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DYNAMIC MODELLING AND VIBRATION ANALYSIS OF A FLEXIBLE CABLE-STAYED BEAM STRUCTURE

R.-F. FUNG

Department of Mechanical and Automation Engineering, National Kaohsiung First University of Science and Technology, University Road, Yuanchau, Kaohsiung, Taiwan 824, *Republic of China*
E-mail: *rffung@ccms.nkfust.edu.tw*

L.-Y. LU

Department of Construction Engineering, National Kaohsiung First University of Science and Technology, University Road, Yuanchau, Kaohsiung, Taiwan 824, *Republic of China*

AND

S.-C. HUANG

Department of Mechanical Engineering, Chung Yuan Christian University, Chung-Li, Taiwan 32023, *Republic of China*

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In this paper, the non-linear vibration of a cable-stayed beam with time-varying length and tension in the cable is investigated. A set of non-linear, time-varying differential equations describing this coupling system is derived by Hamilton's principle and the finite element method. According to the results of numerical simulation, the tension of the cable is related to the cable length, which in turn is a function of the longitudinal and transverse displacements of the cable. Furthermore, it is shown that the tension and length of the cable can be considerably different by using linear and non-linear models.

1. INTRODUCTION

Cable is a lightweight and strong tension structure, and is applied on civil engineering structures and mechanical systems extensively since they are economic means for such applications as stationing large and small structures. In civil engineering, the time-varying length problem is almost not found. However, it is a complete analysis of the cable vibration in mechanical systems, such as lift, platforms for sonar sensors and similar devices. These fields of engineering applications require accurate analysis to predict the static and dynamic behavior of the structures, which may be non-linear and include large deformation of the flexible cable.

The vibration problem of structural element whose effective length varies with time has been the subject of recent research interest. Wei and Wang [1] analyzed the vibration in a moving flexible robot arm. Yuh and Young [2] studied the dynamic modelling of an axially moving beam in rotation. Fung and Cheng [3] investigated the free vibration of a non-linear-coupled string/slider system with moving boundary. The cables of cable-stayed bridges subjected to combined parametric and forced excitations was studied by Uhrig [4],

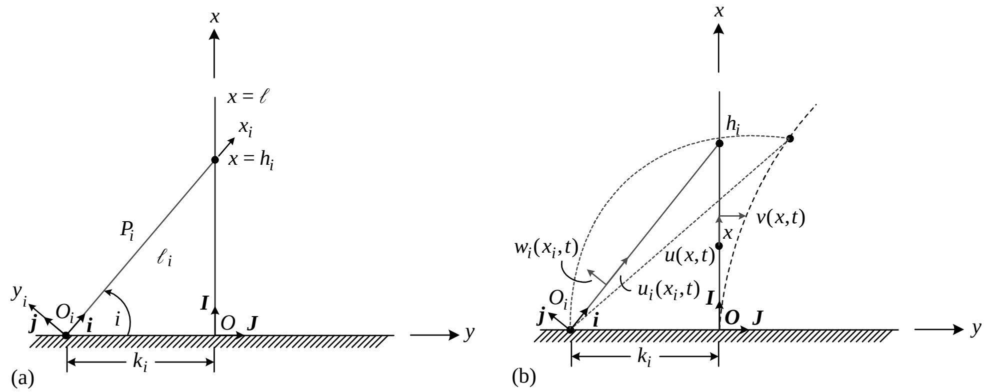

Figure 1. Schematic of the flexible cable-stayed beam structure. (a) The undeformed configuration. (b) The deformed configuration.

but they did not describe the constraint of the tied point between the beam and the cable in detail. Non-linear decentralized control of a flexible cable-stayed beam structure was studied by Magaña *et al.* [5], but the studied model was not a continuous system. Brownjohn *et al.* [6] studied the dynamic behavior of a curved cable-stayed bridge of a 100-m span, which was constructed in Singapore, by an in-field full-scale test and also by an analytical model. Tuladhar *et al.* [7] studied the seismic response of a cable-stayed bridge and concluded that the seismic response is a result of the interaction between the input earthquake characteristics and the bridge properties like mass, stiffness, and support conditions. Stylianou and Tabarrok [8] studied an axially moving beam and used the finite element method to obtain numerical solutions. Al-Bedoor and Khulief [9] studied the motion of an elastic beam with the prismatic and revolute joints by using Lagrangian approach in conjunction with the assumed modes technique.

The objective of the present work is to develop a general dynamic model for an elastic cable-stayed beam structure, in which one end of the cable is tied to the beam at a certain location. The investigation begins with the development of a continuum model representing the cable-stayed beam structure. A set of non-linear partial differential equations governing the motion of the cable-stayed beam is derived by Hamilton's principle. After the finite element techniques, the numerical integration was performed by using Runge–Kutta method. It is found that the cable tension is relative to the cable length, which is a function of the longitudinal and transverse displacements of the cable. Also, the tension and length of the cable can be considerably different in linear and non-linear models.

2. FORMULATION OF DYNAMIC EQUATIONS

The schematic representations of the undeformed and deformed configurations of a flexible cable-stayed beam structure are shown in Figure 1(a) and (b) respectively. The Lagrangian formula is employed to describe a system consisting of m cables and an Euler–Bernoulli beam. The fixed (Oxy) and ($O_i x_i y_i$) co-ordinates are adopted to describe the deformations of the beam and the ith cable respectively. One end of the ith cable is fixed on the ground with a distance k_i from the fixed end of the beam, while the other end of the cable is tied to the point h_i on the beam. During the vibration of the structure, the length of the cable may vary with the time.

2.1. KINETIC AND STRAIN ENERGIES

The beam is modelled as an Euler–Bernoulli beam with a length ℓ, density ρ, uniform cross-sectional area A, modulus of elasticity E and moment of inertia I. On the other hand, the ith cable has a length ℓ_i, density ρ_i, uniform cross-sectional area A_i, modulus of elasticity E_i, and initial tension P_{i0}. Additionally, the angle between the beam and the ith cable is θ_i, which may change with the deflection of the beam and the cable.

The displacement field of an Euler–Bernoulli beam may be written as

$$\mathbf{u}(x, t) = u(x, t)\,\mathbf{I} + v(x, t)\,\mathbf{J}, \tag{1}$$

where $u(x, t)$ and $v(x, t)$ represent the axial and transverse deflections respectively. $\mathbf{I}$ and $\mathbf{J}$ are the unit vectors of the fixed co-ordinate (Oxy). Accordingly, the position vector of an arbitrary point on the beam after deformation may be expressed as

$$\mathbf{R}_b(x, t) = [x + u(x, t)]\,\mathbf{I} + v(x, t)\,\mathbf{J}. \tag{2a}$$

Also, let $u_i(x_i, t)$ and $w_i(x_i, t)$ be the axial and transverse displacement functions of the ith cable, so the position vector of an arbitrary point on the cable may be written as

$$R_c(x_i, t) = [\ell_i + u_i(x_i, t)]\,\mathbf{i} + w_i(x_i, t)\,\mathbf{j}, \tag{2b}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the co-ordinate $(0, x_i, y_j)$.

Therefore, the total kinetic energy of the whole system including a uniform beam and m cables is

$$T = \frac{1}{2}\int_0^{\ell} \rho A[\dot{u}^2(x, t) + \dot{v}^2(x, t)]\,\mathrm{d}x + \sum_{i=1}^{m} \frac{1}{2}\int_0^{\ell_i} \rho_i A_i[\dot{u}_i^2(x_i, t) + \dot{w}_i^2(x_i, t)]\,\mathrm{d}x_i. \tag{3}$$

The Lagrangian strains of the flexible cable-stayed beam structure in the corresponding directions are

$$\varepsilon_{xx} = u_x - yv_{xx} + \tfrac{1}{2}\,v_x^2, \qquad \varepsilon_{xy} = 0, \qquad \varepsilon_{yy} = 0, \tag{4–6}$$

where $\frac{1}{2}\,v_x^2$ is the non-linear term due to the large deformation in the transverse direction. Moreover, the total strain energy can be written as

$$\begin{aligned} U = {} & \frac{1}{2}\int_0^{\ell} \left\{EA\left[u_x^2(x, t) + \tfrac{1}{4}\,v_x^4(x, t) + u_x(x, t)v_x^2(x, t)\right] + EIv_{xx}^2(x, t)\right\}\mathrm{d}x \\ & + \sum_{i=1}^{m} \frac{1}{2}\int_0^{\ell_i} \left\{E_iA_i\left[u_{i,x_i}^2(x_i, t) + \tfrac{1}{4}\,w_{i,x_i}^4(x_i, t) + u_{i,\,x_i}(x_i, t)w_{i,x_i}^2(x_i, t)\right]\mathrm{d}x_i \right. \\ & \left. + P_i(t)w_{i,x_i}^2(x_i, t)\right\}, \end{aligned} \tag{7}$$

where

$$P_i(t) = P_{i0} + E_iA_i\,\frac{\ell_i' - \ell_i}{\ell_i} \tag{8}$$

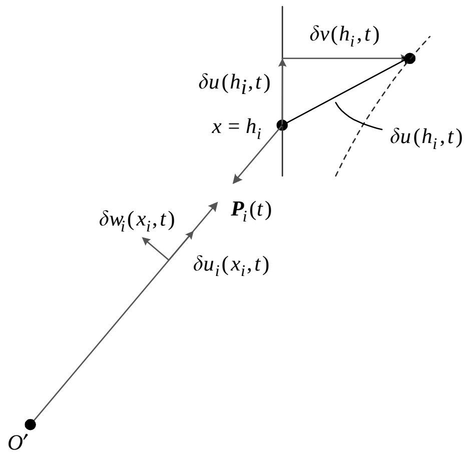

Figure 2. The virtual displacement at the tied point of the ith cable.

is the current tension in the ith cable. The second term of equation (8) is due to the change of the cable length, which is

$$\ell_i' = \{[h_i + u(h_i, t)]^2 + [k_i + v(h_i, t)]^2\}^{1/2}$$
$$= [\ell_i + u_i(\ell_i, t)]^2 + w_i^2(\ell_i, t). \tag{9}$$

Equation (9) describes the relationship of the deflections between the beam at $x = h_i$ and the ith cable at $x_i = \ell_i$. In addition, the virtual work done by the tension P_i associated with the virtual displacements $\delta u(x, t)$ and $\delta v(x, t)$ shown in Figure 2 is

$$\delta W = \sum_{i=1}^{m} P_i \cdot \delta \mathbf{R}(h_i, t)$$
$$= \int_0^{\ell} \sum_{i}^{m} - P_i(t)\delta(x - h_i)(\sin\theta_i\, \delta u(x, t) + \cos\theta_i \delta v(x, t))\, \mathrm{d}x, \tag{10}$$

where $\delta(x - h_i)$ is the Dirac-delta function, $\theta_i = \tan^{-1}((h_i + u(h_i, t))/(k_i + v(h_i, t)))$ and $\delta\mathbf{R}(h_i, t) = \delta\mathbf{u}(h_i, t)$.

2.2. GOVERNING EQUATIONS

Substituting equations (3), (7) and (10) into Hamilton's principle [10]

$$\int_{t_1}^{t_2} [\delta(T - U) + \delta W]\, \mathrm{d}t = 0 \tag{11}$$

taking variation, applying integration by parts and finally collecting the like terms, one obtains the governing equations for the whole system:

$$u{:}\, \rho A\ddot{u}(x, t) - EA\frac{\partial}{\partial x}\left[u_x(x, t) + \tfrac{1}{2}v_x^2(x, t)\right]$$
$$= \sum_{i=1}^{m} - P_i(t)\sin\theta_i \cdot \delta(x - h_i), \quad 0 < x < \ell, \tag{12}$$

$$v\!:\rho A\ddot{v}(x,t) - EA\,\frac{\partial}{\partial x}\left[\tfrac{1}{2}\,v_x^3(x,t) + u_x(x,t)v_x(x,t)\right] + EIv_{xxxx}(x,t)$$

$$= \sum_{i=1}^{m} -P_i(t)\cos\theta_i\cdot\delta(x-h_i),\quad 0<x<\ell, \tag{13}$$

$$u_i\!:\rho_iA_i\ddot{u}_i(x_i,t) - E_iA_i\,\frac{\partial}{\partial x}\left[u_{i,x_i}(x_i,t) + \tfrac{1}{2}\,w_{i,x_i}^2(x_i,t)\right] = \sum_{i=1}^{m} P_i(t)\cdot\delta(x-h_i),$$

$$0<x_i<\ell_i,\ i=1,2,\ldots,m \tag{14}$$

$$w_i\!:\rho_iA_i\ddot{w}_i(x_i,t) - \frac{\partial}{\partial x}\left\{E_iA_i\left[\tfrac{1}{2}\,w_{i,x_i}^3(x_i,t) + u_{i,x_i}(x_i,t)\,w_{i,x_i}(x_i,t)\right]\right.$$

$$\left. + P_i(t)\,w_{i,x_i}(x_i,t)\right\} = 0,\quad 0<x_i<\ell_i,\ i=1,2,\ldots,m \tag{15}$$

and also the associated boundary conditions:

$$u(0,t)=0,\quad u_x(\ell,t)+\tfrac{1}{2}\,v_x^2(\ell,t)=0,\quad v(0,t)=0,\quad v_x(0,t)=0, \tag{16a–j}$$

$$v_{xx}(\ell,t)=0,\quad EA\,[\tfrac{1}{2}\,v_x^3(\ell,t)+u_x(\ell,t)v_x(\ell,t)] - EIv_{xxx}(\ell,t)=0,$$

$$u_i(0,t)=0,\quad u_{i,x_i}(\ell_i,t)+\tfrac{1}{2}\,w_{i,x_i}^2(\ell_i,t)=0,$$

$$w_i(0,t)=0,\quad P_i(t)w_{i,x_i}(\ell_i,t)+E_iA_i\,[\tfrac{1}{2}\,w_{i,x_i}^3(\ell_i,t)+u_{i,x_i}(\ell_i,t)\cdot w_{i,x_i}(\ell_i,t)]=0.$$

By using equation (16b), equation (16f) can be rewritten as $v_{xxx}(\ell,t)=0$.

2.3. DISCUSSION

The dynamic equations of the flexible cable-stayed beam structure, which includes a two-dimensional Euler–Bernoulli beam and cables have been derived previously. From the governing dynamic equations and the boundary conditions, several important observations can be made:

(1) The time-varying tension $P_i(t)$ of the transverse vibration cable appears in the governing equations (12–15) and the boundary condition (16j).
(2) The axial displacement $u(x,t)$ and the transverse displacements $v(x,t)$ of the Euler–Bernoulli beam in the governing equations (12, 13) are coupled and apparent.
(3) The deflection $w_i(x_i,t)$ of the ith cable is coupled with the axial and transverse displacements of the beam by the tension $P_i(t)$ and is consequential.
(4) In equation (8), the cable tension depends on the pretension and its current length. Moreover, the cable length depends on the longitudinal and transverse displacements of the flexible beam as shown in equation (9). In addition, the angle between the cable and the horizon also depends on the longitudinal and transverse displacements.
(5) In equations (12) and (13), the longitudinal displacements are directly coupled with the transverse deflections.
(6) The flexible vibrations of the cable-stayed beam and the cables are consequentially coupled by the cable tension. The governing equations (12–15) of the flexible cable-stayed beam are all non-linear.

(7) In the four governing equations, the differences between the linear and non-linear models are contributed by the following non-linear terms: $\frac{1}{2}v_x^2(x,t)$ in equation (12), $\frac{1}{2}v_x^3(x,t)+u_x(x,t)v_x(x,t)$ in equation (13), $\frac{1}{2}w_{i,x_i}^2(x_i,t)$ in equation (14), $\frac{1}{2}w_{i,x_i}^3(x_i,t)+u_{i,x_i}(x,t)\,w_{i,x_i}(x_i,t)$ in equation (15). The cable tension $p_i(t)$ is also a non-linear term, since it is a function of $u_i^2(\ell_i,t)$ and $w_i^2(\ell_i,t)$, which could be seen from equations (8) and (9).

(8) If the non-linear terms are ignored, the governing equations in u and v directions are still coupled with the cable dynamic equation through the tension $P_i(t)$.

3. FINITE ELEMENT DISCRETIZATION

In the finite element method, the structural response is usually approximated by discretized nodal displacements. In this paper, the non-linear effects of the beam and cable are adopted in the finite element formulation. The beam and cable are divided into n elements. Each node of the beam has three degrees of freedom and that of the cable has one degree of freedom. The usual approach in the finite element method is to assume that each unknown deformation $u(x,t)$, $v(x,t)$ and $w_i(x_i,t)$ is approximated by a finite series in the following form:

$$u(x,t)=\sum_{j=1}^{n}N_{uj}(x)\,q_j(t),\qquad v(x,t)=\sum_{j=1}^{n}N_{vj}(x)\,q_j(t),\tag{17, 18}$$

$$u_i(x_i,t)=\sum_{j=1}^{n}N_{u_ij}(x_i)\,q_j(t),\qquad w_i(x_i,t)=\sum_{j=1}^{n}N_{w_ij}(x_i)\,q_j(t),\tag{19, 20}$$

where N_{uj}, N_{vj}, N_{u_ij} and N_{w_ij} are Hermite shape functions [11] and $q_j(t)$ are the generalized co-ordinates. The non-linear terms of the flexible cable-stayed beam system appear in the potential energy function in which v_x and $w_{i,x}$ can be calculated at the previous time step [12]. This is a simplified numerical technique in the finite element method for non-linear problems.

By using the finite element technique and assembling the equation of motion, one obtains the ordinary differential equations for the coupled system

$$\mathbf{M}\ddot{\mathbf{Q}}+\mathbf{K}\mathbf{Q}=\mathbf{P}(\mathbf{Q}),\tag{21}$$

where $\mathbf{Q}$ is the global displacement vector, $\mathbf{M}$ and $\mathbf{K}$ are the global mass and stiffness matrices, respectively, and $\mathbf{P}(\mathbf{Q})$ is the time-varying force vector as follows:

$$\mathbf{Q}=[q_1\ \cdots\ q_j\ \cdots\ q_{4n}]^{\mathrm{T}_{1\times 4n}},$$

$$\mathbf{M}=[\mathbf{M}^*],\ \mathbf{K}=[\mathbf{K}^*],\ \mathbf{P}(\mathbf{Q})=[\mathbf{P}^*],\tag{22a–d}$$

where

$$\mathbf{M}^*=\sum_{j=1}^{N_e}\left[\rho A(\mathbf{N}_{uj}^{\mathrm{T}}\mathbf{N}_{uj}+\mathbf{N}_{vj}^{\mathrm{T}}\mathbf{N}_{vj})+\sum_{i=1}^{m}\rho_iA_i(\mathbf{N}_{u_ij}^{\mathrm{T}}\mathbf{N}_{u_ij}+\mathbf{N}_{w_ij}^{\mathrm{T}}\mathbf{N}_{w_ij})\right],\tag{23}$$

$$\begin{aligned}\mathbf{K}^*=\sum_{j=1}^{N_e}\Bigg[&EA\,(\mathbf{N}_{u,x}^{\mathrm{T}}\mathbf{N}_{u,x}+\tfrac{1}{4}v_x^2\cdot\mathbf{N}_{v,x}^{\mathrm{T}}\mathbf{N}_{v,x}+\tfrac{1}{2}v_x\cdot(\mathbf{N}_{u,x}^{\mathrm{T}}\mathbf{N}_{u,x}+\mathbf{N}_{v,x}^{\mathrm{T}}\mathbf{N}_{v,x}))\\&+EI\mathbf{N}_{v,xx}^{\mathrm{T}}\mathbf{N}_{v,xx}+\sum_{i=1}^{m}P_i(t)\mathbf{N}_{w_i,x}^{\mathrm{T}}\mathbf{N}_{w_i,x}+E_iA_i(\mathbf{N}_{u_i,x_i}^{\mathrm{T}}\mathbf{N}_{u_i,x_i}\\&+\tfrac{1}{4}w_{i,x_i}^2\cdot\mathbf{N}_{w_i,x}^{\mathrm{T}}\ \mathbf{N}_{w_i,x_i}+\tfrac{1}{2}w_{i,x_i}\cdot(\mathbf{N}_{u_i,x_i}^{\mathrm{T}}\mathbf{N}_{u_i,x_i}+\mathbf{N}_{w_i,x}^{\mathrm{T}}\mathbf{N}_{w_i,x_i}))\Bigg],\end{aligned}\tag{24}$$

$$\mathbf{P}^* = \sum_{i=1}^{m} - P_i(t) \sin \theta_i \mathbf{N}_u(\ell) \; - \; P_i(t) \cos \theta_i \mathbf{N}_v(\ell). \tag{25}$$

4. NUMERICAL RESULTS

In order to speed up the computational time for the numerical simulation, it was assumed that there is only one cable tied to the flexible beam in the simulation. The parameters of the flexible cable-stayed beam system were selected as: $\ell = 2{\cdot}5$ m, $\ell_1 = 2{\cdot}5$ m, $\rho = 7860\ \mathrm{kg/m^3}$, $\rho_1 = 7400\ \mathrm{kg/m^3}$, $A = 10^{-2}\ \mathrm{m^2}$, $A_1 = 10^{-8}\ \mathrm{m^2}$, $E = 2{\cdot}06 \times 10^{11}\ \mathrm{N/m^2}$, $E_1 = 1{\cdot}706 \times 10^{11}\ \mathrm{N/m^2}$, $I = 8{\cdot}3 \times 10^{-6}\ \mathrm{m^4}$ and the ends of the cable are fixed on $h_1 = 2$ m and $k_1 = 1{\cdot}5$ m. In that case, the angle between the cable and the horizontal plane is about 53° ($\approx 0{\cdot}925$ rad) and the pretension which is acting on the cable is 682 N before the cable is deformed. For economy in computing process, a time interval of 0·2 s and a desired accuracy of 10^{-9} are taken [12].

In the simulations, the initial longitudinal and transverse displacements of the flexible beam and cable were taken as (0, 0·5) m and (0·0001, 0·01) m respectively. The first-mode shape of a uniform beam was used as the initial lateral displacement of the flexible beam, i.e.,

$$v(x, 0) = C_0[\sin(\beta x) - \sinh(\beta x) - \alpha(\cos(\beta x) - \cosh(\beta x))], \tag{26}$$

where $\beta\ell = 1{\cdot}875104$, $\alpha = ((\sin(\beta\ell) + \sinh(\beta\ell))/(\cos(\beta\ell) + \cosh(\beta\ell)))$, and the coefficient $C_0 = 0{\cdot}1835$ was assigned for the initial displacement. With the above parameters, fundamental vibration period of the flexible beam is $(2\pi/\beta^2)\sqrt{\rho A/EI} = 0{\cdot}0756$ s. In addition, the lateral first-mode shape of the cable may be given by

$$w_1(x, 0) = C_1[\sinh(\beta_1 x) - \sin(\beta_1 x) + \alpha_1(\cosh(\beta_1 x) - \cos(\beta_1 x))], \tag{27}$$

where $\beta_1\ell_1 = 4{\cdot}730041$, $\alpha_1 = ((\sinh(\beta_1\ell) - \sin(\beta_1\ell))/(\cos(\beta_1\ell) - \cosh(\beta_1\ell)))$, and the coefficient $C_1 = -\ 0{\cdot}0062$ was assigned for the initial displacement.

The transient responses of the linearly and non-linearly flexible beam and cable are compared in Figure 3. In the simulation, the flexible cable-stayed beam system was considered with a light damping, i.e., $\mathbf{C} = 0.001\mathbf{M} + 0.0001\mathbf{K}$. Figures 3(a) and 3(c) show, respectively, the longitudinal displacement of the beam at the free end and that of the cable at the mid-point. The displacements are negative in Figure 3(a) but are positive in Figure 3(b) because the beam is under compression but the cable is under tension. In the figures, a dash line denotes the response of a linear model while a solid line represents the response of a non-linear model. It is seen that the high-frequency amplitudes are absorbed by damper after $t = 0{\cdot}03$ s. Figures 3(b) and 3(d) show, respectively, the transverse deflections of the beam and the cable at the same points of Figures 3(a) and 3(b). The longitudinal displacement of the flexible beam for the linear model is much smaller when compared with that in the non-linear case. However, the transverse displacements for the linear and non-linear models are almost the same as shown in Figure 3(b). It is also observed that in Figures 3(a) and 3(b) the longitudinal displacements of the beam are much smaller than those in the transverse direction. Figure 3(e) shows the time history of the cable tension, which includes the pretension and the effect due to the change of the cable length within the vibration process. Figure 3(f) depicts the time history of the angle change between the cable end and horizon. This change varies with the longitudinal and transverse

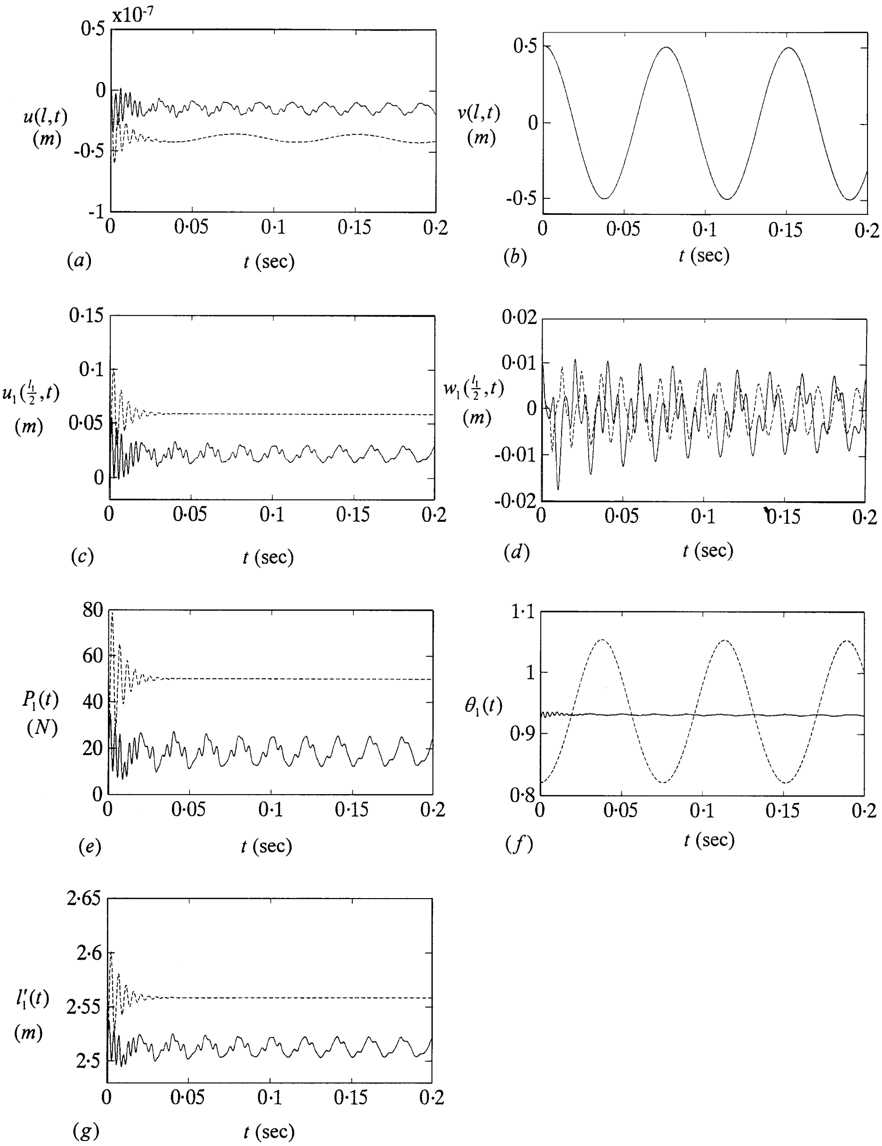

Figure 3. The transient responses of the flexible cable-stayed beam system. (a) The free-end longitudinal displacements of the beam. (b) The free-end transverse displacements of the beam. (c) The middle-point longitudinal displacements of the cable. (d) The middle-point transverse displacements of the cable. (e) The time history of the cable tension. (f) The current angle between the cable and the horizon. (g) The current length of the cable: ---, linear; —, non-linear.

displacements. Figure 3(g) shows the change of the cable length, it also depends on the longitudinal and transverse displacements of the cable at $x = \ell_i$ as in equation (9).

Also, from Figure 3 it is seen that the responses of cable-stayed beam structure predicted by a linear model are larger than those by the non-linear model, except Figures 3(b) and 3(d). This implies that the linear model may overestimate the structural responses and cable tension, and thus lead to an over-conservative member design for the structure. Therefore, using the non-linear model can be economically beneficial for engineering applications.

5. CONCLUSIONS

The non-linear vibration of the flexible cable-stayed beam was studied in this paper. The derivation of the dynamic formulation via Hamilton's principle was based on the expressions of the kinetic energy, strain energy and the virtual works done by the cable tension. At any time instance, the tension of the cable includes the pretension and the effect due to the changes of its length. From the dynamic formulations and the numerical simulation results, the following conclusions can be drawn:

(1) In the linear and non-linear cases, the longitudinal displacements are always much smaller than those in the transverse direction.
(2) The longitudinal displacement in the non-linear case is much larger than that in the linear case.
(3) The non-linear terms in the formulation are due to the large geometric deformation in the transverse direction. If the selected initial transverse deflection is small enough and the longitudinal displacement of the flexible cable-stayed beam system can be neglected, the non-linear terms of the beam and cable are negligible.
(4) The longitudinal displacement of the flexible beam is always negative because a pretension is applied on the cable.
(5) In equations (8) and (9), the tension is related to the cable length which is in turn a function of the longitudinal and transverse displacements of the cable. Furthermore, the tension and length of the cable can be considerably different in the linear and non-linear models.
(6) The angle between the cable and the horizon is related to the longitudinal and transverse displacements of the flexible cable-stayed beam. The time histories of the angle are different in the linear and non-linear cases.

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APPENDIX A: NOMENCLATURE

A	the uniform cross-sectional area of the beam
A_i	the uniform cross-sectional area of the ith cable
$\mathbf{C}$	a light damping matrix
E	modulus of elasticity of the beam
E_i	modulus of elasticity of the ith cable
h_i	the distance between the other end of the cable tied on the beam and ground
I	moment of inertia of the beam
$\mathbf{I}$, $\mathbf{J}$	the unit vectors of the fixed co-ordinate Oxy
$\mathbf{i}$, $\mathbf{j}$	the unit vectors of the fixed co-ordinate O$_i x_i y_i$
k_i	the distance between the cable and the beam
$\mathbf{K}$	the global stiffness matrix
ℓ_i	the length of the ith cable
ℓ	the length of the beam
ℓ'_i	the change of the cable length
m	numbers of cables
$\mathbf{M}$	the global mass matrix
N_{uj}, N_{vj}	Hermite shape functions
$N_{u_i j}$, $N_{w_i j}$	Hermite shape functions
Oxy	the global co-ordinates fixed on the beam
O$_i x_i y_i$	the global co-ordinates fixed on the cable
P_{i0}	the initial tension
$P_i(t)$	the current tension in the ith cable
$\mathbf{P}(\mathbf{Q})$	the time-varying force vector
$q_j(t)$	the generalized co-ordinates
$\mathbf{Q}$	the global displacement vector
$R_b(x, t)$	the position vector of an arbitrary point on the beam after deformation
$R_c(x_i, t)$	the position vector of an arbitrary point on the cable after deformation
T	the total kinetic energy of the whole system
U	the total potential energy of the whole system
$u(x, t)$	the displacement field of an Euler–Bernoulli beam
$u(x, t)$, $v(x, t)$	the axial and transverse deflections respectively
$u_i(x_i, t)$, $w_i(x_i, t)$	the axial and transverse displacement functions of the ith cable
ε_{xx}, ε_{xy}, ε_{yy}	the Lagrangian strains of the flexible cable-stayed beam structure
θ_i	the angle between the beam and the ith cable
ρ	the density of the beam
ρ_i	the density of the ith cable