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STRUCTURE-BORNE NOISE AND VIBRATION OF CONCRETE BOX STRUCTURE AND RAIL VIADUCT

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In this study, the vibration and acoustic resonance, and dominant frequency range of simple concrete box and viaduct are examined from the measurement results. A narrow band analysis—fast Fourier transform (FFT) method is used to analyze the measurement results and finite element method (FEM) is used to validate resonance frequencies for noise and vibration. The experiment of the concrete box structure is a preliminary study of analyzing resonance frequency radiated from the vibrating concrete structure since railway viaduct is a concrete box structure too. According to their noise and vibration spectra, it shows that the vibration resonance is more significant than the acoustics resonance.

Based on the measurement results of the rail viaduct structure-borne noise and vibration, the relationship in terms of transfer function and coherence between noise and vibration are evaluated. They show that the dominant frequency range for noise and vibration of concrete viaduct is between 20 and 157 Hz, the resonance frequencies are 43 and 54 Hz and have significant tonal noise characteristics. The experimental results are in good agreement with the theoretical relationship between sound and vibration.

1. INTRODUCTION

The extension of the railway system becomes one of the major transport system in the world to cope with the growing demand for public transportation, significantly reducing vehicle exhaust emissions and stimulating extensive land development. In addition, speed and frequency of trains and number of viaducts will be increased through the developed area. It is inevitable that there is greatly increased noise nuisance to the nearby residents especially for the structure-borne noise radiated from the concrete viaduct. Therefore, structure-borne noise radiated from the concrete viaduct should be controlled for the new railway. One of the examples is about the West Rail project in Hong Kong, Crockett and Pyke [1] reported that targeted level maximum overall A-weighted level of the structure-radiated and direct train noise, taken independently, was thus set at 61 dB(A). During the design stage, a finite element model (FEM) of the structure, trackform and vehicle was also developed to determine the modal responses of the structural vibration during train passbys. Then an analytical model takes these vibration levels as input and determines the wayside structure-radiated noise. This latest report is one of the firsts to develop structure-radiated noise prediction method based on the vibration analysis of structure.

There were many investigations about structure-borne sound from rail traffic, both theoretical and experimental. However, there is a lack of research on the correlation study of structure-borne noise and vibration along the railway system and concrete box structure.

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Bovey [2] had only determined the vibration transfer characteristics of railway installations. He concluded that impact method is a reliable, and controlled method for providing quantitative data to determine the vibration transfer characteristics of railway installations.

There were separate studies [3–5] treating the structure- and ground-borne noise. Most of them had only concluded the dominant frequency range of sound and vibration from their spectra separately. Moritoh *et al.* [3] undertook a site measurement of concrete bridge structure noise below the bridge structure. It showed that the spectra had marked peaks at frequencies around 50 Hz as the train speed was 240 km/h. Stüber [4] discussed the air- and structure-borne railway noise obtained from the measurement. Bridge noise with ballast-bed track from trains travelling at 80 km/h measured at 25 m away and 1·6 m above the ground level showed that the peak noise level is at 50 Hz, but no vibration level at the same position was recorded. Only the vibration levels at the base of the rail, at the tunnel boundaries and outside the tunnel in the open air are investigated. Morii [5] investigated the vibration–isolation techniques of Shinkansen and focused on the control of the magnitude of the vibration. One of the figures in that paper showed that the peak vibration of viaduct girder without ballast mat was in the range between 40 and 60 Hz.

Heckl *et al.* [6] used an *in situ* measurement to show that the dominant frequency range of wheel/track resonance lies between 40 and 100 Hz. Vibration transmission caused by rail traffic was found to be a low-frequency problem. Vibration and sound caused by suburban trains running at 60 km/h in a tunnel were measured, and the peaks of vibration was found at 40 Hz while the peak noise level were at 50 Hz. Their peak level of sound and vibration are quite familiar with our measurement results but they did not validate the source of peak level.

In fact, the vibration of materials or particles will generate noise and they are interrelated, so the relationship between them is important for analyzing the sources and characteristics of structure-borne noise. In this paper, a preliminary study of the correlation between narrow band noise and vibration spectra was carried out in the simple concrete box structure as both box and viaduct are concrete box structures. Their bending mode shape, noise and vibration spectra should have some similarity, and box structure is a simple and basic structure to ensure that the general experimental techniques and analytical method are applicable. On the other hand, this experimental and theoretical analysis of the box in the laboratory had better accuracy and more supporting information than the site work of viaduct. Therefore, it was analyzed first and then a railway viaduct was measured for analyzing the peak frequency response of structure-borne noise and vibration with the help of the FEM program.

It should also be noted that the box structure could represent an apartment or room inside a building and the structure of railway station. The air pump, sewerage pump and chiller on the floor will vibrate the concrete floor to generate the structure-borne noise and the isolator is designed for preventing the vibration transmission at around 11 Hz only. Moreover, most of the paper was investigating the longitudinal bending wave of viaduct or building [7, 8] and the rigid body motion instead of sectional bending mode shape. Kim *et al.* [7] studied the dynamic problem occurred at the long-span suspension bridge that exceeds 1000 m span length. In order to approach dynamic problem such as wind, vehicle or earthquake-induced oscillation, they developed a simple method of determining accurate natural frequencies and mode shapes along its span only. That longitudinal wave was below 20 Hz, which would not cause any structure-borne problem, is not important from the view of noise control.

Hans *et al.* [8] determined the first two flexural modes of a civil engineering building (in both longitudinal and transverse directions) from experimental time responses. Then they used the wavelet-logarithmic decrement procedure to estimate the non-dimensional damping ratio of the fundamental and the first harmonic bending mode in both the longitudinal and transverse directions, according to harmonic/shock test responses, since they were concerned with the whole building structure instead of the individual apartment inside the building. These are usually <20 Hz and thus not radiating noise as the cross-section do not change shape in resonance modes. A short investigation of bending wave of concrete box section with higher frequency inside building and viaduct was conducted in the previous studies.

2. REVIEW OF STRUCTURE-BORNE NOISE CONTROL

In the past, the control measure of air-borne noise generated from railway lines were well established such as extensive barriers and enclosures. Generally, the existing barrier design focuses on the attenuation of high-frequency noise instead of low-frequency noise. Therefore, structure-radiated noise is significant when a sound barrier effectively controls the wheel/rail noise and the car auxiliary equipment noise. Recently, most of the noise problems are relevant to the low-frequency noise of the modern concrete viaduct structures, so many complaints are reported although it is within the noise limit. It is because the noise limit is using A-weighted level, which tends to underestimate the annoyance quality of low-frequency sound.

Structure-borne noise is a serious problem as it is a low-frequency tonal noise due to vibration, which cannot be controlled by convention method of barriers and window insulation. Its effect will be much more pronounced inside buildings adjacent to the viaduct structure. The building walls and ceiling do not reduce low-frequency noise as efficiently as high-frequency sound. On the other hand, prediction of its spectrum is quite difficult due to the complexity of structure. Then more detailed analysis should be done to solve this low-frequency noise problem.

3. PREDICTION METHOD FOR CONCRETE STRUCTURE

For our concrete structure, it can be assumed as a beam simply supported at both ends and use the beam theorem to obtain the natural frequency by [9]

$$\omega_r = \left(\frac{r\pi}{L}\right)^2 \left(\frac{EI}{\rho A}\right)^{1/2}, \tag{1}$$

where r is the number of the mode, L is the length (m), E is Young's modulus (N/m^2), ρ is the panel density (kg/m^3) and A is the area per unit length of section. However, the above assumption does not consider the presence of walls at the beam end, it is too simple to predict its natural frequency. These four estimated results are 9·8, 39·3, 88·5 Hz and 157·3 Hz for box structure whereas 12·7, 50·8, 114·4 and 203·3 Hz for viaduct. All these values are lower than FEM and measurement results. Since the concrete structure is assumed as a frame and not as a beam its assumption is more realistic in the FEM method. Therefore, it is more accurate for estimating the first four vibration modes of the concrete structure and it is used in this paper for vibration mode prediction.

In the concrete box structure, the sound power is the maximum as the velocity of concrete plate, radiation efficiency and the response of room cavity are the highest. FEM, equations (4) and (5) are used for calculating vibration resonance (peak plate vibration), coincidence frequency (peak radiation efficiency) and room resonance respectively.

For the simple concrete box structure, the finite element model using four plate elements of a structure is used to predict the vibration resonance frequencies of the concrete floor. The system matrices K and M are assembled as though all joints of the floor and walls are rigidly connected. This leads to a singular stiffness matrix K and the system has rigid-body freedom. Thus the axial displacement is not considered and the moment is used only in the formulation. The details and formula of FEM is shown in references [10, 11] and the equations are as follows:

$$M_{aa}\ddot{U}_a + K_{aa}U_a = P_a, \tag{2}$$

$$M_{bb}\ddot{U}_b + K_{bb}U_b = P_b, \tag{3}$$

where M_{aa}, M_{bb} = mass matrix, $\ddot{U}$ = acceleration vector, K = singular stiffness matrix, U = displacement vector, P = load vector, aa = axial, bb = bending.

For the coincidence frequency prediction, panel bending wavelength corresponds to the trace wavelength of an acoustic wave at grazing incidence. A sound wave incidence from any direction at grazing incidence, and of frequency is equal to the panel. Alternatively, a panel excited in flexure at the critical frequency will strongly tend to radiate the corresponding acoustic wave. This critical frequency can be calculated by [12, 13]

$$f_c = 0{\cdot}55c^2\sqrt{\frac{\rho_m}{E}} \times \frac{1}{h}. \tag{4}$$

In the preceding equations, c is the speed of sound in air (m/s), h is the panel thickness (m), ρ_m is the material density (kg/m^3) and E is Young's modulus (Pa).

In the low-frequency range of room modes, enclosure response is dominated by standing waves at certain characteristic frequencies. When paths of travel can repeat upon themselves to form normal modes of vibration, waves travelling around such paths will arrive back at any point along the path in phase. Amplification of the wave disturbance will result and the normal mode will be resonant. When the frequency of the source equals one of the resonance frequencies of a normal mode, resonance occurs and the interior space of the enclosure responds strongly being only limited by the absorption present in the enclosure. The resonance frequencies due to the enclosure can be calculated by equation (5) that all surfaces are reflective [12, 13]:

$$f_n = \frac{c}{2}\sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2}. \tag{5}$$

In this equation, the subscript n on the frequency variable f indicates that "eigenfrequency" of the equation are functions of the particular mode numbers n_x, n_y and n_z and l_x, l_y and l_z are the dimensions of the room in m.

3.1. PREDICTION RESULTS FOR SIMPLE CONCRETE BOX STRUCTURE

The simple concrete box structure is shown in Figure 1, all the concrete slab and ceiling are reflective, so this structure is an enclosure. The first four vibration resonance frequencies for bending wave (mode shape shown in Figure 2) by finite element method (FEM) are found to be 10·208, 32·043, 54·645 and 81·116 Hz. According to equation (4), the coincidence frequency (f_{11}) of the concrete box structure is 187 Hz. For the normal modes of vibration in room, the axial modes of (1,0,0), (0,1,0), (0,0,1) and (0,0,2) that component waves move parallel to an axis (one-dimensional) are 35·4, 25, 47 and 94 Hz. And the tangential modes of

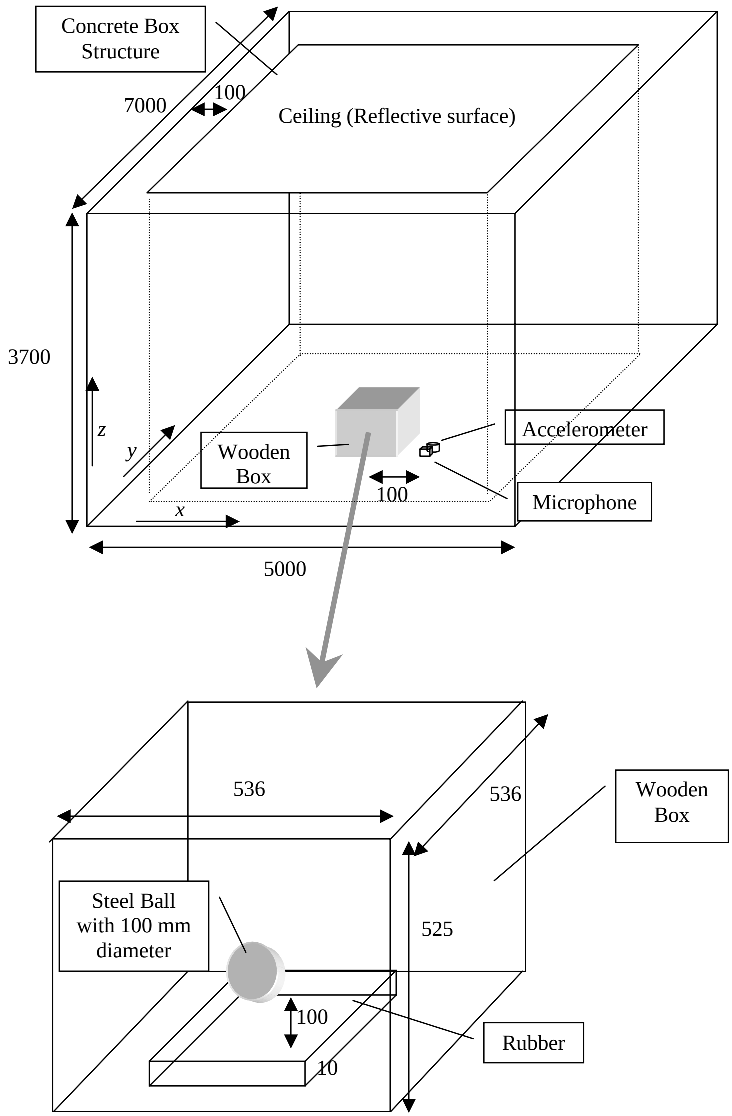

Figure 1. The set-up of the concrete box structure (measure in mm and not in scale).

(1,1,0), (1,0,1) and (0,1,1) that component waves are tangential to one pair of reflective surface, but are oblique to the other two pairs (two-dimensional), are 43·4, 59 and 53·5 Hz.

3.2. PREDICTION RESULTS FOR CONCRETE VIADUCT

For the concrete viaduct structure shown in Figure 3, the first four vibration resonance frequencies for bending wave are 47·869, 144·968, 230·371 and 341·986 Hz and their bending mode shape is the same as concrete box structure. These transverse bending mode are related to noise radiation since the frequencies are >20 Hz and coupled with the excitation force from the rail on one side of the viaduct with two tracks. The predicted coincidence frequency for the bottom concrete plate of the viaduct is 107 Hz. In site measurement, the

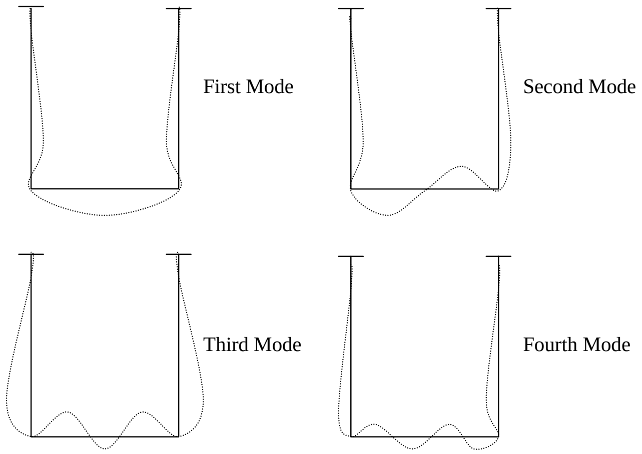

Figure 2. First four mode shapes of bending waves in concrete box/viaduct structure. —— Original shape; ······ deformed shape.

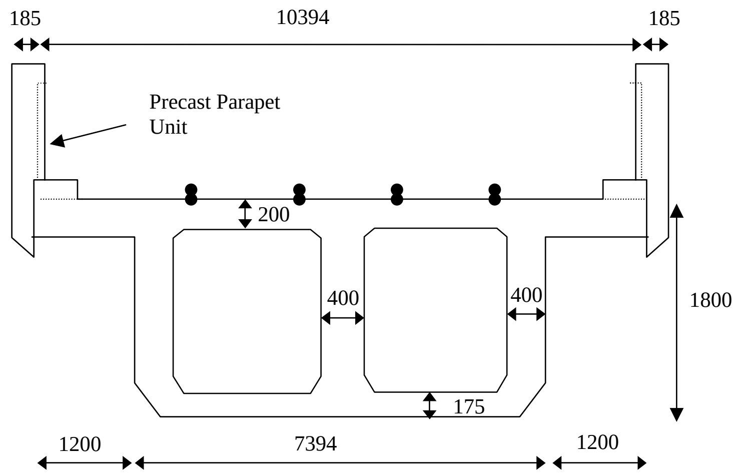

Figure 3. Dimensions of the concrete railway viaduct (measured in mm) and span is 40 m (not in scale).

cylinder tube with absorptive wedge encloses the microphone (Figure 11). Then only the viaduct surface is reflective; no room mode should be predicted for this system.

4. MEASUREMENT SET-UP AND RESULTS

4.1. SIMPLE CONCRETE STRUCTURE

The aim of this experiment is to determine dominant frequency of structure-borne noise and vibrations generated by impacting the concrete box structure. This is the preliminary study of analyzing structure-borne noise generated from the concrete structure. This structure is mounted on pad isolators; a falling steel ball generates a force pulse with very high amplitude and very short duration. A low-frequency component cannot be excited efficiently, so a resilient surface is used to decrease the slowdown rate and extend the impact

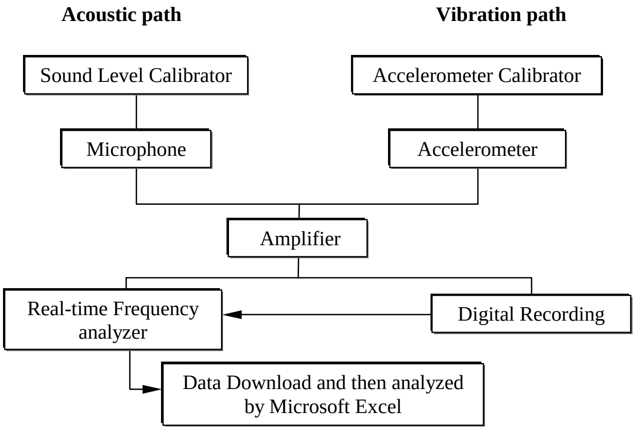

Figure 4. Diagram of the measurement set-up for simulations recording of vibration and noise for simple concrete box and viaduct structure.

duration. Consequently, a larger portion of low frequencies can be generated during impact [11, 13]. In this experiment, only structure-borne noise is being measured and the air-borne noise must be avoided. Therefore, a wooden box (536 mm × 536 mm × 525 mm) with absorption material is used to cover the impact area, the air-borne noise due to the impact sound from the steel ball will be reduced greatly. The position of the accelerometer and microphone is 1 m away from the center of the impact point and its measurement set-up is shown in Figure 1. The measurement flowchart for simultaneous recording of vibrations and noise is illustrated in Figure 4.

Van Tol and Van Lier [14] studied the contribution of the bridge by the input impedance that is using the FFT method for analyzing their magnitude and phase angle in the range of 0–300 Hz. Amick *et al.* [15] used the frequency response function (FRF) to confirm the resonance frequencies of structure. They used the magnitude and phase of the FRF, and coherence function to identify the resonance frequency in the range of 0–200 Hz. In this study, the same measurement method is used for identification of resonance frequency of the concrete viaduct structure. Two-channel spectrum analyzers can be configured to obtain response spectrum automatically using the FRF. The "response" signal is measured simultaneously with the "input" signal, these two signals are digitized and an FFT calculation is used to produce the magnitude and phase of FRF. The relationship between vibration and acoustic pressure in the concrete box structure is measured by acquiring the vibration and acoustic signal during impact.

Their narrowband spectra between 20 and 300 Hz are recorded and shown in Figures 5 and 6, there is a peak level at 30 Hz for all spectra in sound pressure level, acceleration and velocity. The first important resonance 31·5 Hz is the bending mode of the beam. This shows the second-mode vibration resonance of the simple concrete box structure that has a good agreement with the prediction results of FEM (32·043 Hz). The other peak is at 65 Hz (shown in Figure 6); this should be the third-mode vibration resonance that is close to 54·645 Hz in FEM. The mode shapes of these two resonance peaks are illustrated in Figure 2. Based on the theory for multiple input–single output linear systems, the surface velocity and frequency response function is proportional to acoustic pressure. In Figure 6, velocity spectrum of the concrete box shows that the vibration magnitude is decreasing rapidly for higher frequency. Therefore, the structure-borne noise level is significant for the low-frequency range.

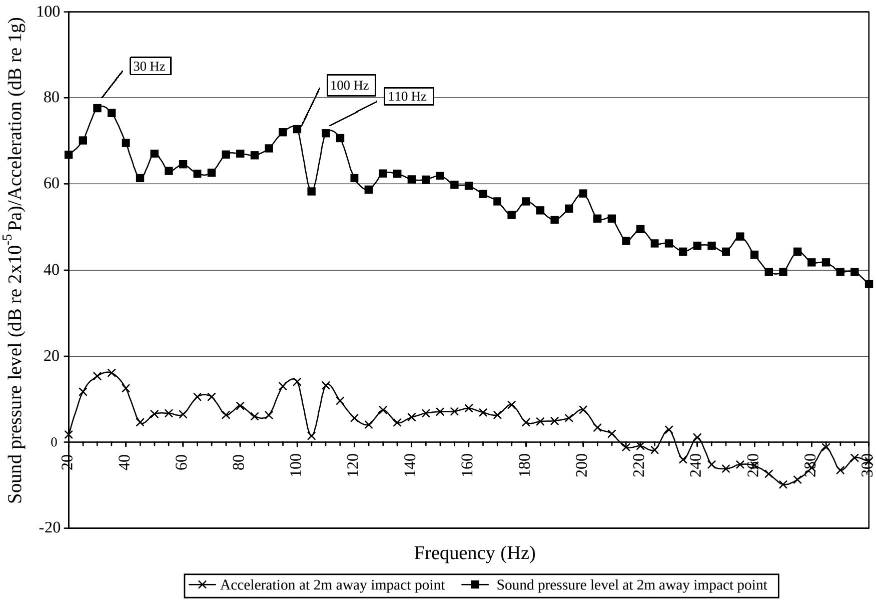

Figure 5. Magnitude of sound pressure level and acceleration of concrete box structure at 2 m away from the impact point (Ball excitation).

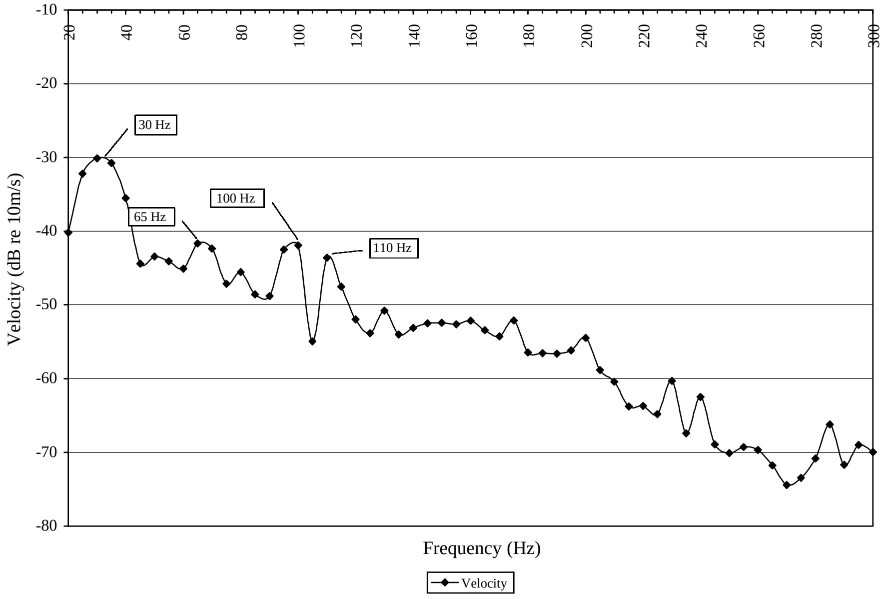

Figure 6. Velocity spectrum of the concrete box structure at 2 m away from the impact point (Ball excitation).

For the ball excitation measurement, the input signal is the vibration of concrete box structure and the output response is the sound pressure. Based on the FFT measurement, the acoustic resonant behavior should be identified from the frequency response and coherence function. However, the phase angle of frequency response (Figure 7) cannot

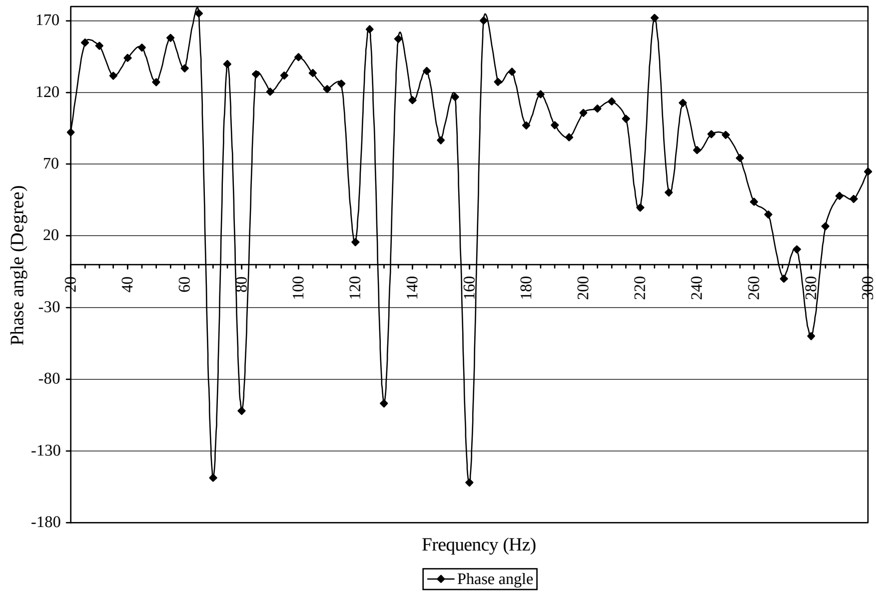

Figure 7. Phase angle of frequency response of the concrete structure at 2 m away from the impact point (Ball excitation).

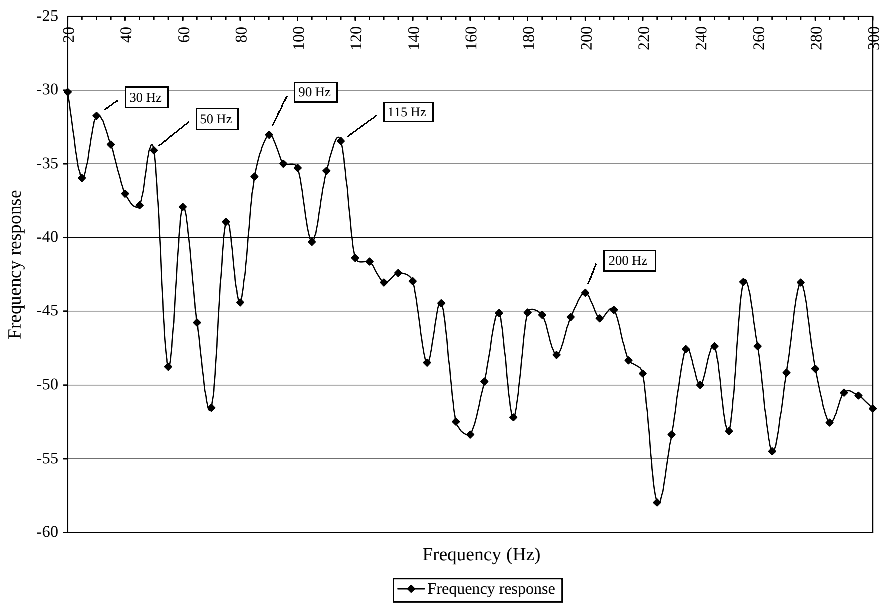

Figure 8. Magnitude of frequency response of the concrete structure at 2 m away from the impact point (Ball excitation).

provide a clear 90° phase angle. This is because there are multiple responses that will cause the overlapping of the phase angle. Also, there is an absorption material to reduce the magnitude of output response and the isolators on the bottom of the concrete structure will reduce the vibration amplitude. Therefore, resonance frequencies are identified by the magnitude of frequency response and with the help of coherence function. The peak

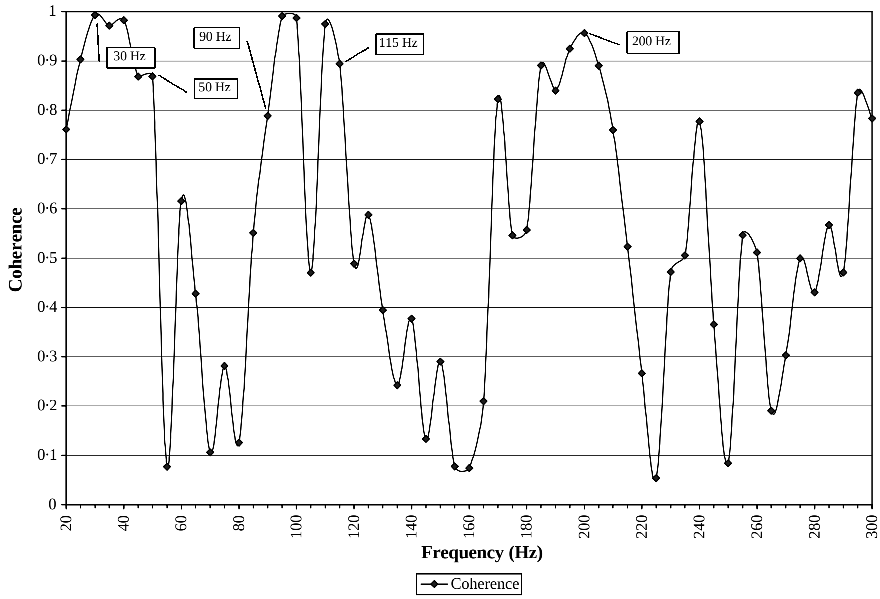

Figure 9. Coherence of the concrete structure at 2 m away from the impact location (Ball excitation).

TABLE 1

Summary of the predicted and measured resonance and coincidence frequencies of concrete viaduct structure

	Predicted frequency (Hz)	Measured value			
		Frequency (Hz)	Frequency response	Coherence	Velocity
Vibration resonance	32·043 (second mode)	30	Peak	0·99	Peak
	54·645 (third mode)	65	—	0·43	Peak
		110	—	0·89	Peak
Coincidence frequency	187	200	Peak	0·96	Peak
Room mode	47 (0,0,1) 53·5 (0,1,1)	50	Peak	0·87	—
	94 (0,0,2)	90	Peak	0·79	—

response shows resonant points and high coherence value means that sound pressure level at these frequencies is totally radiated from the vibration of concrete structure.

From the magnitude of frequency response (Figure 8), 200 Hz should be the coincidence frequency because it is the local peak and the coherence is 0·96. In addition, it is quite close to the prediction results in section 3·1 that is 187 Hz. Also, there are several peaks at 30, 50, 90 and 115 Hz and their coherence (shown in Figure 9) are 0·99, 0·87, 0·79 and 0·89 respectively. Frequencies at 30 and 115 Hz should correspond to vibration resonance and 50 and 90 Hz belong to resonance room mode. The predicted and measured values of resonance and coincidence frequencies are shown in Table 1.

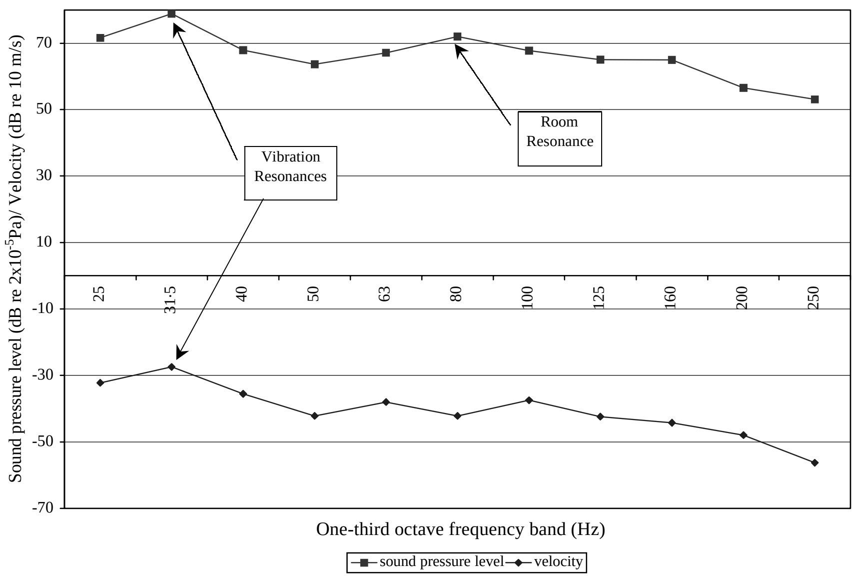

Figure 10. Structure-borne noise and vibration spectra generated from concrete box structure.

The structure-borne noise and vibration spectra are plotted in one-third octave bands in Figure 10, both frequency spectra above 160 Hz are decreasing and have local peaks at the low-frequency range. As compared with the results of FEM, the peak at 31·5 Hz of both spectra represents vibration resonance and peak at 80 Hz of noise spectrum represents room modes for the concrete box structure. The low-frequency component is relatively significant when a falling steel ball impact the structure, which implies that the vibrating concrete structure radiate low-frequency sound and vibration resonance is more significant than the acoustics one.

4.2. CONCRETE VIADUCT

For the site measurement of the running trains on concrete viaduct, the measurement set-up is equal to the concrete box structure illustrated in Figure 4. At location 1 measurement, an anechoic cylinder method [16] is used to directly measure sound pressure level attributable to viaduct radiation and reduce the background noise influence that contaminate the measurement data. The dimensions and measurement points of viaduct are at the bottom (location 1) and 4·8 m below the viaduct (location 2) that are illustrated in Figure 3, photo 1 and Figure 11 respectively.

When the train run on rails, the vibrating motion of rails is transferred to the supporting structures such as tracks and concrete structures (bridges). The structure-borne sound excited by train traffic are generally in the lower frequency range than those of the rolling noise, according to reference [3], the dominant frequency range is between 40 and 100 Hz. From the narrowband measurement of trains travelling at 140 km/h, the dominant frequency of both noise and vibration should be in the range of 40–157 Hz that contains significant tonal noise characteristics. In addition, the coherence (Figure 15) between 40 and 157 Hz are especially high that means the radiated noise is almost totally due to the vibration of the concrete viaduct system.

Photo 1. Measurement locations of vibration and structure-borne noise measurement of the concrete railway viaduct.

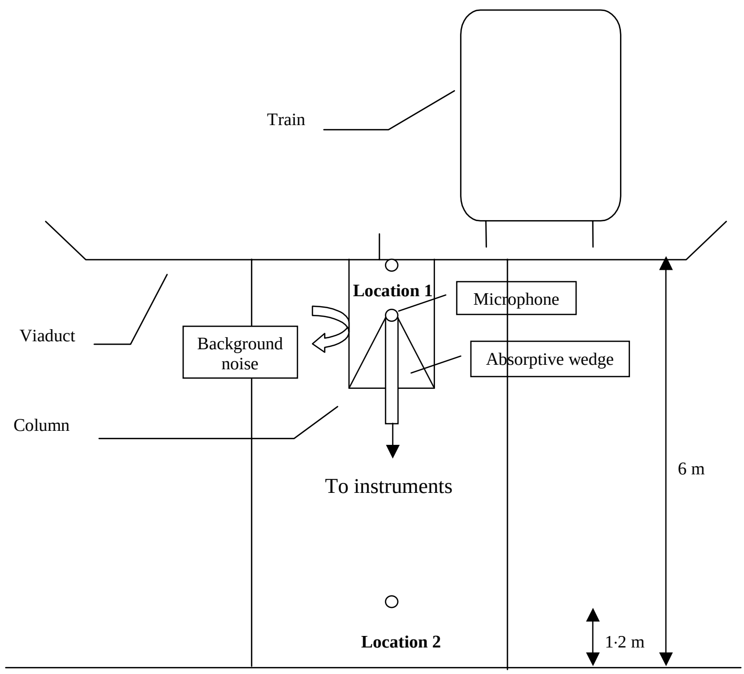

Figure 11. Measurement locations of vibration and structure-borne noise measurement of the concrete railway viaduct (not in scale).

Based on the measurement results, vibration resonance of the viaduct are found to be 43 Hz and 54 Hz as they are the peak responses in the spectra of acceleration (Figure 12) and velocity (Figure 13). Accordingly, the amplification of the vibration at 43 and 54 Hz band is influenced by resonance of the combination of viaduct and train, and the vibration response should be caused by wave motion propagating upon the concrete elevated

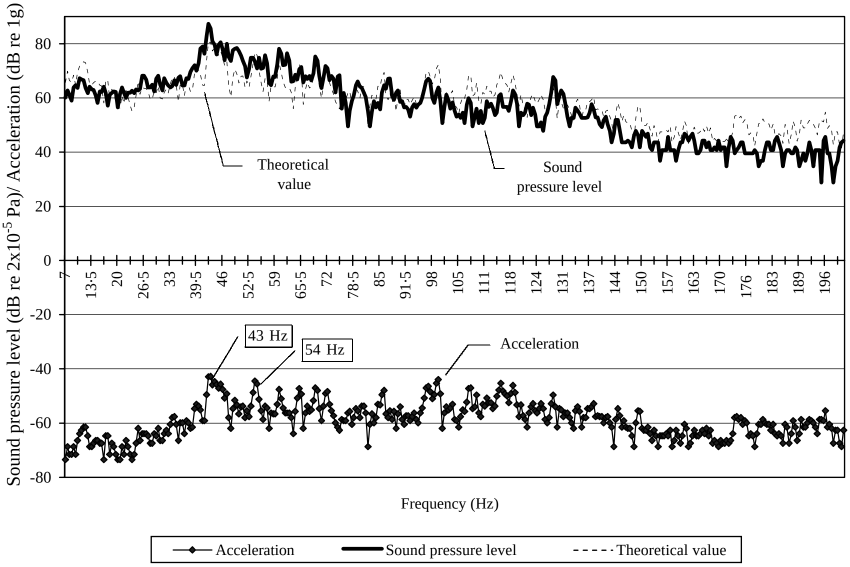

Figure 12. Magnitude of sound pressure level, theoretical value and acceleration at location 1.

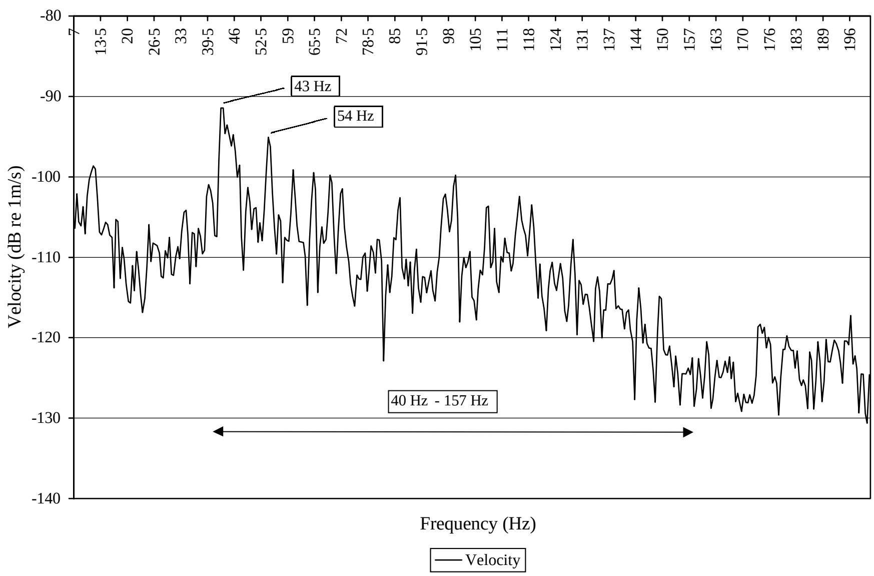

Figure 13. Velocity spectrum at location 1.

structure. Theoretically, the average relationship of simultaneous measurements of vibration and noise level during train passbys is [17]

$$L_p = L_a - 20\log(f) + 36, \tag{6}$$

where L_p = sound pressure level (dB), L_a = r.m.s. vibration acceleration level for the floor (dB re 10^{-6} g), f = frequency, either octave band or 1/3 octave band center frequency (Hz).

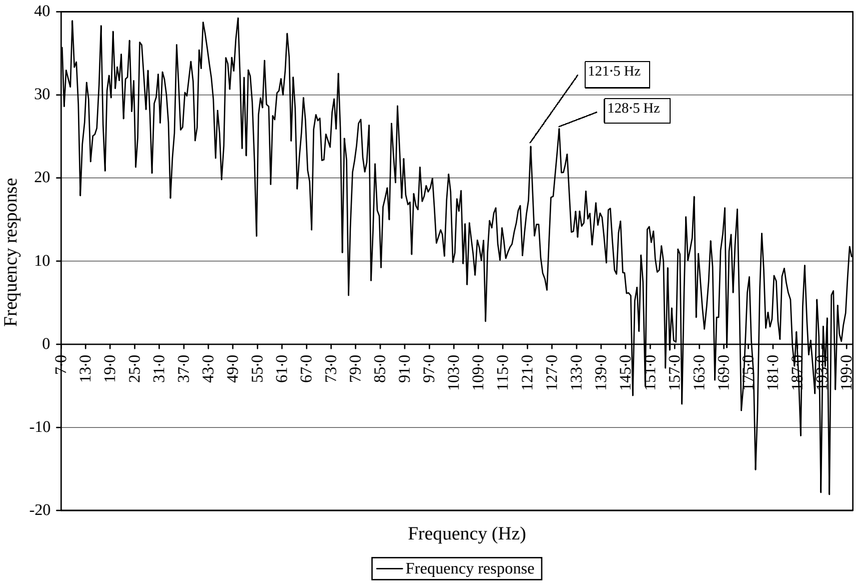

Figure 14. Magnitude of frequency response (acoustic/vibration) at location 1.

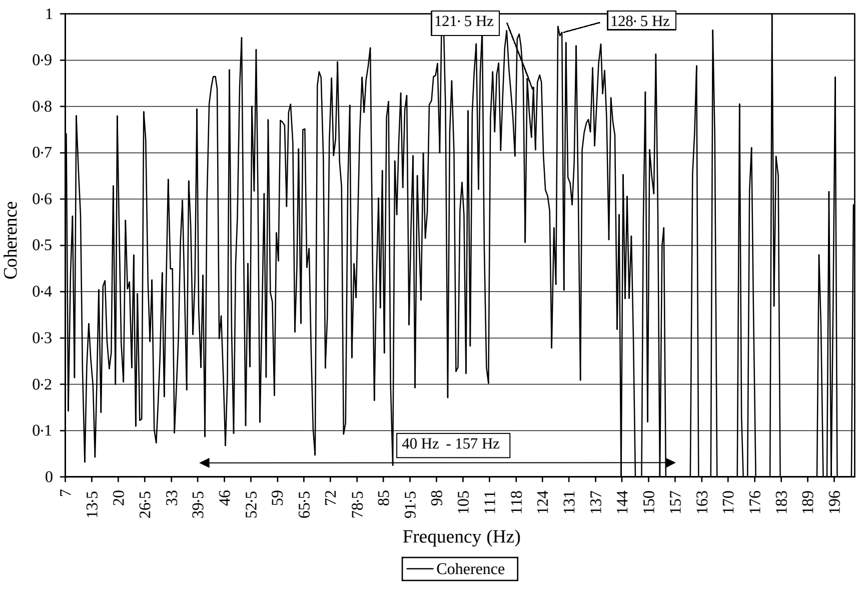

Figure 15. Coherence (acoustic/vibration) at location 1.

To validate the experiment data, equation (6) is used for comparison, which is shown in Figure 12. The measured results have a good agreement with theoretical one; this measurement method is quite reliable and acceptable. According to Figure 13, its vibration spectrum is similar to that of the concrete box structure (Figure 6), both of their vibration magnitude is decreasing rapidly for higher frequency. The higher vibration level in 40–157 Hz can radiate the high level of structure-borne noise since the surface velocity is proportional to the acoustic pressure.

From Figures 14 and 15, the acoustics coincidence phenomena should have occurred at 121·5, 128·5 Hz, whose frequency response is local peaks and its coherence is 0·84 and 0·96

TABLE 2

Summary of the predicted and measured resonance and coincidence frequencies of concrete viaduct structure

	Predicted frequency (Hz)	Measured value			
		Frequency (Hz)	Frequency response	Coherence	Velocity
Vibration	47·869	43	—	0·86	Peak
Resonance	(first mode)	54	—	0·92	Peak
Coincidence	107	121·5	Peak	0·84	—
frequency		128·5	Peak	0·96	—

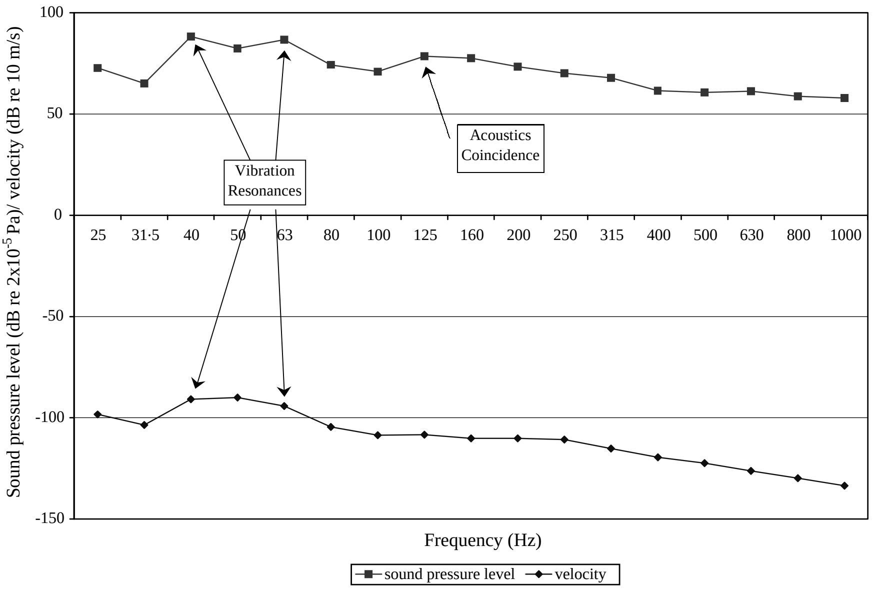

Figure 16. Structure-borne noise and vibration spectra generated from railway viaduct at location 2.

respectively. According to equation (4), the calculated coincidence frequency is 107 Hz for 175 mm thick concrete plate. Then it is more confirmed that 121·5 and 128·5 Hz should be the coincidence frequency for section II concrete viaduct. As compared with vibration resonance, the magnitude of sound pressure level at a structure resonance of 43 Hz is 23 dB higher than that at the acoustic coincidence of 121·5 Hz. This may imply that the structure resonance is more significant than acoustic coincidence. The summary of the predicted and measured values of vibration resonance and coincidence frequency is illustrated in Table 2.

According to the one-third octave band spectrum in Figure 16, the two peaks at 40 and 63 Hz represents the vibration resonance and the peak at 125 Hz represents the acoustics coincidence of viaduct system. Also, acoustics level at 125 Hz is 8–10 dB below that of 40 Hz showing that vibration resonance is more significant than acoustics coincidence in the structure-borne noise radiation.

5. CONCLUSIONS

The analysis of the correlation of noise and vibration indicates that the peak level of structure-borne noise is mainly due to the vibration resonance of the concrete structure. The frequency spectrum, magnitude and phase angles of frequency response and coherence function are important parameters for resonance frequency analysis. From the measurement results, the first- and second-mode resonance frequencies of concrete structures have a good agreement with the prediction results from finite element model. The frequency range of structure-borne noise generated from concrete box and railway viaduct is between 20 and 120 Hz based on both measurement results. In both cases, vibration resonance is more significant than room resonance/acoustic coincidence. Therefore, vibration resonance is one of the important parameters to be analyzed to reduce the structure-borne noise problem in viaduct.

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APPENDIX A: NOMENCLATURE

M_{aa}, M_{bb}	mass matrix
$\ddot{U}$	acceleration vector
K	singular stiffness matrix
U	displacement vector
P	load vector
θ	rotation
L	length
aa	axial
bb	bending
c	speed of sound in air, m/s
ρ_m	material density, kg/m^3
E	Young's modulus, N/m^2
I	moment of inertia, m^3
h	panel thickness, m
n_x, n_y, n_z	particular mode numbers
f_n	frequency variable, Hz
l_x, l_y, l_z	dimensions of the room, m