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APPLICATION OF MODIFIED VLASOV MODEL TO FREE VIBRATION ANALYSIS OF BEAMS RESTING ON ELASTIC FOUNDATIONS

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The purpose of this paper is to apply the modified Vlasov model to the free vibration analysis of beams resting on elastic foundations and to analyze the effects of the subsoil depth, the beam length, their ratio and the value of the vertical deformation parameter within the subsoil on the frequency parameters of beams on elastic foundations. This analysis has been carried out by the aid of a computer program based on the finite element method. The first ten frequency parameters are presented in tabular and graphical forms to evaluate the effects of the parameters considered in this study. Then mode shapes corresponding to the first six of the frequency parameters are given in figures. It is concluded that the effect of the subsoil depth on the frequency parameters of beams on an elastic foundation is generally larger than those of the other parameters considered in this study.

1. INTRODUCTION

Beams resting on elastic foundations are very common technical problems in structural and geotechnical engineering. For this reason, numerous works have been concerned with such problems in the technical literature. In these kinds of problems, the structural system should be analyzed by using a realistic model for soil–structure interaction.

Many researchers use the Winkler model for soil–structure interaction in the static and dynamic analysis of beams resting on elastic foundations, where the vertical surface displacement of the beam is assumed to be proportional at every point to the contact pressure at that point [1]. In the Winkler model, it is assumed that the foundation soil consists of linear elastic springs which are closely spaced and independent of each other. One of the most important shortcomings of this model is that it assumes no interaction between the springs or discontinuity of the foundation.

In order to overcome this problem, several two-parameter models have been suggested by many researchers. The model proposed by Filonenko and Borodich acquires continuity between the individual spring elements in the Winkler model by connecting them to a thin elastic membrane under a constant tension. In the model proposed by Hetenyi, interaction

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between the independent spring elements is accomplished by incorporating an elastic plate in three-dimensional problems, or an elastic beam in two-dimensional problems. Another model proposed by Pasternak acquires shear interaction between springs by connecting the ends of the springs to a layer consisting of incompressible vertical elements which deform by lateral shear only. Vlasov developed a two-parameter model that accounts for the effect of the neglected shear strain energy in the soil and shear forces that come from surrounding soil by introducing an arbitrary parameter, γ, to characterize the vertical distribution of the deformation in the subsoil [2].

All these models are shown to lead to the same differential equation. Basically, all these models are equivalent and differ only in the definition of the second parameter. The Vlasov model requires the estimation of the γ parameter. Jones and Xenophontos [3] established a relationship between the γ parameter and the displacement characteristics, but did not suggest a computational method. Recently, for a beam on elastic foundation, Vallabhan and Das [4] determined the γ parameter as a function of the characteristic of the beam and the foundation, using an iterative procedure. They named this model a modified Vlasov model. Doyle and Povlovic [5] studied a beam having only a portion of its span supported by the Winkler elastic foundation and investigated the effect of such partial support of a beam element on its natural frequencies. Ding [6] solved the problem of vibrations of beams on an elastic foundation using the Winkler model and found the natural frequencies by giving several examples for simply supported beams on a variable Winkler elastic foundation. Franciosi and Masi [7] used a two-parameter model to solve the free vibration problem of beams on an elastic foundation. The results presented by them are depended on the foundation parameters, k and k_1. Eisenberger [8] obtained the natural frequencies of beams resting on a variable one- and two-parameter elastic foundation. He presented the foundation parameters in terms of parts of the beam length. De Rosa [9] examined the free vibration frequencies of Timoshenko beams on a two-parameter elastic foundation. Kukla [10] considered the problem of the free vibration of a Bernoulli–Euler beam supported on a step-like varying Winkler elastic foundation. Lee and Kes [11] carried out a study to determine the natural frequencies of non-uniform Bernoulli–Euler beams resting on non-uniform elastic foundation with general elastic end restraints. Thambiratnam and Zhuge [12] analyzed the free vibration problem of beams on an elastic foundation. Pavlovic and Wylie [13] investigated the natural response of a beam supported by an elastic foundation for the case when the Winkler foundation modulus varies linearly along the span of the beam. Ayvaz and Daloğlu [14] applied the modified Vlasov model to earthquake analysis of beams resting on elastic foundations, and analyzed the effect of the subsoil depth, the beam length and their ratio on its responses. Vallabhan and Das [15] presented the values of the foundation parameters and the vertical deformation parameter in terms of the soil depth, the ratio of the soil depth to the beam length, and external load type. They do not deal with the free vibration analysis. However, no references have been found for the free vibration analysis of beams on elastic foundations by using the modified Vlasov model.

The purpose of this paper is to apply, not to introduce, the modified Vlasov model to the free vibration analysis of beams resting on elastic foundations and to analyze the effects of the subsoil depth, the beam length, their ratio and the value of the vertical deformation parameter, γ, within the subsoil on the frequency parameters of beams on elastic foundations. For this purpose, a computer program is coded to obtain the stiffness and mass matrices of the beam–soil system. The finite element method is used to construct the stiffness and mass matrices. Then, to obtain the the solution of the generalized eigenvalue problem including these two matrices, the program, MATLAB for Windows 4.0, is used.

2. FINITE ELEMENT MODELLING

The governing equation for a beam subjected to the free vibration with no damping is

$$[\mathbf{M}]\{\ddot{\mathbf{w}}\} + [\mathbf{K}]\{\mathbf{w}\} = \mathbf{0} \tag{1}$$

where **K** is the stiffness matrix of the beam-soil system, **M** is the mass matrix of the beam–soil system, **w** *and* $\ddot{\mathbf{w}}$ are the displacement and acceleration of the beam respectively. For a beam resting on an elastic foundation, evaluation of the stiffness and mass matrices are given in the following sections.

2.1. EVALUATION OF THE STIFFNESS MATRIX

The subsoil considered has a finite depth with a rigid boundary at the bottom (Figure 1). The potential energy in the soil–structure system for the unloaded case may be written as

$$\Pi = \int_0^L \frac{E_b I_b}{2}\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)^2 \mathrm{d}x + \int_{-\infty}^{+\infty}\int_0^H \frac{b_w}{2}(\sigma_x \varepsilon_x + \sigma_z \varepsilon_z + \tau_{xz}\gamma_{xz})\,\mathrm{d}z\mathrm{d}x, \tag{2}$$

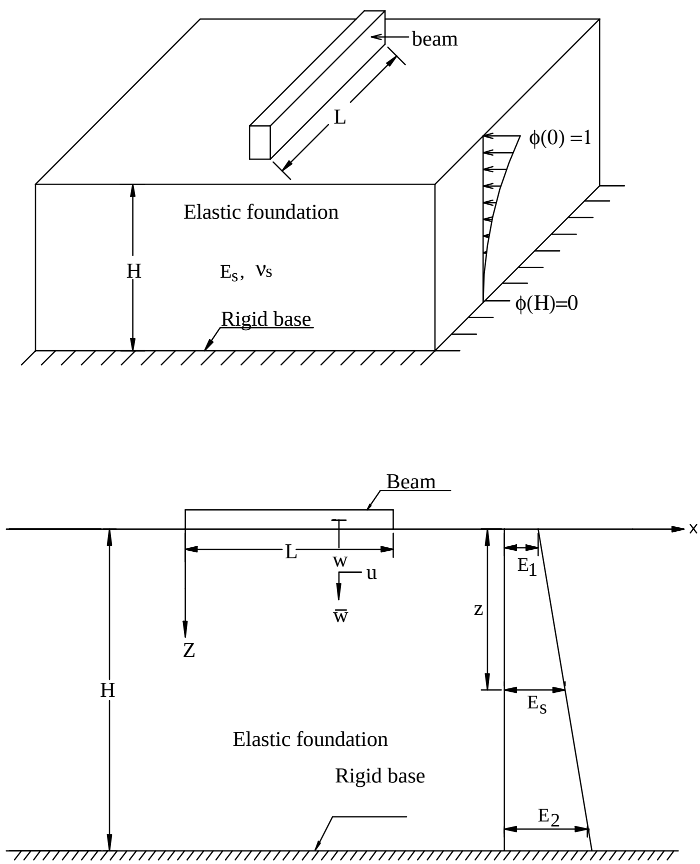

Figure 1. A simple beam on elastic foundation.

where σ_x, σ_z, τ_{xz}, ε_x, ε_z and γ_{xz} are the stresses and the corresponding strains in the subsoil, w, $E_b I_b$, L, b_w are the lateral displacement, the flexural rigidity, the length, and the width of the beam, respectively, and H is the height of the subsoil. By using constitutive relations and strain–displacement equations of elasticity, the stresses at any point in the foundation can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \frac{E_s(1-v_s)}{(1+v_s)(1-2v_s)} \begin{bmatrix} 1 & \dfrac{v_s}{(1-v_s)} & 0 \\ \dfrac{v_s}{(1-v_s)} & 1 & 0 \\ 0 & 0 & \dfrac{(1-2v_s)}{2(1-v_s)} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial \bar{w}}{\partial z} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial \bar{w}}{\partial x} \end{Bmatrix}, \tag{3}$$

where E_s and v_s are the modulus of elasticity and the Poisson ratio of subsoil, respectively. If the assumptions of

Vertical displacement

$$\bar{w}(x,z) = w(x)\phi(z) \text{ for } \phi(0) = 1 \text{ and } \phi(H) = 0 \quad \text{and} \tag{4}$$

horizontal displacement

$$u(x,z) = 0 \tag{5}$$

are made, and if equations (3)–(5) are substituted into equation (2), the following equation can be obtained:

$$\Pi = \int_0^L \frac{E_b I_b}{2} \left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \right)^2 \mathrm{d}x$$

$$+ \frac{E_s b_w}{2} \int_{-\infty}^{+\infty} \int_0^H \left[\frac{(1-v)}{(1+v)(1-2v)} w^2 \left(\frac{\mathrm{d}\phi}{\mathrm{d}z} \right)^2 + \frac{1}{2(1+v)} \left(\frac{\mathrm{d}w}{\mathrm{d}x} \right)^2 \phi^2 \right] \mathrm{d}z\, \mathrm{d}x. \tag{6}$$

In these equations, $\phi(z)$ and u are the mode shapes defining the variation of the deflection $\bar{w}(x,z)$ in the z direction and the displacement of the subsoil in the x direction respectively [15, 16].

By applying variations in Π due to variations in w and ϕ and using variational calculus, one can get

$$k = \int_0^H \frac{E_s b_w (1-v)}{(1+v)(1-2v)} \left(\frac{\mathrm{d}\phi}{\mathrm{d}z} \right)^2 \mathrm{d}z \tag{7}$$

$$2t = \int_0^H \frac{E_s b_w}{2(1+v)} \phi^2 \, \mathrm{d}z \tag{8}$$

where

$$\phi(\mathrm{z}) = \frac{\sinh \gamma (1 - z/H)}{\sinh \gamma} \tag{9}$$

and

$$\left(\frac{\gamma}{H}\right)^2 = \frac{n}{m} = \frac{1-2v}{2(1-v)} \frac{\int_{-\infty}^{+\infty} (\mathrm{d}w/\mathrm{d}x)^2\,\mathrm{d}x}{\int_{-\infty}^{+\infty} w^2\,\mathrm{d}x}. \tag{10}$$

In these expressions, k, $2t$ and γ are the Winkler foundation modulus, shear foundation modulus and vertical deformation parameter within the subsoil respectively. The other terms were previously defined.

As can be seen from equation (10), the value of γ varies with the displacement of the beam and the depth of the subsoil. Therefore, the variables w, k, $2t$, H and γ are all connected to each other for a beam on an elastic foundation.

If equations (7) and (8) are substituted into equation (6), the potential energy of the beam–soil system for the unloaded case (see Figure 1) can be rewritten in the following form:

$$\Pi = \frac{1}{2}\int_0^L E_b I_b \left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^L k w^2\,\mathrm{d}x + \frac{1}{2}\int_0^L 2t\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2 \mathrm{d}x. \tag{11}$$

The element stiffness matrices can be evaluated by using the cubic displacement function that is standard in the finite element beam theory. A cubic field, interpolated from nodal degrees of freedoms (d.o.f.) $\{\mathbf{w}_e\} = \{w_{i,i}, w_{j,j}\}$, is

$$w(x) = [N_1 \quad N_2 \quad N_3 \quad N_4]\,\{\mathbf{w}_e\}, \tag{12}$$

in which N_1–N_4 are as stated in reference [17].

By substituting equation (12) into equation (11), the stiffness matrices of the beam–soil system can be evaluated as

$$\Pi = \tfrac{1}{2}\{\mathbf{w}_e\}^{\mathrm{T}}([\mathbf{k}_b] + [\mathbf{k}_w] + [\mathbf{k}_v])\{\mathbf{w}_e\}, \tag{13}$$

where $[\mathbf{k}_b]$, $[\mathbf{k}_w]$ and $[\mathbf{k}_v]$ are the beam stiffness matrix, the Winkler foundation stiffness matrix and the second parameter foundation stiffness matrix, respectively, and are given as follows [14, 17]:

$$[k_b] = \frac{EI}{l^3}\begin{bmatrix} 12 & -6l & -12 & -6l \\ & 4l^2 & 6l & 2l^2 \\ & & 12 & 6l \\ \text{Sym.} & & & 4l^2 \end{bmatrix}, \tag{14}$$

$$[k_w] = k\begin{bmatrix} \frac{13}{35}l & -\frac{11}{210}l^2 & \frac{9}{70}l & \frac{13}{420}l^2 \\ & \frac{1}{105}l^3 & -\frac{13}{420}l^2 & -\frac{1}{140}l^3 \\ & & \frac{13}{15}l & \frac{11}{210}l^2 \\ \text{Sym.} & & & \frac{1}{105}l^3 \end{bmatrix}, \tag{15}$$

$$[k_v] = 2t\begin{bmatrix} \frac{6}{5l} & -\frac{1}{10} & -\frac{6}{5l} & -\frac{1}{10} \\ & \frac{2}{15}l & \frac{1}{10} & -\frac{1}{30}l \\ & & \frac{6}{5l} & \frac{1}{10} \\ \text{Sym.} & & & \frac{2}{15}l \end{bmatrix}. \tag{16}$$

By assembling each element stiffness matrix obtained from the above equations, the system stiffness matrix is obtained.

2.2. EVALUATION OF THE MASS MATRIX

According to Hamilton's variational principle, the total kinetic energy of the beam–soil system may be written as

$$\Pi_k = \frac{1}{2}\int_{\Omega} \dot{\mathbf{w}}^{\mathrm{T}}\boldsymbol{\mu}\,\dot{\mathbf{w}}\,\mathrm{d}\Omega, \tag{17}$$

where $\boldsymbol{\mu}$ is the mass density matrix and $\dot{\mathbf{w}}$ represents the partial derivative of the vector of generalized displacement with respect to the time variable. The consistent mass matrix, $\mathbf{M}$, is obtained by substituting $\mathbf{w} = \mathbf{N}_1\mathbf{w}_e$ into equation (17):

$$\mathbf{M} = \int_{\Omega} \mathbf{N}_1^{\mathrm{T}}\boldsymbol{\mu}\mathbf{N}_1\,\mathrm{d}\Omega. \tag{18}$$

The mass matrix for the beam–soil system needs to be analyzed. The matrix $\boldsymbol{\mu}$ in equation (18) is a square symmetric matrix of the form

$$\boldsymbol{\mu} = \begin{bmatrix} \rho_b h + \frac{1}{13}\rho_s \mathrm{H} & 0 \\ 0 & \frac{1}{12}\rho_b \mathrm{h}^3 \end{bmatrix}, \tag{19}$$

where ρ_b is the beam mass density, h is the depth of the beam cross-section and $\rho_s$is the mass density of the soil. In view of equation (12), the following expression can be written for each finite piece:

$$[\mathbf{N}_1] = \begin{bmatrix} N \\ -\,\mathrm{d}N/\mathrm{d}x \end{bmatrix}. \tag{20}$$

The consistent mass matrix of the beam and the soil can be evaluated after substituting equation (20) into equation (18) and integrating it from zero to l. By assembling the element mass matrix obtained, the system mass matrix is evaluated [14].

As mentioned before, the governing equation for a beam subjected to a free vibration with no damping is represented by equation (1). After substituting $\mathbf{w} = \mathbf{W}\sin\omega t$ into this equation, one can obtain

$$([\mathbf{K}] - \lambda[\mathbf{M}])\{\mathbf{W}\} = 0, \tag{21}$$

where $\{\mathbf{W}\}$ is a vector of mode shape of vibration and $\lambda\,(=\omega^2$, ω is the circular frequency) is the frequency parameter. The eigenvalue solution of this equation yields the frequency parameters and corresponding mode shapes.

3. NUMERICAL EXAMPLES

3.1. DATA FOR NUMERICAL EXAMPLES

In this study, different values of H, H/L and γ are used for the parametric study of the free vibration analysis of beams on elastic foundations. The values of the vertical deformation parameter, γ, are taken to be 1–8. The depths, H, of the subsoil are taken to be 5, 10 and 15 m for each γ parameter considered, and the ratios H/L used are 0·25, 0·50, 0·75 and 1·00 for each subsoil depth. In the calculation of the mass matrix, the mass densities of beam and subsoil are taken to be 2500 and 1700 kg/m^3, respectively. The properties of the beam–soil system are as follows: The width of the beam is 30 cm; the depth of the beam is 50 cm; the modulus of elasticity of the beam is $2{\cdot}7 \times 10^9$ N/m^2; the modulus of elasticity of the subsoil is 2×10^7 N/m^2 and the Poisson ratio of the subsoil is 0·2.

For the sake of the accuracy in the results, rather than starting with a finite element mesh size, the mesh size required to produce the desired accuracy is determined. To find out the required mesh size, convergence of the frequency parameters is checked for different mesh sizes. In this way, it is concluded that the results have acceptable error when using equally spaced 20 elements for a 10 m beam. The element length is kept constant for different lengths of the beam.

3.2. RESULTS

In this study, the first 10 frequency parameters of beams considered for several subsoil depths, beam lengths, their ratios and the values of the vertical deformation parameter within the subsoil are presented in Table 1. In order to see the effects of the changes in these parameters better on the first six frequency parameters, they are given in Figures 2–9 for $\gamma = 1$–8 respectively.

As seen from Table 1 and Figures 2–9 the values of the frequency parameters for a constant value of H increase as H/L ratio increases, but the values of the frequency parameters for a constant H/L ratio decrease as the value of H increases. The values of the frequency parameters for a constant value of H and H/L increase as the value of γ increases.

It should be noted that the increase in the frequency parameters with increasing H/L ratios for a constant value of H gets larger for larger values of the frequency parameters.

The decrease in the frequency parameters with increasing value of H for a constant H/L ratio gets less for larger values of H. This behavior is understandable in that a beam on an elastic foundation with a larger subsoil depth becomes more flexible and has smaller frequency parameters.

The decrease or increase occurring in the frequency parameters with increasing subsoil depth for a constant value of H/L ratio is larger than the increase or increase occurring in the frequency parameters with increasing H/L ratios for a constant value of H.

These observations indicate that the effects of the change in the subsoil depth on the frequency parameter of the beam on an elastic foundation are always larger than those of the change in the other parameters considered in this study.

TABLE 1

The first 10 frequency parameters of beams on elastic foundations for different values of H, H/L and γ

			Frequency parameters, λ									
γ	H(m)	H/L	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
		0·25	45·17	58·68	89·68	166·28	366·18	808·79	1647·92	3074·02	5314·85	8635·29
	5	0·50	52·86	93·82	286·37	1221·60	4139·36	10 858·35	23 785·93	49 507·91	80 782·93	13 2542·12
		0·75	61·13	133·48	917·71	5440·28	19 880·09	53 304·84	117 699·42	227 953·90	401 964·07	660 805·38
		1·00	68·87	179·07	2475·25	16 491·92	61512·95	165 736·38	366 442·38	710 203·56	1 252 984·53	2 055 523·34
1	10	0·50	13·23	21·62	42·24	92·08	211·07	461·69	926·00	1705·75	2922·66	4718·55
		0·75	15·10	32·41	87·28	276·07	806·88	1990·12	4236·73	8058·81	14 069·59	22 983·66
		1·00	17·18	45·02	169·23	703·83	2305·01	5946·46	12 921·66	24 843·59	43 647·26	71 596·03
	15	0·50	5·93	9·00	15·89	29·25	53·89	97·81	172·07	290·55	469·80	729·01
		0·75	6·67	13·28	30·25	70·21	159·79	340·95	669·56	1215·06	2060·55	3302·86
		1·00	7·50	18·74	52·28	149·08	395·97	925·22	1912·30	3575·88	6178·06	10 024·51
2	5	0·25	51·91	64·39	92·58	164·22	358·10	793·68	1624·48	3040·88	5270·63	8578·59
		0·50	59·26	95·60	276·45	1194·17	4089·77	10 781·18	23 675·77	45 759·43	80 590·87	13 201·32
		0·75	67·08	129·47	888·27	5372·61	19 763·08	53 126·49	117 448·01	227 618·06	401 532·79	660 268·00
		1·00	74·34	167·65	2419·49	16 368·99	61 303·25	165 419·26	365 998·13	709 613·57	1 252 232·05	2 054 596·63
	10	0·50	14·85	22·35	40·44	85·50	198·14	441·17	869·53	1665·89	2870·96	4653·53
		0·75	16·63	31·83	80·34	258·46	775·68	1942·22	4168·88	7967·71	13 951·97	22 836·22
		1·00	18·62	42·53	155·18	671·23	2248·64	5860·58	12 800·45	24 681·22	43 437·97	71 334·06
	15	0·50	6·65	9·29	15·21	26·74	48·60	88·92	158·96	272·61	446·39	699·51
		0·75	7·32	13·08	27·59	62·90	146·07	319·42	638·83	1173·68	2007·03	3235·71
		1·00	8·11	17·81	46·57	134·91	370·66	886·30	1857·18	3501·93	6082·60	9904·88
3	5	0·25	66·72	78·64	105·01	172·89	361·95	791·84	1615·92	3024·49	5245·31	8543·24
		0·50	74·01	107·51	279·60	1173·26	4061·02	10 730·22	23 598·26	45 651·07	80 447·45	132 118·69
		0·75	81·77	137·44	876·57	5330·15	19 680·92	52 994·95	117 257·70	227 359·81	401 197·75	659 847·83
		1·00	88·90	170·34	2387·44	16 282·90	61 147·34	165 176·92	365 653·50	709 151·68	1 251 639·42	2 053 863·76
	10	0·50	18·64	25·59	41·85	83·13	190·68	427·62	875·79	1636·81	2832·35	4604·22
		0·75	20·40	34·23	77·88	247·44	753·77	1906·89	4117·51	7897·66	13 860·60	22 720·90
		1·00	22·41	43·57	147·22	648·40	2206·73	5794·95	12 706·42	24554·12	43 273·14	71 126·90
	15	0·50	8·35	10·67	15·91	26·04	45·64	83·06	149·71	259·49	428·90	677·11
		0·75	8·98	14·16	26·82	58·39	136·40	303·50	615·54	1141·84	1965·45	3183·17
		1·00	9·75	18·40	43·44	124·96	351·80	856·52	1814·41	3444·02	6007·42	9810·27

4	5	0·25	85·64	97·29	122·64	188·13	374·04	800·24	1619·95	3023·45	5238·46	8529·88
		0·50	92·97	124·88	291·49	1186·02	4052·20	10 706·99	23 557·80	45 590·60	80 364·22	132 010·01
		0·75	100·78	152·47	879·06	5312·69	19 637·69	52 919·68	117 144·28	227 202·30	400 990·41	659 585·17
		1·00	107·90	182·15	2376·98	16 237·37	61 056·50	165 029·98	365 440·19	708 862·27	1 251 265·17	2 053 398·47
	10	0·50	23·57	30·20	45·36	84·20	188·44	421·43	864·96	1620·57	2809·95	4574·88
		0·75	25·32	38·37	78·95	242·95	742·23	1886·67	4086·90	7854·95	13804·06	22 648·84
		1·00	27·36	46·89	144·79	636·32	2182·30	5755·16	12 648·23	24 474·50	43 169·07	70 995·39
	15	0·50	10·57	12·67	17·52	26·77	44·89	80·42	144·87	252·15	418·73	663·78
		0·75	11·16	16·01	27·53	56·66	131·32	294·38	601·64	1122·41	1939·71	3150·34
		1·00	11·93	19·97	42·65	119·72	340·78	838·45	1787·93	3407·74	5959·94	9750·21
5	5	0·25	105·90	117·39	142·11	206·07	389·96	813·77	1630·67	3030·89	5242·17	8529·39
		0·50	113·27	144·21	307·33	1195·97	4054·69	10 700·19	23 539·90	45 559·80	80 318·76	131 948·16
		0·75	121·12	170·33	888·88	5309·65	19 618·05	52 879·40	117 079·43	227 109·05	400 865·06	659 424·18
		1·00	128·26	197·97	2378·51	16 216·30	61 006·25	164 943·59	365 311·03	708 684·10	1 251 032·33	2 053 106·98
	10	0·50	28·87	35·32	49·80	87·06	189·17	419·63	860·16	1612·30	2797·71	4558·18
		0·75	30·63	43·20	81·83	242·23	736·99	1875·85	4069·41	7829·67	13 769·90	22 604·69
		1·00	32·70	51·21	145·42	630·74	2168·79	5731·77	12 613·00	24 425·49	43 104·35	70 913·03
	15	0·50	12·97	14·93	19·54	28·24	45·40	79·70	142·73	248·40	413·17	656·19
		0·75	13·53	18·18	28·98	56·53	129·02	289·49	593·69	1110·91	1924·15	3130·22
		1·00	14·29	21·97	43·14	117·32	334·68	827·82	1771·89	3385·40	5930·42	9712·60
6	5	0·25	126·51	137·89	162·20	225·12	407·62	829·82	1644·81	3042·81	5251·55	8535·92
		0·50	133·90	164·20	324·95	1209·59	4063·24	10 702·44	23 534·61	45 545·75	80 294·75	131 913·01
		0·75	141·80	189·30	902·43	5314·46	19 611·59	52 858·92	117 042·25	227 052·58	400 786·78	659 321·69
		1·00	148·95	215·59	2386·42	16 208·86	60 978·99	164 891·78	365 230·18	708 569·94	125 0881·04	2 052 915·84
	10	0·50	34·29	40·62	54·65	90·83	191·48	420·23	858·74	1608·53	2791·25	4548·70
		0·75	36·04	48·31	85·62	243·57	735·26	1870·34	4059·38	7814·36	13 748·57	22 576·60
		1·00	38·15	55·96	147·68	628·78	2161·46	5717·75	12 590·97	24 394·11	43 062·32	70 859·03
	15	0·50	15·42	17·28	21·74	30·07	46·57	80·03	142·11	246·69	410·23	651·87
		0·75	15·96	20·47	30·81	57·27	128·29	287·00	589·13	1103·94	1914·44	3117·41
		1·00	16·72	24·15	44·30	116·51	331·37	821·45	1761·85	3371·10	5911·24	9687·94
7	5	0·25	147·20	158·51	182·52	244·69	426·21	847·24	1660·86	3057·26	5264·18	8546·50
		0·50	154·62	184·46	343·50	1225·26	4075·28	10 709·94	23 536·69	45 541·53	80 283·37	131 893·63
		0·75	162·55	208·87	918·05	5323·80	19 612·83	52 850·07	117 021·41	227 017·86	400 736·38	659 253·89
		1·00	169·70	234·14	2397·99	16 209·40	60 965·28	164 860·42	365 177·93	708 493·75	1 250 778·16	2 052 784·32
	10	0·50	39·75	45·99	59·70	95·10	194·71	422·23	859·29	1607·39	2788·19	4543·47
		0·75	41·50	53·54	89·91	246·11	735·60	1867·97	4053·76	7804·96	13 734·85	22 558·03
		1·00	43·62	60·94	150·87	628·95	2157·78	5709·27	12 576·74	24 373·19	43 033·75	70 821·89

TABLE 1

Continued

γ	H(m)	H/L	Frequency parameters, λ									
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
	15	0·50	17·88	19·68	24·04	32·10	48·12	80·98	142·37	246·18	408·84	649·49
		0·75	18·41	22·83	32·83	58·51	128·47	285·92	586·58	1099·66	1908·19	3108·95
		1·00	19·16	26·43	45·85	116·64	329·70	817·59	1755·37	3361·56	5898·21	9670·98
8	5	0·25	167·93	179·18	202·97	264·58	445·34	865·50	1678·08	3073·28	5278·83	8559·60
		0·50	175·37	204·85	362·61	1242·21	4089·48	10 720·73	23 543·39	45 543·49	80 279·94	131 884·16
		0·75	183·31	228·73	934·95	5335·97	19 618·90	52 848·55	117 010·84	226 996·85	400 703·57	659 207·96
		1·00	190·48	253·25	2411·85	16 214·95	60 960·09	164 841·94	365 143·72	708 441·51	1 250 705·80	2 052 690·39
	10	0·50	45·22	51·40	64·87	99·68	198·51	425·11	861·08	1607·92	2787·27	4540·92
		0·75	46·97	58·85	94·51	249·38	737·23	1867·56	4050·92	7799·28	13 725·93	22 545·46
		1·00	49·10	66·06	154·65	630·45	2156·39	5704·28	12 567·43	24 358·85	43 013·68	70 795·38
	15	0·50	20·36	22·10	26·38	34·24	49·91	82·31	143·19	246·42	408·42	648·33
		0·75	20·87	25·23	34·97	60·07	129·23	285·74	585·28	1097·07	1904·12	3103·22
		1·00	21·62	28·77	47·64	117·36	329·08	815·32	1751·13	3355·03	5889·05	9658·87

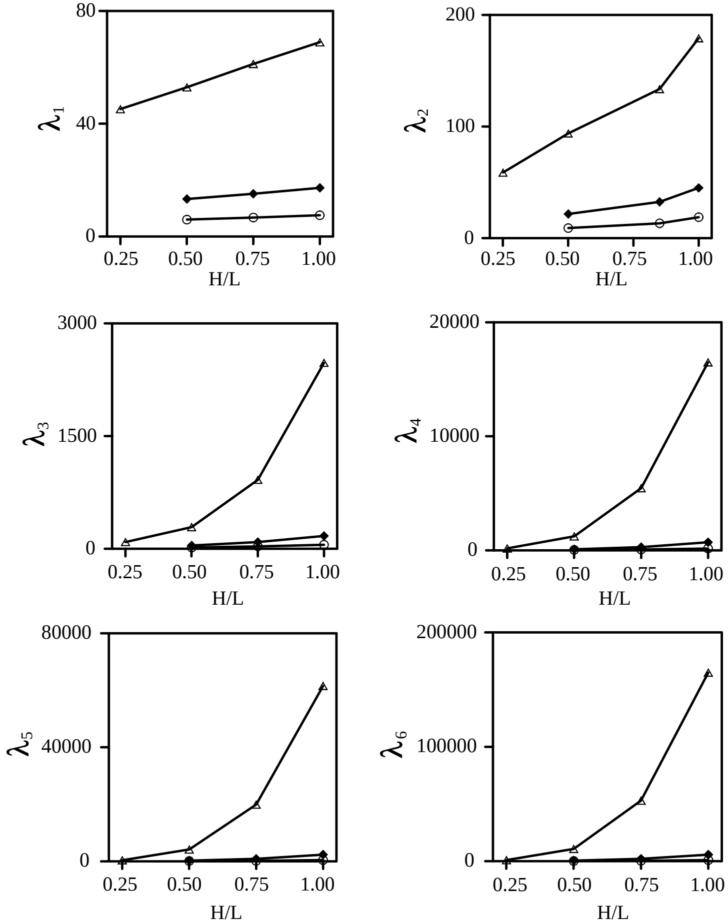

Figure 2. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 1$. Key for H values: —Δ—, 5 m; —◆—, 10 m; —o—, 15 m.

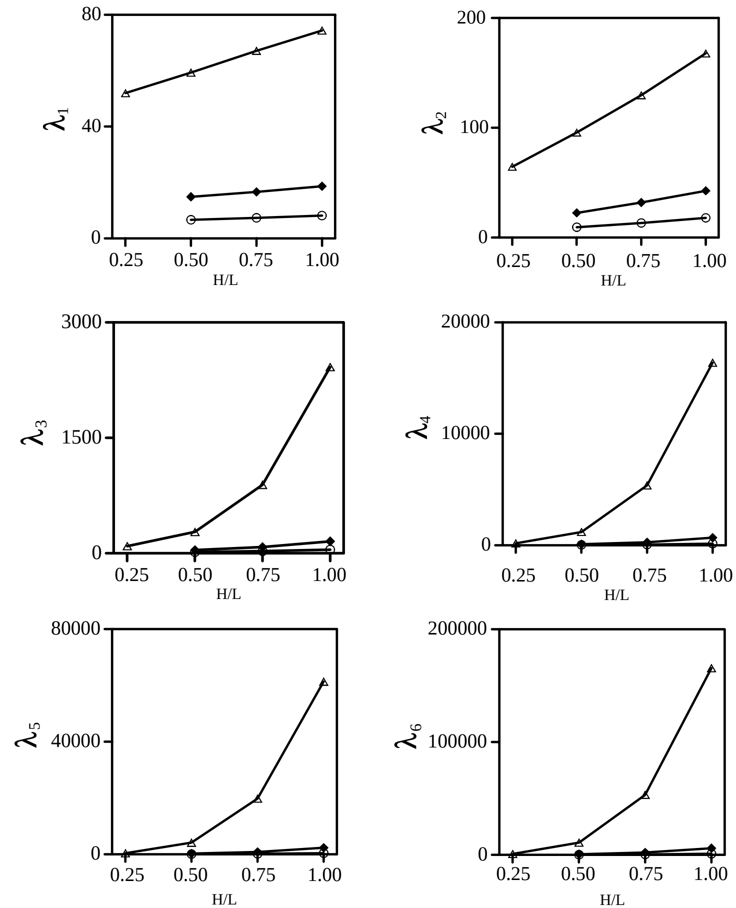

Figure 3. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 2$. Key as for Figure 2.

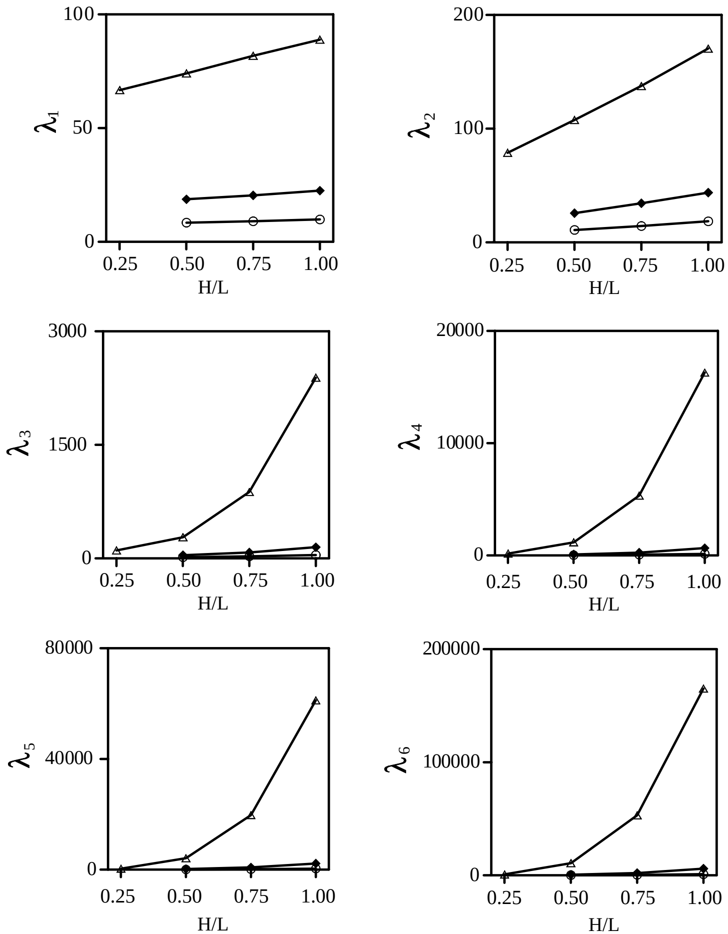

Figure 4. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 3$. Key as for Figure 2.

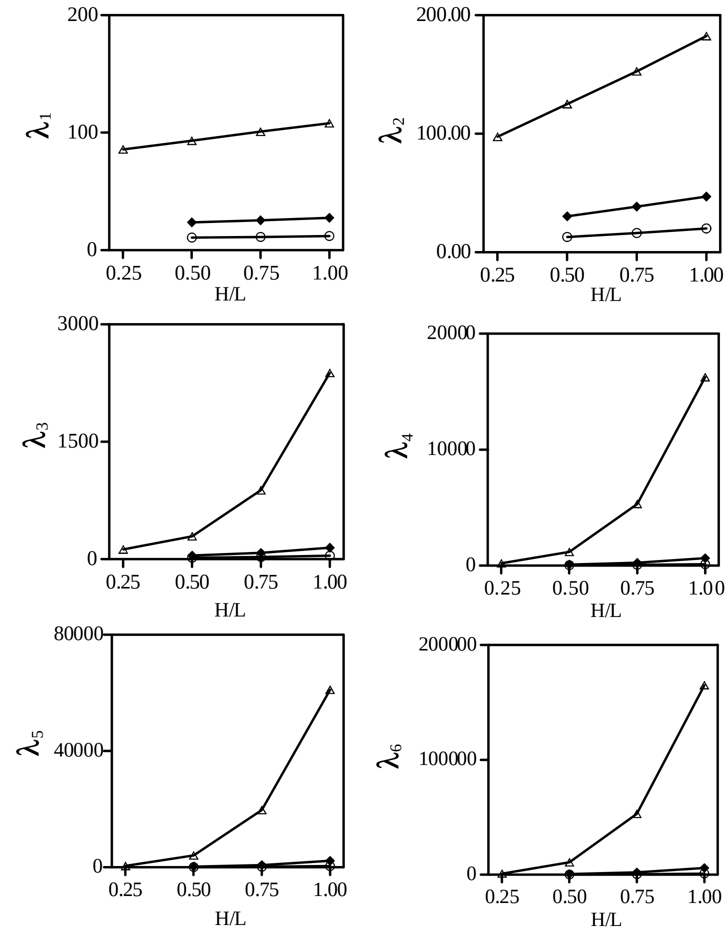

Figure 5. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 4$. Key as for Figure 2.

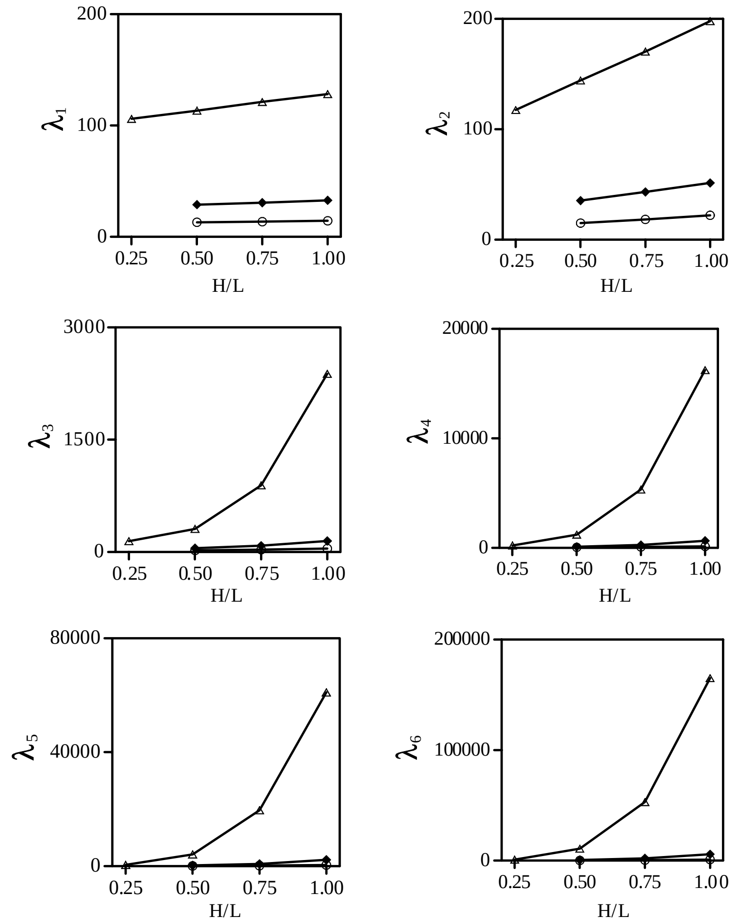

Figure 6. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 5$. Key as for Figure 2.

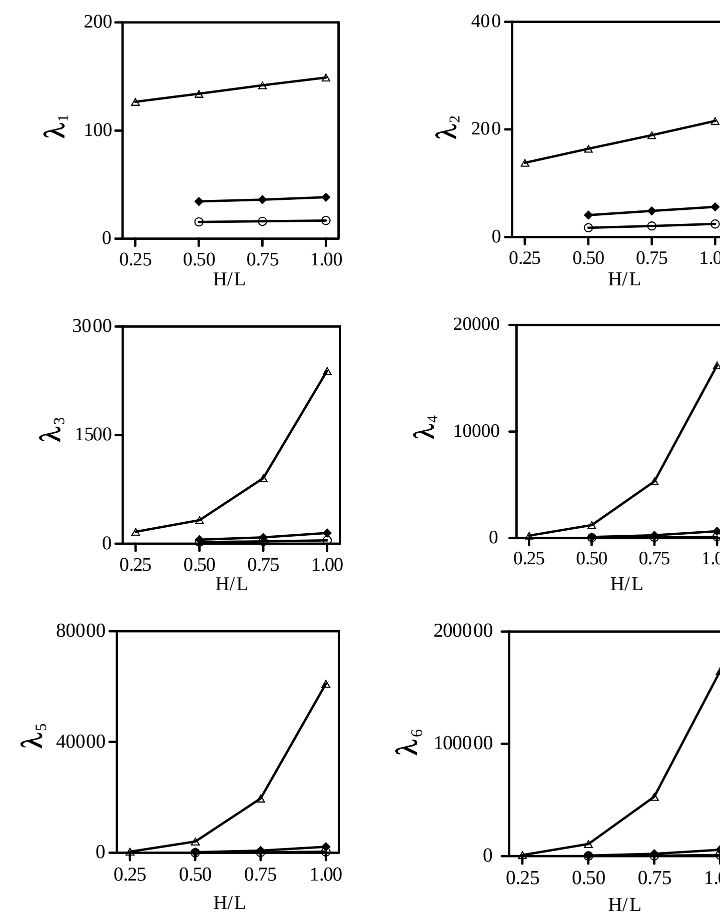

Figure 7. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 6$. Key as for Figure 2.

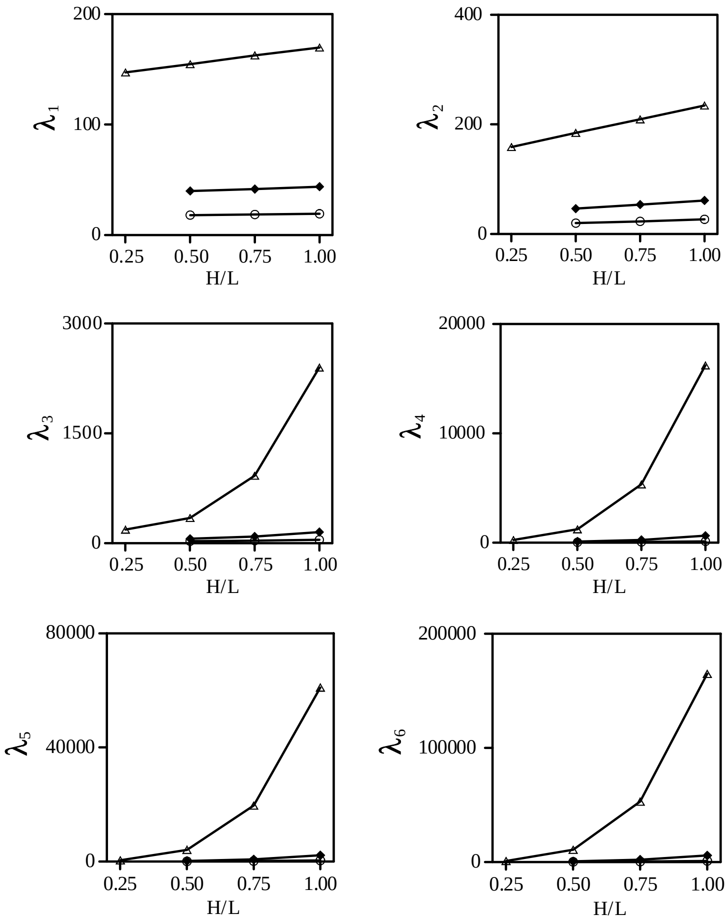

Figure 8. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 7$. Key as for Figure 2.

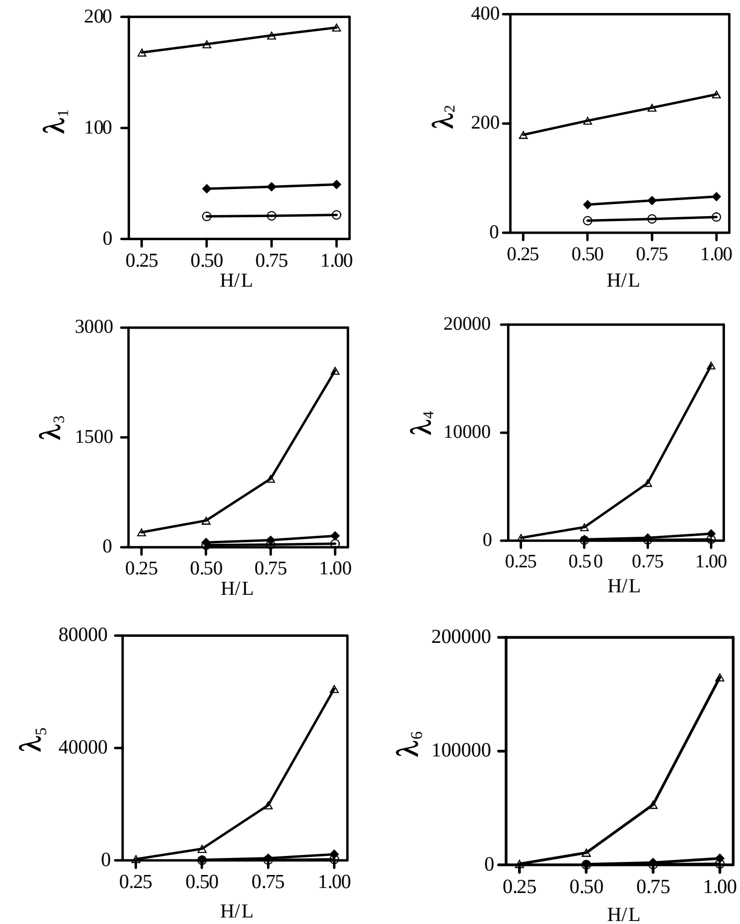

Figure 9. The effects of different values of H and H/L on the first six frequency parameters of the beam on elastic foundations for $\gamma = 8$. Key as for Figure 2.

As can also be seen from Figures 2–9, the curves for a constant value of H/L get fairly closer to each other as the value of H increases. This shows that the curves of the frequency parameters will almost coincide with each other when the value of H increases more. In other words, the increase in the subsoil depth will not affect the frequency parameters after a determined value of H. In addition, variations occurring in the frequency parameters increases as the value of H/L ratio increases.

As can also be seen from these figures, depending on the increase in H/L ratio, the increase occurring in the frequency parameters for the larger values of the vertical deformation parameters, γ, gets less as γ increases.

In this study, the mode shapes of the beams on an elastic foundation are also obtained for all parameters considered. Since presentation of all of these mode shapes would take up excessive space, only the mode shapes corresponding to the six lowest frequency parameters of the beam for $\gamma = 1$, $H = 5$ and $H/L = 0{\cdot}25$ are presented. These mode shapes are given in Figure 10. In order to make the visibility better, the mode shapes are plotted with exaggerated amplitudes.

As seen from this figure, the number of half waves is proportional to the mode number. It should be noted that appearances of the mode shapes not given, corresponding to the six lowest frequency parameters for the other values of the parameters H, H/L and γ, are similar to the mode shapes presented here.

It should also be noted that the results obtained by using the modified Vlasov model are not compared with the results of the Winkler model, which is simpler, because the stiffness parameter, k, is calculated within the program coded depending on the assumed values of γ, but these parameters in the Winkler model should be given to the program as data.

The results obtained in this study are not compared with the results given by Franciosi and Masi [7], because the results given by them are dependent on the foundation parameters k and k_1, but, in this study, the foundation parameters are not given as data, they are calculated by the program and are dependent on the vertical deformation parameter within the subsoil. Therefore, comparison of the results of both studies will not be appropriate.

4. CONCLUSION

The modified Vlasov model has been applied effectively to the free vibration analysis of beams resting on elastic foundations. Two soil parameters are calculated in terms of the parameters, γ.

In addition, the following conclusions can be drawn from the results obtained in this study.

- The frequency parameter always increases with increasing H/L ratio for any values of subsoil depth.
- The frequency parameter always decreases as the subsoil depth increases for any values of H/L.
- The frequency parameter always increases with increasing γ values for any values of H and H/L ratio.
- In general, the effects of the change in the subsoil depth on the frequency parameters of the beams on elastic foundations are larger than those of the other parameters considered in this study.

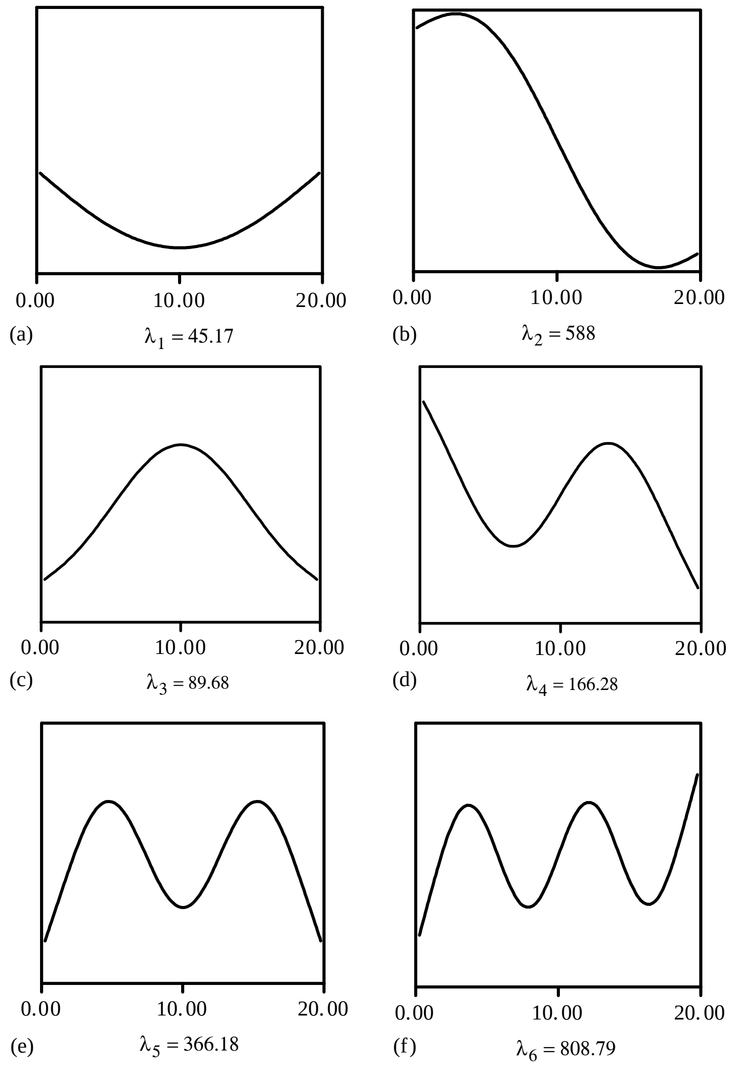

Figure 10. The first six mode shapes of the beam on elastic foundations for $\gamma = 1$, $H = 5$ m and $H/L = 0{\cdot}25$. (a) First mode shape; (b) second mode shape; (c) third mode shape; (d) fourth mode shape; (e) fifth mode shape; and (f) sixth mode shape.

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