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REPLY TO "COMMENTS ON 'VIBRATION ANALYSIS OF THIN CYLINDRICAL SHELLS USING WAVE PROPAGATION APPROACH'"†

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I thank the authors for their valuable comments on our paper. I agree with them that the wave propagation approach is an interesting method in calculating the natural frequencies of cylindrical shells for various boundary conditions. I think the discussions will help us to know more about this simple and effective method.

In our wave propagation approach [1], only the wavenumbers in the axial direction of the shell are needed for calculating the natural frequencies of cylindrical shells. These wavenumbers are approximately obtained from the wavenumbers of an equivalent beam with the same boundary conditions. No beam functions are needed for the displacement amplitudes in the axial direction of the shell.

The displacements of the shell were expressed in a general format of wave propagation and defined by

$$u = U_m \cos(n\theta) e^{(i\omega t - ik_m x)}, \quad v = V_m \sin(n\theta)\, e^{(i\omega t - ik_m x)}, \quad w = W_m \cos(n\theta)\, e^{(i\omega t - ik_m x)}, \tag{1}$$

where U_m, V_m and W_m are, respectively, the wave amplitudes in the x, θ, z directions, k_m the axial wavenumber and n the circumferential modal parameter.

In their wave approach [2], the displacements of the shell were expressed in the form of standing waves and defined by

$$\begin{aligned} \tilde{u}_z &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_{mnz} \alpha_m(z) \cos(n\theta - \phi_r)\, e^{j\omega t}, \\ \tilde{u}_\theta &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_{mn\theta}\, \beta_m(z) \sin(n\theta - \phi_r)\, e^{j\omega t}, \\ \tilde{u}_r &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_{mnr}\, \gamma_m(z) \cos(n\theta - \phi_r)\, e^{j\omega t}, \end{aligned} \tag{2}$$

where $\alpha_m(z)$, $\beta_m(z)$, $\gamma_m(z)$ are the amplitude distributions of the three displacements along the z direction. Subscripts m and n denote the mode number in the z and the θ directions, respectively. They actually used equation (12) of reference [2] for calculating the natural frequencies of the shell for three kinds of boundary conditions, namely, the simply supported–simply supported (SS–SS), clamped–clamped (C–C) and free–free (F–F) boundary conditions. The wavenumbers were given in equations (17), (18) and (19) of the paper, respectively, for these boundary conditions.

† C. Wang and J. C. S. Lai 2002 *Journal of Sound and Vibration* **249**, 1011–1015. Comments on "Vibration analysis of thin cylindrical shells using the wave propagation approach" (doi: 10.1006/jsvi.2001.3763).

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It is my personal opinion that our method could be easily extended for calculating the natural frequencies of complex shell structures. It has been extended for a cylindrical panel [3], in which the wavenumbers are needed for the axial as well as the circumferential directions of the panel. The wavenumbers for different boundary conditions were listed in Table 1 of reference [3]. The method can be also extended to a laminated composite cylindrical shell [4], where the difference between references [4] and [1] is that the governing equations of shell are further complex. Generally speaking, this method can be used for complex shells, like the shear deformation should be considered for a thick shell. In this case the right shell theory and the right wavenumber in the corresponding beam should be chosen for the analysis of natural frequencies with the wave propagation approach. This method can even be extended to a coupled fluid-structure analysis, as for a fluid-filled pipe structure [5]. This is possible because our solutions for the shell are expressed in a wave propagation format (equation (1)), which can be easily coupled with the fluid medium expressed in the same wave propagation format.

I should stress that this method is a simple and effective analytic method. With it, various boundary conditions can be easily handled. Furthermore, it can be combined with numerical methods, like FEM, and experimental methods to treat very complex boundary conditions. Because only the wavenumbers in the axial direction of the shell are needed, if they aren't be analytically available, like from Table 1 of reference [3], they can be found numerically or experimentally by studying the wavenumbers of the same beam with the same boundary conditions. It is not difficult to investigate experimentally the wave propagation in a beam with complex boundary conditions and get the wavenumbers of the beam. It may be difficult to model analytically and numerically the damping effects at the boundaries, but not difficult to measure the wave reflection from the boundaries.

I note that the authors did comparisons of frequency–wavenumber relationship between wave approach and FEM for shells with $a/h = 20$ and 5 in the comments and reference [2]. However in my personal view, these comparisons would be more appropriate if they are made between the exact solution and wave approach, then the validity and accuracy of the wave approach could be better evaluated. I hope to elaborate on it more.

The system characteristic equation of a shell (equation (5) of reference [1]) is

$$F(k_m, \omega) = 0, \tag{3}$$

where $F(k_m, \omega)$ is a polynomial function for the thin shell. This characteristic function can be used to investigate the wave propagation in the shell as well as the natural frequency of the shell. The exact frequency–wavenumber relation (dispersion curve) can be obtained for each given frequency ω from equation (3). This curve is continuous. With wave propagation approach, discrete wavenumbers are given. The corresponding resonant frequencies are also calculated from equation (3). This dispersion cure is discrete from the wave propagation approach. These two dispersion curves can be compared at discrete wavenumber points to see the validity and accuracy of the wave propagation approach. If the discrete dispersion curves are compared between the wave approach and FEM, it might be difficult to validate the wave approach. As we know the accuracy of FEM results is dependent on the number of mesh elements and nodes. The FEM result should be validated itself. This is a tough work especially for parametric analyses, for example, if we need to calculate the natural frequencies of shell for different thicknesses and use the results as reference. Maybe we need to validate the FEM results for different thicknesses. One kind of meshing might be fine for one thickness, but generates large errors for another thickness. Comparison of curves may be instructive but difficult for quantitative evaluation. In reference [1] we directly compared the natural frequencies of a shell between the wave propagation approach and other methods available in the literature for different shells with

SS–SS, C–C and clamped–simply supported (C–SS) boundary conditions. These comparisons were used for validation of wave propagation approach. We also compared the natural frequencies between the wave propagation approach with FEM and found the difference between them was within 2% after doing some meshing tests.

REFERENCES

1. X. M. Zhang, G. R. Liu and K. Y. Lam 2001 *Journal of Sound and Vibration* **239**, 397–403. Vibration analysis of thin cylindrical shells using wave popagation approach.
2. C. Wang and J. C. S. Lai 2000 *Applied Acoustics* **59**, 385–400. Prediction of natural frequencies of finite length circular cylindrical shells.
3. X. M. Zhang, G. R. Liu and K. Y. Lam 2001 *Applied Acoustics* **62**, 527–543. Frequency analysis of cylindrical panels using a wave propagation approach.
4. X. M. Zhang, 2001 *Applied Acoustics* **62**, 1221–1228. Vibration analysis of cross-ply laminated composite cylindrical shells using wave propagation approach.
5. X. M. Zhang, G. R. Liu and K. Y. Lam 2001 *Applied Acoustics* **62**, 229–243. Coupled vibration analysis of fluid-filled cylindrical shells using the wave propagation approach.