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INSTABILITY OF A BOGIE MOVING ON A FLEXIBLY SUPPORTED TIMOSHENKO BEAM

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The stability of vibration of a bogie uniformly moving along a Timoshenko beam on a viscoelastic foundation has been studied. The bogie has been modelled by a rigid bar of a finite length on two identical supports. Each support consists of a spring and a dashpot connected in parallel. The upper ends of the supports are attached to the bar, whilst the lower ends are mounted onto concentrated masses through which the supports interact with the beam. It is assumed that the masses and the beam are always in contact. It is shown that when the velocity of the bogie exceeds the minimum phase velocity of waves in the beam, the vibration of the system may become unstable. The instability region is found in the space of the system parameters with the help of the D-decomposition method and the principle of the argument. An extended analysis of the effect of the bogie parameters on the model stability has been carried out.

1. INTRODUCTION

It is most probable that in future, the high-speed trains will travel faster than the elastic waves that propagate in the railway structure [1–5]. If this happens, the train–railway interaction would become qualitatively different from that in the case of a "slow" motion of the train. The crucial difference is contained in the effect of the waves that are generated in the railway by a "fast" train. These waves, that in the literature are referred to as "anomalous Doppler waves" [6], instead of stabilizing the train vibrations by means of the radiation damping, destabilize these vibrations by transferring the energy of the horizontal progressive train motion into the energy of the vertical train vibration.

The effect of the wave-induced instability of a mechanical object that moves on a distributed elastic system was for the first time independently described by Denisov *et al.* [7] and Bogacz *et al.* [8]. In these papers, it was demonstrated that there exists a critical velocity of the object such that, having been exceeded, instability of the object's vibrations occurs. In reference [9] it was found that this velocity in the undamped case is equal to the minimum phase velocity of waves in the elastic system. In the same paper, the physical background of the instability phenomenon was uncovered. By using the energy and wave momentum variation laws it was shown that anomalous Doppler waves cause the instability.

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The instability phenomenon remained of purely academic interest for quite a long time, for it was believed that the minimum phase velocity of waves in a railway structure is much higher than that which is reachable for existing trains. Recently, it was found, however, that there exist at least two mechanisms that can substantially lower the phase velocity in a railway track. These mechanisms are the axial stress in continuously welded rails [10–12] and railway–subsoil interaction [13]. In both cases, the critical velocity which, being exceeded, may shift the instability to the range of realistic train velocities of the order 200–400 km/h.

To guarantee safety of high-speed train passengers, the mechanical properties of the train have to be chosen such that the instability is prevented. To this end, stability analysis of vibrations of a moving train interacting with flexible rails has to be carried out. In recent papers, the stability analysis has been performed for relatively simple models of the train such as a moving oscillator [13–15] and a set of two oscillators [16].

In the present paper, a more sophisticated model for the bogie of a wagon has been studied, composed of a rigid bar and two identical supports connecting the bar and a Timoshenko beam on viscoelastic foundations serving as a model for the railway. This model can be considered as an extension to the model considered by Wolfert *et al.* [16]. It employs a Timoshenko model for the beam instead of the Euler–Bernoulli one and allows the bogie supports to interact not only through the beam but also by means of a rigid bar free to move and rotate in the vertical plane. Although the extension appears minor, the instability regions obtained within the current model crucially differ from that found in reference [16]. The difference originates in the interaction between the bogie supports through the bar, which was neglected in reference [16].

The instability region in the space of the model parameters is found in the following manner. Firstly, the governing equations are written in the reference system that moves (uniformly) with the bogie. Secondly, the Laplace transform with respect to time is applied to these equations. The transformed equations are then evaluated to give the characteristic equation that determines the natural frequencies of vibrations of the moving bogie as it interacts with the beam. The criterion for the instability to occur is that one of the roots of the characteristic equation possesses a positive real part. Since the characteristic equation contains an integral, a special procedure is needed to determine whether such a root exists. In this paper, it is shown that this can be conveniently done by subsequent application of the D-decomposition method [7, 9, 12–17] and the principle of the argument [18]. Once the method of finding the instability domain was established, an extended parametric analysis of this domain has been carried out. Finally, the instability domain obtained for the current model is compared to those that are obtained for less sophisticated models presented in references [15, 16].

2. MODEL AND CHARACTERISTIC EQUATION

One considers a bogie that uniformly moves on a flexibly supported Timoshenko beam, as depicted in Figure 1. Assuming that the supports of the bogie are always in contact with the beam, the governing equations for the model may be written as

$$\rho F \frac{\partial^2 u}{\partial t^2} - \chi G F \frac{\partial^2 u}{\partial x^2} + \chi G F \frac{\partial \varphi}{\partial x} + k_f u + v_f \frac{\partial u}{\partial t}$$

$$= - \delta(x - Vt - d)\left(m \frac{\mathrm{d}^2 u^{01}}{\mathrm{d}t^2} + \left(k_0 + \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\right)(u^{01} - u^0 - \mathrm{d}\Theta/2)\right)$$

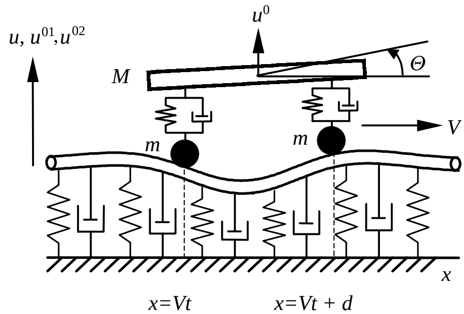

Figure 1. Uniform motion of a bogie over a Timoshenko beam on a viscoelastic foundation.

$$-\delta(x-Vt)\left(m\frac{\mathrm{d}^2u^{02}}{\mathrm{d}t^2}+\left(k_0+\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\right)(u^{02}-u^0+\mathrm{d}\Theta/2)\right),$$

$$\rho I\frac{\partial^2\varphi}{\partial t^2}-EI\frac{\partial^2\varphi}{\partial x^2}+\chi GF\left(\varphi-\frac{\partial u}{\partial x}\right)=0,$$

$$M\frac{\mathrm{d}^2u^0}{\mathrm{d}t^2}+\left(k_0+\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\right)(2u^0-u^{01}-u^{02})=0,$$

$$J\frac{\mathrm{d}^2\Theta}{\mathrm{d}t^2}+\frac{d}{2}\left(k_0+\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\right)(d\Theta-u^{01}+u^{02})=0,$$

$$u^{01}(t)=u(Vt+d,t),\qquad u^{02}(t)=u(Vt,t),\qquad \lim_{|x-Vt|\to\infty}u(x,t)=0,\qquad \lim_{|x-Vt|\to\infty}\varphi(x,t)=0. \tag{1}$$

where $u(x,t)$, $u^{01}(t)$, $u^{02}(t)$, and $u^0(t)$ are the vertical deflection of the beam, the vertical displacement of the mass of the right support, the vertical displacement of the mass of the left support and the vertical displacement of the centre of mass of the bar; $\varphi(x,t)$ is the angle of rotation of the cross-section of the beam; Θ is the angle of rotation of the bar around the centre of mass; M and J are the mass and the moment of inertia of the bar; E and G are the Young's modulus and the shear modulus of the beam material; ρ and I are the mass density of the beam material and the moment of inertia of the beam cross-section; F is the cross-sectional area of the beam, χ is the Timoshenko factor; k_f and v_f are the stiffness and the viscosity of the foundation per unit length; k_0 and ε_0 are the stiffness and the damping factor of the supports of the bar and $\delta(\cdot)$ is the Dirac delta-function. The units of the parameters are ρ (kg/m^3), E,G (N/m^2), I (m^4), k_f (N/m^2), v_f (N s/m^2), F (m^2), m,M (kg), ε_0 (N s/m), k_0 (N/m), J (Kg m^2), d (m), $\delta(x)$ (1/m).

The problem can be conveniently analyzed (1) by introducing the dimensionless variables and parameters $U=u\omega_0/c$, $U^{01}=u^{01}\omega_0/c$, $U^{02}=u^{02}\omega_0/c$, $\tilde{t}=\omega_0 t$, $y=x\omega_0/c$, $\kappa=c^2F/(\omega_0^2 I)$, $\gamma=c_p^2/c^2$, $v=v_f/(\rho F\omega_0)$, $M_L=m\omega_0/(\rho Fc)$, $M_U=M\omega_0/(\rho Fc)$, $K=k_0/(\rho F\omega_0 c)$, $\varepsilon=\varepsilon_0/(\rho Fc)$, $D=d\omega_0/c$, $J_1=2J\omega_0^2/(\rho Fdc^2)$, $\alpha=V/c$, $c=\sqrt{\chi c_s}$, where $c_p=\sqrt{E/\rho}$ and $c_S=\sqrt{G/\rho}$ are the compressional and the shear wave velocities in the beam, $\omega_0=\sqrt{k_f/(\rho F)}$ is the cut-off frequency of the beam on the elastic foundation. Employing these parameters and introducing a reference system $\{\xi = y - \alpha\tilde{t},\ \tau = \tilde{t}\}$ moving with the velocity of the bogie, the governing equations can be rewritten as

$$\frac{\partial^2 U}{\partial \tau^2} - 2\alpha \frac{\partial^2 U}{\partial \xi \partial \tau} + (\alpha^2 - 1)\frac{\partial^2 U}{\partial \xi^2} + \frac{\partial \varphi}{\partial \xi} + U + v\left(\frac{\partial U}{\partial \tau} - \alpha \frac{\partial U}{\partial \xi}\right)$$

$$= -\,\delta(\xi - D)\left(M_L \frac{\mathrm{d}^2 U^{01}}{\mathrm{d}\tau^2} + \left(K + \varepsilon \frac{\mathrm{d}}{\mathrm{d}\tau}\right)(U^{01} - U^0 - D\Theta/2)\right)$$

$$-\,\delta(\xi)\left(M_L \frac{\mathrm{d}^2 U^{02}}{\mathrm{d}\tau^2} + \left(K + \varepsilon \frac{\mathrm{d}}{\mathrm{d}\tau}\right)(U^{02} - U^0 + D\Theta/2)\right),$$

$$\frac{\partial^2 \varphi}{\partial \tau^2} - 2\alpha \frac{\partial^2 \varphi}{\partial \xi \partial \tau} + (\alpha^2 - \gamma)\frac{\partial^2 \varphi}{\partial \xi^2} + \kappa\left(\varphi - \frac{\partial U}{\partial \xi}\right) = 0,$$

$$M_U \frac{\mathrm{d}^2 U^0}{\mathrm{d}\tau^2} + \left(K + \varepsilon \frac{\mathrm{d}}{\mathrm{d}\tau}\right)(2U^0 - U^{01} - U^{02}) = 0,$$

$$J_1 \frac{\mathrm{d}^2 \Theta}{\mathrm{d}\tau^2} + \left(K + \varepsilon \frac{\mathrm{d}}{\mathrm{d}\tau}\right)(D\Theta - U^{01} + U^{02}) = 0,$$

$$U(D, \tau) = U^{01}(\tau), \qquad U(0, \tau) = U^{02}(\tau), \qquad \lim_{|\xi| \to \infty} U(\xi, \tau) = 0, \qquad \lim_{|\xi| \to \infty} \varphi(\xi, \tau) = 0. \tag{2}$$

To obtain the characteristic equation for vibrations of the moving bogie on the beam, integral transforms are applied to the system of equations (2):

$$\tilde{U}_s^{01,02}(s) = \int_0^\infty U^{01,02}(\tau)\exp(-s\tau)\,\mathrm{d}\tau, \qquad \tilde{U}_s^{0}(s) = \int_0^\infty U^{0}(\tau)\exp(-s\tau)\,\mathrm{d}\tau,$$

$$\tilde{\Theta}_s(s) = \int_0^\infty \Theta(\tau)\exp(-s\tau)\,\mathrm{d}\tau,$$

$$\tilde{U}_s(\xi, s) = \int_0^\infty U(\xi, \tau)\exp(-s\tau)\,\mathrm{d}\tau, \qquad \tilde{\varphi}_s(\xi, s) = \int_0^\infty \varphi(\xi, \tau)\exp(-s\tau)\,\mathrm{d}\tau,$$

$$\tilde{\tilde{U}}_{k,s}(k, s) = \int_{-\infty}^\infty \tilde{U}_s(\xi, s)\exp(-\mathrm{i}k\xi)\,\mathrm{d}\xi, \qquad \tilde{\tilde{\varphi}}_{k,s}(k, s) = \int_{-\infty}^\infty \tilde{\varphi}_s(\xi, s)\exp(-\mathrm{i}k\xi)\,\mathrm{d}\xi.$$

By assuming that the initial conditions to problem (2) are trivial (initial conditions do not influence the system stability since the governing equations are linear), application of the transforms results in the system of algebraic equations

$$\tilde{\tilde{U}}_{k,s}(k,s)F(k,s) = -(\tilde{U}_s(0,s) + \tilde{U}_s(D,s)\exp(-\mathrm{i}kD))(M_L s^2 + K + \varepsilon s)$$

$$+ \tilde{U}_s^0(\exp(-\mathrm{i}kD) + 1)(K + \varepsilon s) + D\tilde{\Theta}_s/2(\exp(-\mathrm{i}kD) - 1)(K + \varepsilon s),$$

$$\tilde{U}_s^0(M_U s^2 + 2K + 2\varepsilon s) - (\tilde{U}_s(D,s) + \tilde{U}_s(0,s))(K + \varepsilon s) = 0,$$

$$\tilde{\Theta}_s(J_1 s^2 + DK + D\varepsilon s) - (\tilde{U}_s(D,s) - \tilde{U}_s(0,s))(K + \varepsilon s) = 0,$$

$$F(k,s) = (A(k,s)B(k,s) - \kappa k^2)/A(k,s), \qquad A(k,s) = s^2 - 2\alpha\mathrm{i}ks - k^2(\alpha^2 - \gamma) + \kappa,$$

$$B(k,s) = s^2 - 2\alpha\mathrm{i}ks - k^2(\alpha^2 - 1) + \nu s - \nu\alpha\mathrm{i}k + 1, \tag{3}$$

from which $\tilde{U}_s^{01}(s)$, $\tilde{U}_s^{02}(s)$ and $\tilde{\tilde{\varphi}}_{k,s}(k,s)$ can be eliminated by employing equalities $\tilde{U}_s(D,s) = \tilde{U}_s^{01}(s)$, $\tilde{U}_s(0,s) = \tilde{U}_s^{02}(s)$ and $\tilde{\tilde{\varphi}}_{k,s}(k,s) = \mathrm{i}\kappa\tilde{\tilde{U}}_{k,s}(k,s)/A(k,s)$.

The next step in obtaining the characteristic equation for the bogie vibrations is application of the inverse Fourier transform over k to the system of equations (3). This yields

$$\tilde{U}_s(\xi,s) = -\frac{(M_L s^2 + K + \varepsilon s)}{2\pi}\left(\tilde{U}_s(0,s)\int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{F(k,s)} + \tilde{U}_s(D,s)\int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}k(\xi - D))\,\mathrm{d}k}{F(k,s)}\right)$$

$$+ \frac{\tilde{U}_s^0}{2\pi}\left(\int_{-\infty}^{\infty}\frac{(\mathrm{i}k(\xi - D))\,\mathrm{d}k}{F(k,s)} + \int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{F(k,s)}\right)(K + \varepsilon s)$$

$$+ \frac{D\tilde{\Theta}_s}{4\pi}\left(\int_{-\infty}^{\infty}\frac{(\mathrm{i}k(\xi - D))\,\mathrm{d}k}{F(k,s)} - \int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{F(k,s)}\right)(K + \varepsilon s),$$

$$\tilde{U}_s^0(M_U s^2 + 2K + 2\varepsilon s) - (\tilde{U}_s(D,s) + \tilde{U}_s(0,s))(K + \varepsilon s) = 0,$$

$$\tilde{\Theta}_s(J_1 s^2 + DK + D\varepsilon s) - (\tilde{U}_s(D,s) - \tilde{U}_s(0,s))(K + \varepsilon s) = 0. \tag{4}$$

Eliminating $\tilde{U}_s^0$ and $\tilde{\Theta}_s$ from these equations and then subsequently setting $\xi = 0$ and $\xi = D$, the following two algebraic equations with respect to the Laplace-displacements of the contact points $\tilde{U}_s(0,s)$ and $\tilde{U}_s(D,s)$ are obtained:

$$\begin{bmatrix} 1 + Z_0 I_0 - Z_1(I_- + I_0) + Z_2(I_- - I_0) & Z_0 I_- - Z_1(I_- + I_0) - Z_2(I_- - I_0) \\ Z_0 I_+ - Z_1(I_0 + I_+) + Z_2(I_0 - I_+) & 1 + Z_0 I_0 - Z_1(I_0 + I_+) - Z_2(I_0 - I_+) \end{bmatrix}$$

$$\times \begin{bmatrix} \tilde{U}_s(0,s) \\ \tilde{U}_s(D,s) \end{bmatrix} = 0 \tag{5}$$

with

$$Z_0 = M_L s^2 + K + \varepsilon s, \qquad Z_1 = \frac{(K+\varepsilon s)^2}{M_U s^2 + 2K + 2\varepsilon s}, \qquad Z_2 = \frac{(K+\varepsilon s)^2 D}{2(J_1 s^2 + DK + D\varepsilon s)},$$

$$I_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\mathrm{d}k}{F(k,s)}, \quad I_+ = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\exp(\mathrm{i}kD)\,\mathrm{d}k}{F(k,s)}, \quad I_- = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\exp(-\mathrm{i}kD)\,\mathrm{d}k}{F(k,s)}. \tag{6}$$

The system of equations (5) has a non-trivial solution if and only if the following characteristic equation is satisfied:

$$\left(R + C_3 I_0 - \frac{1}{C_1}(I_- + I_0) + \frac{1}{2C_2}(I_- - I_0)\right)\left(R + C_3 I_0 - \frac{1}{C_1}(I_0 + I_+) - \frac{1}{2C_2}(I_0 - I_+)\right)$$
$$-\left(C_3 I_+ - \frac{1}{C_1}(I_0 + I_+) + \frac{1}{2C_2}(I_0 - I_+)\right)\left(C_3 I_- - \frac{1}{C_1}(I_- + I_0) - \frac{1}{2C_2}(I_- - I_0)\right) = 0, \tag{7}$$

where

$$C_1 = M_U s^2 R + 2, \qquad C_2 = J_1 s^2 R/D + 1, \qquad C_3 = M_L s^2 R + 1, \qquad R = 1/(K+\varepsilon s). \tag{8}$$

The model stability is studied by investigating the eigenvalues of the characteristic equation (7) in the following sections.

3. ANALYSIS OF THE ROOTS OF THE CHARACTERISTIC EQUATION

The criterion for the instability of the bogie vibration is that at least one of the roots of the characteristic equation (7) has a positive real part. This equation, however, is an integral equation with respect to the Laplace variable s and the problem of finding its complex roots is not trivial. To solve this problem, avoiding laborious calculations, it is customary to apply the so-called D-decomposition method [17], which for the first time has been applied to a similar problem in reference [7] (see also references [9, 12–17]). The idea of this method is to map the imaginary axis of the complex (**s**)-plane onto the plane of one of the system parameters, which should be temporarily considered as complex. The mapping rule follows from the characteristic equation, which can be rewritten to express the chosen parameter explicitly. Once the mapping is accomplished, one obtains a mapped line (the D-decomposition curve), which divides the parameter plane into domains with differing number of roots possessing a positive real part.

In this paper, the D-decomposition is performed in the K-plane. The positive half of the real axis of this plane corresponds with the stiffness of the bogie supports. In accordance with equation (8) the mapping rule onto this plain reads

$$K = 1/R(\Omega) - \mathrm{i}\varepsilon\Omega. \tag{9}$$

Parameter Ω in equation (9) should be varied from minus infinity to plus infinity, while complex function $R(\Omega)$ is to be found from the characteristic equation (7) with $s = \mathrm{i}\Omega$. To find $R(\Omega)$ equation (7) is rewritten as

$$\frac{R(\Omega)}{(M_U s^2 R(\Omega) + 2)(J_1 s^2 R(\Omega)/D + 1)}(Q_0(\Omega) + Q_1(\Omega)R(\Omega) + Q_2(\Omega)R^2(\Omega)) = 0, \tag{10}$$

with

$$Q_0(\Omega) = -4 + 2J_1\Omega^2(2I_0 - I_+ - I_- - 2M_L I_0^2)/D + \Omega^2(M_U(2I_0 + I_+ + I_-) + 8M_L I_0)$$
$$\times 2J_1\Omega^4(2M_L I_+ I_- - M_U(I_0^2 - I_+ I_-))/D - 2\Omega^4 M_L(M_U + 2M_L)(I_0^2 - I_+ I_-),$$

$$Q_1(\Omega) = 2\Omega^2(M_L + 2J_1/D - 2I_0\Omega^2(M_L M_U + J_1(M_U + 2M_L)/D)$$
$$+ 2M_L J_1 \Omega^4(M_L + M_U)(I_0^2 - I_+ I_-)/D + M_U M_L^2 \Omega^4(I_0^2 - I_+ I_-)),$$

$$Q_2(\Omega) = 2\Omega^4 M_U J_1(M_L\Omega^2(2I_0 - M_L\Omega^2(I_0^2 - I_+ I_-)) - 1)/D,$$

where

$$I_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{F(k,\Omega)}, \quad I_+ = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}kD)\,\mathrm{d}k}{F(k,\Omega)}, \quad I_- = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(-\mathrm{i}kD)\,\mathrm{d}k}{F(k,\Omega)},$$

$$F(k,\Omega) = \frac{(-\Omega^2 + 2\alpha k\Omega - k^2(\alpha^2 - \gamma) + \kappa)(-\Omega^2 + 2\alpha k\Omega - k^2(\alpha^2 - 1) + \mathrm{i}v_1\Omega - \mathrm{i}v_1\alpha k + 1) - \kappa k^2}{(-\Omega^2 + 2\alpha k\Omega - k^2(\alpha^2 - \gamma) + \kappa)}.$$

The only relevant roots $R(\Omega)$ of equation (10) are those of the equation

$$Q_0(\Omega) + Q_1(\Omega)R(\Omega) + Q_2(\Omega)R^2(\Omega) = 0, \tag{11}$$

which can be easily solved with respect to $R(\Omega)$ once the coefficients $Q_0(\Omega)$, $Q_1(\Omega)$ and $Q_2(\Omega)$ have been calculated. To calculate these coefficients, the integrals I_0, I_+, and I_- need to be evaluated. This is done by the method of contour integration [18], according to which the integration result may be written as (employed contours of integration are shown in Figure 2)

$$I_0 = \mathrm{i}\sum_n \left.\frac{A(k_n, \mathrm{i}\Omega)(k - k_n)}{(k-k_1)(k-k_2)(k-k_3)(k-k_4)}\right|_{k=k_n},$$

$$I_+ = \mathrm{i}\sum_n \left.\frac{A(k_n, \mathrm{i}\Omega)(k - k_n)\exp(\mathrm{i}k_n D)}{(k-k_1)(k-k_2)(k-k_3)(k-k_4)}\right|_{k=k_n},$$

$$I_- = -\mathrm{i}\sum_m \left.\frac{A(k_m, \mathrm{i}\Omega)(k - k_m)\exp(-\mathrm{i}k_m D)}{(k-k_1)(k-k_2)(k-k_3)(k-k_4)}\right|_{k=k_m}, \tag{12}$$

where k_n are the roots of the equation $A(k, \mathrm{i}\Omega)B(k, \mathrm{i}\Omega) - \kappa k^2 = 0$ with a positive imaginary part, whereas k_m are the roots of this equation with a negative imaginary part.

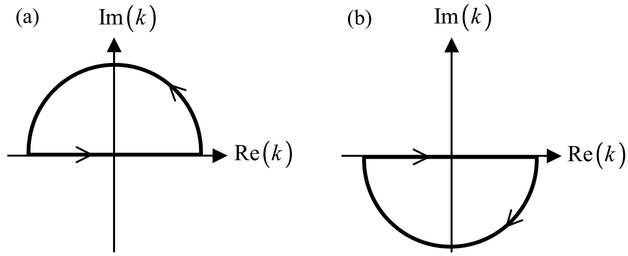

Figure 2. Contour of integration for (a) I_0 and I_+; (b) I_-.

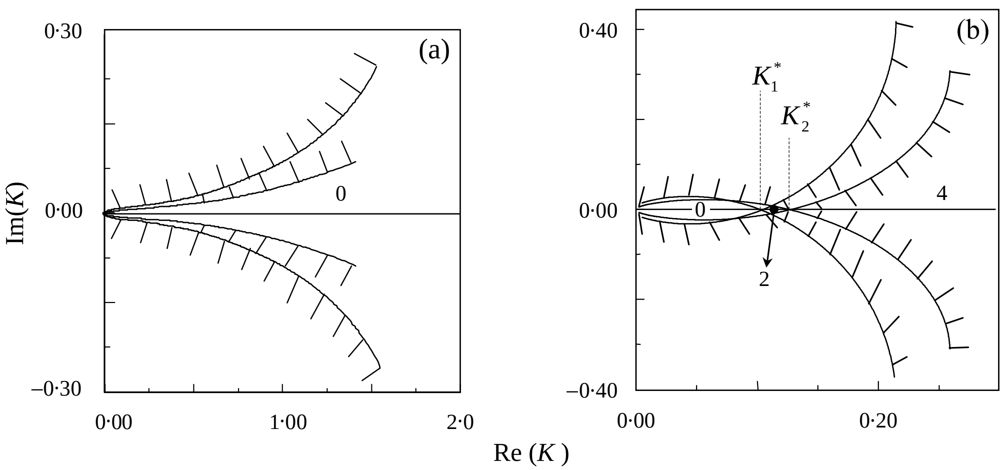

Figure 3. D-decomposition curves for (a) $\alpha < \alpha^*$; (b) $\alpha > \alpha^*$.

Now the D-decomposition curves may be analyzed. Obviously, two curves should be considered, each corresponding to one of the two roots of the quadratic equation (11). Parametric analysis shows that the D-decomposition curves may have two qualitatively different shapes, which are depicted in Figures 3(a) and 3(b). The former figure corresponds to the sub-critical case $\alpha < \alpha^*$ when the velocity of the bogie is smaller than a certain critical velocity α^*. The latter figure is related to the super-critical case $\alpha > \alpha^*$. In the absence of damping in the beam foundation ($v = 0$), the critical velocity α^* reads

$$\alpha^* = \sqrt{\gamma - \gamma\kappa - 2\kappa + 2\kappa\sqrt{\gamma\kappa + 1 - \gamma}}/(\kappa - 1) \tag{13}$$

and represents the minimum phase velocity of waves in the Timoshenko beam on an elastic foundation. Increasing the damping coefficient of the foundation gives a larger magnitude of α^*.

The fundamental difference between Figures 3(a) and 3(b) is that Figure 3(a) shows no crossing points of the D-decomposition curves and the positive part of the real axes, whereas Figure 3(b) exhibits two such points. This implies that in the sub-critical case ($\alpha < \alpha^*$, Figure 3(a)) the number of "unstable" roots (roots with a positive real part) of the characteristic equation does not vary with the stiffness K of the bogie supports. On the contrary, in the super-critical case ($\alpha > \alpha^*$, Figure 3(b)) the number of unstable roots

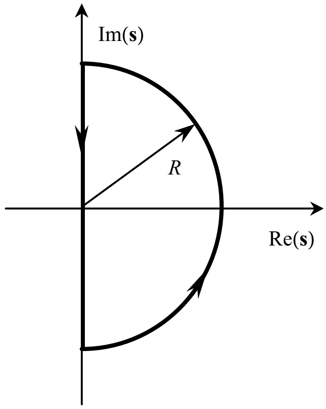

Figure 4. Contour in the (s)-plane that is used for the principle of the argument.

changes once K passes the critical values K_1^* and K_2^*. Using the shading of the D-decomposition curves, which implies that by crossing a curve in the direction of shading one extra unstable root is gained, it can be concluded that if the number of the unstable roots were $N = n_a$ for $0 < K < K_1^*$, then for $K_1^* < K < K_2^*$ and $K > K_2^*$ it would be $N = n_a + 2$ and $N = n_a + 4$ respectively.

Thus, employment of the D-decomposition method allowed one to determine a relative variation of the number of unstable roots N with the stiffness K, but not the number itself. The remaining task, therefore, is to determine the number of unstable roots for any single value of the stiffness K. This can be conveniently done by using the principle of the argument [18]. To apply this principle, equation (11) should be considered as a function $P(s)$ of the complex argument s, i.e.:

$$P(s) = Q_0(-\mathrm{i}s) + Q_1(-\mathrm{i}s)R(s) + Q_2(-\mathrm{i}s)R^2(s) \quad \text{with} \quad R(s) = \frac{1}{(K + \varepsilon s)}.$$

This function is then mapped and parametrically plotted in the complex (P)-plane by varying s such that it describes a closed contour in the complex (s)-plane. Once the mapping is accomplished, the number of rotations of the mapped line around the origin of the complex (P)-plane is to be counted. In accordance with the principle of the argument, this number will be equal to the difference between the number of roots of equation $P(s) = 0$ and the number of poles of this equation that are located inside the chosen closed contour.

Thus, to determine the number of unstable roots of the equation $P(s) = 0$, the contour should be chosen such that it surrounds the right half-plane of the complex variable s, see Figure 4. It can be shown that equation $P(s) = 0$ has no poles in the half-plane $\mathrm{Re}(s) > 0$ and, therefore, the number of rotations of the mapped line will be equal to the number of unstable roots. Thus, application of the principle of the argument completes the mathematical procedure of the stability analysis of the bogie vibrations.

Distribution of the "unstable roots" over the K-plane that is found by subsequent application of the D-decomposition method and the principle of the argument is shown in Figure 3. As has been mentioned, the only physically relevant information in this figure is related to the positive semi-axes $\mathrm{Re}(K) > 0$, since K is the stiffness of the bogie support, which is real and positive. Figure 3(a) shows that the bogie is always stable in the sub-critical case $\alpha < \alpha^*$. On the contrary, in the super-critical case $\alpha > \alpha^*$, the bogie vibrations become unstable if $K_1^* < K$.

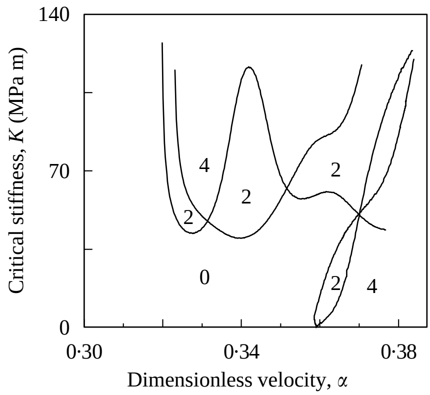

Figure 5. Separation of (α, K)-plane into domains with different number of unstable roots.

Dependence of the critical stiffnesses K_1^* and K_2^* on the velocity of the bogie is quite complex but can be found straightforwardly by application of the D-decomposition method. This dependence is shown in Figure 5 for the following set of the model parameters (the bogie is assumed to represent a wagon of a train, whilst the beam and the visco-elastic foundation model the rail properties and the ground reaction respectively):

$$\rho = 7849\ \text{kg}, \quad F = 7{\cdot}687 \times 10^{-3}\ \text{m}^2, \quad I = 3{\cdot}055 \times 10^{-5}\ \text{m}^4, \quad \chi = 0{\cdot}82,$$

$$E = 2 \times 10^{11}\ \text{N/m}^2, \quad G = 7{\cdot}813 \times 10^{10}\ \text{N/m}^2, \quad k_f = 10^8\ \text{N/m}^2, \quad v_f = 10^4\ \text{Ns/m}^2,$$

$$J = 598000\ \text{kg m/s}^2, \quad d = 15{\cdot}7\ \text{m}, \quad m = 1000\ \text{kg}, \quad M = 20000\ \text{kg},$$

$$\varepsilon_0 = 86000\ \text{Ns/m}. \tag{14}$$

Since K_1^* and K_2^* exist only in the super-critical case, Figure 5 presents the (α, K)-plane for $\alpha > \alpha^* \approx 0{\cdot}31$. It is seen that the dependencies $K_1^*(\alpha)$ and $K_2^*(\alpha)$ form a sophisticated pattern dividing the plane into domains with the number of unstable roots N equal to zero, two and four. The stability domain corresponds to $N = 0$ and this is the boundary of the domain that will be analyzed in the following sections.

Before starting with the parametric study of the model stability, it is worth emphasizing that in this section it has been shown that vibrations of the bogie may become unstable if the bogie moves super-critically. An extended explanation for the instability phenomenon has been presented in references [9, 12, 13]. As shown in these papers, the energy needed for amplification of the bogie vibration is supplied by an external source that maintains the uniform motion of the bogie. This energy is transferred to the vibrational energy by means of the anomalous Doppler waves [6, 9]. These waves, which can be radiated only in the super-critical regime, cause a vertical reaction of the beam to the moving bogie such that it equates to the reaction of a dashpot with a negative damping factor. Such a reaction, obviously, may lead to instability of the model vibrations.

4. PARAMETRIC STUDY OF THE STABILITY DOMAIN

In this section, a parametric study of the (in)stability domain is performed with emphasis on the effect of (1) damping in the bogie supports $\varepsilon_0(\varepsilon)$, (2) mass of the bar $M(M_U)$,

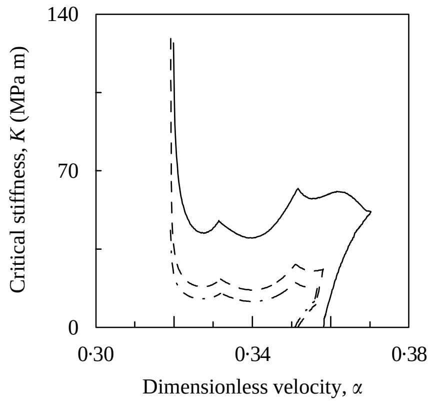

Figure 6. Boundaries of the stability domain for three different magnitudes of the damping in the bogie supports. Key: ε_0 (kN s/m), ——, 86; – – –, 17; – · –, 8·6.

(3) damping in the beam foundation $v_f(v)$, (4) mass of the bogie supports $m(M_L)$ and (5) bogie wheelbase $d(D)$. In all calculations parameter set (14) is used by varying one parameter at a time.

4.1. EFFECT OF THE VISCOUS DAMPING IN THE SUPPORTS OF THE BOGIE

In Figure 6, the boundary of the stability domain is depicted for three different magnitudes of the damping factor ε_0. This boundary divides the parameter plane into two domains. In the domain below the boundary the model vibrations are stable, whereas in the domain that is located above the boundary the vibrations are unstable. As it is to be expected, the figure shows that on increasing the damping in the supports the stability domain expands. This expansion mainly takes place along the K-axis, which implies that a higher damping factor allows the use of stiffer supports to keep the system stable. The effect of the damping in the supports on the velocity above which the model becomes unconditionally unstable ($\alpha \approx 0{\cdot}37$) is minor.

4.2. EFFECT OF THE MASS OF THE BOGIE BAR

Figure 7 is plotted for three different magnitudes of the mass M. Obviously, the effect of this mass is negligible in the considered range.

4.3. EFFECT OF THE DAMPING IN THE BEAM FOUNDATION

Two values for the viscous damping v_f are considered in order to study the effect of the viscous damping in the beam foundation. The result is presented in Figure 8. The figure shows that this damping influences the stability domain crucially. The first effect is that with decreasing v_f, the stability domain visibly shrinks along the velocity axes. This is related to the fact that the critical velocity α^* becomes smaller as v_f decreases. The second effect is that the boundary of the stability domain for the smaller magnitude of v_f possesses a well-defined peak at $\alpha \approx 0{\cdot}335$. This peak is concerned with the reflection of waves in the beam occurring between the bogie supports. This reflection plays a crucial role when the

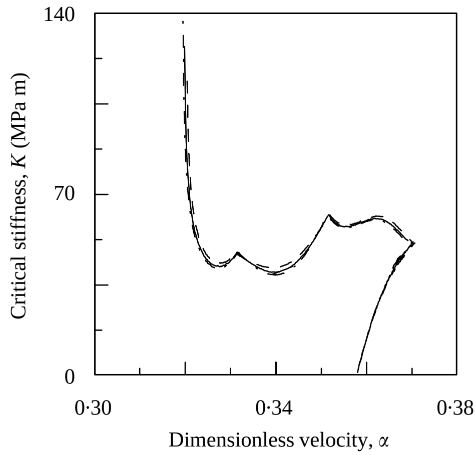

Figure 7. Effect of the mass of the bogie bar. Key for mass, M (kg): - - -, 15 000; ——, 20 000; – · –, 25 000.

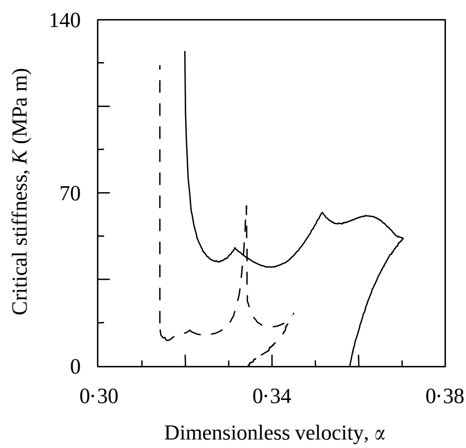

Figure 8. Effect of the damping in the beam foundation. Key for damping, ν_f (N s/m^2): - - -, 5000; ——, 10 000.

damping in the beam foundation is small enough to allow standing waves to be formed between the supports.

4.4. EFFECT OF THE MASS OF THE BOGIE SUPPORTS

Figure 9 is plotted for three different values of m. The figure shows an expansion of the stability domain towards higher velocities as m decreases. Additionally, for the lowest shown value of the mass, the expansion takes place towards higher values of the stiffness K. Thus, although the mass m of the supports is much lower than the mass M of the bar ($m/M \approx 0{\cdot}05$), the former plays a more important role in the system stability and, therefore, must be included in the model.

4.5. EFFECT OF THE WHEELBASE

Figure 10 presents the stability domain for three magnitudes of the wheelbase d. It is seen that for the chosen parameters of the model, the wheelbase influences the stability domain

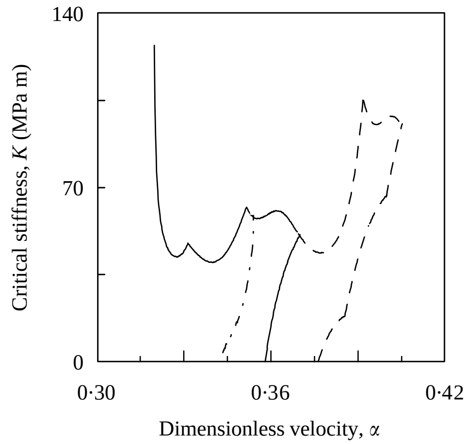

Figure 9. Effect of the mass of the bogie supports. Key for mass, m (kg): – – –, 500; ——, 1000; – · –, 2000.

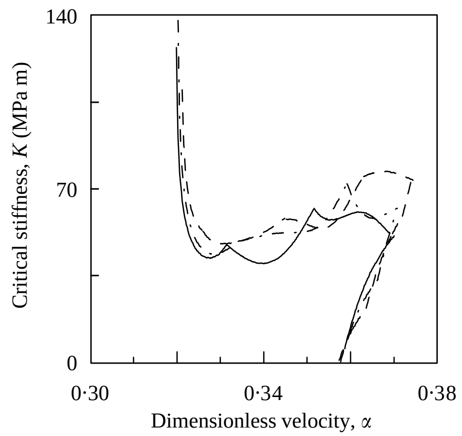

Figure 10. Effect of the wheelbase. Key for wheelbase length, d (m): – – –, 10·7; ——, 15·7; – · –, 20·7.

only slightly. One should be aware, however, that if the damping in the beam foundation were smaller, the effect of the wheelbase could become more pronounced. This would be related to the possible formation of standing waves between the bogie supports.

Summarizing the results of the parametric study of the model stability, one can say that among the parameters considered the most influential ones are the damping factors ε_0 (damping in the supports) and ν_f (damping in the foundation). The former can significantly change the critical stiffness of the supports, whereas the latter is responsible for the critical velocity causing instability to occur.

5. COMPARISON WITH SIMPLER MODELS

In this section, the stability domain for the bogie is compared with that for simpler models, which have been analyzed in references [15, 16].

5.1. COMPARISON WITH A TWO-MASS OSCILLATOR

In reference [15] the stability analysis was carried out for a two-mass oscillator on a flexibly supported Timoshenko beam. The oscillator was composed of two masses connected by a spring and a dashpot. The difference between the oscillator and the bogie is that the former has only one contact point with the beam and possesses no moment of inertia of the upper mass. For comparison, the lower mass of the oscillator was taken to be equal to the mass of the bogie support, while the upper mass of the oscillator was considered as equal to the half of the mass of the bogie bar. The damping coefficient of the dashpot of the oscillator was assumed to be equal to that of the bogie support. The parameters of the beam were taken from set (14).

The boundary of the stability domain for both cases is presented in Figure 11. The figure shows that the stability domains in these two cases are quite similar. The stability domain for the bogie, however, is slightly smaller and much less monotonic. The latter is obviously concerned with the wave reflection between the bogie supports.

The most important conclusion to be made from this comparison is that the simple two-mass oscillator model can be used for a rough estimation of the stability of the bogie, since their stability domains do not differ much. One should be aware, however, of possible deviations of the stability domain for the bogie from that for the oscillator, especially in the case of a small foundation viscosity when the cross-influence of the bogie supports that takes place by means of waves in the beam becomes pronounced.

5.2. COMPARISON WITH A SIMPLIFIED MODEL FOR THE BOGIE

In reference [16], a simplified model for the bogie was considered in which the upper ends of the bogie supports were assumed to move horizontally thereby excluding the interaction between the supports through the bogie bar.

The boundary of the stability domain for the simplified bogie is shown in Figure 12 by a dashed line (there are some more boundaries for this model but they are related to such a high value of the stiffness K that it becomes irrelevant to take these boundaries into consideration). The domain that is located to the left of this dashed line is stable. Figure 12

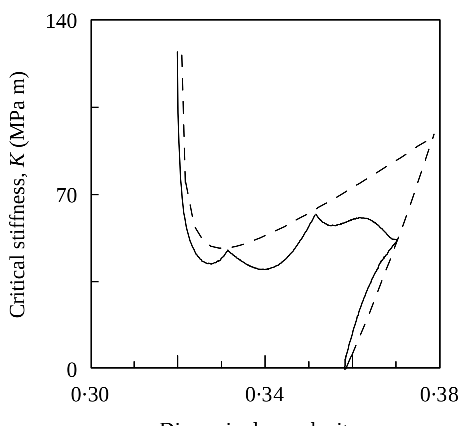

Figure 11. Comparison of the stability domain for the bogie (——) and a two-mass oscillator (– – –).

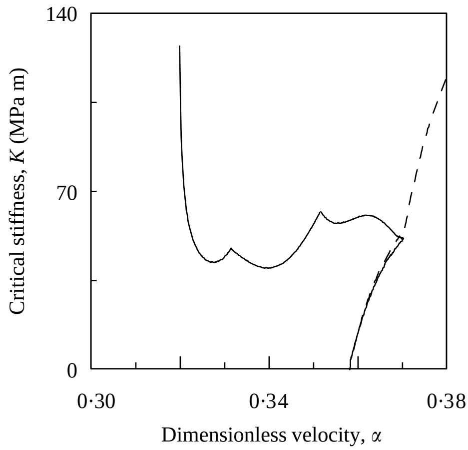

Figure 12. Comparison of the stability domain for the bogie (——) and the model by Wolfert *et al.* (simplified bogie) (- - -).

shows that the simplified bogie can be used for estimation of the critical velocity after which the instability arises but not for estimation of the critical stiffness.

6. CONCLUSIONS

The vertical vibrations of a bogie uniformly moving along a Timoshenko beam on a viscoelastic foundation has been considered. It has been shown that the amplitude of these vibrations may grow exponentially in time, which implies that the vibrations become unstable. The necessary condition for the instability to occur is that the velocity of the bogie exceeds a certain critical velocity. In the absence of viscosity in the beam foundation, this velocity is equal to the minimum phase velocity of waves in the beam on the elastic foundation.

Stability of the model is determined by the roots of the characteristic equation for the bogie interacting with the beam. To study these roots the D-decomposition method and the principle of the argument have been subsequently used. The D-decomposition method has been applied to divide the space of the model parameters into domains with different number of "unstable roots". Then the principle of the argument has been employed for a specific set of the model parameters to define the number of unstable roots in one of these domains. The authors believe that combination of these two methods allows one to study the stability of much more complex models that include, for example, a set of bogies or wagons and a three-dimensional foundation for the beam.

Parametric study of the stability domain has been carried out with emphasis on the effect of (1) damping in the bogie supports, (2) mass of the bogie bar, (3) damping in the beam foundation, (4) mass of the bogie supports and (5) bogie wheelbase. It has been shown that the stability of the model most crucially depends on the damping in the supports and the damping in the foundation. The mass of the bogie bar has been found to be the least influential factor.

The stability domain for the bogie has been compared with that for simpler models, namely for a two-mass oscillator considered in reference [15] and for a "simplified bogie" studied in reference [16]. It has been found that the two-mass oscillator model can be used for a rough estimation of the instability domain for the bogie, while the "simplified bogie" cannot be employed even for such an estimation.

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