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MULTIPLE DAMAGE LOCATION WITH FLEXIBILITY CURVATURE AND RELATIVE FREQUENCY CHANGE FOR BEAM STRUCTURES

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The purpose of this paper is to seek efficient methods for multiple damage location in beam structures. Two methods are studied in terms of finite element model (FEM) simulations on beam structure. Firstly, a finite element model of reinforced concrete beam is established to compute changes in flexibility and flexibility curvature of the beam with various damage patterns, and sensitivities of flexibility and flexibility curvature for closely distributed damage patterns of various extents are also compared. Due to its high sensitivity to closely distributed damages, flexibility curvature is recommended for multiple damage location. Secondly, the full correlation between the relative frequency changes and the analytical frequency changes are confirmed with FEM analyses: therefore, the relative frequency changes, rather than the frequency changes themselves, are used in the multiple damage location assurance criterion (MDLAC) to increase the location efficiency. Numerous examples are presented to verify the two methods.

1. INTRODUCTION

Damages are generally detected indirectly through changes in modal parameters and structural parameters. So it is important to find the most sensitive parameters to various damage patterns in structures. Early damage detection methods are generally for detection and location of single damage. In terms of the finite element model (FEM), Cawley [1] first studied damage location method based on frequency changes. Contursi [2] and Williams [3] proposed the multiple damage location assurance criterion (MDLAC) method for multiple damage location, which are also based on frequency changes. This method assumes that one frequency change pattern matches one damage pattern, and the actual damage can be located by examining the correlation between the frequency change and the analytical frequency change. However, it is difficult to locate multiple damages by frequency changes only, because damages at different locations may result in the same frequency changes. Pandey [4] presented a method based on changes in flexibility, and applied it for multiple damage locations [5]. Though this method can locate single structural damage with only a few lower mode parameters, it remains difficult to locate multiple damages, as mentioned by the author [5].

This paper is organized as follows. In section 2, sensitivities of changes in flexibility and flexibility curvature to various damage patterns are first studied by computations on a beam structure with FEM. By comparing the sensitivities, the flexibility curvature is chosen for multiple damage location, especially for closely distributed damages. In section 3, the correlation of the relative frequency change and the analytical frequency change is

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studied with the FEM analyses on the beam structure. As a result, the relative frequency changes, rather than the frequency changes themselves, are used in conjunction with MDLAC [3] method. Examples are presented to illustrate the efficiency of the two methods in section 4.

2. CHANGE IN FLEXIBILITY AND FLEXIBILITY CURVATURE TO DAMAGE DETECTION

With the normalized modes $\boldsymbol{\phi}_i$, $\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i = 1$ $(i = 1, \ldots, n)$, the stiffness matrix $\mathbf{K}$ and flexibility matrix $\mathbf{F}$ of the model can be expressed by mode expansion [4]

$$\mathbf{K} = \mathbf{M}\boldsymbol{\Phi}\boldsymbol{\Omega}\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M} = \mathbf{M}\left(\sum_{i=1}^{n} \omega_i^2 \boldsymbol{\phi}_i \boldsymbol{\phi}_i^{\mathrm{T}}\right)\mathbf{M}, \tag{1}$$

$$\mathbf{F} = [f_{i,j}] = \mathbf{K}^{-1} = \boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^{\mathrm{T}} = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \boldsymbol{\phi}_i \, \boldsymbol{\phi}_i^{\mathrm{T}}, \tag{2}$$

where $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_n]$ is an $n_n \times n$ dimensional mode shape matrix, n_n is the number of nodes. $\boldsymbol{\Omega} = \mathrm{diag}[\omega_1^2, \ldots, \omega_i^2, \ldots, \omega_n^2]$ is the eigenvalue matrix and ω_i is the ith natural frequency, n is the order of the system and $f_{i,j}$ is the ith row and jth column element of the flexibility matrix $\mathbf{F}$. As can be seen from equations (1) and (2), the modal contribution to the stiffness matrix increases as the frequency ω_i increases. On the other hand, the modal contribution to the flexibility matrix decreases as the frequency increases, so the flexibility matrix converges rapidly with several (generally, the first 2 or 3) lower modes. Therefore, a good estimate of the flexibility matrix can be approximated with lower modes. In this paper, only the first 3 modes are used to estimate the flexibility matrix.

Denote the change of flexibility matrix as

$$\Delta = \mathbf{F}^{(0)} - \mathbf{F}^{(d)} = [\delta_{ij}], \tag{3}$$

where $\mathbf{F}^{(0)}$ and $\mathbf{F}^{(d)}$ are the flexibility matrices of the intact and the corresponding damaged structure, respectively. δ_{ij} is the ith row and jth column element of matrix Δ. The damages result in stiffness reduction and the flexibility increment in the corresponding elements near the damages, especially in the corresponding diagonal elements. So if we define flexibility change vector as

$$\boldsymbol{\delta} = \{\bar{\delta}_{11} \cdots \bar{\delta}_{jj} \cdots \bar{\delta}_{n_n n_n}\}^{\mathrm{T}} \tag{4}$$

then it can be used as a measure of change in flexibility at each measurement location to detect and locate damages in structure [4]. The location of damage is considered to be at the degree of freedom k which makes $\bar{\delta}_k = \max_j(\bar{\delta}_{jj})$.

When locating multiple damages, $\bar{\delta}_k$ is taken as local maximum absolute values of the elements in the flexibility change vector $\boldsymbol{\delta}$.

However, methods based on change in flexibility matrix refer to the flexibility matrix of intact structure as baseline. In practice, the availability of parameters for intact structure is generally not guaranteed. The method may also give false damage prediction when the damaged positions are not well-separated or when different positions are damaged to different extents, due to which local maximum changes in flexibility may not occur at the damaged regions.

Flexibility curvature can be approximated by a difference scheme as follows:

$$f_i^{(c)} = \frac{f_{i-1,i-1} - 2f_{i,i} + f_{i+1,i+1}}{\Delta l^2}, \quad i = 2, \ldots, n_n - 1, \tag{5}$$

where $f_{i,i}$ and $f_i^{(c)}$ are the ith diagonal element of the flexibility matrix and the ith item of the flexibility curvature vector, respectively, and Δl is the length of the elements.

As to the continuous structure without damages, the flexibility curvature vector will possess a smooth curve shape. So the local peak on the curve can be used to indicate abnormal flexibility/stiffness changes at that position, i.e., it normally means damages occur at the corresponding positions. In this way, the local peak positions of flexibility curvature can be used to locate multiple damages in structure. Thus, the multi-damage detection method based on the flexibility curvature of the current structure is developed.

The most practical advantage of the method based on the flexibility curvature is that its computation does not refer to the parameters of intact structure as baseline. This is because a good estimate of the flexibility matrix of the damaged structure can be obtained by using several lower modes, either from experimental modal analysis or from finite element analysis. According to equation (5), the corresponding lower modes are sufficient to estimate flexibility curvature vector, by which the damage status of the structure can be detected.

3. THE PATTERN CORRELATION OF RELATIVE FREQUENCY CHANGE WITH ANALYTICAL FREQUENCY CHANGE

In the MDLAC method the sensitivity of the ith frequency f_i to damage extent at the jth position D_j is defined as [3]

$$S_{ij} = \frac{\partial f_i}{\partial D_j} = \frac{1}{8 f_i^0 \pi^2} \frac{\{\varphi_i^0\}^{\mathrm{T}}[K_j^0]\{\varphi_i^0\}}{\{\varphi_i^0\}^{\mathrm{T}}[M^0]\{\varphi_i^0\}}, \tag{6}$$

where $[K_j^0]$, $[M^0]$ and $\{\varphi_i^0\}$ are, respectively, the global formulation of the stiffness matrix of element j, the global mass matrix and the ith mode shape. Superscript "0" represents the intact structure. As to beam-like structures subjected to bending load, D_j is defined as $(B_j^{(0)} - B_j^{(d)})/B_j^{(0)}$, where $B_j^{(d)}$ and $B_j^{(0)}$ are the bending stiffness of the damaged beam element j and the corresponding intact beam element j respectively.

The pattern correlation of frequency change $\{\Delta f\}$ and analytical frequency change $\{\delta f\}$ is a measure of the confidence to which the actual damage pattern matches the analytical one. Williams [3] defined MDLAC as

$$\mathrm{MDLAC}(\delta D) = \frac{|\{\Delta f\}^{\mathrm{T}} \cdot \{\delta f(\{\delta D\})\}|^2}{(\{\Delta f\}^{\mathrm{T}} \cdot \{\Delta f\}) \cdot (\{\delta f(\{\delta D\})\}^{\mathrm{T}} \cdot \{\delta f(\{\delta D\})\})}, \tag{7}$$

where $\{\delta f\} = [S]\{\delta D\}$ is the analytical frequency change vector, $\{\delta D\}$ is the element stiffness reduction factor vector of the structure. If element 1 is damaged to the extent of 0·5, then $\{\delta D\} = \{0{\cdot}5 \;\; 0 \;\; \cdots \;\; 0\}^{\mathrm{T}}$. $\{\Delta f\} = \{f\} - \{f^0\}$ is the frequency change vector, where $\{f\}$ and $\{f^0\}$ are the frequency column vectors for the damaged and the corresponding intact structures respectively.

In this paper, the relative frequency change is used in the MDLAC, instead of the absolute frequency change $\{\Delta f\}$ in equation (7). Define the relative frequency change vector as

$$\{\Delta f^{(r)}\} = \{\Delta f_1^{(r)}, \cdots \Delta f_i^{(r)}, \cdots \Delta f_m^{(r)}\}^{\mathrm{T}},$$

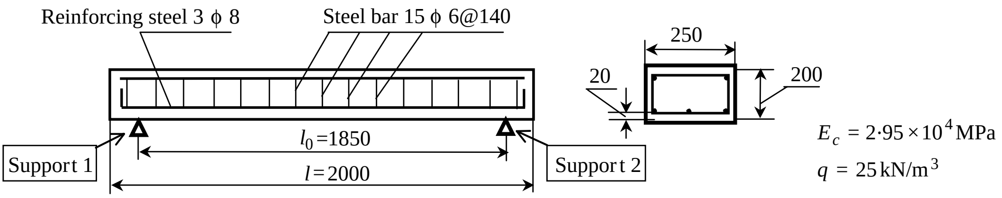

Figure 1. A simply supported reinforced concrete beam, length unit: mm. E_c and q are the elastic modulus and weight density of the concrete.

where $\Delta f_i^{(r)} = \Delta f_i, / f_i^0$, Δf_i and f_i^0 represent the ith term in the frequency change vector $\{\Delta f\}$ and the frequency vector$\{f^0\}$, respectively, i is the mode number, m is the interest order of the system. The MDLAC in this paper is defined as

$$\mathrm{MDLAC}(\delta D) = \frac{|\{\Delta f^{(r)}\}^T \cdot \{\delta f(\{\delta D\})\}|^2}{(\{\Delta f^{(r)}\}^T \cdot \{\Delta f^{(r)}\}) \cdot (\{\delta f(\{\delta D\})\}^T \cdot \{\delta f(\{\delta D\})\})}. \tag{8}$$

Conventional optimization methods can be used for pattern matching of $\{\Delta f^{(r)}\}$ and $\{\Delta f\}$ with $\{\delta f\}$.

4. VERIFICATION WITH A REINFORCED CONCRETE BEAM

A simply supported reinforced concrete beam, as shown in Figure 1, is used to study the validity of the proposed methods. The beam is modelled with 61 identical FEM beam elements. The damage extent D_j of element j is defined as $D_j = (B_j^{(0)} - B_j^{(d)})/B_j^{(0)}$, where $B_j^{(d)}$ and $B_j^{(0)}$ are the bending stiffnesses of the damaged model and the corresponding intact model of the beam respectively. Different damage patterns are simulated by FEM. The dynamical parameters calculated are used to study the damage detection capability of the proposed methods for different simulated damage patterns, which are listed in Table 1.

4.1. EFFECTIVENESS OF CHANGE IN FLEXIBILITY AND FLEXIBILITY CURVATURE TO MULTIPLE DAMAGE DETECTION

According to equation (2), good estimate of flexibility should be obtained so that flexibility change method and flexibility curvature method can be used for damage detection. The vector formed by diagonal elements of the flexibility matrix of the intact beam model is approximated with the first 1, 2, 3 and 5 modes in Figure 2 respectively. It is clear that the vector converged rapidly with the first 2 or 3 modes, so in this paper the flexibility matrices of different damage patterns of the simply supported beam are all estimated with the first 3 modes.

Curves of the flexibility change and the flexibility curvature for different damage patterns listed in Table 1 are plotted in Figures 3–6 and 7–12 respectively. Damages identified by using flexibility change method and flexibility curvature method are also listed in Table 1 for comparison.

It can be clearly seen that the flexibility change method is good at detecting sparsely distributing damages: even when the damage extent of the element drops to 0·1 and 0·05, damages can still be clearly located, as can be seen from identification results for damage pattern 1 in Table 1. However, the flexibility change is not a good damage indicator for

Table 1

Damage patterns

Damage pattern no.		1	2	3	4
Damaged element no.		27, 35	23, 31, 39	29, 31, 33	31, 46
Damage extent		0·05, 0·05	0·05, 0·05, 0·05	0·05, 0·05, 0·05	0·05, 0·05
Flexibility change method	Damage identified	27, 35	31	16, 20, 44, 47	31
	Figure no.	Figure 3	Figure 4	Figure 5	Figure 6
Flexibility curvature method	Damage identified	30, 33	30, 33	None	None
	Figure no.	Figure 7	Figure 8	Figure 10	Figure 11
Damage extent		0·1, 0·1	0·1, 0·1, 0·1	0·1, 0·1, 0·1	0·1, 0·1
Flexibility change method	Damage identified	27, 35	31	16, 20, 44, 47	31
	Figure no.	Figure 3	Figure 4	Figure 5	Figure 6
Flexibility curvature method	Damage identified	26, 30, 33, 36	25, 29, 32, 34, 36	None	None
	Figure no.	Figure 7	Figure 8	Figure 10	Figure 11
Damage extent		0·2, 0·2	0·2, 0·2, 0·2	0·2, 0·2, 0·2	0·2, 0·2
Flexibility change method	Damage identified	27, 35	31	31	31
	Figure no.	Figure 3	Figure 4	Figure 5	Figure 6
Flexibility curvature method	Damage identified	None	24, 31, 39	17, 30, 33, 46	31
	Figure no.	Figure 7	Figure 8	Figure 10	Figure 11
Damage extent			0·3, 0·3, 0·2		0·3, 0·3
Flexibility change method	Damage identified		31		31
	Figure no.		Figure 4		Figure 6
Flexibility curvature method	Damage identified		23, 31, 39		31, 46
	Figure no.		Figure 9		Figure 12
Damage extent			0·5, 0·5, 0·3		0·4, 0·4
Flexibility change method	Damage identified		31		
	Figure no.		Figure 4		
Flexibility curvature method	Damage identified		23, 31, 39		31, 46
	Figure no.		Figure 9		Figure 12
Damage extent		0·5, 0·5	0·5, 0·5, 0·5	0·5, 0·5, 0·5	
Flexibility change method	Damage identified	27, 35	31	31	
	Figure no.	Figure 3	Figure 4	Figure 6	
Flexibility curvature method	Damage identified	27, 35	23, 31, 39	29, 31, 33	
	Figure no.	Figure 7	Figure 9	Figure 10	

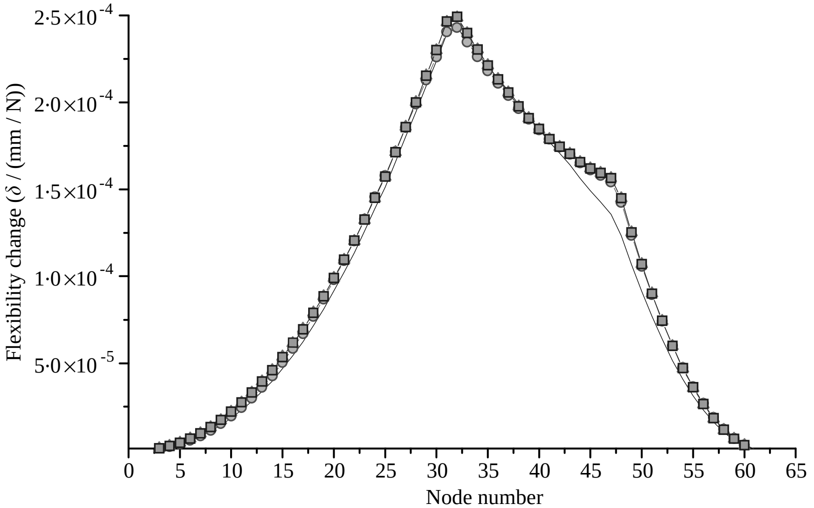

Figure 2. Flexibility change estimations for damages at elements 31 and 46 to the extent of 0·5. —— with the first mode; —◎— with the first 2 modes; —△— with the first 3 modes; —▦— with the first 5 modes.

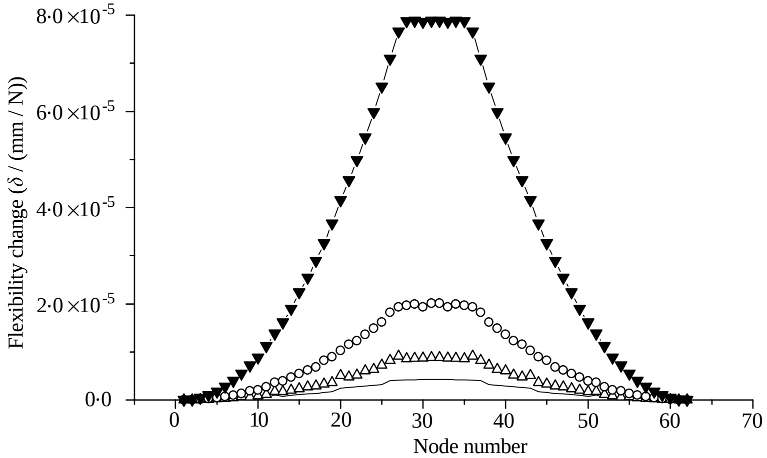

Figure 3. Flexibility changes with damages at elements 27 and 35 to extents: 0·05, 0·1, 0·2 and 0·5. —— 27, 35 $D_j = 0{\cdot}05, 0{\cdot}05$; —△— 27, 35 $D_j = 0{\cdot}1, 0{\cdot}1$; —○— 27, 35 $D_j = 0{\cdot}2, 0{\cdot}2$; —▼— 27, 35 $D_j = 0{\cdot}5, 0{\cdot}5$.

closely distributing damages, even when the damages extent of the element rises to 0·5, not all the damages are located, as can be seen from the identification results for damage pattern 2, 3, and 4 in Table 1.

Although the flexibility change is not able to locate every position of the closely distributing damages, at least one distinguishable local peak can be seen within the damage areas for all the damage patterns in Table 1. The local peak value increases with the increasing damage extent of the element. See also Figures 3–6. So, the flexibility change is a good damage index for indicating damage existence in structures being tested. Suppose there is a local peak in the curve on element 31. We cannot conclude that the damage definitely takes place on element 31, because damages on element 23, 31 and 39, or 29, 31

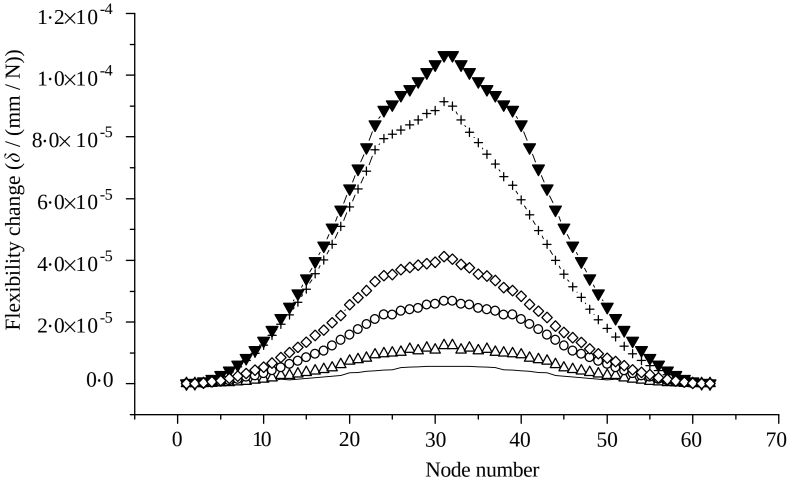

Figure 4. Flexibility changes with damages at elements 23, 31 and 39 to different extents. —— 23, 31, 39 $D_j = 0{\cdot}05, 0{\cdot}05, 0{\cdot}05$; —△— 23, 31, 39 $D_j = 0{\cdot}1, 0{\cdot}1, 0{\cdot}1$; —○— 23, 31, 39 $D_j = 0{\cdot}2, 0{\cdot}2, 0{\cdot}2$; —◇— 23, 31, 39 $D_j = 0{\cdot}3, 0{\cdot}3, 0{\cdot}2$; —+— 23, 31, 39 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}3$; —▼— 23, 31, 39 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}5$.

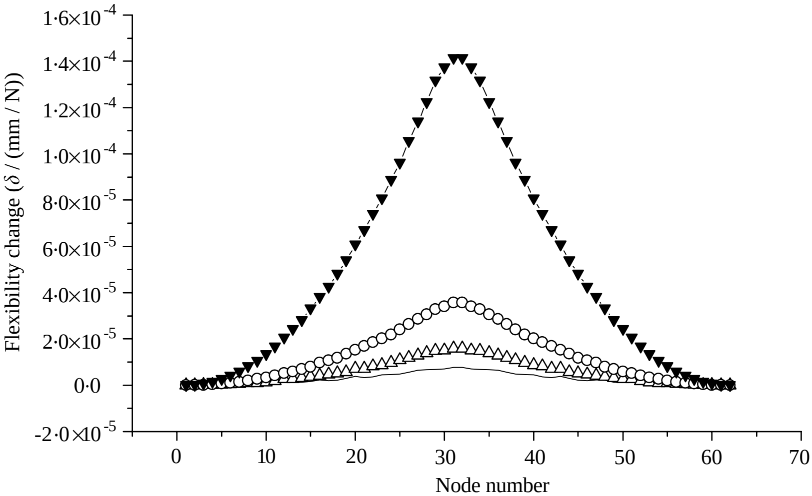

Figure 5. Flexibility changes with damages at elements 29, 31 and 33 to extents: 0·05, 0·1, 0·2 and 0·5. —— 29, 31, 33, 33 $D_j = 0{\cdot}05, 0{\cdot}05, 0{\cdot}05$; —△— 29, 31, 33 $D_j = 0{\cdot}1, 0{\cdot}1, 0{\cdot}1$; —○— 29, 31, 33 $D_j = 0{\cdot}2, 0{\cdot}2, 0{\cdot}2$; —▼— 29, 31, 33 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}5$.

and 33 and the like may result in similar local peak in the curve on element 31 (see also Figures 4 and 5). However, we can conclude that damages did take place in the structure and damages are in the vicinity of element 31. From the identified results listed in Table 1, it is clear that the flexibility change method can be used to determine whether or not there are damages in the structure with very high sensitivity. It can even detect damages to the extent of 0·05, see also Figures 3–6.

In contrast with the flexibility change method, the flexibility curvature is not a sensitive indicator for small damages: even when the damage extent of the element rises to 0·2, it is

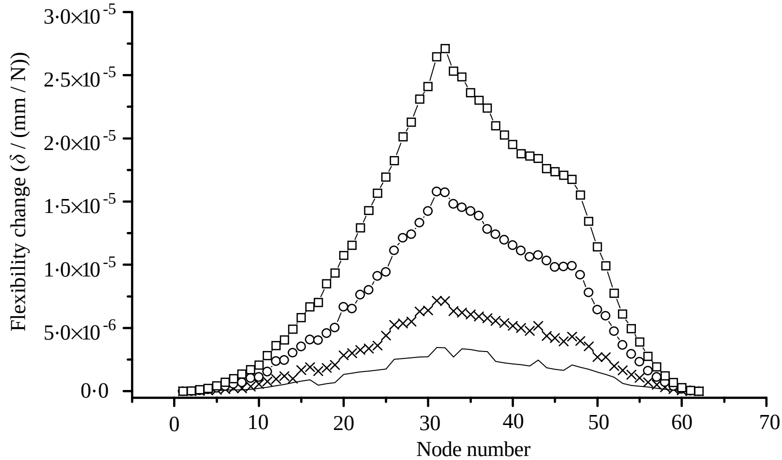

Figure 6. Flexibility changes with damages at elements 31 and 46 to extents: 0·05, 0·1, 0·2 and 0·3. —— 31, 46 $D_j = 0{\cdot}05, 0{\cdot}05$; —×— 31, 46 $D_j = 0{\cdot}1, 0{\cdot}1$; —○— 31, 46 $D_j = 0{\cdot}2, 0{\cdot}2$; —□— 31, 46 $D_j = 0{\cdot}3, 0{\cdot}3$.

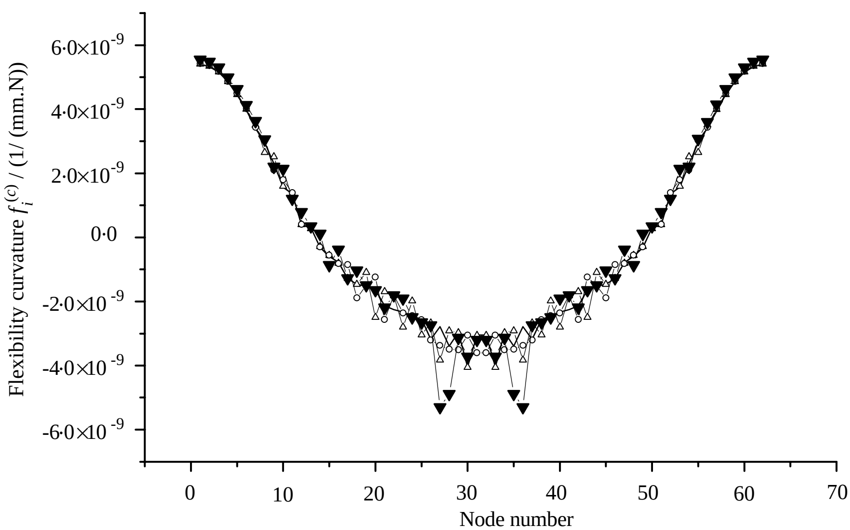

Figure 7. Flexibility curvatures with damages at elements 27 and 35 to extents: 0·05, 0·1, 0·2 and 0·5. —— 27, 35 $D_j = 0{\cdot}05, 0{\cdot}05$; —△—27, 35 $D_j = 0{\cdot}1, 0{\cdot}1$; —○— 27, 35 $D_j = 0{\cdot}2, 0{\cdot}2$; —▼— 27, 35 $D_j = 0{\cdot}5, 0{\cdot}5$.

not able to precisely indicate all the damage locations, see Figures 7, 8, 10 and 11. However, it is a good damage indicator when damage extent of the element is greater than 0·3. It can precisely indicate almost all the locations of the closely distributing damages, as can be seen from the identification results for damage patterns 1–4 with damage extent greater than 0·3 in Table 1, see also Figures 7, 9, 10 and 12. Especially in pattern 3 in Table 1, the distances between damage locations are only 2 elements in length.

4.2. MDLAC BASED ON RELATIVE FREQUENCY CHANGE

For the case of element 35 being damaged to the extent of 0·5, the analytical frequency change, the frequency change and relative frequency change from the intact model are

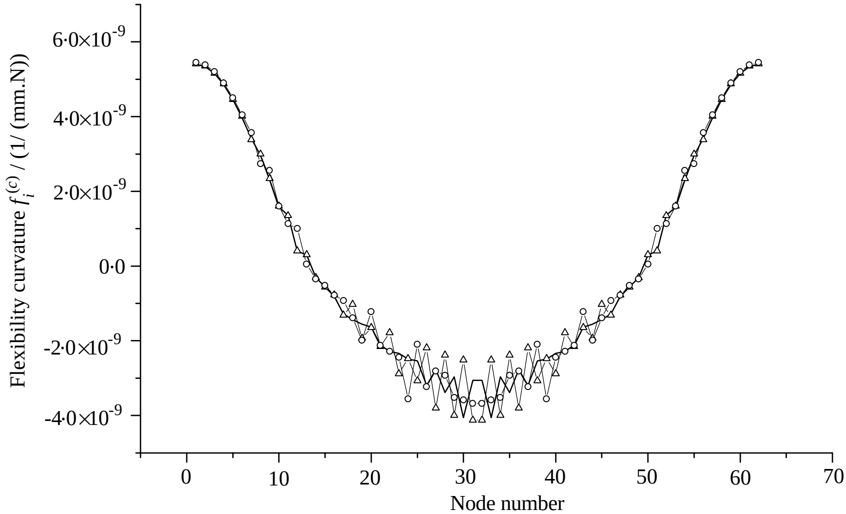

Figure 8. Flexibility curvatures with damages at elements 23, 31 and 39 to extents: 0·05, 0·1 and 0·2. —— 23, 31, 39 $D_j = 0{\cdot}05, 0{\cdot}05, 0{\cdot}05$; —△— 23, 31, 39 $D_j = 0{\cdot}1, 0{\cdot}1, 0{\cdot}1$; —○— 23, 31, 39 $D_j = 0{\cdot}2, 0{\cdot}2, 0{\cdot}2$.

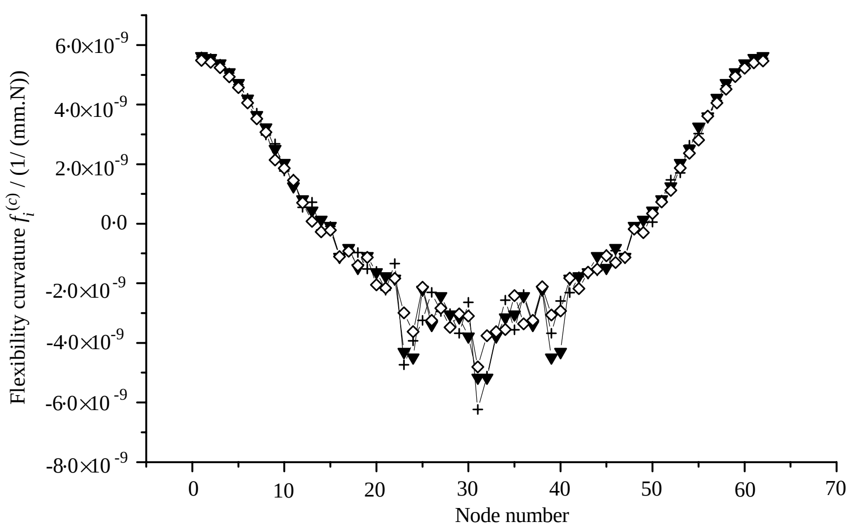

Figure 9. Flexibility curvatures with damages at elements 23, 31 and 39 to extents: 0·2, 0·3 and 0·5. —◇— 23, 31, 39 $D_j = 0{\cdot}3, 0{\cdot}3, 0{\cdot}2$; —+— 23, 31, 39 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}3$; —▼— 23, 31, 39 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}5$.

shown in Figure 13. It is clear that the analytical frequency change curve is fully correlated with the relative frequency change. So it is of advantage to use the relative frequency changes in the MDLAC.

The first 15 modal frequencies for the model with both elements 15 and 47 damaged to the extent of 0·5 and the corresponding intact model are depicted in Table 2. According to William [3], in order to improve the accuracy of damage detection, damage-insensitive frequencies (frequencies with smaller changes due to damages) should be eliminated from the frequency change vector and the corresponding analytical frequency change vector, and

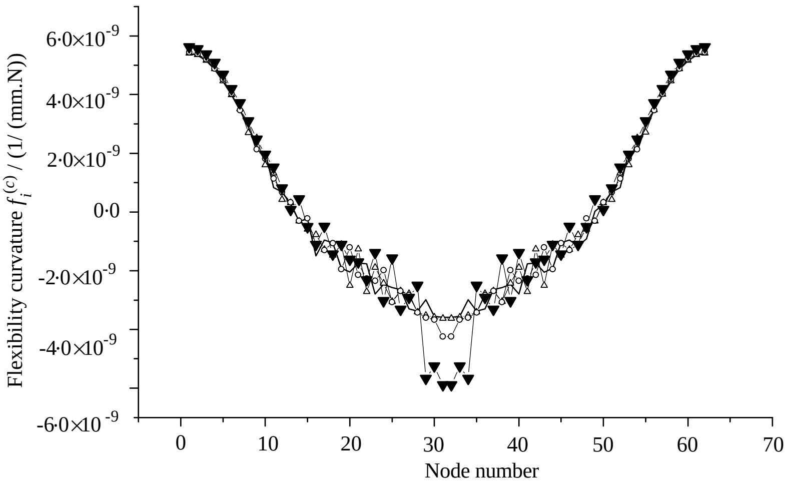

Figure 10. Flexibility curvatures with damages at element 29, 31 and 33 to extents: 0·05, 0·1, 0·2 and 0·5. —— 29, 31, 33 $D_j = 0{\cdot}05, 0{\cdot}05, 0{\cdot}05$; —△— 29, 31, 33 $D_j = 0{\cdot}1, 0{\cdot}1, 0{\cdot}1$; —○— 29, 31, 33 $D_j = 0{\cdot}2, 0{\cdot}2, 0{\cdot}2$; —▼— 29, 31, 33 $D_j = 0{\cdot}5, 0{\cdot}5, 0{\cdot}5$.

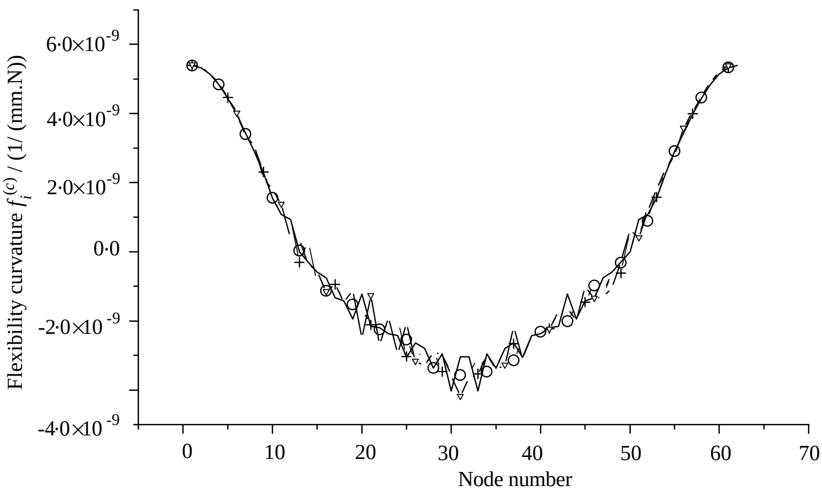

Figure 11. Flexibility curvatures with damages at elements 31 and 46 to extents: 0·0, 0·05, 0·1 and 0·2. —— intact; —○— 31, 46 $D_j = 0{\cdot}05, 0{\cdot}05$; —+— 31, 46 $D_j = 0{\cdot}1, 0{\cdot}1$; —▽— 31, 46 $D_j = 0{\cdot}2, 0{\cdot}2$.

the candidate damage searching positions in the vicinity of the real-life damage region are recommended. In practice, the region in the vicinity of the damage locations identified with all possible positions can be used as the candidate damage regions, or you may use other simple methods such as the flexibility change method to determine the candidate damage regions. In this paper the same procedure is followed. As shown in Figures 14 and 15, for damage pattern 3 in Table 1 with extents 0·05 and 0·5, respectively, damages cannot be predicted with all the first 15 modal frequencies and the full 61 searching positions. On the contrary, damages in elements 23, 31 and 39 are accurately detected by using 12 dominating frequencies and by searching 20 candidate positions with the Daviden–Fletcher–Pellew algorithm in MATLAB, see Figures 16–18. This example demonstrates that the MDLAC

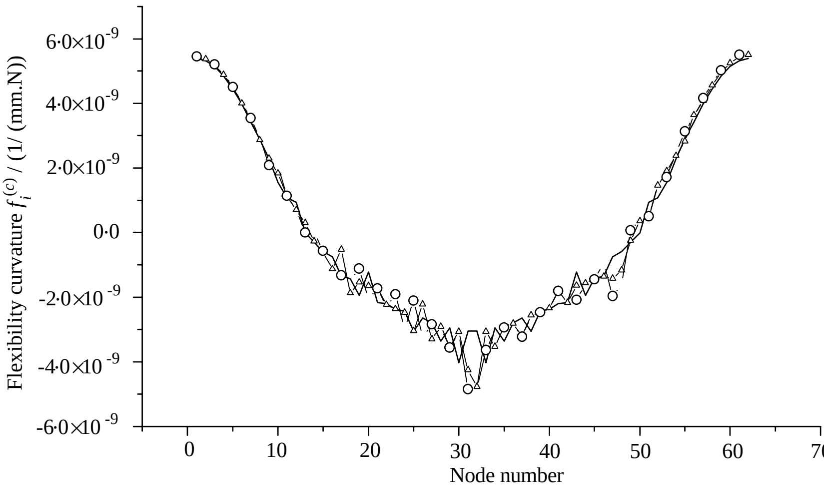

Figure 12. Flexibility curvatures with damages at elements 31 and 46 to extents: 0·0, 0·3 and 0·4. —— intact; —△— 31, 46 $D_j = 0{\cdot}3, 0{\cdot}3$; —○— 31, 46 $D_j = 0{\cdot}4, 0{\cdot}4$.

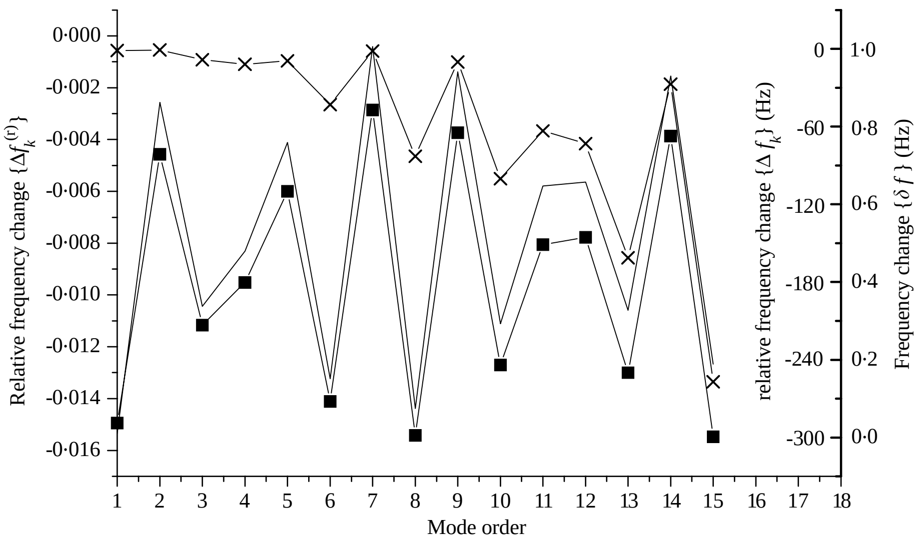

Figure 13. Frequency change, relative frequency change and analytical frequency change with element 35 damaged to extent 0·5. —■— analytical frequency change $\{\delta f\} = [S]\cdot\{D\}$; —×— frequency change; —— relative frequency change.

Table 2

The first 15 *modal frequencies in Hertz of the damage model* (*both elements* 15 *and* 47 *are damaged to the extent of* 0·5) *and the corresponding intact model*

Intact	90·2	360·8	811·8	1443·2	2255·0	3247·2	4419·7	5772·6
	7305·9	9019·5	10913·3	12987·3	15241·3	17675·3	20289·1	
Damaged	88·8	349·6	796·3	1442·0	2233·3	3156·2	4322·0	5754·7
model	7264·6	8803·1	10653·0	12906·0	15192·0	17331·0	19798·0	

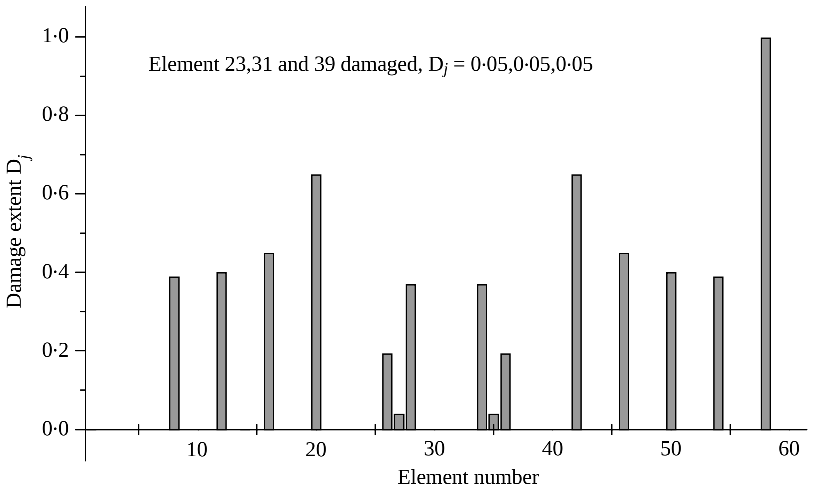

Figure 14. Damage detection results with 15 modes, 61 searching positions, for elements 23, 31 and 39 all damaged to extent 0·05.

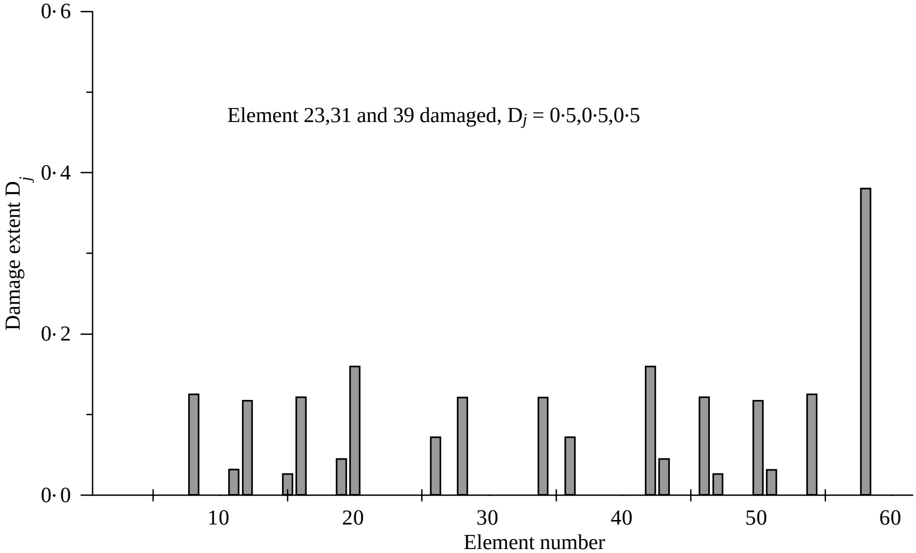

Figure 15. Damage detection results with 15 modes, 61 searching positions, for elements 23, 31 and 39 all damaged to extent 0·5.

method can be used to locate small (in this example, on element damaged to the extent of 0·05 is still detectable) multiple damages in the structure.

5. CONCLUDING REMARKS

The presented examples show that changes in flexibility are very sensitive to local damages and the damage has been successfully indicated for a reduction in bending stiffness of only 5%. However, changes in flexibility are difficult to locate in closely distributed

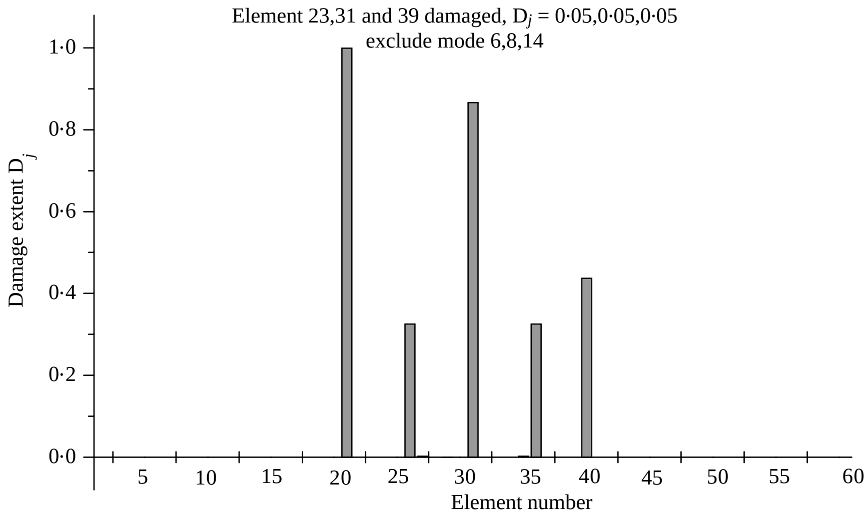

Figure 16. Damage detection results with 12 modes and 20 searching positions, for elements 23, 31 and 39 all damaged to extent 0·05.

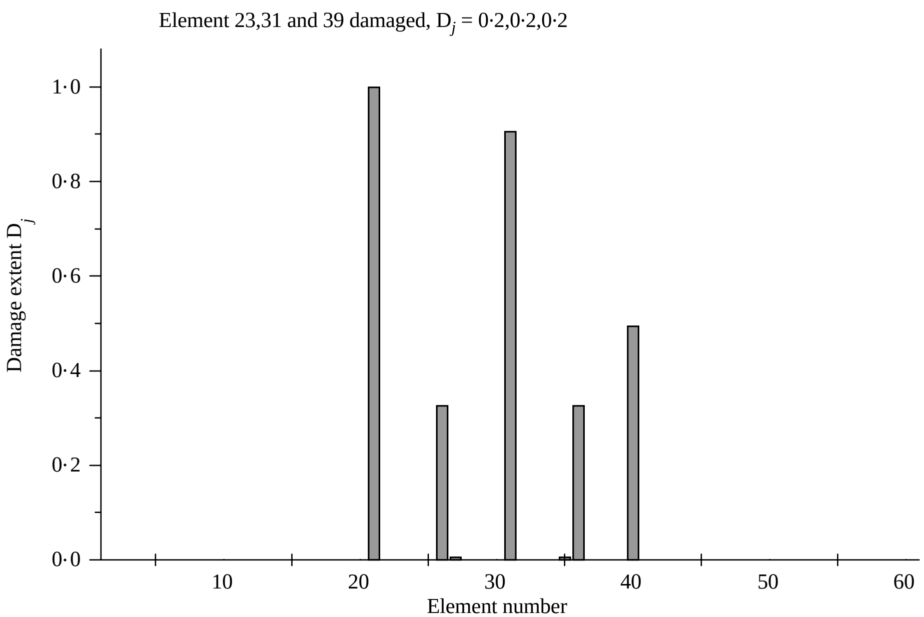

Figure 17. Damage detection results with 12 modes and 20 searching positions, for elements 23, 31 and 39 all damaged to extent 0·2.

damages because peaks for different damages merge together and are difficult to identify. It is found that the flexibility curvature method is effective for locating very closely distributed damages, provided the damage extent of the element is $> 0{\cdot}3$. The most attractive feature of the flexibility curvature method is that it can be implemented without the initial curvature parameters of intact structure. However, data at a large number of positions need to be

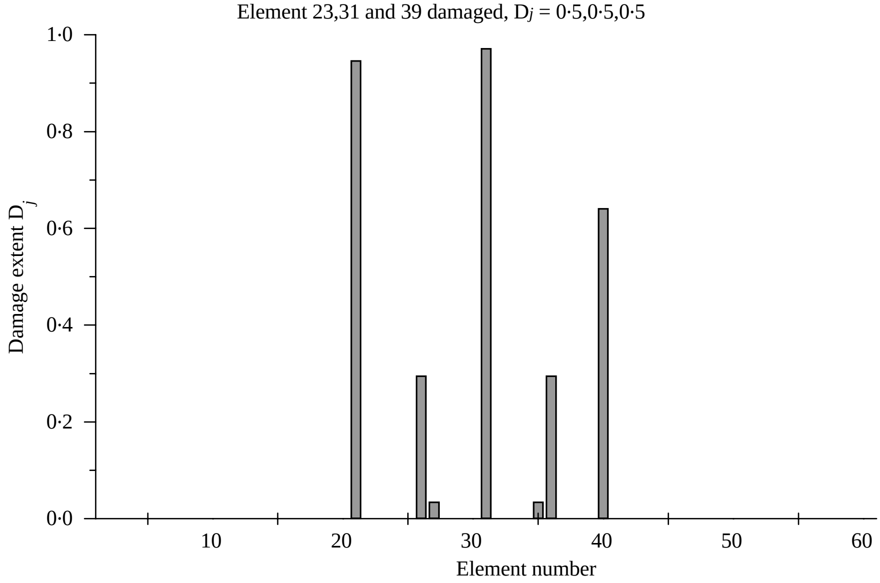

Figure 18. Damage detection results with 12 modes and 20 searching positions, for elements 23, 31 and 39 all damaged to extent 0·5.

measured for estimating the flexibility curvature. Hopefully, the newly appeared line-distribution transducers such as laser fiber strain transducer or laser fiber curvature transducer can be adapted for such use in future applications.

The presented example shows the full correlation between the relative frequency change and the analytical frequency change. As a result, MDLAC using the relative frequency change is very effective for local damages. It is also found that the result of this method can be improved by restricting the searching positions and by eliminating damage-insensitive frequencies. However, this method requires that higher order frequencies be measured.

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