Journal of Sound and Vibration (2002) **254**(5), 1025–1027
doi:10.1006/jsvi.5101, available online at http://www.idealibrary.com on IDEAL®

BOOK REVIEW

THE TREFFTZ FINITE AND BOUNDARY ELEMENT METHOD, 2001, by Q.-H. QIN. Southampton: WIT Press, viii + 282 pp. Price £118.00, US$183.00, €188.00. ISBN 1-85312-855-4

This book is the first monograph describing in detail the modern version of the Trefftz approach to numerical modelling. The original idea of Erich Trefftz, published by him in 1926, consisted of application of trial functions (called T-functions) identically satisfying the governing partial differential equations of a problem. He proposed to calculate the unknown coefficients of these functions from the problem-boundary conditions. However, the direct application of the Trefftz concept encounters some difficulties, especially for more complex regions, because of problems with conditioning. Hence, it is suggested to apply the global version of the Trefftz method only to the modelling of objects of a rather simple form. In the case of complex structures, it is necessary to divide the considered region into sub-areas to avoid the difficulties mentioned above. This book is mainly dedicated to such sub-structuring in the modern Trefftz method.

Chapter 1 of the book, Introduction, deals with the basic formulations and the history of the method development. There were two considerable steps in this history—finding the deep mathematical background of the Trefftz-type complete functional systems (Herrera, 1980) and the first proposal of the hybrid Trefftz finite elements (Jirousek and Leon, 1977). The history is presented in detail with special attention paid to the works of Jirousek and his group (18 quoted papers). Following earlier publications, the author introduces the names of T-complete functions and T-elements to the notions mentioned above. He presents a definition of the classical Jirousek's T-element with the Trefftz functional field in its internal region and the polynomial "frame" function defined only along its boundary. The tables include a comparison between the hybrid Trefftz hierarchical p-element formulation and its conventional p-version equivalent. After a short presentation of the boundary element method (BEM), we notice that the T-elements stand between this approach and the standard finite element method. Indeed, they are large finite elements, in which stiffness matrices contain only integrals calculated along the element boundaries.

Chapter 2, Potential Problems, discusses not only this particular application of the T-elements but also their general properties. The most important seems to be the observation (after Jirousek) concerning the rank condition of the hybrid T-element stiffness matrix. The number of the internal T-functions m must always be larger or equal to the number k of the nodal degrees of freedom (coefficients of the frame function) minus the number r of the rigid body motion (RBM) terms. In this classical type of the hybrid T-element, the internal field cannot contain the RBM terms. They must be calculated from the conformability conditions between the Trefftz field and the frame function. The classical hybrid Trefftz element (1977) is not the only way of sub-structuring. Hence, the author gives a review of the best known Trefftz-type formulations, the "traction frame" and the frameless T-elements. These considerations are general and can be applied to any elliptical (not only potential) problems. In this chapter, the author also introduces the T-complete systems for the Laplace and Helmholtz equations. Special purpose functions for regions with a singular corner and a circular hole are presented as well.

0022-460X/02/$35.00

In Chapter 3, linear elastostatics is discussed. The respective T-complete sets are introduced, also for singular corners and elliptical holes (solution with conformal mapping). The Piltner way of derivation of the T-functions for 3-D problems is also presented. Chapter 3 is closed by numerical examples of a compressed perforated panel and a stretched skew cracked plate.

Chapter 4 is dedicated to thin plates. It opens with some basic relations of the Kirchhoff theory, the specification of the T-complete systems (also special functions) and different particular solutions resulting from various loads. An interesting extension to thin plates on an elastic foundation is presented. The chapter concludes with numerical examples investigating the element convergence and the sensitivity to mesh distortion.

The problem of thick plates is also taken into account in Chapter 5. The theory of such structures is carefully discussed and the variational formulation of the hybrid T-element is introduced. The T-complete systems consist, in this case, of polynomials and modified Bessel functions. The chapter is closed by extensive numerical investigations of the element convergence.

Chapter 6 deals with transient heat conduction. In its introduction, the history of the problem solution by the finite element approach is presented with a review of respective papers. Then a time-stepping formula is proposed. This short chapter (13 pages) is closed by a numerical example defined for a square domain. The results are compared to the analytical solution and the conventional finite element approach. They are presented without detailed comments, which would be interesting in this case.

Chapter 7 discusses an area, in which the author was personally involved to a large extent (8 papers quoted); geometrically, non-linear analysis for plate-bending problems. This relatively large chapter (31pp.) presents investigations of both thin and thick plates. The theory is followed by modified variational principles and iterative schemes. The extension to thin and thick plates on an elastic foundation is included. Seven extensive numerical examples present post-buckling analysis of circular and square plates and also large deflections of such plates on an elastic Winkler foundation.

Chapter 8 (12pp.) signals a possibility of application of the T-elements to physically non-linear problems. Certain basic relations are given, unfortunately not followed by numerical examples. The next chapter (Dynamics of Plate Bending Problems, 8pp.) gives a time-step formulation discussing its accuracy and stability. In spite of its limited size, it includes two numerical examples—a clamped circular plate and a simply supported plate subjected to a pulse loading.

The last chapter presents formulations of the Trefftz boundary element method (T-BEM). The main difference between this approach and the standard BEM consists in the application of the T-complete systems in the place of the fundamental solutions. This results in the basic advantage of the T-BEM; regular integrals in all the calculations. However, the personal view of the reviewer to this approach is less optimistic than the author's. Changing from singular fundamental solutions to the regular T-functions can result in worse conditioning and the limitations indicated in the beginning of the review. The chapter contains two numerical examples solving the potential problem for an L-shape region and an edge crack in a square domain. The Galerkin and collocation approaches are applied and compared. The results of the examples are very accurate.

The book, unfortunately, is not closed by any final conclusions and perspectives of the method, which would be suggested for the next edition. It is supplemented by two appendices: "Numerical Integration" and "Matrix Algebra and Calculus".

Summarizing, this is the first large presentation of this powerful numerical tool of mathematical modelling. In certain calculations, especially those repeating varying linear solutions very many times, e.g., optimization, the introduction of the analytical solutions

(T-functions) results in a considerable decrease of the computational time for the same, or larger accuracy of calculations. Therefore, the method is recommended to all the problems for which the T-complete systems can be derived.

A. P. Zielinski