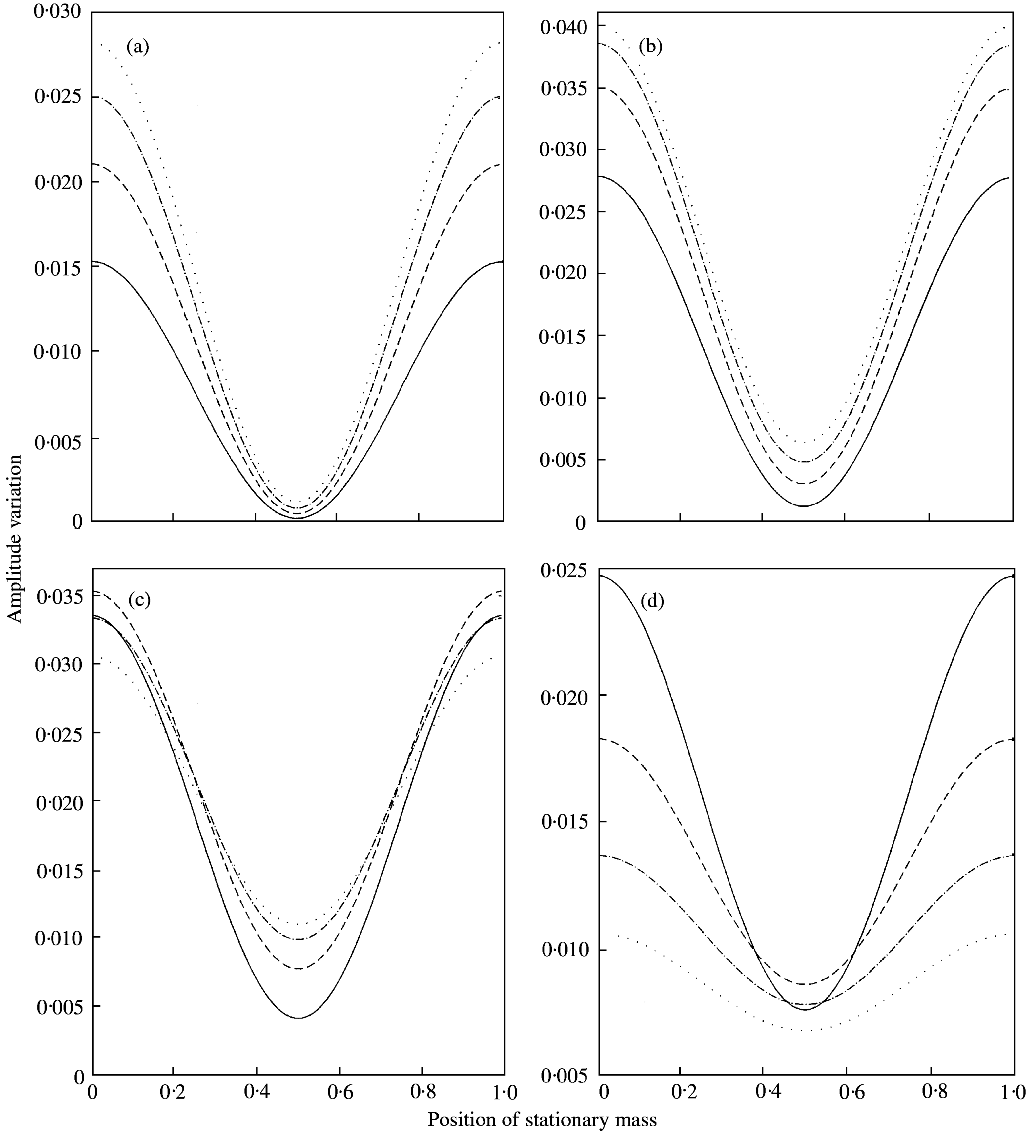

Figure 8. Amplitude variation versus position of stationary mass for sliding–sliding end conditions, $n = 1$, $\alpha = 0{\cdot}1$: ——, $\beta = 0{\cdot}25$; - - -, 0·50; - · - · -, 0·75; · · · ·, 1·00. (a) $v = 0{\cdot}20$, (b) $v = 0{\cdot}40$, (c) $v = 0{\cdot}60$, (d) $v = 0{\cdot}80$.

7. M. PAKDEMİRLİ and A. G. ULSOY 1997 *Journal of Sound and Vibration* **203**, 815–832. Stability analysis of an axially accelerating string.
8. H. R. ÖZ, M. PAKDEMİRLİ and E. ÖZKAYA 1998 *Journal of Sound and Vibration* **215**, 571–576. Transition behaviour from string to beam for an axially accelerating material.
9. H. R. ÖZ and M. PAKDEMİRLİ 1999 *Journal of Sound and Vibration* **227**, 239–257. Vibrations of an axially moving beam with time dependent velocity.
10. M. PAKDEMİRLİ and E. ÖZKAYA 1998 *Mathematical and Computational Applications* **3**, 93–100. Approximate boundary layer solution of a moving beam problem.
11. E. ÖZKAYA and M. PAKDEMİRLİ 2000 *Journal of Sound and Vibration* **234**, 521–535. Vibrations of an axially accelerating beam with small flexural stiffness.

12. H. R. Öz 2001 *Journal of Sound and Vibration* **239**, 556–564. On the vibrations of an axially traveling beam on fixed supports with variable velocity.
13. T. B. Benjamin 1961 *Proceedings of the Royal Society of London* **261A**, 457–499. Dynamics of a system of articulated pipes conveying fluids: I. Theory and II. Experiment.
14. S. Nemat-Nasser, S. S. Prasad and G. Herrmann 1966 *American Institute of Aeronautics and Astronautics Journal* **4**, 1276–1280. Destabilizing effect of velocity-independent forces in non-conservative continuous systems.
15. R. W. Gregory and M. P. Paidoussis 1966 *Proceedings of the Royal Society of London* **239A**, 512–544. Unstable oscillation of tabular cantilevers conveying fluid, I. Theory and II. Experiment.
16. M. P. Paidoussis and G. X. Li 1993 *Journal of Fluids and Structures* **7**, 137–204. Pipes conveying fluid: a model dynamical problem.
17. S. Y. Lee and C. D. Mote Jr 1997 *Journal of Sound and Vibration* **204**, 717–734. A generalized treatment of the energetics of translating continua, Part I: strings and second order tensioned pipes.
18. S. Y. Lee and C. D. Mote Jr 1997 *Journal of Sound and Vibration* **204**, 735–753. A generalized treatment of the energetics of translating continua, Part II: beams and fluid conveying pipes.
19. H. R. Öz and H. Boyaci 2000 *Journal of Sound and Vibration* **236**, 259–276. Transverse vibrations of tensioned pipes conveying fluid with time-dependent velocity.
20. J. L. Hill and C. P. Swanson 1970 *American Society of Mechanical Engineers Journal of Applied Mechanics* **37**, 494–497. Effects of lumped masses on the stability of fluid conveying tubes.
21. T. T. Wu and P. P. Raju 1974 *American Society of Mechanical Engineers Journal of Pressure Vessel Technology* **96**, 154–158. Vibration of a fluid conveying pipe carrying a discrete mass.
22. S. S. Chen and J. A. Jendrzejczyk 1985 *Journal of Acoustical Society of America* **77**, 887–895. General characteristics, transition, and control of instability of tubes conveying fluids.
23. M. Stylianou and B. Tabarrok 1994 *Journal of Sound and Vibration* **178**, 433–453. Finite element analysis of an axially moving beam, Part I: time integration.
24. M. Stylianou and B. Tabarrok 1994 *Journal of Sound and Vibration* **178**, 455–481. Finite element analysis of an axially moving beam, Part II: stability analysis.
25. J. S. Chen 1997 *Journal of Vibration and Acoustics* **119**, 152–157. Natural frequencies and stability of an axially-travelling string in contact with a stationary load system.
26. D. Borglund 1998 *Journal of Fluids and Structures* **12**, 353–365. On the optimal design of pipes conveying fluid.
27. S. Y. Lee and C. D. Mote Jr 1998 *Journal of Sound and Vibration* **212**, 1–22. Travelling wave dynamics in a translating string coupled to stationary constraints: energy transfer and mode localization.
28. M. G. Kang 2000 *Journal of Sound and Vibration* **238**, 179–187. The influence of rotary inertia of concentrated masses on the natural vibrations of fluid conveying pipes.
29. H. R. Öz and C. Evrensel *Journal of Sound and Vibration* (letter to the editor) Natural frequencies of tensioned pipes conveying fluid and carrying a concentrated mass. (to be published).