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STUDY ON THE RADIATION ACOUSTIC FIELD OF RECTANGULAR RADIATORS IN FLEXURAL VIBRATION

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In this paper, the radiation acoustic field of a rectangular radiator in flexural vibration is studied. The radiator is a rectangular thin plate in flexural vibration with simply supported boundary conditions. Based on the theory of Rayleigh integral, the acoustic pressure distribution of the radiator in its far field is obtained analytically and the near acoustic field of the radiator is also computed numerically. In the near field, the three-dimensional acoustic pressure distribution is obtained. In the far field, the two- and three-dimensional acoustic pressure distribution patterns are calculated. The dependence of the radiation acoustic pressure of the radiator on the vibrational order of the plate in flexural vibration is analyzed. It is demonstrated that the radiation acoustic pressure along the central axis of the radiator is always zero in the near and far acoustic fields, and the radiation near acoustic field and the far acoustic field are both directional. This is different from the acoustic field generated by an oscillating piston. On the other hand, when the vibrational order is increased, the acoustic pressure distribution of the radiator becomes complex.

1. INTRODUCTION

There has been increased interest in the use of ultrasonic transducers to radiate in air in recent years. The applications of ultrasound in air include two main categories. One type of applications involves the ultrasound of small signal that is usually a short pulsant signal [1–4]. The applications of this type include ultrasonic gauging, ultrasonic ranging and ultrasonic alarming. The other type of applications involves the ultrasound of large signal that is usually a continuous sine or cosine signal. These applications include ultrasonic drying, ultrasonic defoaming, ultrasonic agglomeration and ultrasonic levitation. With the development of ultrasonic applications in air, the air transducer has received more and more attention. There are many types of air-coupled ultrasonic transducers. The widely used types include the piezoelectric transducer and the static transducer. In the design of piezoelectric air transducers, the main problem is the large impedance mismatch between the piezoelectric ceramic transducer and the air. This problem can be solved partially by using impedance matching layers. Another method to improve the impedance match for air transducers is to use the flexural vibration of a thin piezoelectric ceramic plate or a combination of two thin piezoelectric ceramic plates and a thin metallic plate, which is sandwiched between the piezoelectric ceramic plates. The third method to improve the impedance match is to use the compound transducer of mode conversion. In this type of compound transducer, the radiator of the transducer is a circular or rectangular thin plate that is excited by a longitudinal sandwich transducer. The thin plate undergoes flexural vibration. This type of transducers can be excited by small or large signals and used for ultrasonic non-destructive evaluation or ultrasonic processing.

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The use of high-power ultrasound in gases has also increased and received more attention during the last few years. For the generation of high-power sound in gases, there are mainly two methods. One is the use of air siren [5, 6], and the other involves the use of the piezoelectric transducer. Compared with the applications of small signal ultrasound, the impedance match for high-power piezoelectric ultrasonic air-coupled transducers becomes more crucial for high-power ultrasonic applications in air. As the specific acoustic impedances of gases are low compared with those of the piezoelectric transducers, the applications of high-power ultrasonic energy in gases are limited by the lack of high-power air-coupled transducers of high electro-acoustic efficiency. For traditional ultrasonic transducers, such as Langevin longitudinal ultrasonic transducers, their output ultrasonic power and the radiating efficiency in air are low because of the severe impedance mismatch.

In order to obtain an efficient transmission of high-power ultrasonic energy it is necessary to achieve a good impedance matching between the transducer and the gas. A new type of ultrasonic transducer of high efficiency, high power and directivity was developed, which can improve the impedance matching with the gas and can produce high power and intense ultrasound in the medium [7–13]. Since such compound transducers combine the large radiation surface of the thin plate in flexural vibration and the high power and high electro-acoustic efficiency of the traditional Langevin longitudinal transducer, they are more suitable for high-power ultrasonic applications in air.

On the other hand, it should be pointed out that the acoustic attenuation in air is large, especially for the case of high-power acoustic radiation. Considering this factor, the radiation distance of high-power ultrasound in air is confined. In practical applications, such as ultrasonic drying and ultrasonic agglomeration, the processing distance is not very large. This is a problem in applications involving high-power sound in air.

The radiation ultrasonic field is very important in practical applications for both small signal and large signal air-coupled ultrasonic transducers. In our previous paper [14], the acoustic pressure distribution of a circular thin plate radiator in flexural vibration was analyzed theoretically. The directivity pattern was derived. In this paper, the radiation ultrasonic field of the thin rectangular plate in flexural vibration is calculated, the acoustic pressure in the near field and the far field is obtained and the directivity function is derived. The relationship between the acoustic pressure and directivity function and the vibrational order of the flexural rectangular plate is analyzed. It can be seen that when the vibrational order of the rectangular thin plate in flexural vibration is increased, the distribution of the near field and far field radiation acoustic pressure becomes more complex.

2. FLEXURAL VIBRATION OF RECTANGULAR THIN PLATES WITH SIMPLY SUPPORTED BOUNDARY CONDITIONS

The flexural vibration of rectangular thin plates is a traditional subject. The solution to the flexural wave equation depends on the boundary conditions of the plate. Apart from the plate with simply supported boundary conditions, the analytical solutions to the wave equation of thin plates with other boundary conditions are difficult to find. Since the radiation acoustic field depends on the vibrational distribution of the rectangular thin plate, the radiation acoustic field cannot be found analytically for some boundary conditions. For simplicity, the article deals with the rectangular thin plate in flexural vibration with simply supported boundary conditions. For the flexural vibration of a rectangular thin plate with simply supported boundary conditions, the complete analytical solution to the wave equation can be easily obtained [15, 16],

$$\eta_{mn}(x, y) = A \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{W}, \tag{1}$$

where $\eta_{mn}(x, y)$ is the flexural vibration displacement, A is a constant, L and W are the length and width of the rectangular plate, and m and n are the vibrational orders of the plate in flexural vibration. Based on the simply supported boundary conditions, the natural frequency equation of the plate can be obtained,

$$\omega_{mn} = \pi^2 \left(\frac{m^2}{L^2} + \frac{n^2}{W^2} \right) \sqrt{\frac{D}{\bar{m}}}, \tag{2}$$

where $\omega_{mn} = 2\pi f_{mn}$ is the natural frequency, $D = ET^3/12(1 - v^2)$ is the rigidity of flexure of the plate, $\bar{m} = \rho T$ is the mass of unit area of the plate, T is the thickness of the plate, ρ is the mass density of the plate, E and v are Young's modulus and the Poisson ratio of the material. In this case, there are $m + 1$ and $n + 1$ nodal lines on the plate, which are parallel to the length and the width of the plate. From equation (2), the velocity distribution on the surface of the plate can be obtained as

$$u_A(x, y) = \mathrm{j}\omega A \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{W}. \tag{3}$$

3. RADIATION ACOUSTIC FIELD OF RECTANGULAR THIN PLATE IN FLEXURAL VIBRATION

Assume the rectangular thin plate radiator in flexural vibration is mounted on a flat rigid baffle of infinite extent. The co-ordinates are sketched in Figure 1. The acoustic pressure at the field point P can be obtained by dividing the radiating surface of the flexural plate into infinitesimal elements, each of which acts like a baffled simple source of strength $\mathrm{d}Q = u_A\, \mathrm{d}s$. The acoustic pressure generated by one of these infinitesimal sources at field point P is given by

$$\mathrm{d}p = \mathrm{j} \frac{k\rho_0 C_0}{2\pi H} u_A \,\mathrm{d}s \exp[\mathrm{j}(\omega t - kH)], \tag{4}$$

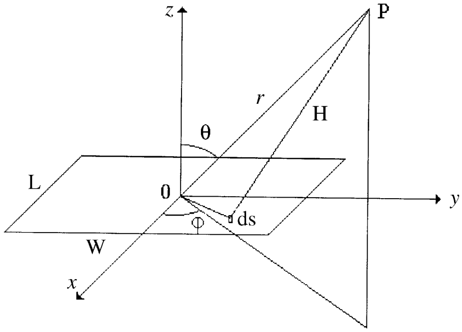

Figure 1. A rectangular thin plate in flexural vibration and its co-ordinates.

where $k = \omega/C_0$, C_0 is the speed of sound in air, ρ_0 is the density of air and H is the distance between the observation point P and the infinitesimal element. Based on the theory of Rayleigh integral, the total acoustic pressure is

$$p(r, \theta, t) = \iint_s \mathrm{j}\,\frac{k\rho_0 C_0}{2\pi H}\, u_A \exp[\mathrm{j}(\omega t - kH)]\,\mathrm{d}s, \tag{5}$$

where $s = LW$, $\mathrm{d}s = \mathrm{d}x\,\mathrm{d}y$, s is the radiating area of the plate, u_A is the speed with which the plate vibrates flexurally. Let the Cartesian co-ordinates of the observation point P be X_0, Y_0, Z_0, its spherical co-ordinates be r, θ, φ, r is also the distance between the center of the co-ordinates system and the observation point, $\mathrm{d}s$ is an infinitesimal element on the rectangular plate, its Cartesian co-ordinates are x and y. From Figure 1, the distance H can be expressed as

$$H = \sqrt{(X_0 - x)^2 + (Y_0 - y)^2 + Z_0^2}. \tag{6}$$

Substituting the expression of the velocity distribution u_A into equation (5) yields

$$p = -\frac{k\rho_0 c_0}{2\pi}\,\omega A \iint \sin(m\pi x/L)\sin(n\pi y/W)/H\mathrm{e}^{\mathrm{j}(\omega t - kH)}\,\mathrm{d}s. \tag{7}$$

Equation (7) is the expression of the radiation acoustic pressure of a rectangular thin plate in flexural vibration. In the following analysis, the far and near acoustic fields of the rectangular radiator are analyzed according to equation (7) respectively.

3.1. ACOUSTIC PRESSURE DISTRIBUTION IN THE FAR FIELD

For the far field, i.e., $r \rangle\rangle L, W$, the effect of the difference in distance on the amplitude of the radiation acoustic pressure can be ignored, while its effect on the phase cannot be ignored. Using the far field approximation, the distance H in the acoustic pressure amplitude can be approximated as r. For the distance in the phase of the acoustic pressure, it can be expressed as approximately

$$H = r - \frac{X_0 x + Y_0 y}{r}. \tag{8}$$

On the other hand, the position of the observation point can be also expressed in its directional cosines,

$$\cos\alpha = X_0/r, \quad \cos\beta = Y_0/r, \quad \cos\gamma = Z_0/r, \tag{9}$$

where α, β and γ are the directional cosines of the observation point. Based on the above analysis, the radiation acoustic pressure in the far field can be expressed as

$$p = -\frac{k\rho_0 c_0}{2\pi r}\,\omega A\mathrm{e}^{\mathrm{j}(\omega t - kr)} \iint \sin(m\pi x/L)\sin(n\pi y/W)\mathrm{e}^{\mathrm{j}k\cos\alpha x}\mathrm{e}^{\mathrm{j}k\cos\beta y}\,\mathrm{d}x\,\mathrm{d}y. \tag{10}$$

Integrating equation (10) yields

$$p = -\frac{k\rho_0 c_0}{2\pi r}\,\omega A\mathrm{e}^{\mathrm{j}(\omega t - kr)} P(x)P(y). \tag{11}$$

In equation (11), $P(x)$ and $P(y)$ are two introduced functions. Their expressions are

$$P(x) = \int_{-L/2}^{L/2} \sin(m\pi x/L) e^{jk\cos\alpha x}\,dx$$

$$= 2j\frac{k\cos(kL\cos\alpha/2)\cos\alpha\sin(m\pi/2) - \sin(kL\cos\alpha/2)m\pi/L\cos(m\pi/2)}{(m\pi/L)^2 - k^2(\cos\alpha)^2}, \qquad (12)$$

$$P(y) = \int_{-W/2}^{W/2} \sin(n\pi y/W) e^{jk\cos\beta y}\,dy$$

$$= 2j\frac{k\cos(kW\cos\beta/2)\cos\beta\sin(n\pi/2) - \sin(kW\cos\beta/2)n\pi/W\cos(n\pi/2)}{(n\pi/W)^2 - k^2(\cos\beta)^2}. \qquad (13)$$

Using the relation between the spherical co-ordinates and the directional cosines, we can get the following equations:

$$\cos\alpha = \sin\theta\cos\varphi, \qquad \cos\beta = \sin\theta\sin\varphi. \qquad (14)$$

Substituting equation (14) into equations (12) and (13) yields

$$P(x) = \int_{-L/2}^{L/2} \sin(m\pi x/L) e^{jk\cos\alpha x}\,dx$$

$$= 2j\frac{k\cos(kL\sin\theta\cos\varphi/2)\sin\theta\cos\varphi\sin(m\pi/2) - \sin(kL\sin\theta\cos\varphi/2)m\pi/L\cos(m\pi/2)}{(m\pi/L)^2 - k^2(\sin\theta\cos\varphi)^2},$$

$$P(y) = \int_{-W/2}^{W/2} \sin(n\pi y/W) e^{jk\cos\beta y}\,dy$$

$$= 2j\frac{k\cos(kW\sin\theta\sin\varphi/2)\sin\theta\sin\varphi\sin(n\pi/2) - \sin(kW\sin\theta\sin\varphi/2)n\pi/W\cos(n\pi/2)}{(n\pi/W)^2 - k^2(\sin\theta\sin\varphi)^2}.$$

Therefore, the radiation acoustic pressure of the rectangular thin plate radiator in flexural vibration in the far field can be obtained as

$$p = \frac{2k\rho_0 c_0}{\pi r}\,\omega A e^{j(\omega t - kr)} PQ, \qquad (15)$$

$$P = \frac{k\cos(kL\sin\theta\cos\varphi/2)\sin\theta\cos\varphi\sin(m\pi/2) - \sin(kL\sin\theta\cos\varphi/2)m\pi/L\cos(m\pi/2)}{(m\pi/L)^2 - k^2(\sin\theta\cos\varphi)^2},$$

$$Q = \frac{k\cos(kW\sin\theta\sin\varphi/2)\sin\theta\sin\varphi\sin(n\pi/2) - \sin(kW\sin\theta\sin\varphi/2)n\pi/W\cos(n\pi/2)}{(n\pi/W)^2 - k^2(\sin\theta\sin\varphi)^2}.$$

It can be seen that the far field radiation acoustic field of the rectangular thin plate in flexural vibration depends not only on the distance, but also on the direction. This means

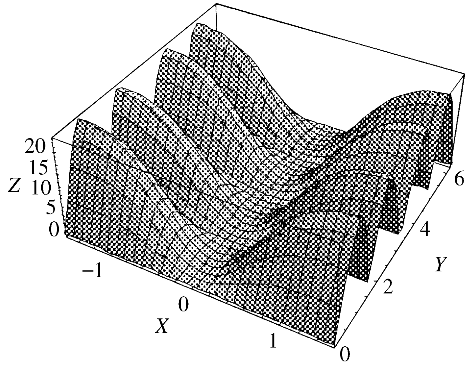

Figure 2. Calculated three-dimensional acoustic pressure directivity pattern of the rectangular thin plate in flexural vibration ($m = n = 2, f = 1599{\cdot}06$ Hz).

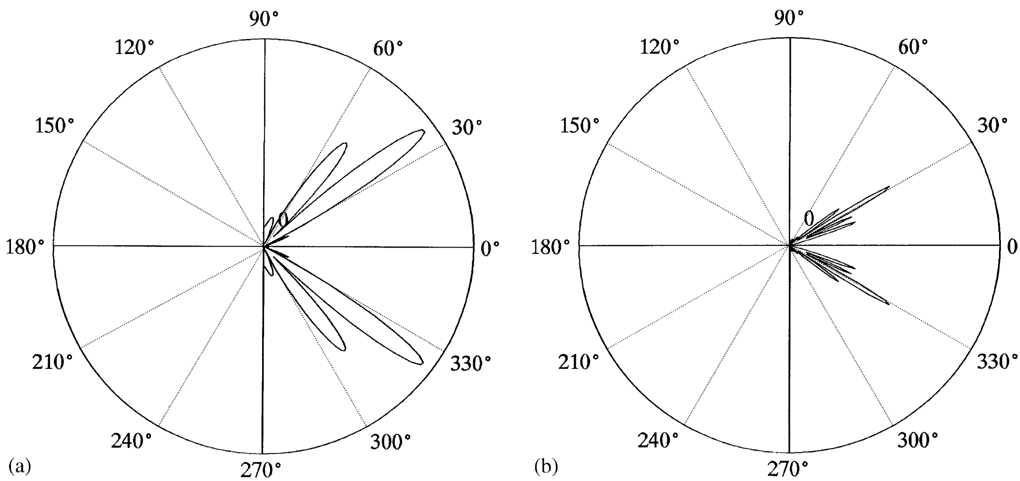

Figure 3. Calculated two-dimensional acoustic pressure directivity pattern of the rectangular thin plate in flexural vibration: Observation plane perpendicular to the rectangular plate: (a) $m = 5$, $n = 5, f = 9994{\cdot}13$ Hz; (b) $m = 7$, $n = 7, f = 19588{\cdot}5$ Hz.

that the radiation acoustic field is directional. From equation (15), the directivity function can be obtained as

$$D(\theta, \varphi) = PQ. \tag{16}$$

From the expression of the directivity function, it can be seen that the acoustic pressure distribution depends on the geometrical dimensions, the vibrational order of the plate, and the frequency. This is somewhat different from the radiation acoustic field of an oscillating piston in an infinite baffle. On the other hand, when $\theta = 0$ and $\varphi = 0$, the acoustic pressure is always zero. This is completely different from the radiation acoustic field of a piston. Figure 2 is the calculated directivity pattern of the rectangular plate in flexural vibration. In the calculation, the material of the plate is stainless steel. Its material parameters are: $\rho = 7{\cdot}8 \times 10^3$ kg/m^3, $E = 1{\cdot}95 \times 10^{11}$ N/m^2, $v = 0{\cdot}28$. E and v are Young's modulus and the Poisson ratio of the material. The speed of sound in the air is $C_0 = 340$ m/s. The length, width and thickness of the plate are: $L = 0{\cdot}24$ m, $W = 0{\cdot}16$ m, $T = 0{\cdot}003$ m.

From Figure 2, it can be seen that the radiation acoustic field of the rectangular thin plate in flexural vibration is complex. In different directions, the acoustic pressure has many

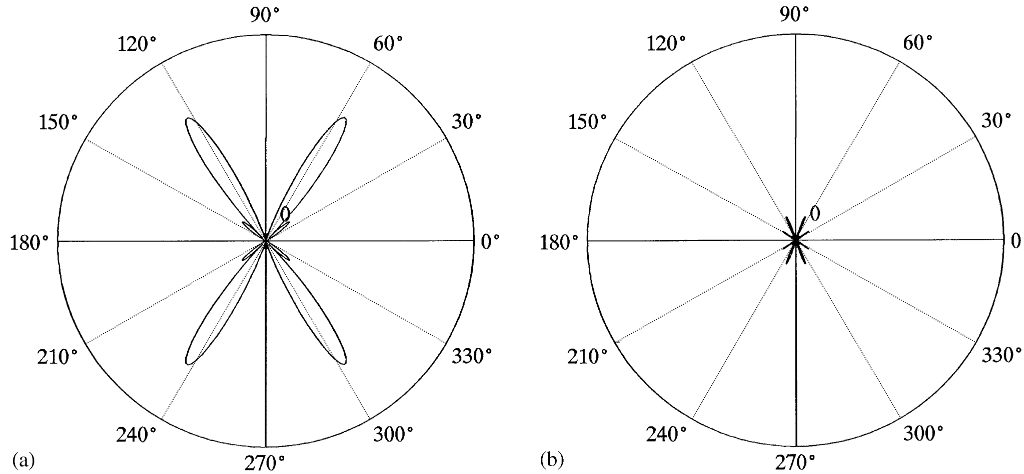

Figure 4. Calculated two-dimensional acoustic pressure directivity pattern of the rectangular thin plate in flexural vibration: Observation plane parallel to the rectangular plate: (a) $m = 5$, $n = 5$, $f = 9994{\cdot}13$ Hz; (b) $m = 7$, $n = 7$, $f = 19588{\cdot}5$ Hz.

nodes and anti-nodes. In Figure 2, $m = n = 2$, $f_{22} = 1599{\cdot}06$ Hz. X and Y denote the spherical co-ordinates θ and φ. Z denotes the directivity function $D(\theta, \varphi)$.

Figure 2 is a three-dimensional directivity pattern of the rectangular thin plate radiator in flexural vibration. In order to display the acoustic pressure distribution intuitionistically, the two-dimensional directivity pattern is also calculated. Figure 3(a), 3(b), 4(a) and 4(b) are the calculated two-dimensional directivity patterns of the radiator in flexural vibration. In Figures 3(a) and 3(b), the directivity pattern describes the acoustic pressure distribution in the plane perpendicular to the rectangular plate. This means that Figure 3(a) and 3(b) describes the relationship between the radiation acoustic pressure and the spherical co-ordinate θ when the spherical co-ordinate φ is a constant. In Figure 4(a) and 4(b), the directivity pattern describes the acoustic pressure distribution in the plane parallel to the rectangular plate. This means that Figure 4(a) and 4(b) describes the relationship between the radiation acoustic pressure and the spherical co-ordinate φ when the spherical coordinate θ is a constant. In Figures 3(a), 3(b), 4(a) and 4(b), A and B denote the different vibrational order of the plate in flexural vibration.

3.2. ACOUSTIC PRESSURE DISTRIBUTION IN THE NEAR FIELD

According to the traditional definition, the critical distance D_c between the near field and the far field of a rectangular thin plate acoustic radiator can be expressed as the following equation approximately:

$$D_c = \frac{L^2}{\lambda}. \tag{17}$$

In equation (17), L is the length of the plate, $\lambda = C_0/f$ is the wavelength in the air; f is the frequency of the radiated sound that is also equal to the resonance frequency of the rectangular thin plate in flexural vibration. It can be seen that the critical distance dividing the far and near fields depends not only on the geometrical dimension of the radiator, but also on the frequency of the radiated acoustic wave and the surrounding medium into which

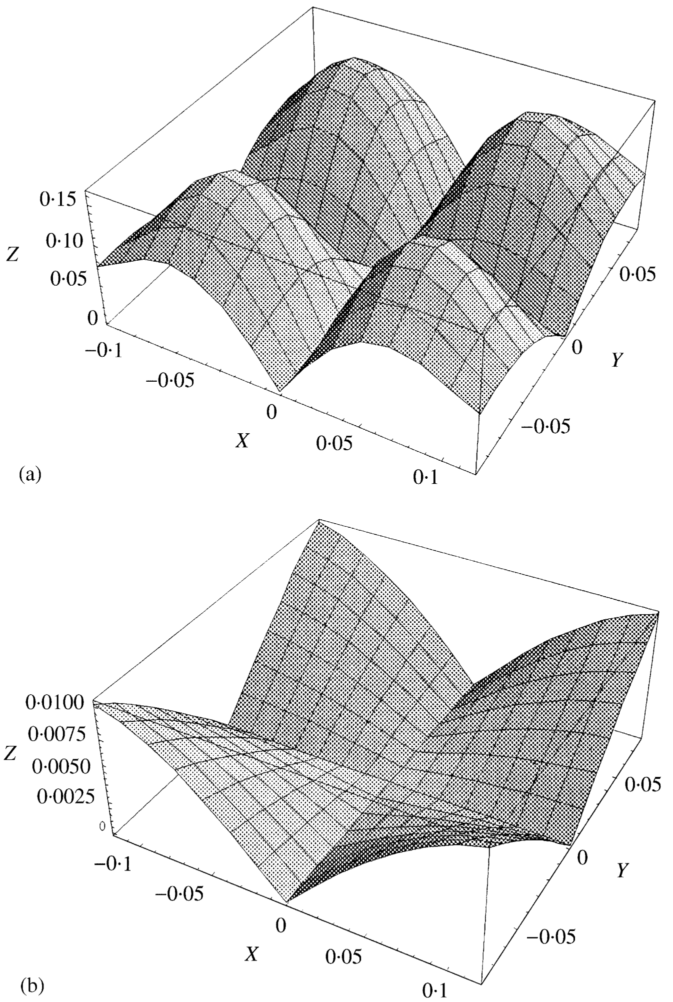

Figure 5. Calculated three-dimensional near field acoustic pressure distribution on the cross-sections perpendicular to the central axis at different distances from the rectangular thin plate in the same flexural vibrational order: (a) $m = 2$, $n = 2$, $f = 1599{\cdot}06$ Hz, $D/\lambda = 0{\cdot}047$, D is the distance of the observation plane from the radiator; (b) $m = 2$, $n = 2$, $f = 1599{\cdot}06$ Hz, $D/\lambda = 1{\cdot}18$.

the sound is radiated. For example, in the case of the same radiator and the surrounding medium, which means that the geometrical dimensions of the radiator and the speed of sound are definite, when the frequency of the radiated acoustic wave is increased, the critical distance is increased.

When the distance between the observation point and the radiator is smaller than the critical distance, i.e., $r < D_c$, the radiation acoustic field is referred to as the near field; in contrast, when the distance between the observation point and the radiator is much larger than the critical distance, i.e., $r \gg D_c$, the radiation acoustic field is referred to as the far field.

In the near field of the rectangular thin plate radiator, since the distance between the observation point and the infinitesimal element on the plate is different, the effect of the variation arising from the distance on the amplitude and the phase of the radiation acoustic pressure cannot be neglected. In this case, the acoustic pressure in the near field cannot be obtained analytically. The reason is that the integral in equation (7) is almost impossible except when using a numerical integral method, and this is why the previous study about the radiation acoustic field of any radiating systems, including the simplest case of an oscillating piston, gives almost the acoustic pressure distribution in the far field.

In order to describe the radiation acoustic field of the rectangular thin plate in flexural vibration with simply supported boundary completely, the radiation acoustic pressure of the rectangular thin plate radiator in its near field is analyzed in the following analysis, and the three-dimensional acoustic pressure distribution in the near field is obtained using a numerical method. Figures 5(a), 5(b), 6(a) and 6(b) give the calculated three-dimensional acoustic pressure distribution in relative value in the near field. Figure 5(a) and 5(b) shows the three-dimensional acoustic pressure distributions on the cross-sections perpendicular to the central axis at different distance from the rectangular thin plate. Corresponding to

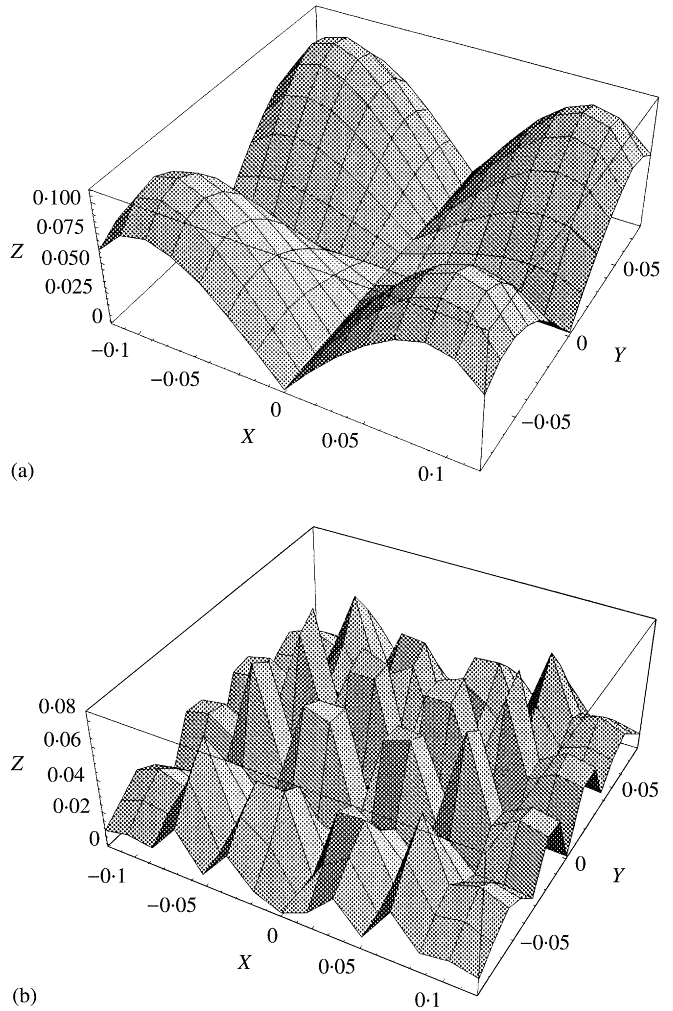

Figure 6. Calculated three-dimensional near field acoustic pressure distribution on the cross-sections perpendicular to the central axis at the same distance from the rectangular thin plate in different flexural vibrational order: (a) $m = 1$, $n = 1$, $f = 399{\cdot}8$ Hz, $D/\lambda = 0{\cdot}047$, D is the distance of the observation plane from the radiator; (b) $m = 5$, $n = 4$, $f = 7503{\cdot}4$ Hz, $D/\lambda = 0{\cdot}047$.

Figure 5(a) and 5(b), the flexural vibrational order of the plate is the same, and therefore the frequency of the radiated sound and the flexural vibrational displacement distribution are invariable. Figure 6(a) and 6(b) shows the three-dimensional acoustic pressure distributions on the cross-section perpendicular to the central axis at the same distance from the rectangular thin plate. However, corresponding to Figure 6(a) and 6(b), the flexural vibrational order of the rectangular thin plate is different, which means that the flexural vibrational displacement distribution on the rectangular thin plate and the frequency of the radiated acoustic wave are different. In Figures 5(a), 5(b), 6(a) and 6(b), X and Y denote the Cartesian co-ordinates, Z denotes the calculated relative acoustic pressure.

From these figures, it can be seen that in the near field, the radiation acoustic pressure on the cross-section perpendicular to the central axis at different distances from the radiator is different. For the acoustic field adjacent to the radiator, the radiation acoustic field is mostly confined to the area that is equal to that of the radiator. When the distance is increased, the acoustic pressure distribution becomes divergent and directional. On the other hand, when the vibrational order of the rectangular thin plate in flexural vibration is increased, the acoustic pressure distribution becomes complex. The reason is that when the vibrational order is increased, the flexural vibrational displacement distribution on the surface of the radiator becomes complex and the frequency of the radiation acoustic wave is increased.

It is noted that for the radiation acoustic field of a rectangular thin plate radiator in flexural vibration, its near acoustic field characteristic is different from that generated by an oscillating piston. For an oscillating piston, its nearfield acoustic pressure distribution on the cross-section perpendicular to the central axis is nearly uniform, while for the rectangular thin plate radiator in flexural vibration, its near field acoustic pressure distribution on the cross-section perpendicular to the central axis is complex. In some definite planes, the radiation acoustic pressure is always zero, while in other directions the radiation acoustic pressure is directional. Therefore, it can be concluded that for the radiation acoustic field generated by a rectangular thin plate in flexural vibration, the nearfield and the far field are both directional.

4. CONCLUSIONS

In this paper, the radiation acoustic field of the rectangular thin plate in flexural vibration with simply supported boundary condition is studied. The near field and the far field acoustic pressure distribution and the three-dimensional directivity pattern for the far field are obtained theoretically. From the analysis, it is demonstrated that the radiation acoustic field of the rectangular thin plate in flexural vibration is directional. The acoustic field depends not only on the geometrical dimensions and the vibrational distribution of the plate, but also on the resonance frequency and the vibrational order of the thin plate in flexural vibration. When the vibrational order of the plate is increased, the directivity pattern becomes complex. On the other hand, the radiation acoustic pressure of the plate in flexural vibration on the central axis is always zero. The reason is that there are symmetrical nodal lines on the plate; the vibrational phase on both sides of the nodal line is opposite.

Compared with the radiation acoustic field generated by an oscillating piston, the radiation acoustic field of a rectangular thin plate in flexural vibration is more complex. The acoustic pressure on the central axis and some definite planes are always zero. On the other hand, it is shown that the near field and the far field of the rectangular thin plate radiator in flexural vibration are both directional, and this is a completely different concept from the traditional one.

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