Journal of Sound and Vibration (2002) **255**(1), 201–202
doi:10.1006/jsvi.2001.4131, available online at http://www.idealibrary.com on IDEAL®

AUTHOR'S REPLY

P. D. Cha

Department of Engineering, Harvey Mudd College, Claremont, CA 91711, *U.S.A.*
E-mail: *philip_cha@hmc.edu*

(*Received* 11 *January* 2002)

The author wishes to thank Gürgöze for his interest in the derivation presented in the appendix of reference [1]. The approach proposed by Gürgöze is indeed a viable alternative to determining the closed-form expression for the eigenvalues of the following characteristic determinant:

$$\det\{[\mathbf{K}^d] - \omega^2[\mathbf{M}^d] + \sigma\mathbf{u}\mathbf{u}^{\mathrm{T}}\} = 0, \tag{1}$$

where $[\mathbf{K}^d]$ and $[\mathbf{M}^d]$ are diagonal matrices whose ith elements are K_i and M_i, respectively, and ω^2 represent the eigenvalues of the system. His approach is certainly more direct, and yields the same characteristic equation that was given in the appendix of reference [1]. However, in spite of the author's assertion that the following formula:

$$\det([\mathbf{A}] + \alpha\mathbf{b}\mathbf{b}^{\mathrm{T}}) = \det[\mathbf{A}](1 + \alpha\mathbf{b}^{\mathrm{T}}[\mathbf{A}]^{-1}\mathbf{b}) \tag{2}$$

is well known, equation (2) is not likely to be familiar to most readers.

Interestingly, there is yet another approach to finding the eigenvalues of equation (1), which requires only elementary theory in linear algebra, though the manipulation is a bit lengthy. Equation (1), after some manipulation, can be shown to be equivalent to the following characteristic equation:

$$\prod_{i=1}^{N}(K_i - \omega^2 M_i) + \sigma\sum_{i=1}^{N} u_i^2 \prod_{j=1,\, j\neq i}^{N}(K_j - \omega^2 M_j) = 0. \tag{3}$$

For $N = 2$, equation (3) can be easily verified by hand; for $N = 3$, the above can be obtained by using the special "basketweaving" method for calculating the determinant of a (3×3) matrix; for $N \geqslant 4$, equation (1) can be expanded by using the formal definition of the determinant to yield equation (3), which can also be written as

$$\prod_{i=1}^{N}(K_i - \omega^2 M_i) + \sigma\sum_{i=1}^{N}\frac{u_i^2}{K_i - \omega^2 M_i}\prod_{j=1}^{N}(K_j - \omega^2 M_j) = 0 \tag{4}$$

or

$$\prod_{i=1}^{N}(K_i - \omega^2 M_i) + \left\{1 + \sigma\sum_{i=1}^{N}\frac{u_i^2}{K_i - \omega^2 M_i}\right\} = 0 \tag{5}$$

which is the same result that was given in the appendix of reference [1].

0022-460X/02/$35.00

The approach to determine the characteristic equation of equation (1) is by no means unique. The approaches outlined in reference [1] and presented here are conceptually simple, requiring only elementary linear algebra, and can be extended to any value of s, though the steps may be algebraically intensive. The scheme proposed by Gürgöze is more direct, though it is strictly valid for the special case of $s = 1$ and requires a fairly specialized formula (see equation (2)). Regardless, for $s = 1$, all three approaches lead eventually to equation (5).

REFERENCES

1. P. D. CHA and W. WONG 1999 *Journal of Sound and Vibration* **219**, 689–706. A novel approach to determine the frequency equations of combined dynamical systems.